⑨ Flow past a circular cylinder: uniform stream $\rightrightarrows$

+ doublet

$$\psi = u_\infty r\sin\theta - \frac{\lambda\sin\theta}{r}$$

$$= \left(u_\infty r - \frac{\lambda}{r}\right)\sin\theta$$

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\left(u_\infty r - \frac{\lambda}{r}\right)\cos\theta = \frac{1}{r^2}\left(u_\infty r^2 - \lambda\right)\cos\theta$$

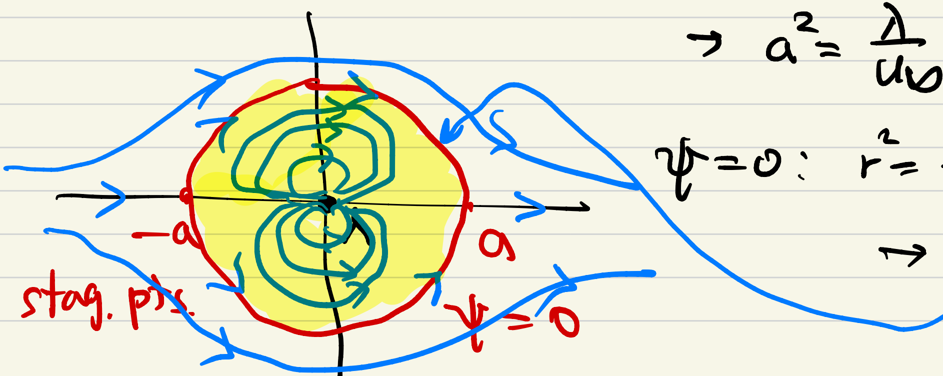
$$u_\theta = -\frac{\partial\psi}{\partial r} = -u_\infty\sin\theta - \frac{\lambda\sin\theta}{r^2} = -\frac{1}{r^2}\left(u_\infty r^2 + \lambda\right)\sin\theta$$

stag. pt. $u_r = u_\theta = 0$: $\theta = 0$ or π

$u_\infty r^2 - \lambda = 0 \rightarrow u_\infty a^2 - \lambda = 0$

$\rightarrow a^2 = \dfrac{\lambda}{u_\infty}$ $(\lambda = u_\infty a^2)$

$u_\infty \rightarrow$



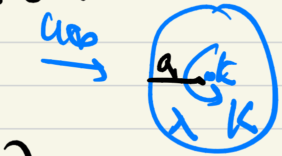
$\psi = 0$: $r^2 = \dfrac{\lambda}{u_\infty} = a^2$

$\rightarrow r = a$

circle

⑩ Flow past a circular cylinder with circulation

→ uniform stream + doublet + vortex



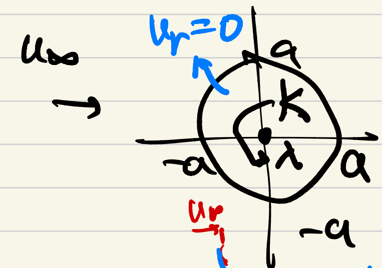
$$\psi = u_\infty r\sin\theta - \frac{\lambda\sin\theta}{r} - k\ln r + k\ln a \qquad (\lambda = a^2 u_\infty)$$

$$@\ r=a,\quad \psi = \underbrace{u_\infty a\sin\theta - \frac{\lambda\sin\theta}{a}}_{=0} - k\ln a + k\ln a = 0$$

$$\Rightarrow \psi = u_\infty \sin\theta\left(r - \frac{a^2}{r}\right) - k\ln\frac{r}{a}$$

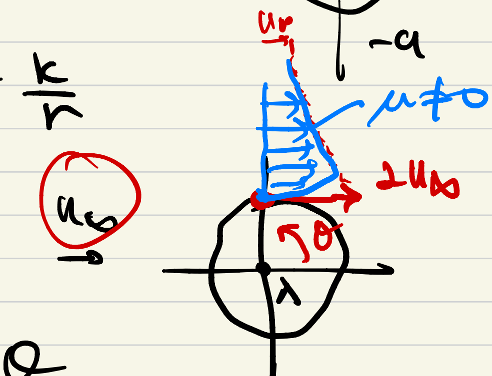
$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = u_\infty\cos\theta\left(1 - \frac{a^2}{r^2}\right)$$

$$u_\theta = -\frac{\partial\psi}{\partial r} = -u_\infty\sin\theta\left(1 + \frac{a^2}{r^2}\right) + \frac{k}{r}$$



$@\ r=a,\ u_r = 0$

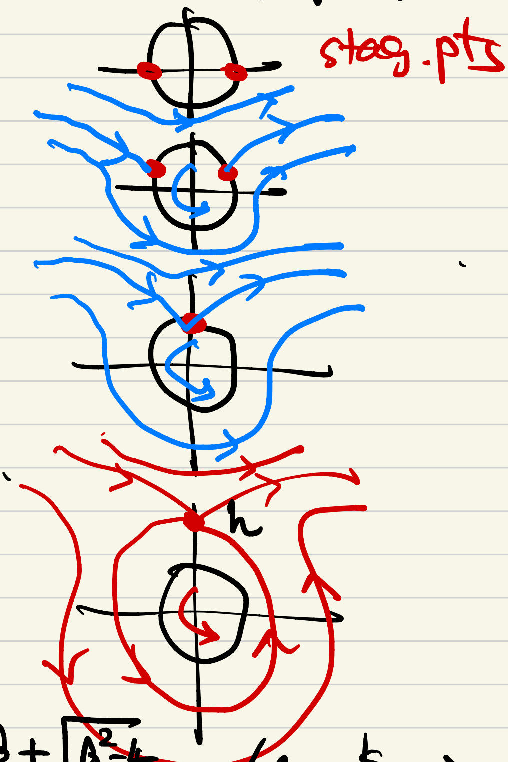
$$\boxed{u_\theta = -2u_\infty\sin\theta + \frac{k}{a}}$$



if $k=0$, $u_\theta(r=a) = -2u_\infty\sin\theta$

$(K \neq 0)$ $u_\theta = 0 \Rightarrow \sin\theta = \dfrac{K/a}{2u_\infty} = \dfrac{K}{2u_\infty a}$

for small K, $\sin\theta_{stag} = \dfrac{K}{2u_\infty a} < 1$: two stag. pts.



$K = 0$: $\sin\theta_s = 0 \rightarrow \theta_s = 0, \pi$

$K = u_\infty a$; $\sin\theta_s = \frac{1}{2} \rightarrow \theta_s = \frac{\pi}{6}, \frac{5\pi}{6}$

$K = 2u_\infty a$; $\sin\theta_s = 1 \rightarrow \theta_s = \frac{\pi}{2}$

$K > 2u_\infty a$: stag. pts does not exist on the cylinder surface.

@ $y = h$, $u_r = u_\theta = 0$

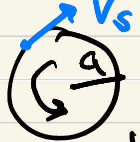
$\hookrightarrow \theta = \cancel{-\frac{\pi}{2}}, \frac{\pi}{2}$

$u_\theta = 0$: $u_\infty\left(1 + \dfrac{a^2}{h^2}\right) = \dfrac{k}{h} \rightarrow \dfrac{h}{a} = \dfrac{\beta + \sqrt{\beta^2 - 4}}{2}$, $\left(\beta = \dfrac{k}{u_\infty a} > 2\right)$

⊛ Kutta–Joukowski theorem

$u_\infty \longrightarrow$ p_∞

p_s V_s

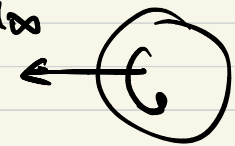


steady

$=$

unsteady

u_∞



$$p_\infty + \frac{1}{2}\rho u_\infty^2 = p_s + \frac{1}{2}\rho V_s^2 \longrightarrow \times$$

$$= p_s + \frac{1}{2}\rho\left(-2u_\infty \sin\theta + \frac{k}{a}\right)^2 \qquad \left(u_r\big|_{r=a} = 0\right)$$

$$\rightarrow\ p_s = p_\infty + \frac{1}{2}\rho u_\infty^2\left(1 - 4\sin^2\theta + 4\beta\sin\theta - \beta^2\right) \qquad (\beta = k/u_\infty a)$$

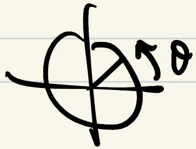
$$\rightarrow\ \boxed{C_p \equiv \frac{p_s - p_\infty}{\frac{1}{2}\rho u_\infty^2} = 1 - 4\sin^2\theta + 4\beta\sin\theta - \beta^2}$$

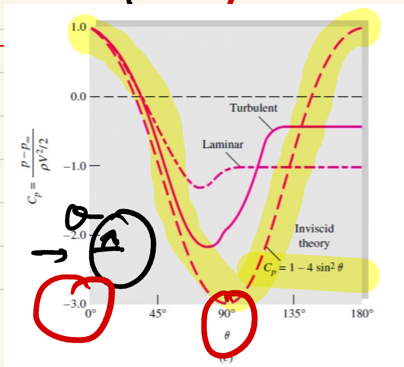
press. coeff

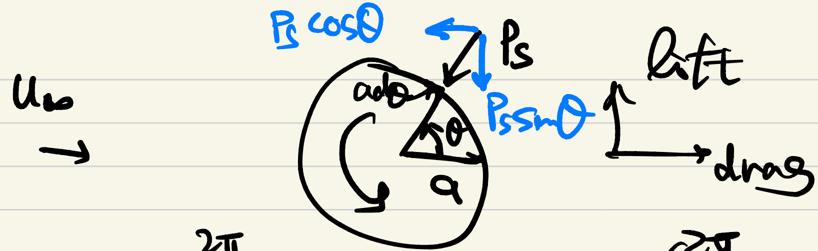
$$k = 0 \ :\ \beta = 0 \ \rightarrow\ \boxed{C_p = 1 - 4\sin^2\theta}$$

stationary cylinder

$\rightarrow$







$$\text{Drag} = D = \int_0^{2\pi} (-P_s\cos\theta)\, b\, a\, d\theta = \int_0^{2\pi} -(P_s - P_\infty)\cos\theta\, b\, a\, d\theta = 0$$

d'Alembert's paradox (1752)

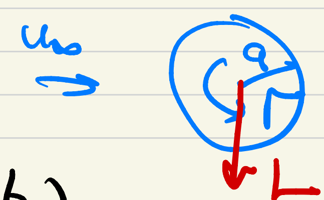
According to inviscid theory, the drag of any body of any shape immersed in a uniform stream is identically zero. → overcome by Prandtl (1904)

$$\text{Lift} = L = \int_0^{2\pi} (-P_s\sin\theta)\, b\, a\, d\theta = \int_0^{2\pi} -(P_s - P_\infty)\sin\theta\, b\, a\, d\theta$$

$$= \cdots = -\rho u_\infty \underbrace{2\pi k}_{\Gamma} \cdot b \neq 0$$

$$\boxed{L/b = -\rho u_\infty \Gamma}$$

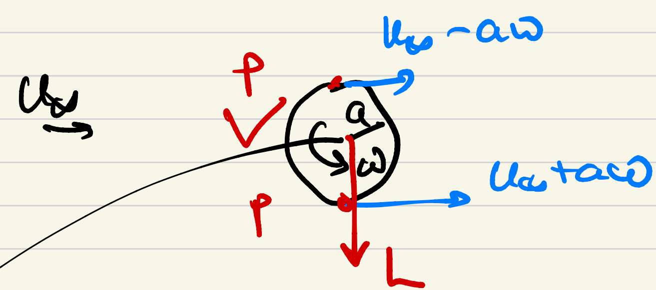
Kutta (1902)
Joukowski (1906)



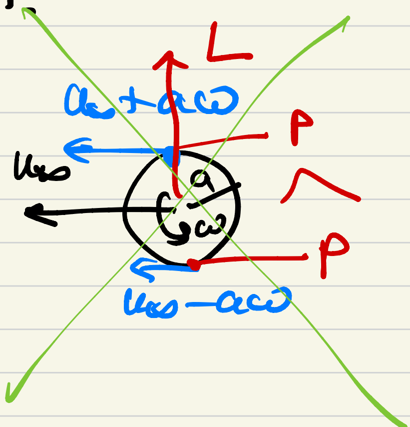
"Kutta-Joukowski lift theorem"

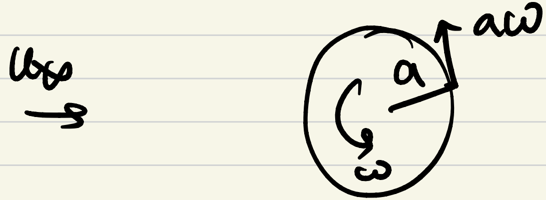
According to inviscid theory, the lift per unit depth of any cylinder of any shape immersed in a uniform stream equals $\rho u_\infty \Gamma$, where Γ is the total net circulation contained within the body shape.

The direction of the lift is 90° from the stream direction, rotating opposite to the circulation.



Magnus effect

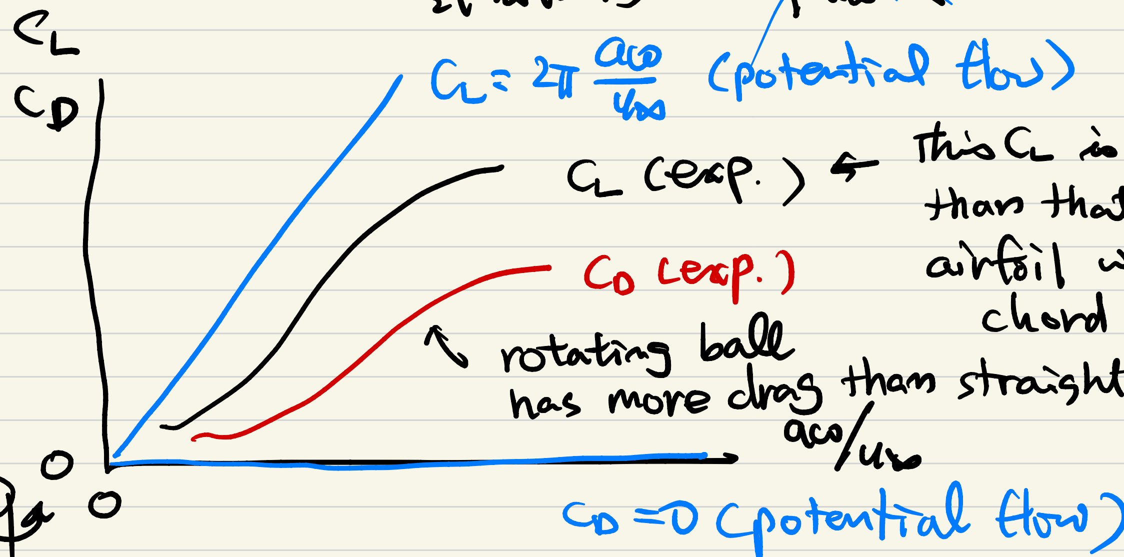




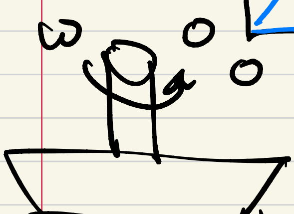
$$\Gamma = \oint_S \underline{u}\cdot d\underline{l} = a\omega \cdot 2\pi a$$

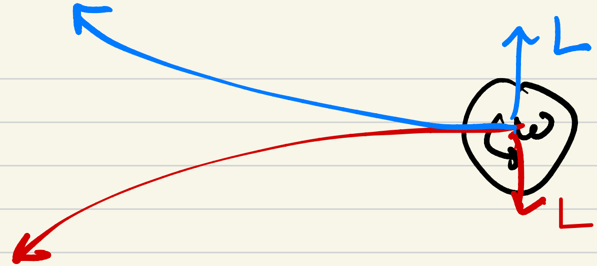
$$L/b = -\rho U_\infty \Gamma = -\rho U_\infty \cdot 2\pi a^2 \omega$$

$$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 (2ab)} = \frac{\cancel{\rho} \cancel{U_\infty}\, 2\pi \cancel{a^2} \omega}{\cancel{\rho}\, U_\infty^{\cancel{2}}\, \cancel{a}} = \frac{2\pi a\omega}{U_\infty}$$



This C_L is higher than that of typical airfoil w/ same chord length.





inverse Magnus effect

Magnus effect

Kim & Choi (JFM)

Inverse Magnus effect on a rotating sphere: when and why

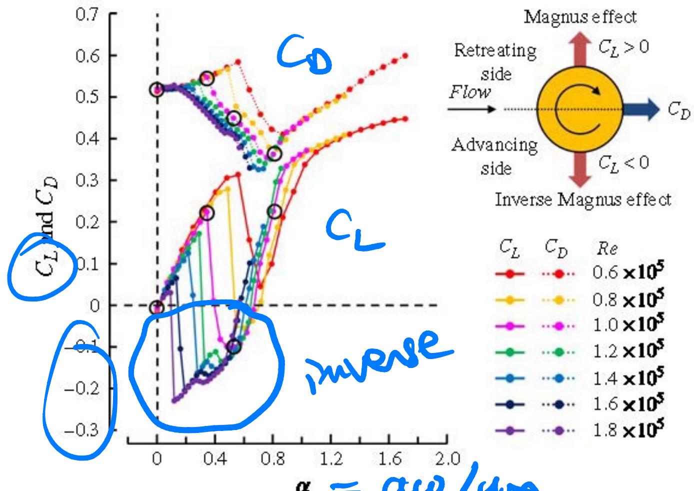
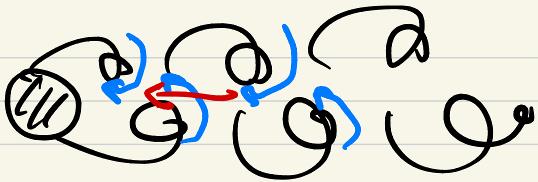


FIGURE 2. Variations of the lift and drag coefficients with the spin ratio at $Re = 0.6 \times 10^5$–1.8×10^5. The lift coefficient is defined to be positive when the lift force is exerted from the advancing to the retreating side (i.e. the Magnus effect occurs) and vice versa. It should be noted that the eight circles on the left of the figure denote the cases investigated in figure 4(*a*–*d*).

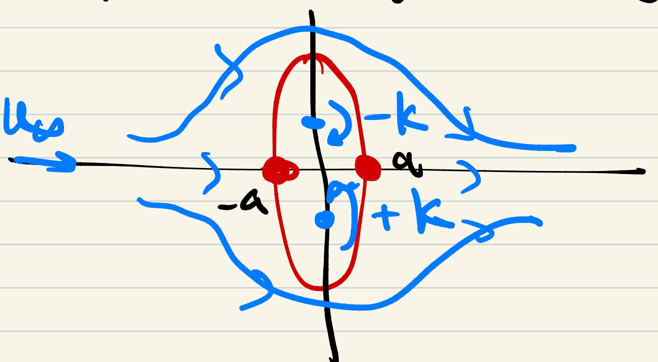


Karman vortex shedding

alternating

inverse Karman vortex shedding

- Kelvin oval : a family of body shapes taller than they are wide

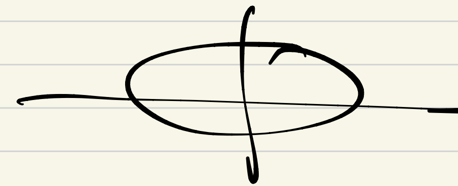


uniform stream

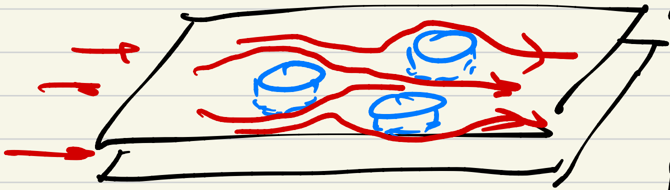
+

vortex pair

Rankine oval



- Potential-flow analogs



Hele-Shaw flow

$h \ll O(1)$

↳ $Re_h \ll 1$

↓

highly viscous flow

↓

no flow separation

↓

looks similar to potential flow