

# 재료의 기계적 거동 (Mechanical Behavior of Materials)

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## Lecture 16 – FRACTURE & FATIGUE

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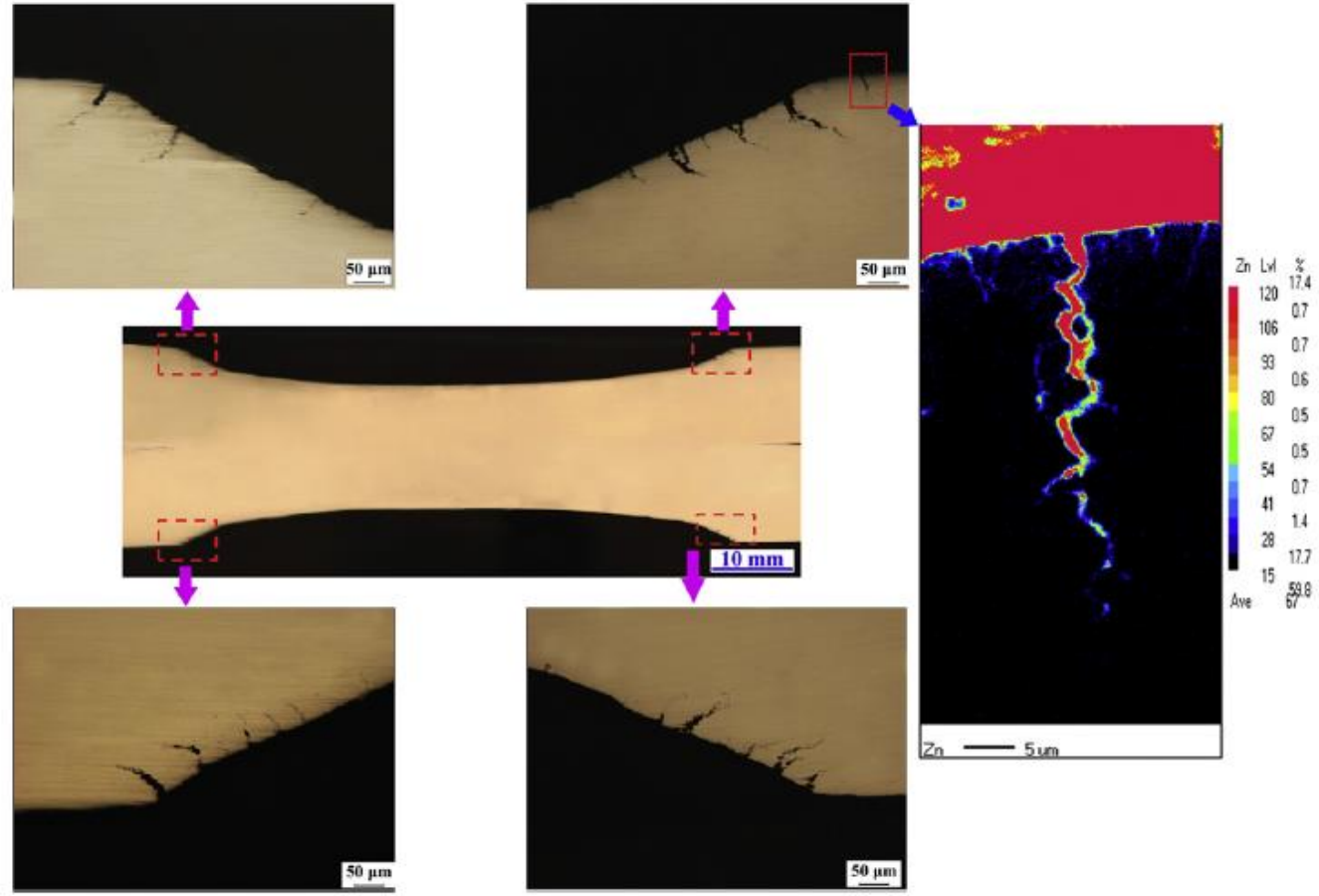
# WHAT IS FRACTURE?

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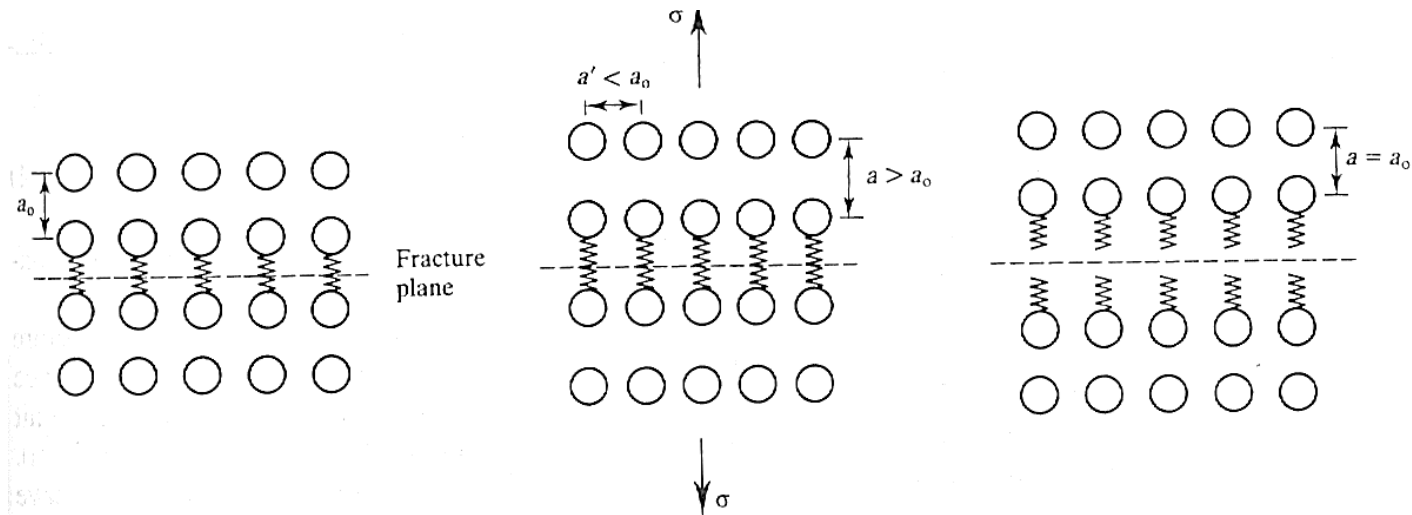
- **The separation or fragmentation of a body under stress.**
- **Fracture proceeds via two processes:**
  - 1. crack initiation**
  - 2. crack propagation**
- **We can loosely categorize fractures as brittle (fast) or ductile.**
  - 1. Brittle: failure with little or no plastic deformation**
  - 2. Ductile: failure with appreciable plastic deformation**



# Liquid metal embrittlement of Zn-coated steel



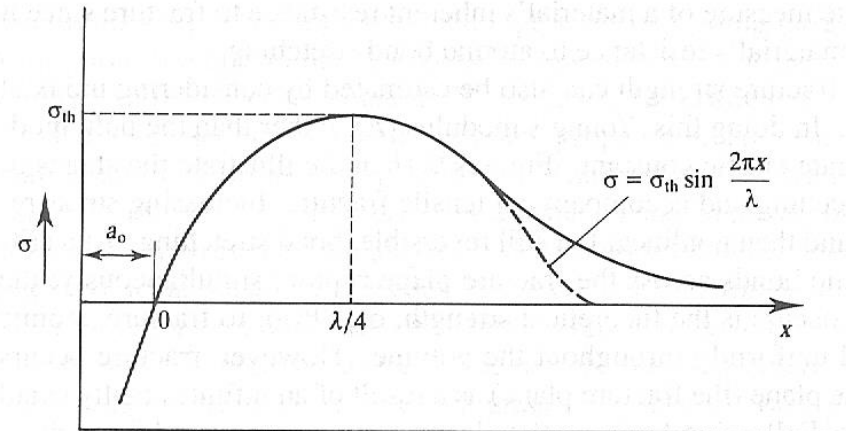
# THEORETICAL FRACTURE STRENGTH



$$\sigma_{th} = \frac{\lambda E}{2\pi a_0} \cong \frac{E}{2\pi} \cong \frac{E}{10}$$

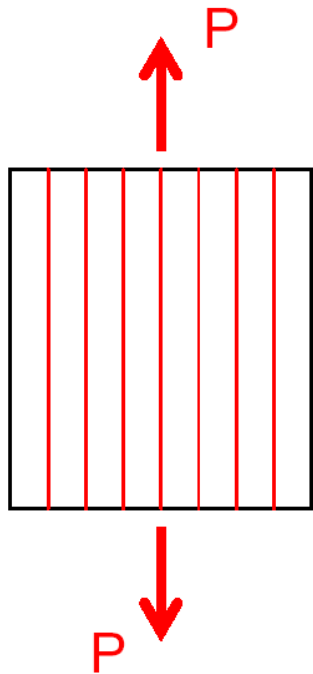
$$\int_0^{\lambda/2} \sigma_{th} \sin \frac{2\pi x}{\lambda} dx = \frac{\lambda \sigma_{th}}{\pi} = 2\gamma (\text{surface energy})$$

$$\sigma_{th} = \left( \frac{\gamma E}{a_0} \right)^{1/2}$$

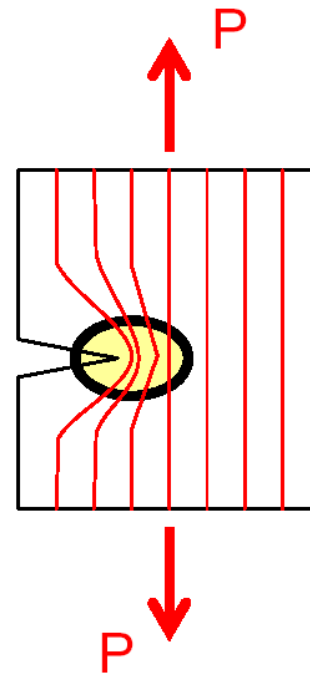


# Why/how do materials fail?

**DEFECTS** concentrate the stress locally to levels high enough to rupture the bonds.



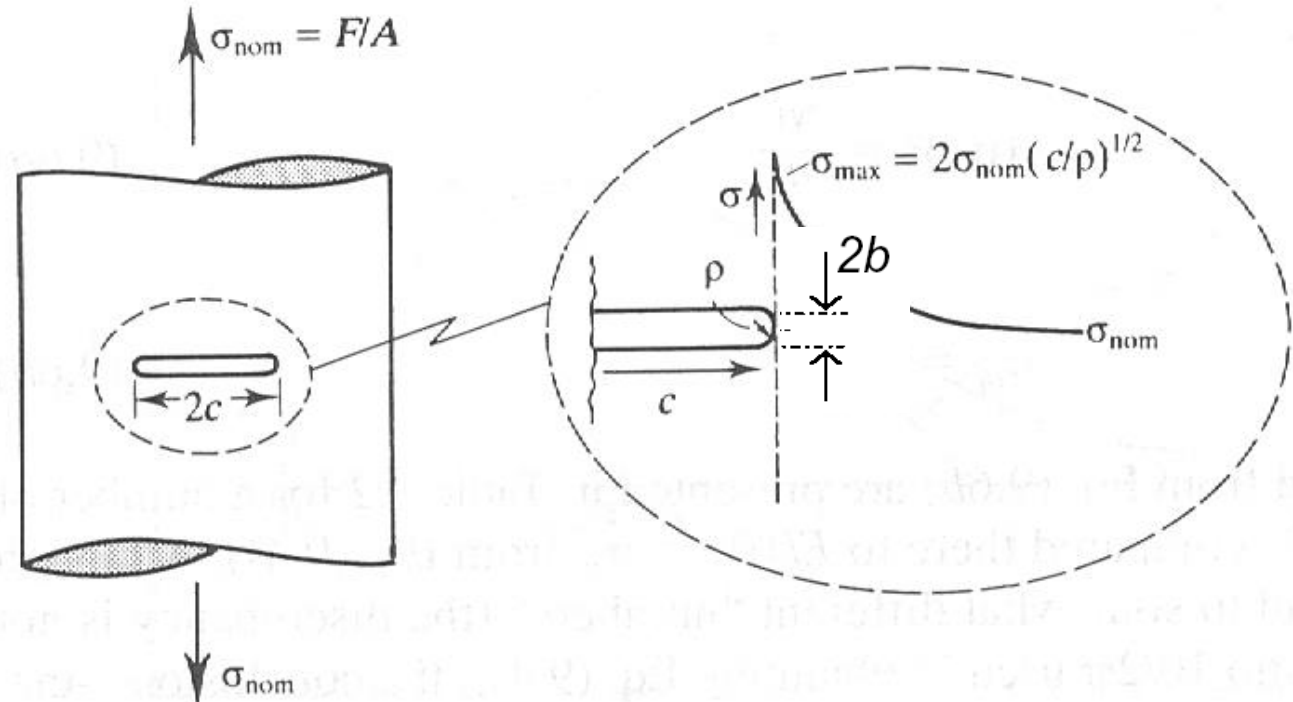
Defect free solid  
Forces are distributed evenly  
Stress distribution is even



Defect in solid  
Forces concentrate in local region  
**STRESS CONCENTRATION**

# Stress Concentration

Surface  
&  
internal  
cracks



**Infinite plate under uniform tension with an elliptical hole (plane stress)**

$$\sigma_{max} = \sigma_{nom} \left( 1 + 2 \frac{c}{b} \right)$$

$$\rho = \frac{b^2}{c} \quad \text{for extremely flat ellipse or very narrow crack}$$

$$\sigma_{max} = \sigma_{nom} \left( 1 + 2 \sqrt{\frac{c}{\rho}} \right) \approx 2\sigma_{nom} \sqrt{\frac{c}{\rho}}$$

# Stress Concentration

**Critical stress intensity approach assumes that fracture occurs when maximum crack tip stress equals the theoretical strength.**

$$\sigma_{\max} = \sigma_{nom} \left( 1 + 2\sqrt{\frac{c}{\rho}} \right) \approx 2\sigma_{nom} \sqrt{\frac{c}{\rho}}$$

$$\sigma_{th} = \left( \frac{\gamma E}{a_0} \right)^{1/2}$$

$$\sigma_F = \left( \frac{\gamma E \rho}{4a_0 c} \right)^{1/2}$$

# Energy approach (Griffith criterion) (plane stress, linear elastic)

As the crack extends, new surface area ( $dc$ ) is created. Correspondingly, the potential energy of the system increases (surface energy  $\gamma$ ). This increase is balanced by a recovery of elastic strain energy in front of the crack tip during crack propagation.

$$4\gamma c$$

Surface energy

$$\begin{aligned} \text{area} &: 2\pi c^2 \\ \text{energy} &: \frac{\sigma^2}{2E} \end{aligned}$$

Restored (released)  
elastic energy

$$U_{Tot} = 4c\gamma - \frac{\pi\sigma^2 c^2}{E}$$

Total energy





# Energy approach (Griffith criterion) (plane stress, linear elastic)

$$\sigma_F = \left( \frac{2\gamma E}{\pi c} \right)^{1/2}$$

- This is the **GRIFFITH** equation.
- With it, we can calculate **the maximum tolerable crack dimension** for a given state of stress.
- Or, the **maximum allowable stress** if the maximum crack dimension is known.
- These equations apply **only to brittle elastic solids**.
- **We must develop other relationships for plastic solids.**



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# Suspension Bridge

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# ELASTO PLASTIC FRACTURE MECHANICS

- IRWIN (1952) suggested that a plastic work term and strain energy release rate should be added to the Griffith equation to make it applicable to metallic materials.

$$G = \frac{\partial U}{\partial c}$$

$$\sigma_F = \left( \frac{G_c E}{\pi c} \right)^{1/2}$$

$G_c$  : material toughness

# ELASTO PLASTIC FRACTURE MECHANICS

The critical value of  $G$  that makes the crack propagate to fracture is  $G_c$ .  $G_c$  is a material parameter called the “critical strain energy release rate”, “*toughness*”, or “crack resistance force”.

Now the conditions for crack growth can be represented as:

$$\sigma_F = \frac{K_c}{(\pi c)^{1/2}}$$

- For plane stress  $K_c = (G_c E)^{1/2}$
- For plain strain  $K_c = \left[ \frac{G_c E}{(1-\nu^2)} \right]^{1/2}$

## Critical stress intensity factor

$K_c$  is a material parameter known as the *fracture toughness*.



# Grain size dependency of fracture toughness

## Hall-Petch equation

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

The dislocation pileup acts similarly to an internal crack with a length that scales with the grain size  $d$ , intensifying the stress in the surrounding grains.

$$\sigma_F = \frac{K_c}{(\pi c)^{1/2}} \quad \longrightarrow \quad \sigma_F = \frac{K_c}{(\pi d)^{1/2}} \quad \sigma_F = k_f d^{-1/2}$$

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# Fatigue - general characteristics

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- **Fatigue can occur under axial (tension/compression), torsional stresses/strains. Cyclical strain (stress) leads to fatigue failure.**
- **Failure can occur at stress less than YS or UTS for static load.**
- **Failure is sudden, without warning, and catastrophic! (90% of metals fail in this mode.)**
- **Fatigue failure is brittle in nature, even in normally ductile metals, due to initiation of crack propagation.**

# Cyclic Strain control

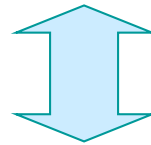
- Constitutive relation for cyclic stress-strain:  $\Delta\sigma = K'(\Delta\varepsilon)^{n'}$
- $n' \approx 0.1-0.2$
- Fatigue life: Coffin Manson relation:  $\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f (2N_f)^c$
- $\varepsilon'_f \sim$  true fracture strain; close to tensile ductility
- $c \approx -0.5$  to  $-0.7$ ,  $c = -1/(1+5n')$ ; large  $n' \rightarrow$  longer life.



# Palmgren-Miner's rule

$$\sum \frac{n_i}{N_i} = 1$$

**Where  $N_i$  is fatigue life at a stress level and  $n_i$  is the number of cycles at this stress.**



**Coaxing**