Ch. 9 Compressible flow 압축성유동

high speed → density change → compressible
ρ

liquid → almost incompressible ⇒ $Ma = \frac{V}{a} \ll 1$
↑ speed of sound

gas → compressible
↳ gas dynamics

9.1 Introduction

$\nabla \cdot \underline{u} = 0$

← mechanical press.

Incompressible flow : $\underline{V}$, p (ρ = const)
③ ① → 4 unknowns

cont. eq. ①
N-S eq. ③
4 eqs

$\frac{\partial \underline{u}}{\partial t} = - - - \nabla p$ - - -

Compressible flow : $\underline{V}$, p, ρ, T ← temperature
③ ① ① ① → 6 unknowns

thermodynamic press.

$\frac{\partial \rho}{\partial t} = - - -$

cont. eq ①

N-S eq. ③

$\frac{\partial u}{\partial t} = - - -$

change in ρ → change in T
↓ ⇓
change in P energy eq ①

⇒ state eq. ①
$p = \rho R T$

$\frac{\partial T}{\partial t} = - - -$

6 eqs.

$$\boxed{Ma = \frac{V}{a}}$$ Mach number

$Ma < 0.3$	:	incomp. flow ($\rho \simeq$ const)
$0.3 < Ma < 0.8$	:	subsonic flow (no shock)
$0.8 < Ma < 1.2$	:	transonic flow (shock)
$1.2 < Ma < 3.0$	:	supersonic flow (no subsonic)
$Ma > 3.0$	:	hypersonic flow

ME

AA

- specific-heat ratio $k = c_p / c_v$ $\quad k = 1.4$ for air

perfect gas, $p = \rho R T$, $R = c_p - c_v = \text{const}$

$$c_p = \frac{k}{k-1} R$$

For real gas, c_p, c_v, k moderately vary with T.

$$du = c_v\, dT \qquad T\,ds = du + p\, d\left(\frac{1}{\rho}\right)$$

$$dh = c_p\, dT \qquad = dh - \frac{1}{\rho}dp = c_p\, dT - \frac{1}{\rho}dp$$

$$\rightarrow ds = \frac{c_p}{T}dT - \frac{1}{\rho T}dp = \frac{c_p}{T}dT - \frac{R}{p}dp$$

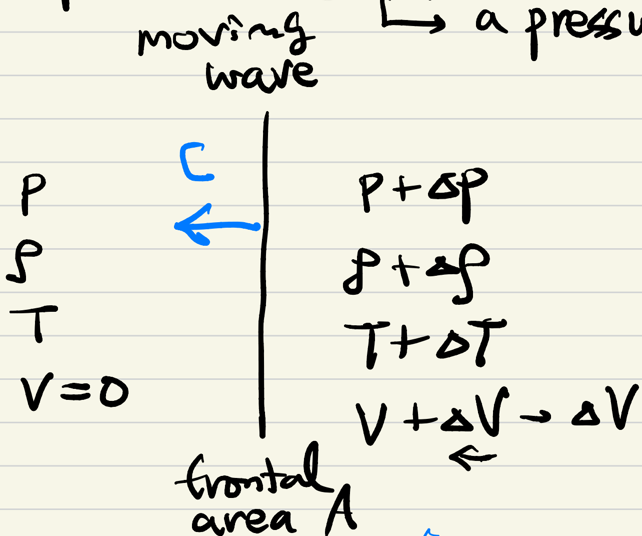
$$\rightarrow s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

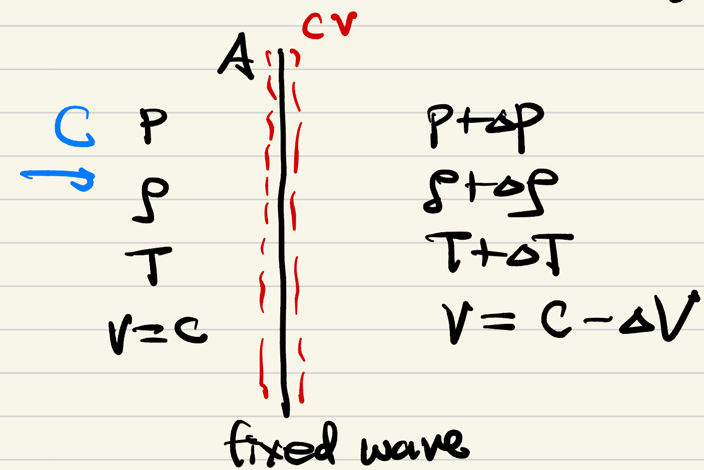
$$= c_v \ln\frac{T_2}{T_1} - R\ln\frac{\rho_2}{\rho_1}$$

* Isentropic process : $ds = 0 \rightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/k-1} = \left(\frac{\rho_2}{\rho_1}\right)^{k}}$

reversible adiabatic

9.2 Speed of sound $\rightarrow$ a pressure pulse of infinitesimal strength





cont: $\rho c \cancel{A} = (\rho+\Delta\rho)(c-\Delta V)\cancel{A} \rightarrow \Delta V = c\dfrac{\Delta\rho}{\rho+\Delta\rho}$ — ①

if $\Delta\rho = O(\varepsilon)$, $\Delta V = O(\varepsilon)$

mtm: $\Sigma F = pA - (p+\Delta p)A$

$= -\rho c^2 A + (\rho+\Delta\rho)(c-\Delta V)^2 A$

$\Rightarrow \Delta p = \rho c \Delta V$ — ②

if $\Delta\rho = O(\varepsilon)$

$\rightarrow \Delta V = O(\varepsilon)$

$\rightarrow \Delta p = O(\varepsilon)$

① → ② : $\boxed{c^2 = \frac{\Delta p}{\Delta \rho}\left(1 + \frac{\Delta \rho}{\rho}\right)}$ $\qquad P = P(\rho, T)$

$$\left.\begin{array}{l}\Delta p \to 0 \\ \Delta \rho \to 0\end{array}\right] \to c^2 = \left.\frac{\partial p}{\partial \rho}\right|_{?} \to a^2 = \left.\frac{\partial p}{\partial \rho}\right|_{?}$$

speed of sound

powerful explosion ($\Delta \rho = O(1)$) waves move much faster than sound wave ($\Delta \rho = O(\varepsilon)$).