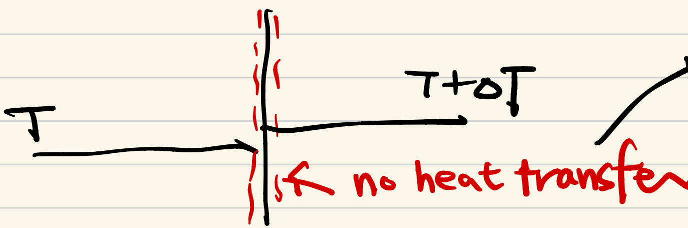
$$c^2 = \frac{\Delta p}{\Delta \rho}\left(1+\frac{\Delta \rho}{\rho}\right) \qquad p = p(\rho, T)$$

$$\left.\begin{matrix}\Delta \rho \to 0 \\ \Delta p \to 0\end{matrix}\right] \to a^2 = \left.\frac{\partial p}{\partial \rho}\right|_{?}$$: speed of sound

Newton (1686) obtained 'a' using the assumption of isothermal process. → 20% error. wrong!



adiabatic process

$\Delta T = O(\varepsilon)$ → reversible process

→ <u>isentropic process</u>

$$\Delta s = 0: \quad p/\rho^k = \text{const} \to \ln p - k \ln \rho = \text{const}$$

$$\frac{dp}{p} - k\frac{d\rho}{\rho} = 0$$

$$\to \left.\frac{\partial p}{\partial \rho}\right|_s = k\frac{p}{\rho} = kRT = a^2 \qquad \therefore \boxed{a = \sqrt{kRT}}$$

@ 1 atm, 15°C, air $a = 340$ m/s

water 1490 m/s

steel 5060 m/s) bulk modulus

water $Ma = 0.3 \to V \doteq 0.3 \times 1500 \doteq 450$ m/s X

$\hookrightarrow$ incomp.

9.3 Adiabatic and Isentropic steady flow

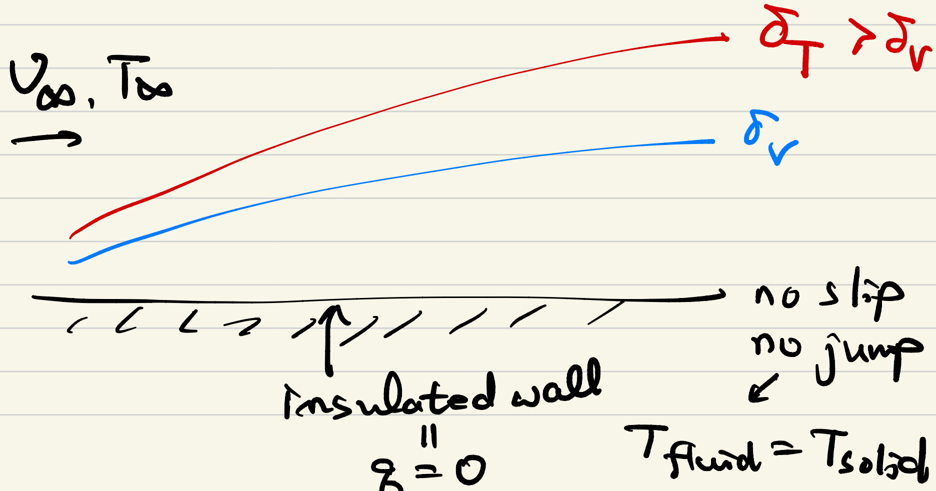
$Pr = \frac{\nu}{\alpha} < 1$ air $Pr = 0.7$ (0.71)

$\to \boxed{\nu < \alpha}$

$\therefore \alpha > \nu$

$\rho \frac{\partial \underline{V}}{\partial t} = \cdots + \mu \nabla^2 \underline{V}$

$\rho c_p \frac{\partial T}{\partial t} = \cdots + \alpha \nabla^2 T$



energy conservation $\quad B = E, \quad \beta = dE/dm = e$

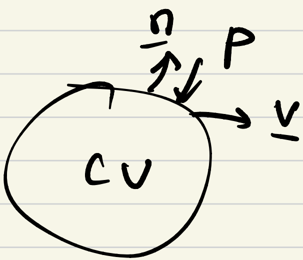
$$e = e_{internal} + e_{kinetic} + e_{potential} + \cdots$$

$$\qquad\; \hat{u} \qquad\qquad \tfrac{1}{2}V^2 \qquad\qquad gz$$

$$\Rightarrow \frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\int_{CV} e\rho \, d\forall + \int_{CS} e\rho(\underline{V}\cdot\underline{n})\, dA$$

$$\dot{W} = \frac{dW}{dt} = \dot{W}_{shaft} + \dot{W}_{press} + \dot{W}_{viscous\ stress}$$

$$= \dot{W}_s + \dot{W}_p + \dot{W}_v$$



$$\dot{W}_p = \int_{CS} p(\underline{V}\cdot\underline{n})\, dA$$

$$\dot{W}_v = -\int_{CS} \underset{\sim}{\tau}\cdot\underline{V}\, dA \qquad \underset{\sim}{\tau} : \text{stress vector on } dA$$

$$\Rightarrow \dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall + \int_{CS} \underline{\left(e + \frac{p}{\rho}\right)} \rho(\underline{V}\cdot\underline{n})\, dA$$

$$= \hat{u} + \tfrac{1}{2}V^2 + gz + \frac{p}{\rho} = h + \tfrac{1}{2}V^2 + gz$$

1D & steady : $\quad 1 \longrightarrow 2$

$$\dot Q - \dot W_s - \dot W_v = -\dot m \left(h_1 + \tfrac{1}{2}V_1^2 + g z_1\right) + \dot m \left(h_2 + \tfrac{1}{2}V_2^2 + g z_2\right)$$

$$\rightarrow h_1 + \tfrac{1}{2}V_1^2 + g z_1 = h_2 + \tfrac{1}{2}V_2^2 + g z_2 - \underbrace{q}_{=\dot Q/\dot m} + \dot W_s/\dot m + \dot W_v/\dot m$$

inside the bdry layer : $q \neq 0$, $\dot W_v \neq 0$

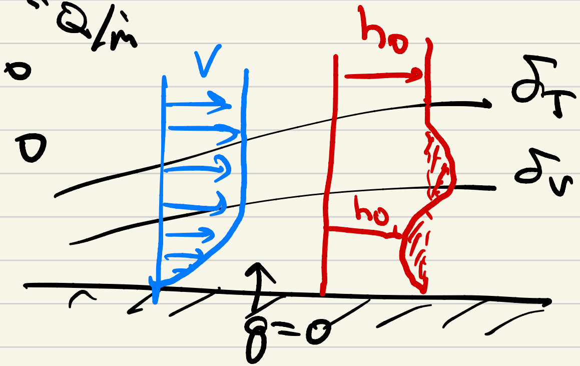
outside " " : $q = 0$, $\dot W_v = 0$

↳ $h_1 + \frac{1}{2}V_1^2 + g z_1 = h_2 + \frac{1}{2}V_2^2 + g z_2$

(neglect $g(z_2 - z_1) \doteq 0$)

$$\rightarrow h_1 + \tfrac{1}{2}V_1^2 = h_2 + \tfrac{1}{2}V_2^2 = \text{const} = \boxed{h_0 = h + \tfrac{1}{2}V^2}$$

↑ stagnation enthalpy



h_0 varies inside the thermal bdry layer, but its average value is the same because of energy conservation

For perfect gas, $h = c_p T \rightarrow \overset{h_0}{c_p T_0} = \overset{h}{c_p T} + \frac{1}{2}V^2$

T_0: stagnation temperature

$$V_{max} = \sqrt{2 c_p T_0} = \left(2\frac{kR}{k-1}T_0\right)^{\frac{1}{2}}$$

$$c_p T_0 = c_p T + \frac{1}{2}V^2 \rightarrow \frac{T_0}{T} = 1 + \frac{V^2}{2c_pT} = 1 + \frac{(k-1)V^2}{2a^2}$$

$$\left(c_p T = \frac{kR}{k-1}T = \frac{a^2}{k-1}\right)$$

$$\rightarrow \boxed{\frac{T_0}{T} = 1 + \frac{k-1}{2}Ma^2}$$

$Ma = V/a$

adiabatic

$$\boxed{\frac{a_0}{a} = \left(\frac{T_0}{T}\right)^{\frac{1}{2}} = \left(1 + \frac{k-1}{2}Ma^2\right)^{\frac{1}{2}}}$$

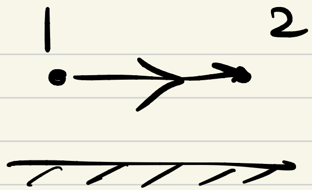
adiabatic

If isentropic process,

$$\boxed{\begin{aligned} \frac{P_0}{P} &= \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} = \left(1 + \frac{k-1}{2}Ma^2\right)^{\frac{k}{k-1}} \\ \frac{\rho_0}{\rho} &= \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}} = \left(1 + \frac{k-1}{2}Ma^2\right)^{\frac{1}{k-1}} \end{aligned}}$$

P_0: stag. press

ρ_0: 〃 density



irrev. adiabatic process

$T_{o_1} = T_{o_2}$ $\quad h_{o_1} = h_{o_2}$

$a_{o_1} = a_{o_2}$

$$p_1 + \frac{1}{2}\rho_1 V_1^2 = p_{o_1} \neq p_{o_2} = p_2 + \frac{1}{2}\rho_2 V_2^2$$

$$\rho_{o_1} \neq \rho_{o_2}$$

isentropic process : $T_{o_1} = T_{o_2}$, $a_{o_1} = a_{o_2}$, $h_{o_1} = h_{o_2}$, $p_{o_1} = p_{o_2}$

$\rho_{o_1} = \rho_{o_2}$

(adiabatic) $h + \frac{1}{2}V^2 = h_o = \text{const}$

$$\rightarrow \boxed{dh + VdV = 0}$$

$$Tds = dh - \frac{1}{\rho}dp = 0 \rightarrow dh = \frac{1}{\rho}dp$$

isentropic

$$\boxed{\frac{dp}{\rho} + VdV = 0 \quad \text{Bernouille eq.}}$$

isentropic process

- (a_0, T_0, p_0, ρ_0) : stagnation values

(a^*, T^*, p^*, ρ^*) : critical values @ Ma = 1
sonic "

↳ vary at irreversible adiabatic process

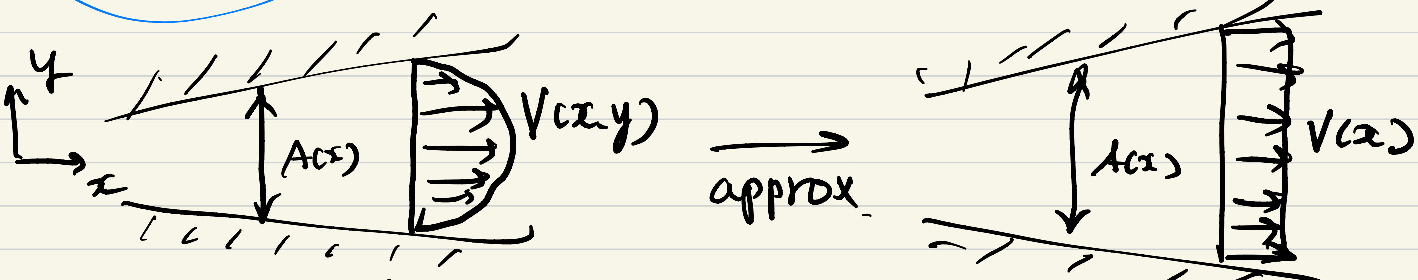
$\rho_{0_1} \neq \rho_{0_2}$
$p_{0_1} \neq p_{0_2}$
$T_{0_1} = T_{0_2}$

critical velocity (Ma=1) $V^* = a^* = \sqrt{kRT^*} = \sqrt{\frac{2k}{k+1} R T_0}$

$$\frac{a^*}{a_0} = \left(\frac{2}{k+1}\right)^{\frac{1}{2}}, \quad \frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}, \quad \frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}, \quad \frac{T^*}{T_0} = \frac{2}{k+1}$$

only ft of k.

9.4 Isentropic flow w/ area change



2-D flow → 1-D flow

cont : $\rho(x)\, V(x)\, A(x) = \dot{m} = \text{const}$

$$d\rho \cdot VA + \rho\, dV \cdot A + \rho V \cdot dA = 0$$

$$\rightarrow \quad \frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

Bernoulli eq : $\dfrac{dp}{\rho} + V dV = 0$

speed of sound ; $dp = a^2 d\rho$

$$\Rightarrow \quad \boxed{\frac{dV}{V} = \frac{dA}{A} \cdot \frac{1}{Ma^2 - 1} = -\frac{dp}{\rho V^2}}$$

isentropic process

- subsonic ($Ma<1$)

$\rightarrow$ $dA > 0$ $\rightarrow$ $dV<0$, $dp>0$, $d\rho>0$

subsonic diffuser

$\rightarrow$ $dA<0$: $dV>0$, $dp<0$, $d\rho<0$

subsonic nozzle

- supersonic ($Ma>1$)

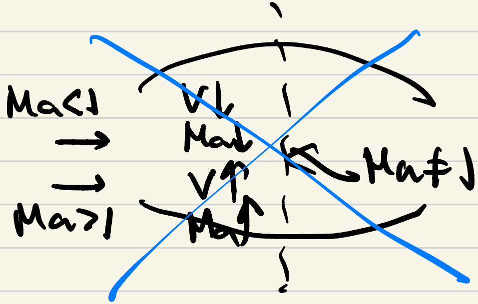
$\rightarrow$ $dA>0$ $\rightarrow$ $dV>0$, $dp<0$, $d\rho<0$

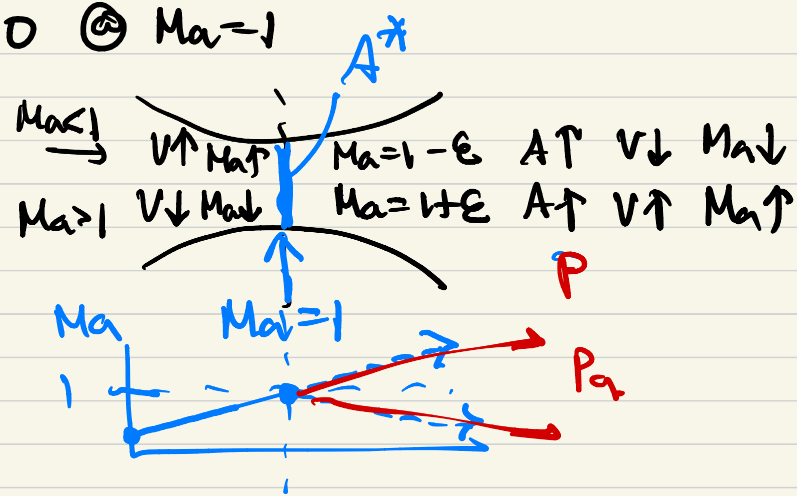
supersonic nozzle

$\rightarrow$ $dA<0$: $dV<0$, $dp>0$, $d\rho>0$

supersonic diffuser

when Ma=1? → $dA = 0$ @ Ma=1





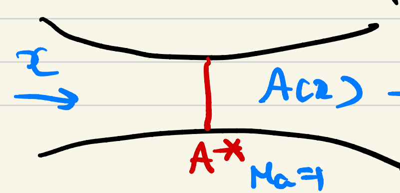
• Perfect gas relations

$$\rho V A = \rho^* V^* A^*$$

$$a^* = \sqrt{kRT^*}$$

$$\rightarrow \frac{A}{A^*} = \frac{\rho^*}{\rho}\frac{V^*}{V} = \frac{\rho^*}{\rho_0}\cdot\frac{\rho_0}{\rho}\cdot\frac{\sqrt{kRT^*}}{V} = \underbrace{\frac{\rho^*}{\rho_0}}_{k}\cdot\underbrace{\frac{\rho_0}{\rho}}_{Ma}\cdot\underbrace{\frac{\sqrt{kRT}}{V}}_{Ma}\cdot\sqrt{\underbrace{\frac{T^*}{T_0}}_{k}\cdot\underbrace{\frac{T_0}{T}}_{Ma}}$$

$$= \frac{1}{Ma}\left[\frac{1+\frac{1}{2}(k-1)Ma^2}{\frac{1}{2}(k+1)}\right]^{\frac{1}{2}\cdot\frac{k+1}{k-1}}$$



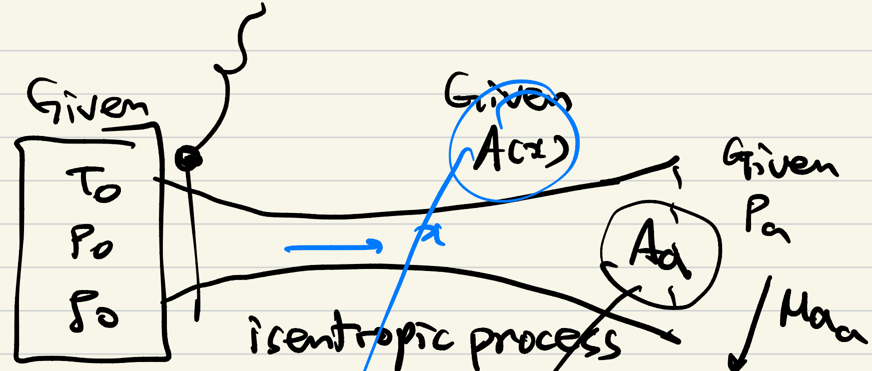
For air ($k=1.4$)

$$\frac{A}{A_*} = \frac{1}{Ma} \cdot \frac{(1+0.2Ma^2)^3}{1.728}$$

$$\frac{T_0}{T} = 1+0.2Ma^2$$

$$\frac{\rho_0}{\rho} = (1+0.2Ma^2)^{2.5}$$

$$\frac{p_0}{p} = (1+0.2Ma^2)^{3.5}$$



$$\frac{p_0}{p_a} = f(Ma) \rightarrow Ma_a = ☆$$

$\hookrightarrow T_a \quad \rho_a$

$T(x)$
$p(x)$
$\rho(x)$ $\leftarrow$ $Ma(x)$ $\leftarrow$ A_* ($Ma=1$)