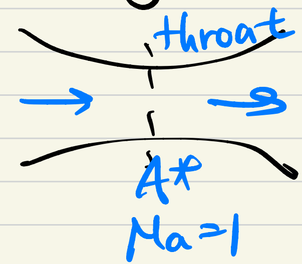
* Choking: for given stagnation conditions, the maximum possible mass flow passes through duct when its throat is at the sonic condition. Then, the duct is said to be choked and carry no additional mass flow.

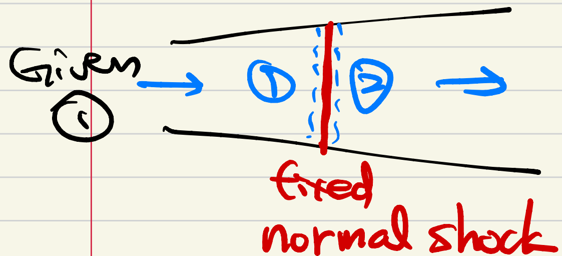


$$\dot{m}_{max} = \rho^* V^* A^* = \rho_0 \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \underbrace{\sqrt{kRT^*}}_{\sqrt{kRT_0 \frac{2}{k+1}}} A^*$$

$$\left(\begin{array}{l} \rho^*/\rho_0 = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \\ T^*/T_0 = 2/k+1 \end{array}\right)$$

$$= k^{\frac{1}{2}} \left(\frac{2}{k+1}\right)^{\frac{1}{2}\cdot\frac{k+1}{k-1}} A^* \underbrace{\rho_0 (RT_0)^{\frac{1}{2}}}_{= P_0/(RT_0)^{\frac{1}{2}}}$$

9.5 Normal-shock wave



⇓

"irreversible" adiabatic process

$$A_1 = A_2 = A$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \Rightarrow \rho_1 V_1 = \rho_2 V_2 = G = \text{const} \quad \text{— (a)}$$

mtm: $(p_1 - p_2)\cancel{A} = \rho_2 V_2^2 \cancel{A} - \rho_1 V_1^2 \cancel{A}$

$$\rightarrow p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 \quad \text{— (b)}$$

(Bernoulli eq: $p_1 + \frac{1}{2}\rho_1 V_1^2 = p_2 + \frac{1}{2}\rho_2 V_2^2$)

Bernoulli eq. IS NOT valid.

energy eq: $h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 = h_0 = \text{const}$ — (c)

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} = R, \quad h = c_p T, \quad k = c_p/c_v = \text{const}$$

5 unknowns, 5 eqs.

→ solve them → two sols. due to V^2 term

→ choose one s.t. $s_2 > s_1$.

ⓐ–ⓒ : $h_2 - h_1 = \frac{1}{2}(P_2 - P_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$: Rankine–Hugoniot relation

Since $h = c_p T = \frac{kR}{k-1} T = \frac{k}{k-1}\frac{P}{\rho}$

$$\frac{\rho_2}{\rho_1} = \frac{1 + \beta\, P_2/P_1}{\beta + P_2/P_1} \quad \text{– ⓓ} \quad \left(\beta = \frac{k+1}{k-1}\right)$$

c.f. $\frac{\rho_2}{\rho_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}}$ isentropic process

$Tds = C_v dT + p\, d\left(\frac{1}{\rho}\right)$

$T = \frac{k}{(k-1)c_p}\frac{P}{\rho}$

$$\rightarrow \frac{s_2 - s_1}{c_v} = \ln\left[\frac{P_2}{P_1}\left(\frac{\rho_1}{\rho_2}\right)^k\right]$$

If $P_2/P_1 < 1$, $s_2 - s_1 < 0 \rightarrow s_2 < s_1$

∴ rarefaction shock is impossible.

$\Rightarrow P_2/P_1 > 1 \rightarrow \boxed{P_2 > P_1} \rightarrow s_2 > s_1$ compression shock

• Mach number relations

ⓐ–ⓒ & $h = \frac{k}{k-1}\frac{P}{\rho}$ $\Rightarrow$ $\boxed{\frac{P_2}{P_1} = \frac{1}{k+1}\left[2k\,Ma_1^2 - (k-1)\right]}$

if $Ma_1 > 1$, $P_2 > P_1$

upstream should be supersonic.

ⓑ & $\rho V^2 = \rho Ma^2 a^2 = \rho Ma^2 kRT = kP\,Ma^2$

$\rightarrow \frac{P_2}{P_1} = \frac{1 + k\,Ma_1^2}{1 + k\,Ma_2^2}$

$\boxed{Ma_2^2 = \frac{(k-1)Ma_1^2 + 2}{2k\,Ma_1^2 - (k-1)}}$

if $Ma_1 > 1$, $\boxed{Ma_2 < 1}$ subsonic

& ⓓ

$\boxed{\frac{\rho_2}{\rho_1} = \frac{(k+1)Ma_1^2}{(k-1)Ma_1^2 + 2} = \frac{V_1}{V_2}}$

→ supersonic | subsonic →

if $Ma_1 > 1$, $\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} > 1$

$\boxed{\frac{T_2}{T_1} = \left[2 + (k-1)Ma_1^2\right]\frac{2k\,Ma_1^2 - (k-1)}{(k+1)^2 Ma_1^2}}$

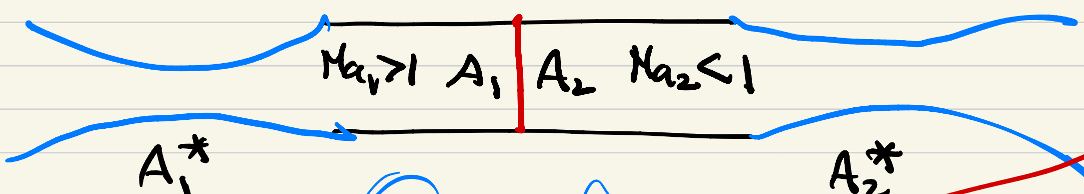
if $Ma_1 > 1$, $T_2 > T_1$

$h_{o_1} = h_{o_2}$, $\boxed{T_{o_1} = T_{o_2}}$

$$\boxed{\frac{P_{o_2}}{P_{o_1}} = \frac{P_{o_2}}{P_2}\frac{P_2}{P_1}\frac{P_1}{P_{o_1}} = \left[\frac{(k+1)Ma_1^2}{2+(k-1)Ma_1^2}\right]^{\frac{k}{k-1}} \left[\frac{k+1}{2kMa_1^2-(k-1)}\right]^{\frac{1}{k-1}}}$$

isentropic relation

if $Ma_1 > 1$, $P_{o_2} < P_{o_1}$



$$\boxed{\frac{A_2^*}{A_1^*} = \frac{A_2^*}{A_2}\underbrace{\frac{A_2}{A_1}}_{=1}\frac{A_1}{A_1^*} = \frac{Ma_2}{Ma_1}\left[\frac{2+(k-1)Ma_1^2}{2+(k-1)Ma_2^2}\right]^{\frac{1}{2}\cdot\frac{k+1}{k-1}}}$$

if $Ma_1 > 1$, $A_2^* > A_1^*$.

Table B.1 isentropic process

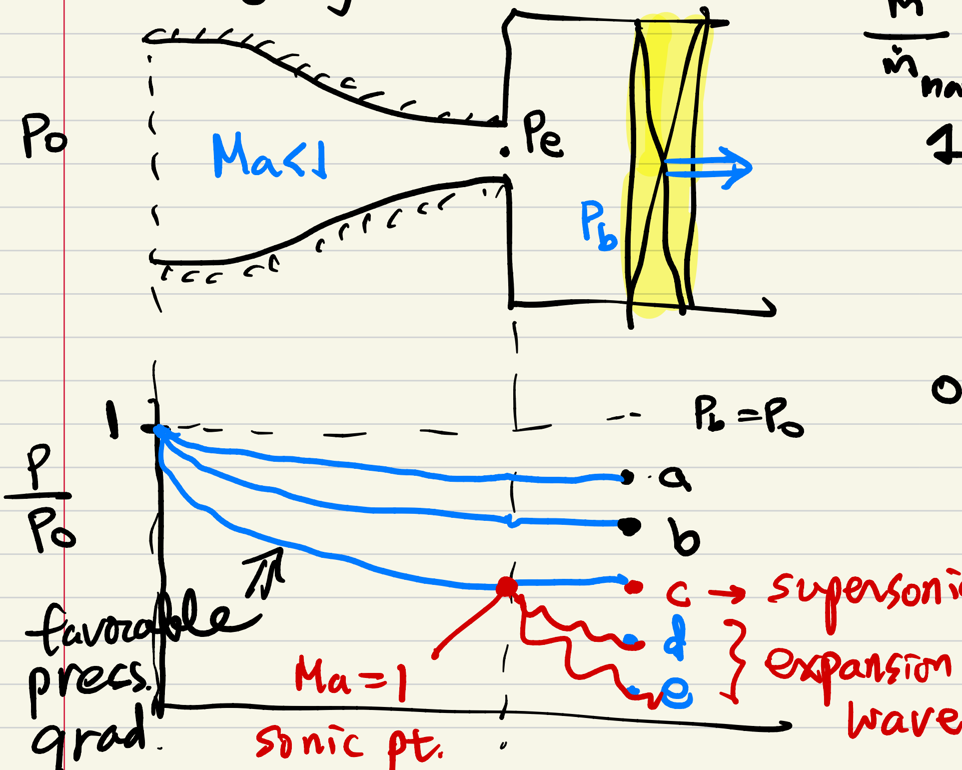
Ma	P/P_0	ρ/ρ_0	T/T_0	A/A^*
0				
⋮				
4.0				

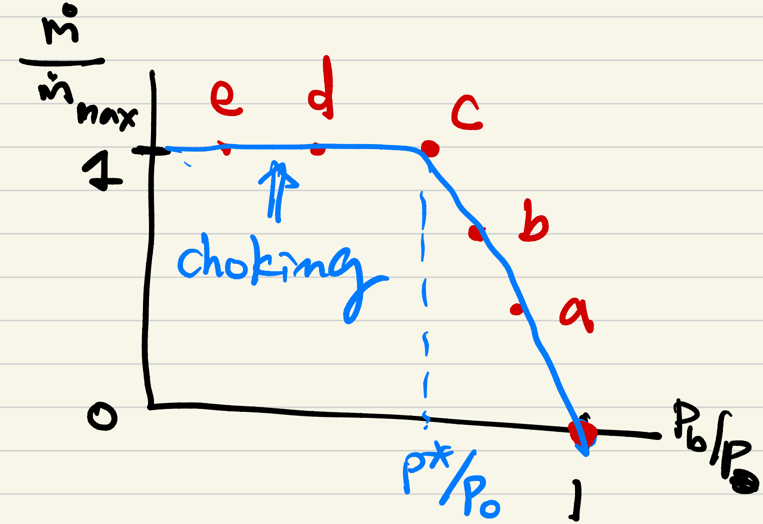
Table B.2 Normal-shock relation ($k=1.4$)

Ma_1	Ma_2	P_2/P_1	$V_1/V_2 = \rho_2/\rho_1$	T_2/T_1	P_{02}/P_{01}	A_2^*/A_1^*
1.0						
⋮						
5.0						

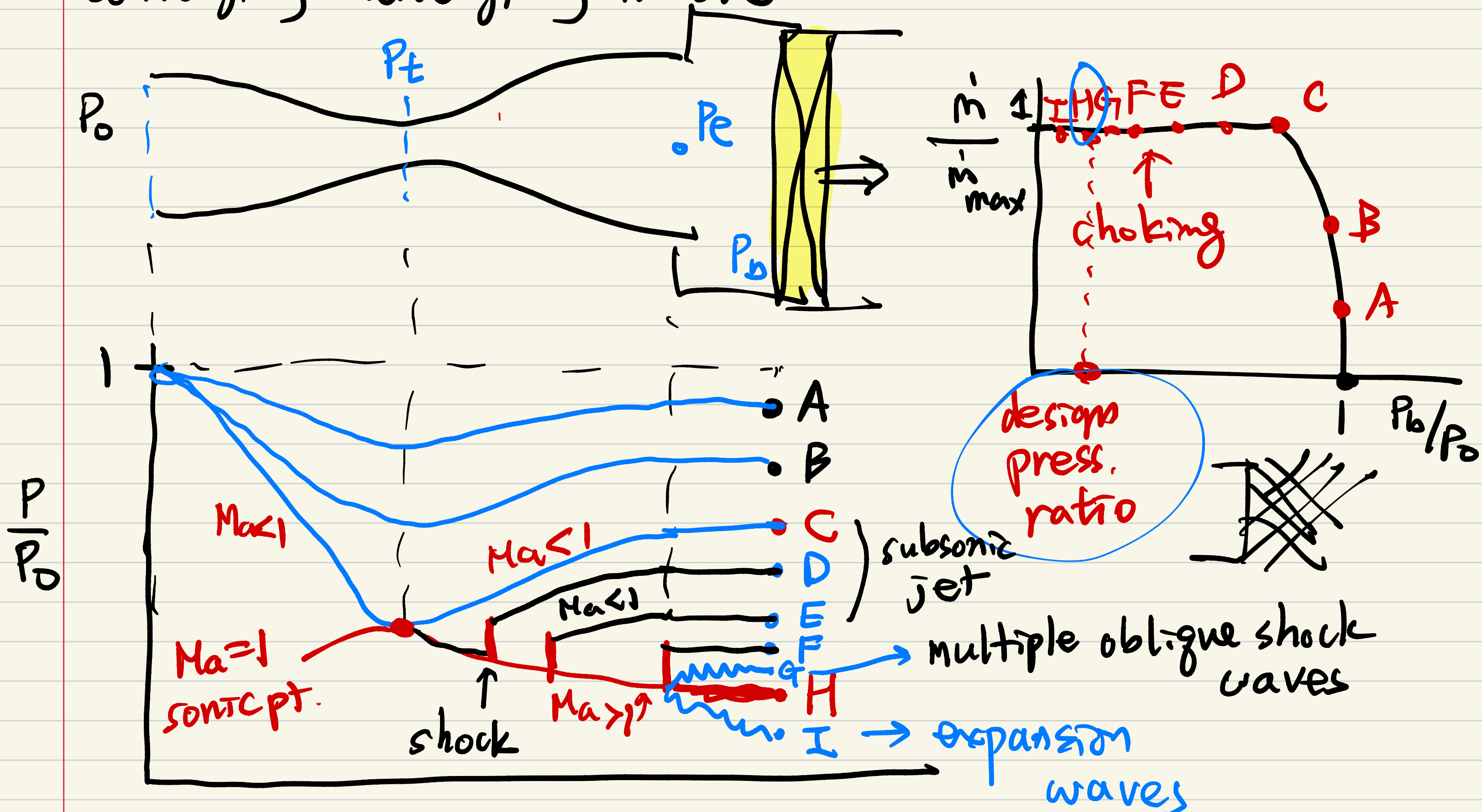
9.6 Operation of converging and diverging nozzles

- Converging nozzle

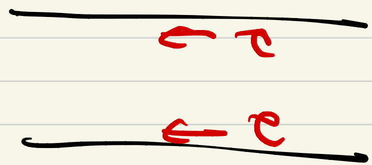




- converging - diverging nozzle



9.7 Compressible duct flow w/ friction



9.8 Frictionless duct flow w/ heat transfer