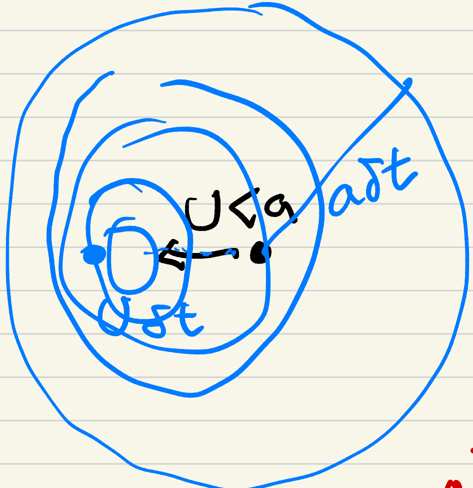
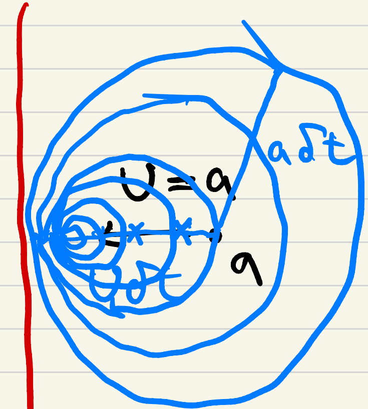
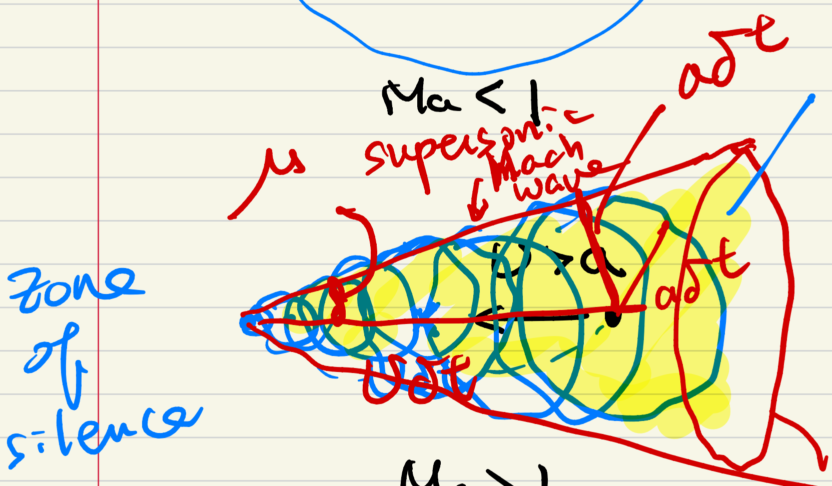
9.9 2D supersonic flow



Ma < 1



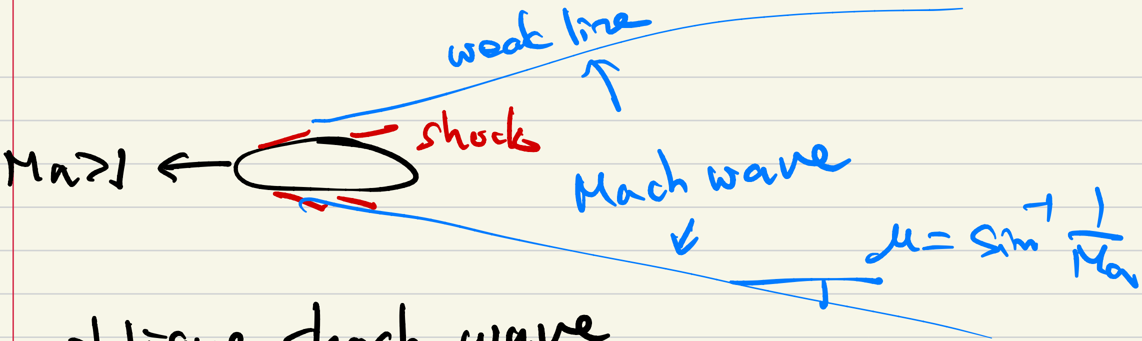
Ma=1



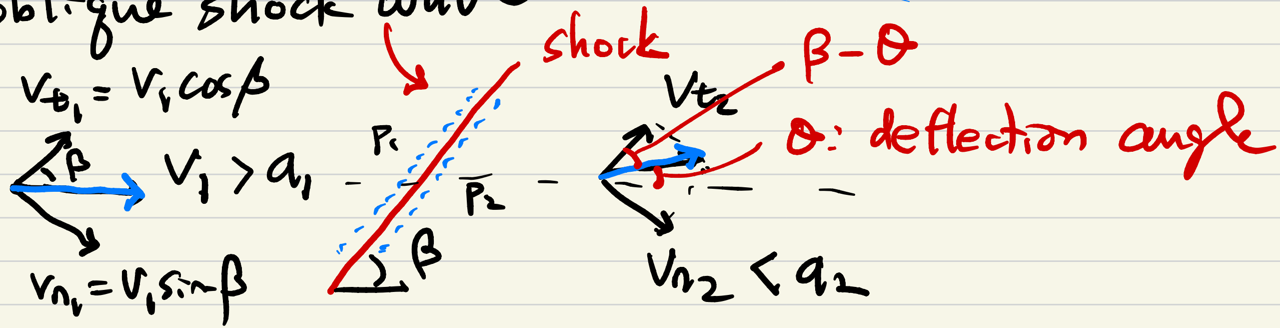
Ma>1

$$\sin\mu = \frac{a\delta t}{U\delta t} = \frac{a}{U}$$

$$\mu = \sin^{-1}\frac{a}{U} = \sin^{-1}\frac{1}{Ma} \text{ : Mach angle}$$



- oblique shock wave



$V_{n_1} > a_1$

cont: $\rho_1 V_{n_1} A = \rho_2 V_{n_2} A$

n-mtm: $(P_1 - P_2) A = \rho_2 V_{n_2}^2 A - \rho_1 V_{n_1}^2 A$

t- " : $0 = \rho_1 V_{n_1} A (V_{t_2} - V_{t_1}) \rightarrow V_{t_2} = V_{t_1} = V_t = \text{const}$

energy: $h_1 + \frac{1}{2} V_{n_1}^2 + \frac{1}{2} V_{t_1}^2 = h_2 + \frac{1}{2} V_{n_2}^2 + V_{t_2}^2 = h_0$

Use normal shock wave formula for V_{n_1}, V_{n_2}, P_1, P_2, ...

$$Ma_{n_1} = \frac{V_{n_1}}{a_1} = Ma_1 \sin\beta \qquad \tan\beta = \frac{V_{n_1}}{V_{t_1}}$$

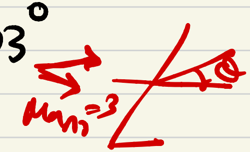
$$Ma_{n_2} = \frac{V_{n_2}}{a_2} = Ma_2 \sin(\beta - \theta) \qquad \tan(\beta-\theta) = \frac{V_{n_2}}{V_{t_2}}$$

deflection angle $\theta = \tan^{-1}\frac{V_{n_1}}{V_t} - \tan^{-1}\frac{V_{n_2}}{V_t}$

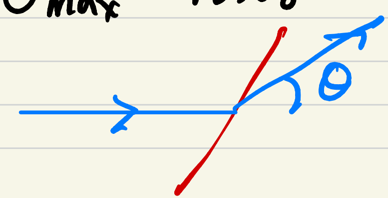
max θ ? $\frac{\partial\theta}{\partial V_t} = 0 \rightarrow \frac{V_t}{V_{n_1}} = \left(\frac{V_{n_2}}{V_{n_1}}\right)^{\frac{1}{2}} = \left(\frac{1}{r}\right)^{\frac{1}{2}}$ $\quad r = \frac{V_{n_1}}{V_{n_2}}$

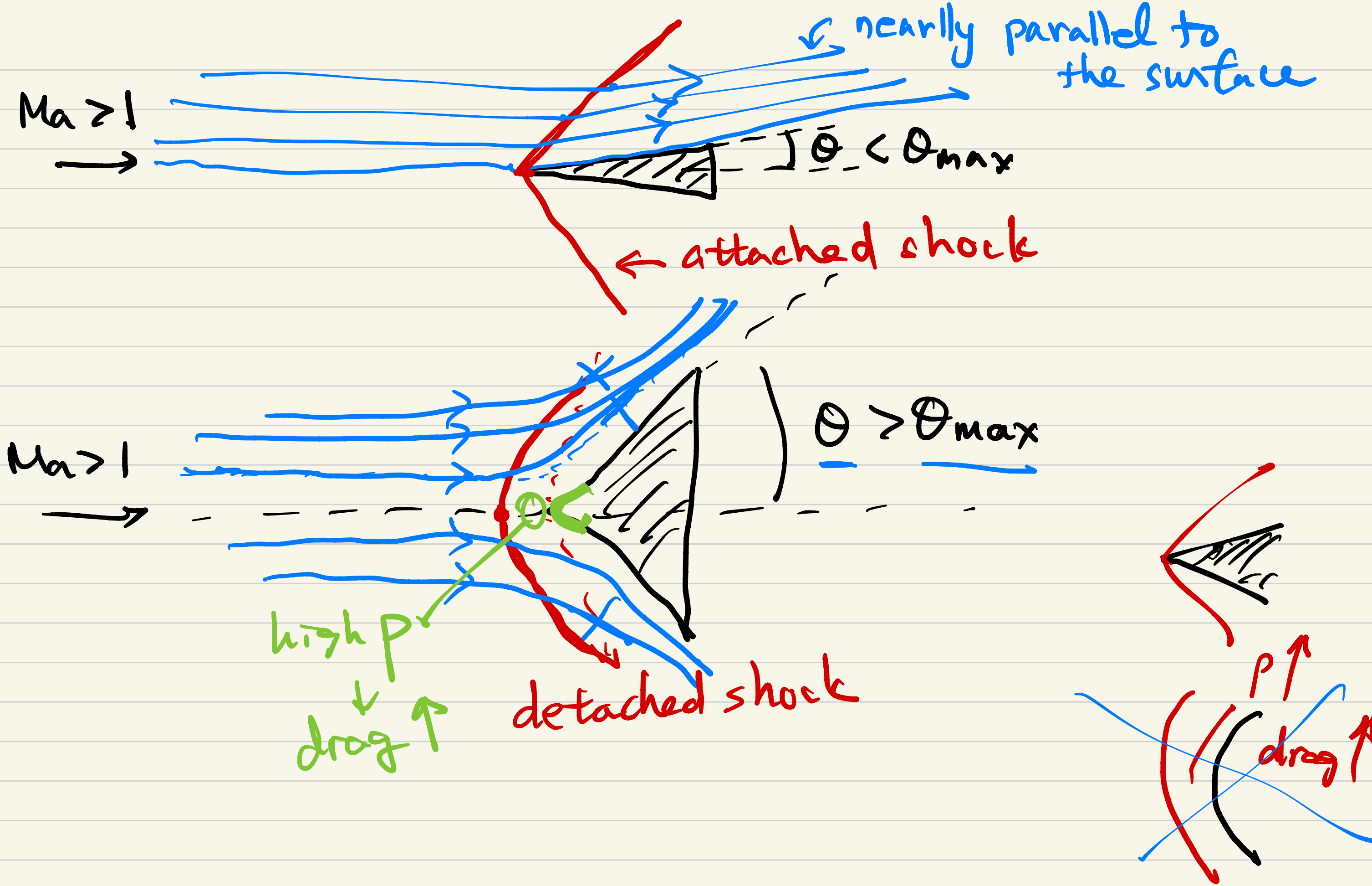
$$\rightarrow \boxed{\theta_{max} = \tan^{-1}\sqrt{r} - \tan^{-1}\frac{1}{\sqrt{r}}}$$

If $Ma_{n_1} = 3$, $r = \frac{V_{n_1}}{V_{n_2}} = 3.8571 \rightarrow \theta_{max} = 36.03°$



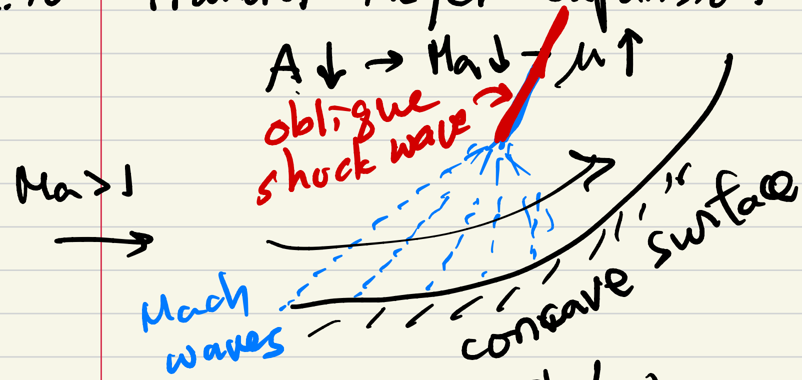
$Ma_{n_1} \rightarrow \infty$, $r = \frac{V_{n_1}}{V_{n_2}} = 6.0 \rightarrow \theta_{max} = 45.58°$




nearlly parallel to the surface
Ma > 1
θ < θmax
attached shock
Ma > 1
θ > θmax
high p
↓
drag ↑
detached shock
p ↑
drag ↑

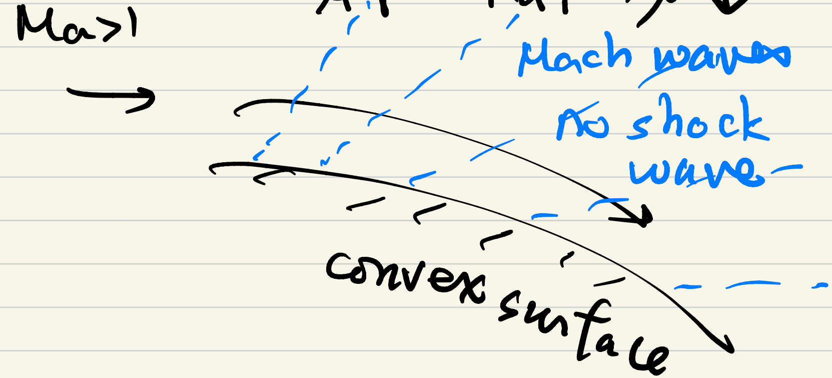
9.10 Prandtl-Meyer Expansion waves

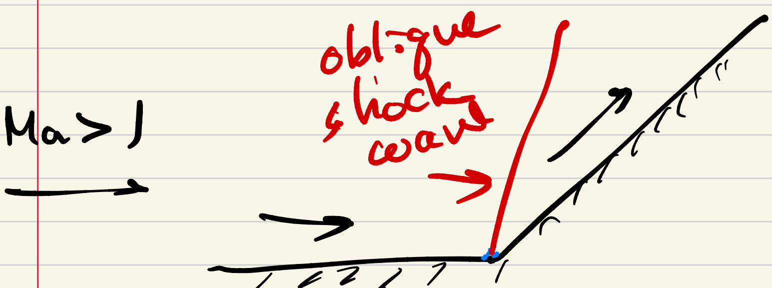
$A\downarrow \rightarrow Ma\downarrow \rightarrow \mu\uparrow$

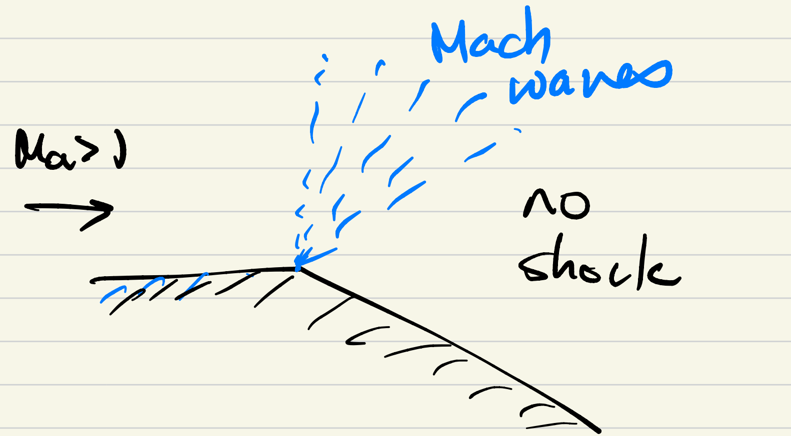


$$\left(\mu = \sin^{-1}\frac{1}{Ma}\right)$$

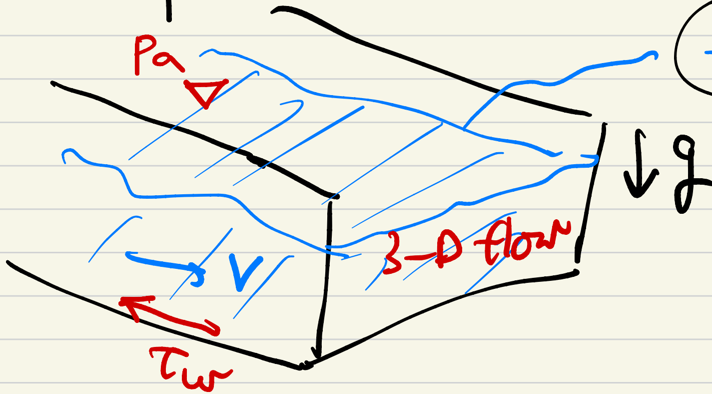
$A\uparrow \rightarrow Ma\uparrow \rightarrow \mu\downarrow$



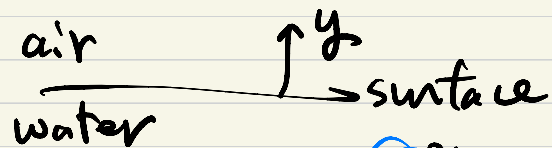




Ch. 10 Open channel flow (개수로 유동)



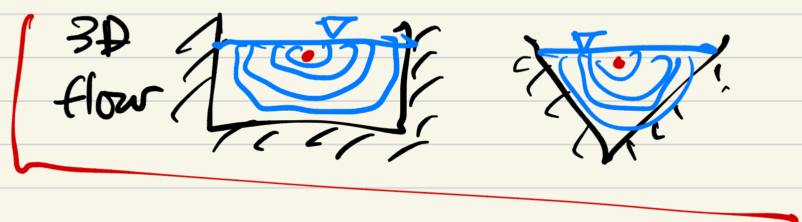
free surface (shape is unknown a priori)

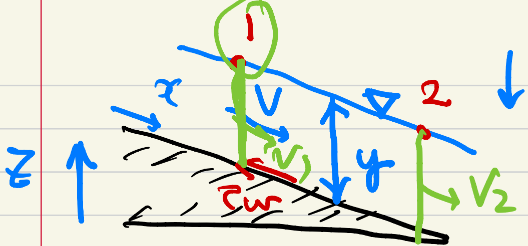


$$\tau = \mu_a \frac{\partial u}{\partial y}\Big|_a = \mu_w \frac{\partial u}{\partial y}\Big|_w \simeq 0 \Leftarrow \text{free shear-stress}$$

$\mu_w \gg \mu_a$

- balance between the gravity force and friction force
- depth profile changes
- 1D approximation





$$P_1 = P_2 = P_a$$

$$\boxed{\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g} + z_2 + h_f}$$

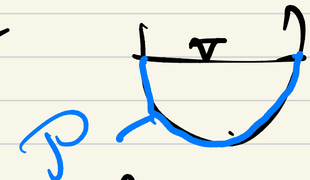
friction head loss

1D approx.

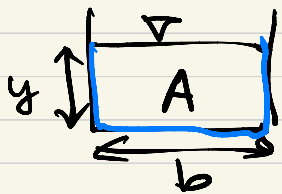
Ch.6 $h_f = f \frac{L}{d} \frac{V^2}{2g} \simeq f \frac{x_2 - x_1}{D_h} \frac{V_{avg}^2}{2g}$

friction factor

$$D_h = \frac{4A}{P}$$



$Re = D_h V_{avg} / \nu \geq 10^5$ mostly turbulent



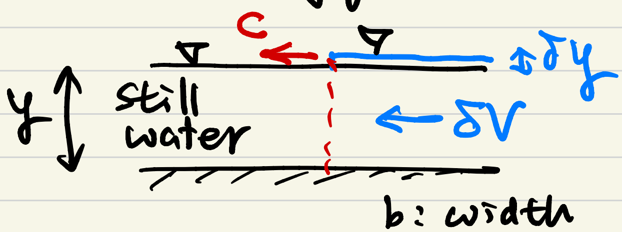
$\left. \begin{array}{l} P = b + 2y \\ A = by \end{array} \right) \; D_h = \frac{4A}{P} = \frac{4by}{b+2y}$

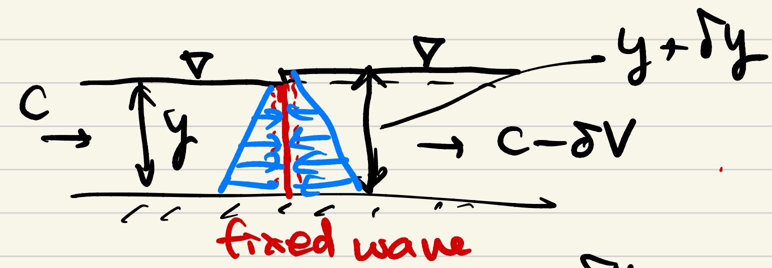
Moody chart ← accurate but seldom used

Manning's formula ← mostly used

- Flow characteristics by Froude number

$$Fr \equiv \frac{V}{\sqrt{gy}}$$ y: water depth $\left(Ma = \frac{V}{a}\right)$





Cont : $\rho c y b = \rho (c-\delta V)(y+\delta y) b \rightarrow \delta V = c \frac{\delta y}{y+\delta y}$ — ①

mtm : $\Sigma F = -\frac{1}{2}\rho g b (y+\delta y)^2 + \frac{1}{2}\rho g b y^2 = \rho c y b (c - \delta V - c)$

$$\rightarrow g\left(1+\frac{1}{2}\frac{\delta y}{y}\right)\delta y = c\,\delta V \quad \text{— ②}$$

① → ② $\boxed{c^2 = gy\left(1+\frac{\delta y}{y}\right)\left(1+\frac{1}{2}\frac{\delta y}{y}\right)}$ wave propagation speed

As $\delta y \uparrow$, $c \uparrow$

As $\delta y \rightarrow 0$, $c^2 = gy \rightarrow c = \sqrt{gy}$: speed of shallow water surface wave

$Fr = \frac{V}{C}$

$Fr < 1$: subcritical flow

$Fr = 1$: critical "

$Fr > 1$: supercritical "