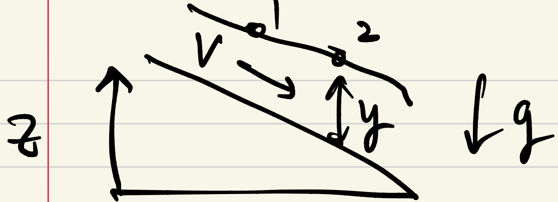
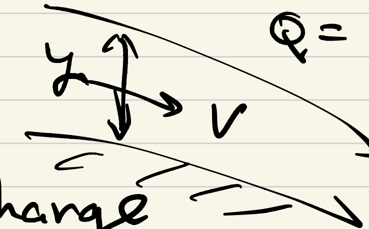


$$\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g} + z_2 + h_f$$

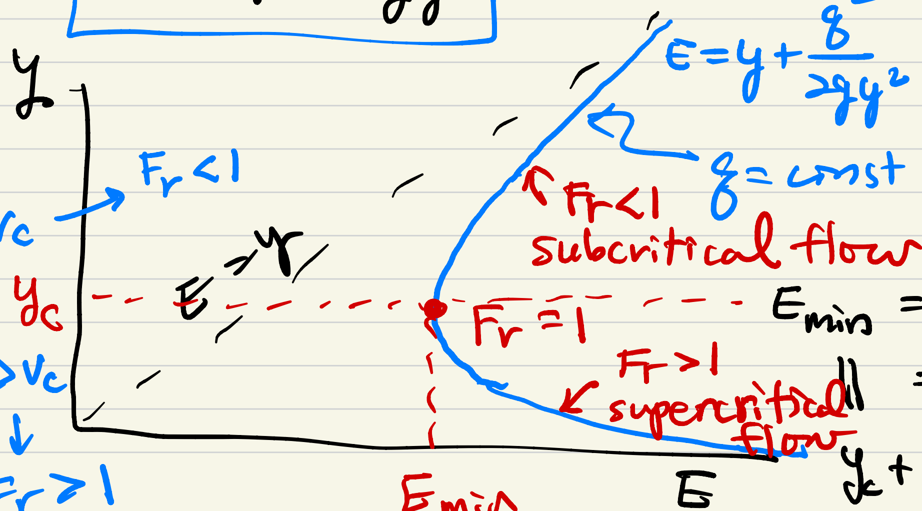
$Q = \text{const}$



- specific energy : $E \equiv y + \dfrac{V^2}{2g}$

$Q = Vyb = qb$, $\quad q = \dfrac{Q}{b}$: discharge per width
$\quad$ " const

$\rightarrow \boxed{E = y + \dfrac{q^2}{2gy^2}}$



$Q = Vyb$
$y > y_c \rightarrow V < V_c \rightarrow Fr < 1$
critical depth $\rightarrow y_c$
$y < y_c \rightarrow V > V_c$
$\downarrow$
$Fr > 1$

y_c s.t. $\dfrac{dE}{dy} = 0$

$\rightarrow y_c = (q^2/g)^{\frac{1}{3}} = \left(\dfrac{Q^2}{b^2 g}\right)^{\frac{1}{3}}$

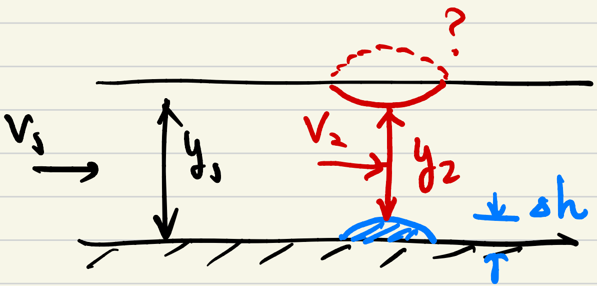
$E_{min} = E(y_c)$

$\quad \| \quad = y_c + \dfrac{q^2}{2gy_c^2} = \dfrac{3}{2} y_c$

$y_c + \dfrac{V_c^2}{2g} \Rightarrow V_c = \sqrt{gy_c} = C$

$Fr_c = 1$

- Frictionless flow over a bump



Cont: $V_1 y_1 = V_2 y_2$

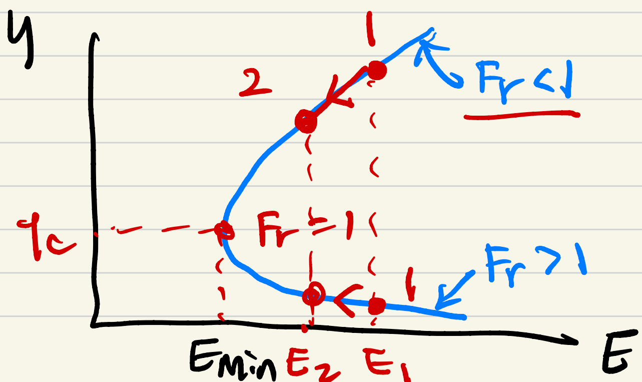
Bernoulli eq: $\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + (y_2 + \Delta h)$

$$E_1 = E_2 + \Delta h$$

$$y_2^3 - E_2 y_2^2 + \frac{1}{2g} V_1^2 y_1^2 = 0$$

$$E_2 = E_1 - \Delta h$$

If Δh is not too large, $Fr_1 = \frac{V_1}{\sqrt{g y_1}} < 1$:



$$(y_2 + \Delta h) - y_1 = \frac{1}{2g}(V_1^2 - V_2^2)$$

$$= \frac{1}{2g}(V_1 + V_2)(V_1 - V_2) < 0$$

$E_2 = E_1 - \Delta h < E_1 \Rightarrow y_2 < y_1$

$\rightarrow V_2 > V_1$

$\Rightarrow y_2 + \Delta h < y_1$ ∴ water level down!

$Fr_1 > 1$: $E_2 < E_1 \rightarrow y_2 > y_1 \rightarrow V_2 < V_1$

$$(y_2 + \Delta h) - y_1 = \frac{1}{2g}(V_1+V_2)(V_1-V_2) > 0$$

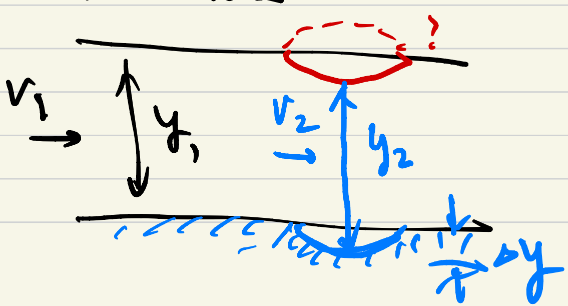
$\rightarrow y_2 + \Delta h > y_1 \Rightarrow$ water level <u>up</u>!

If $\Delta h = \Delta h_{max} = E_1 - E_{min}$, flow at the crest is critical ($Fr_2 = 1$).

If $\Delta h > \Delta h_{max}$, no physical sol.

$\rightarrow$ a bump too large will choke the channel and causes frictional effects, typically a hydraulic jump.

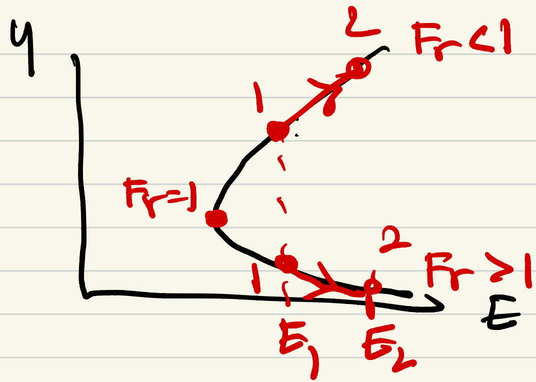
- Frictionless flow over a hollow



cont: $V_1 y_1 = V_2 y_2$

Bernoulli eq: $\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + (y_2 - \Delta h)$

$$E_1 = E_2 - \Delta h$$



$E_1 = E_2 - \Delta h$

$y_2 - \Delta h - y_1 = \frac{1}{2g}(V_1+V_2)(V_1-V_2)$

- $Fr_1 < 1$, $E_2 = E_1 + \Delta h$

  $\rightarrow y_2 > y_1 \rightarrow V_2 < V_1$

  $y_2 - \Delta h - y_1 > 0 \Rightarrow y_2 - \Delta h > y_1$

  water level up.

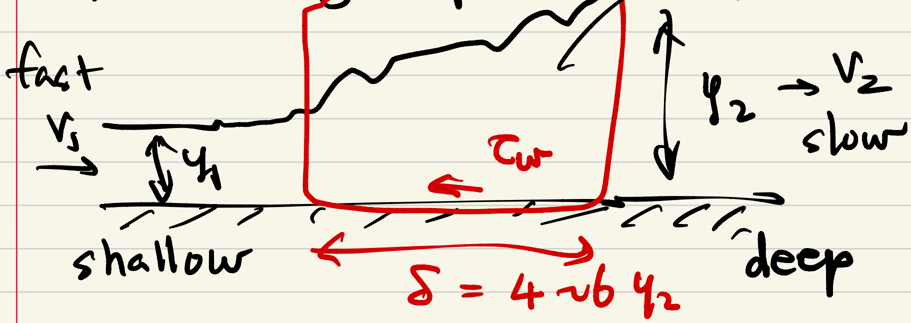
- $Fr_1 > 1$, $y_2 < y_1 \rightarrow V_2 > V_1$

  $y_2 - \Delta h - y_1 < 0 \Rightarrow y_2 - \Delta h < y_1$

  water level down.

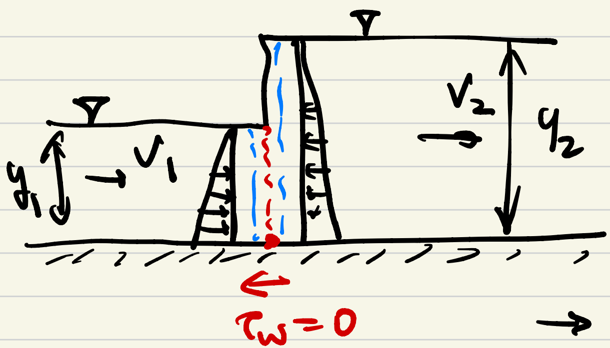
critical flow cannot occur.

- hydraulic jump



extremely turbulent

effective energy dissipator



cont: $\rho V_1 y_1 b = \rho V_2 y_2 b$

mtm: $-\frac{1}{2}\rho g b\,(y_2^2 - y_1^2)$

$= \rho V_1 y_1 b\,(V_2 - V_1)$

$\to V_1^2 = g y_1 \eta \cdot \frac{1}{2}(1+\eta)$, $\eta = \frac{y_2}{y_1}$

$V_2 = V_1\, y_1/y_2 = V_1/\eta$, $Fr_1 = V_1/\sqrt{g y_1}$

$\to \boxed{\eta = \frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8 Fr_1^2}\right)} \to \frac{V_2}{V_1} = \frac{y_1}{y_2} = \frac{1}{\eta} \to Fr_2 \sim ft(Fr_1)$

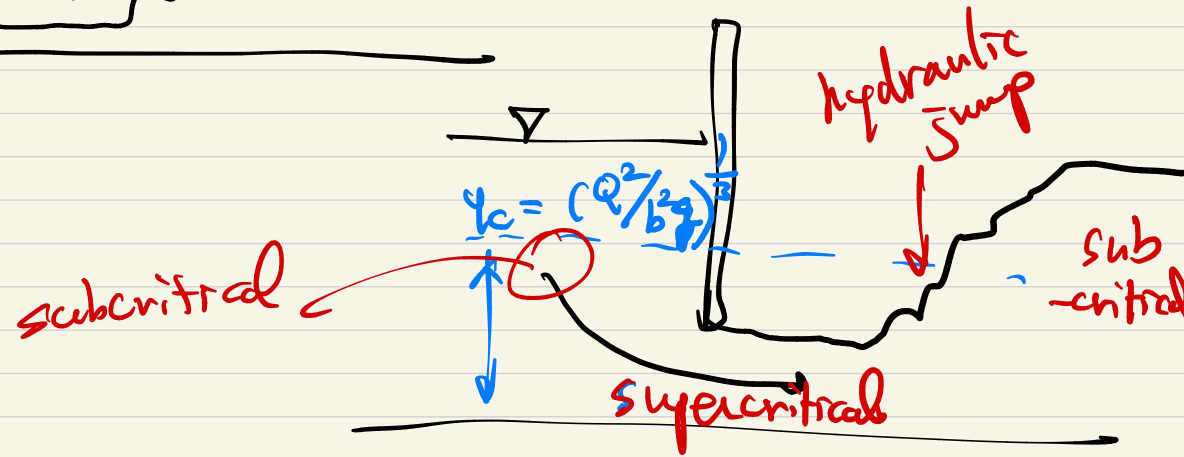
$Fr_2 = \sqrt{\frac{1}{2}(1+\eta)}/\eta < 1$ for $\eta > 1$

$$h_f = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right) = \cdots = \frac{(y_2 - y_1)^3}{4 y_1 y_2} > 0$$

$\rightarrow y_2 > y_1 \rightarrow \eta = \frac{y_2}{y_1} > 1 \rightarrow \boxed{Fr_1 > 1}$ $\therefore$ supercritical flow

$Fr_2 < 1 \leftarrow V_2 < V_1$

subcritical flow



Ch. 11. Turbomachinery (터보기계)

Fluid machinery (유체기계)

① pumps : those which add energy to fluid

② turbines : " " extract " from "

→ usually connected to a rotating shaft

"turbo" machinery

spin, whirl

Pump

- working fluid is liquid → pump

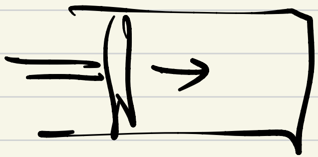
  gas → fan : $\Delta p(\uparrow)$ is small

  blower : Δp is up to 1atm.

  compressor : Δp is above 1atm.

· classification of pumps

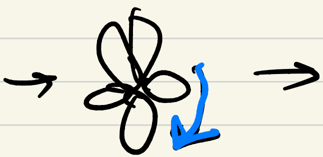
① positive-displacement pump



forces the fluid along one direction
by volume change ⇒ $\Delta p \uparrow$

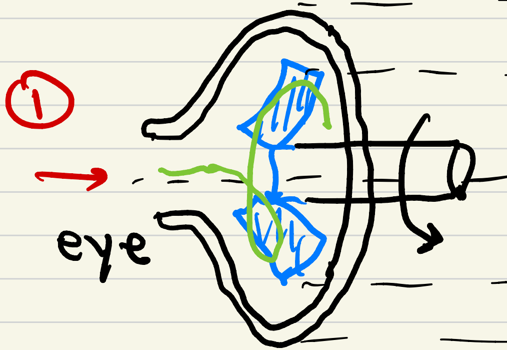
effective in handling high ν fluid

② dynamic pump : adds momentum to fluid by means
of fast moving blades or vanes or
certain special designs

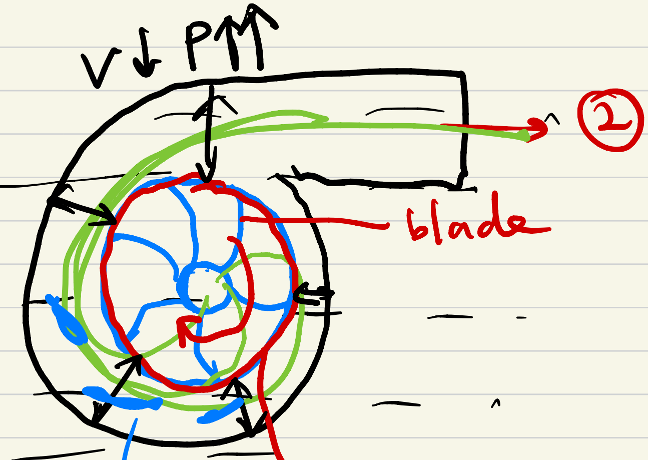


no closed volume
higher flow rate
steady discharge

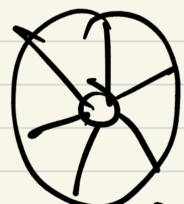
- Centrifugal pump (원심 펌프)



centrifugal compressor
원심 압축기




forward vane


radial vane


backward vane

- Efficiency

Net head : $H = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_2 - \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_1$

$= h_s - h_f$

shaft loss

usually $V_2 \simeq V_1$, $z_2 - z_1 < 1m$ neglect

$\rightarrow H \doteqdot \frac{p_2 - p_1}{\rho g} = \frac{\Delta p}{\rho g}$

Power delivered to fluid $P_w = \rho g H Q$ Q : discharge

(water horse power)

Power required to drive the pump

bhp $= \omega T$

(brake horse power) ω : shaft angular vel.

T : torque

Efficiency $\eta = \frac{P_w}{bhp} = \frac{\rho g H Q}{\omega T} < 1 \quad (\sim 0.9)$

<u>loss</u>