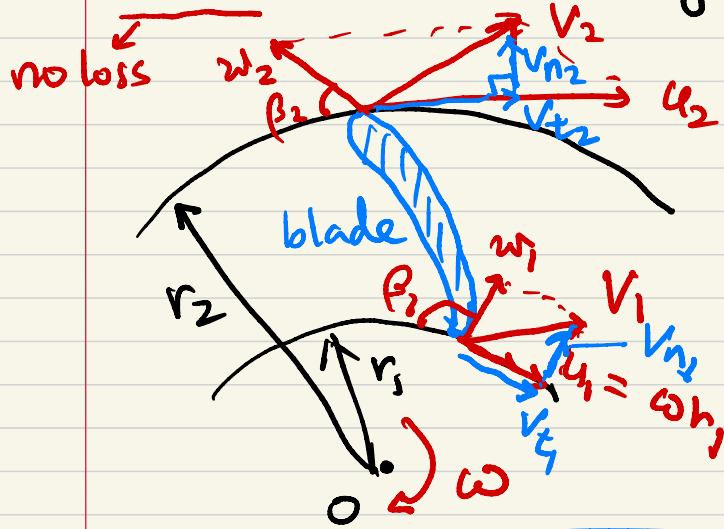


- loss
- ① loss of fluid due to leakage in the impeller-casing clearance
 - ② shock loss at the eye between inlet flow and blade entrance
 - ③ friction loss in the blade passage
 - ④ circulation loss at the exit of the blades

.....
.....

- "Euler" turbomachinery eqs.



1D flow

angular m/m cons.: $u_2 = \omega r_2$

$$\Sigma M_o = T = \rho Q (r_2 V_{t2} - r_1 V_{t1})$$

$$P_w = \rho g H Q = \omega T \quad (\eta = 1)$$

$$= \rho Q (\omega r_2 V_{t2} - \omega r_1 V_{t1})$$

$$= \rho Q (u_2 V_{t2} - u_1 V_{t1})$$

$$\Rightarrow H = \frac{P_w}{\rho g Q} = \frac{1}{g} (u_2 V_{t2} - u_1 V_{t1})$$

↪ not a fct of V_n .

$$V^2 = u^2 + w^2 - 2uw \cos \beta$$

$\underbrace{\quad}_{= u - V_t}$

$$\rightarrow u V_t = \frac{1}{2} (V^2 + u^2 - w^2)$$

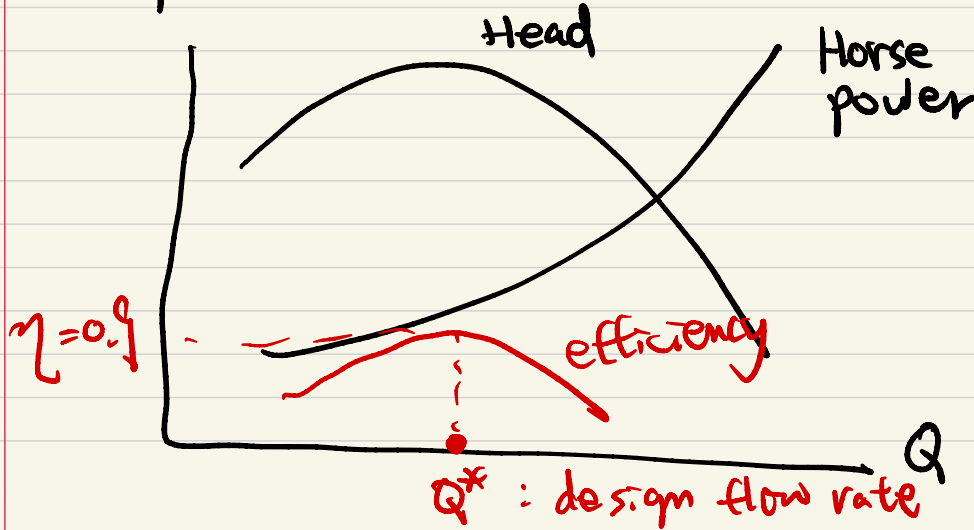
$$\Rightarrow H = \frac{1}{2g} \left[(v_2^2 - v_1^2) + (u_2^2 - u_1^2) - (w_2^2 - w_1^2) \right]$$

$$= \left(\frac{P}{\rho g} + \frac{v^2}{2g} + z \right)_2 - \left(\frac{P}{\rho g} + \frac{v^2}{2g} + z \right)_1$$

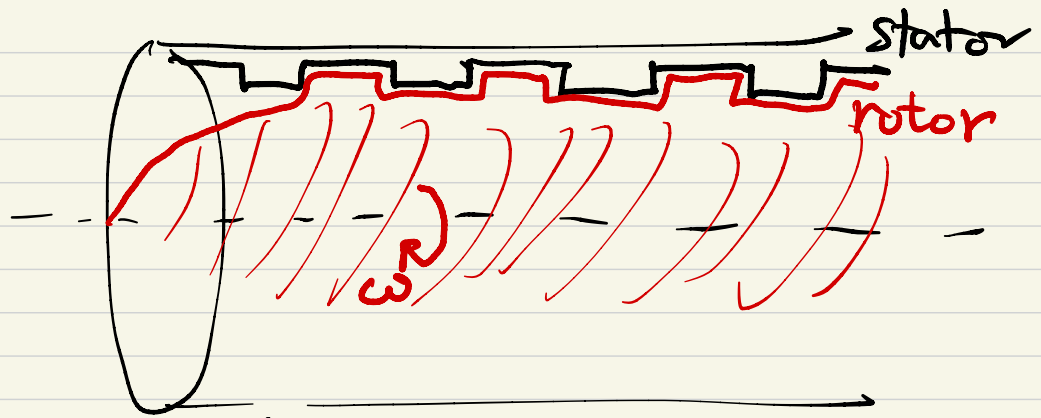
$$\Rightarrow \boxed{\frac{P}{\rho g} + z + \frac{v^2}{2g} - \frac{r^2 \omega^2}{2g} = \text{const}}$$

Bernoulli eq in rotating coord.

- Pump performance curves for constant shaft rotation speed



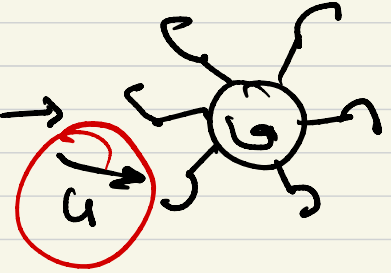
- Axial-flow pump



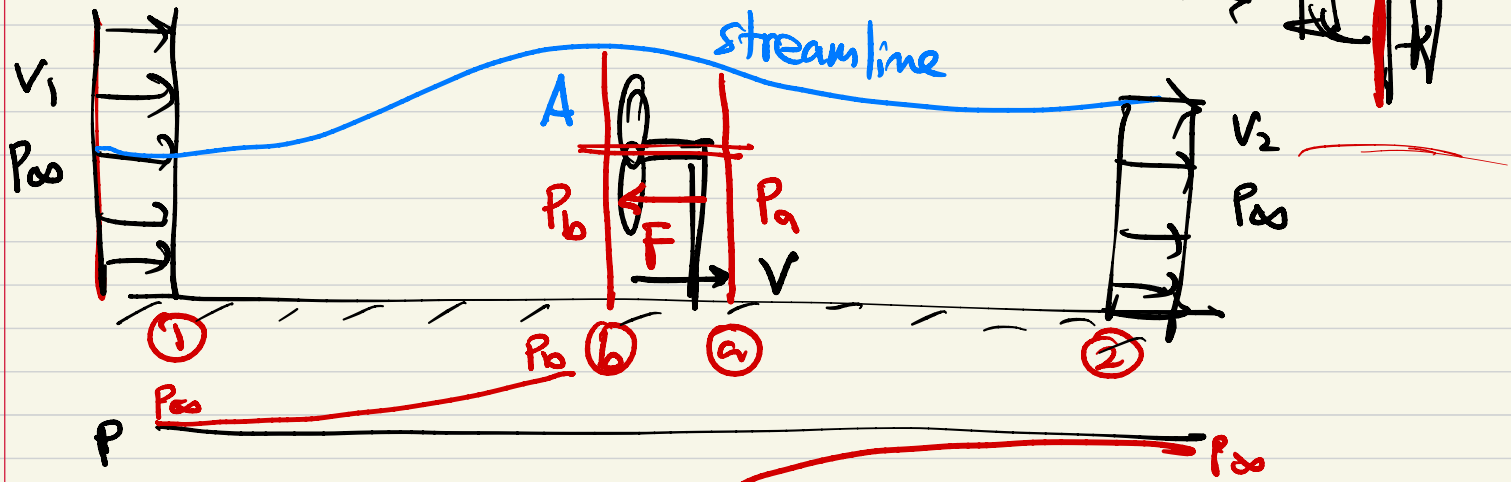
- Turbines

① reaction turbine : blade

② impulse : high-speed stream →



• wind turbine



Idealized wind turbine theory

$$\textcircled{1} - \textcircled{2} : \sum F_x = -F = \dot{m} (V_2 - V_1) \quad V_a \doteq V_b = V \quad (\because A \text{ const})$$

$$\textcircled{b} - \textcircled{a} : \sum F_x = -F + (P_b - P_a) A = \dot{m} (V_a - V_b) = 0$$

$$\rightarrow F = (P_b - P_a) A = \dot{m} (V_1 - V_2) = \rho A V (V_1 - V_2)$$

Bernoulli eq: $P_b + \frac{1}{2} \rho V_1^2 = P_b + \frac{1}{2} \rho V^2$: ① - ①

$P_a + \frac{1}{2} \rho V^2 = P_{a0} + \frac{1}{2} \rho V_2^2$: ② - ②

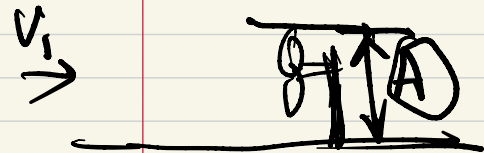
$\rightarrow P_b - P_a = \frac{1}{2} \rho (V_1^2 - V_2^2) = \rho V (V_1 - V_2)$

$\Rightarrow V = \frac{1}{2} (V_1 + V_2)$

Power $\mathcal{P} = F \cdot V = \rho A V (V_1 - V_2) V = \frac{1}{4} \rho A (V_1^2 - V_2^2) (V_1 + V_2)$

Given V_1 , max \mathcal{P} ?

$\frac{\partial \mathcal{P}}{\partial V_2} = 0 \Rightarrow V_2 = \frac{1}{3} V_1 \rightarrow \mathcal{P}_{\max} = \frac{8}{27} \rho A V_1^3$



$\mathcal{P}_{\text{available}} = \frac{1}{2} \dot{m} V_1^2 = \frac{1}{2} \rho A V_1^3$

($\dot{m} = \rho A V_1$)

(Betz number)

Power efficiency $\mathcal{C}_p = \frac{\mathcal{P}}{\mathcal{P}_{\text{avail}}}$

$\mathcal{C}_{p\max} = \frac{8/27}{1/2} = \frac{16}{27} = 0.593$

