

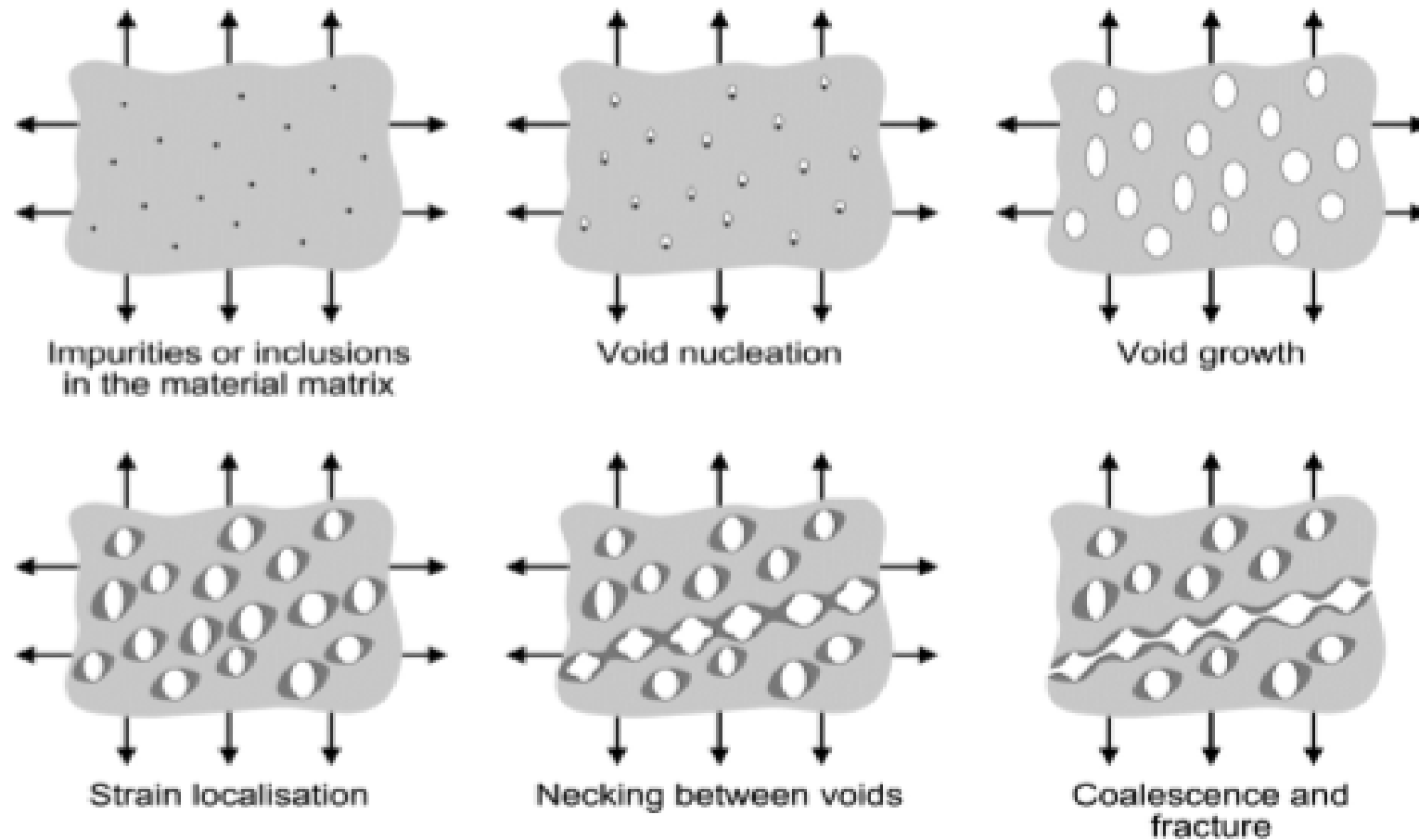
# Damage Modelling in Ductile Materials

Specific research topic: A comparative study on three fracture models for incremental sheet metal forming with Al1050 aluminium alloy.

Degradation of material properties and ability of load carrying capacity of material termed as damage, after appreciable damage rupture/crack formation occurs.

**Ductile damage:** (local approaches or CDM)

Mechanism: void nucleation, growth and coalescence. (sometimes shear fracture in voids may happen in shear loading conditions )



**Fig. 1.** Schematic of nucleation, growth and coalescence of voids.

# Damage Modelling in Ductile Materials

Major models used widely:

1. GTN Model : fully coupled (in sense of yield function and damage variable [f])
2. CDM Model (Lemaitre, Chaboche, Murakami)
3. Uncoupled fracture Models (BW, MMC, HC etc)

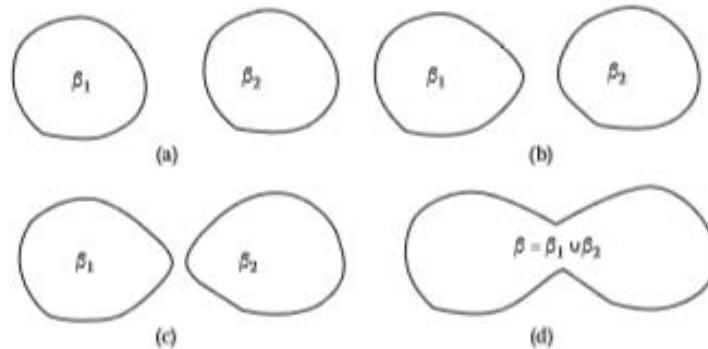


FIGURE 11.3  
Schematic representation of the void coalescence through the process of void impingement.  
(a) Initial configuration, (b) necking starts, (c) necking progresses further and (d) final void impingement.

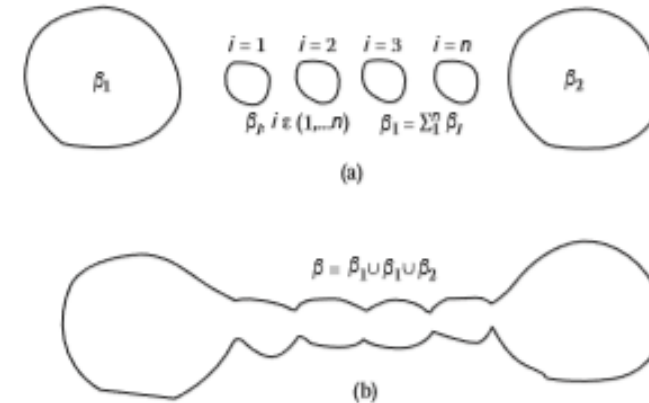


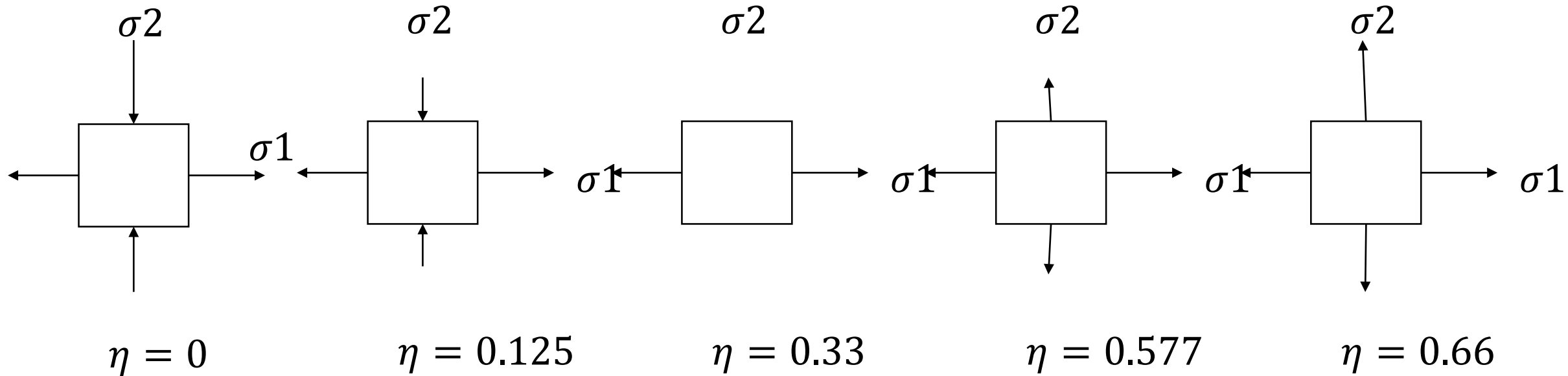
FIGURE 11.4  
Schematic representation of the void coalescence through the process of void sheet formation.  
(a) Initial configuration and (b) after coalescence.

# stress triaxiality

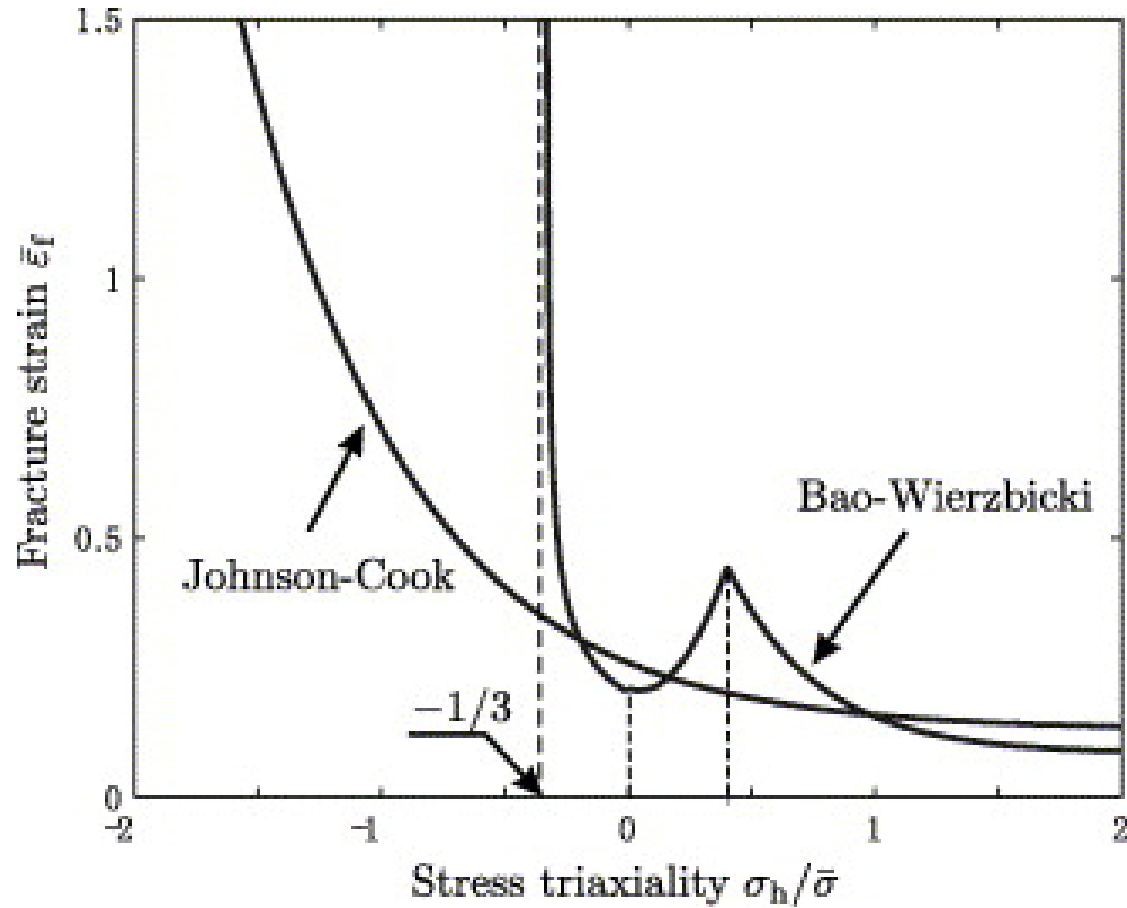
$$\eta = \sigma_m / \sigma_{eq}$$

$$\eta = \frac{1}{3} \frac{1 + \beta}{\sqrt{1 - \beta + \beta^2}}$$

In case of plane stress, where  $\beta$  is stress ratio



## Fracture Model in stress triaxiality space



# BW Model

Damage initiation based on estimating equivalent plastic strain at the onset of fracture wrt triaxiality

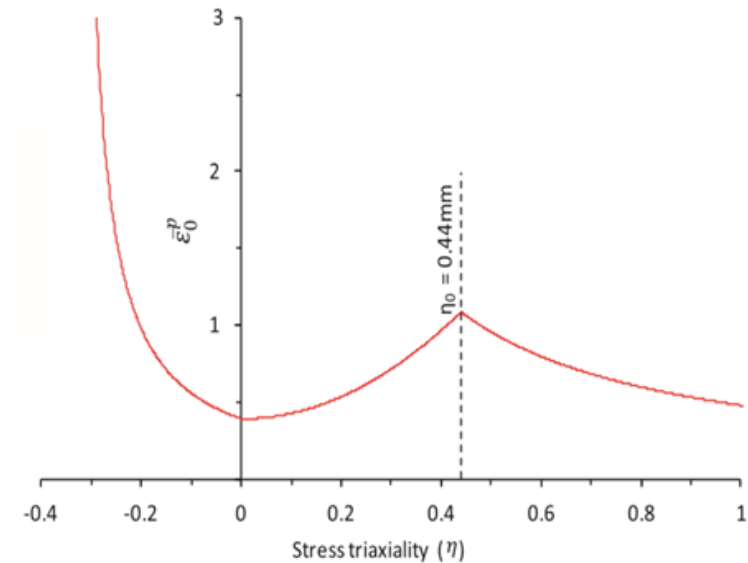
Three pronged model developed by conducting experiments with varying triaxialities and finding point of onset of fracture

$$\bar{\varepsilon}_0^p = \frac{C_1}{(1+3\eta)} \quad \left\{ -\frac{1}{3} \leq \eta \leq 0 \right\}$$

$$\bar{\varepsilon}_0^p = C_1 + (C_2 - C_1)(\eta/\eta_0)^2 \quad \left\{ 0 \leq \eta \leq \eta_0 \right\}$$

$$\bar{\varepsilon}_0^p = C_2(\eta_0/\eta) \quad \left\{ \eta \geq \eta_0 \right\}$$

$$\omega = \int \frac{d\varepsilon^p}{\bar{\varepsilon}_0^p}$$



## BW Model

C1: in pure shear; C2: in uniaxial tension;  $n_0$  average triaxiality in uniaxial tension test

No fracture for triaxialities less than  $-1/3$

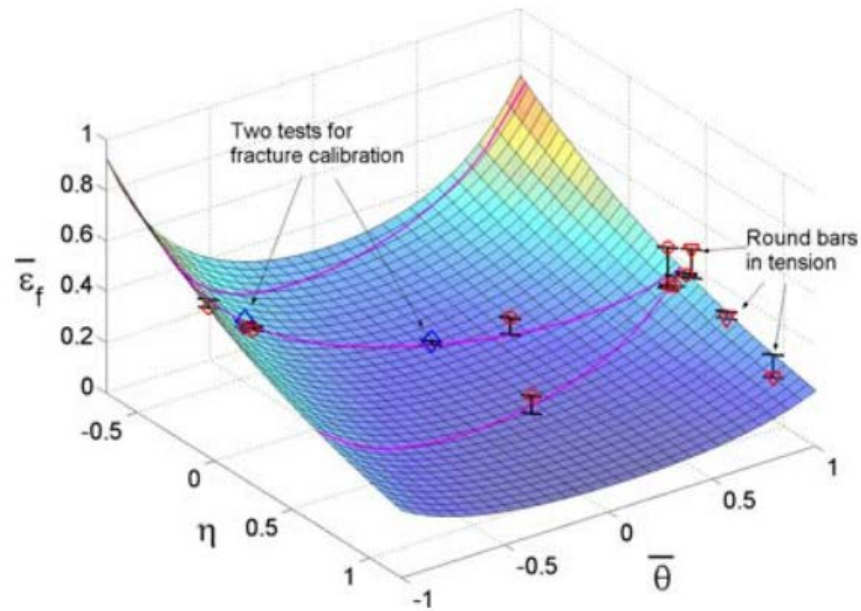
Damage initiation marked by variable  $\omega$  as it reaches unity. Takes care of history of triaxialities in the loading process.

Experimental determination of C2 has two major assumptions:

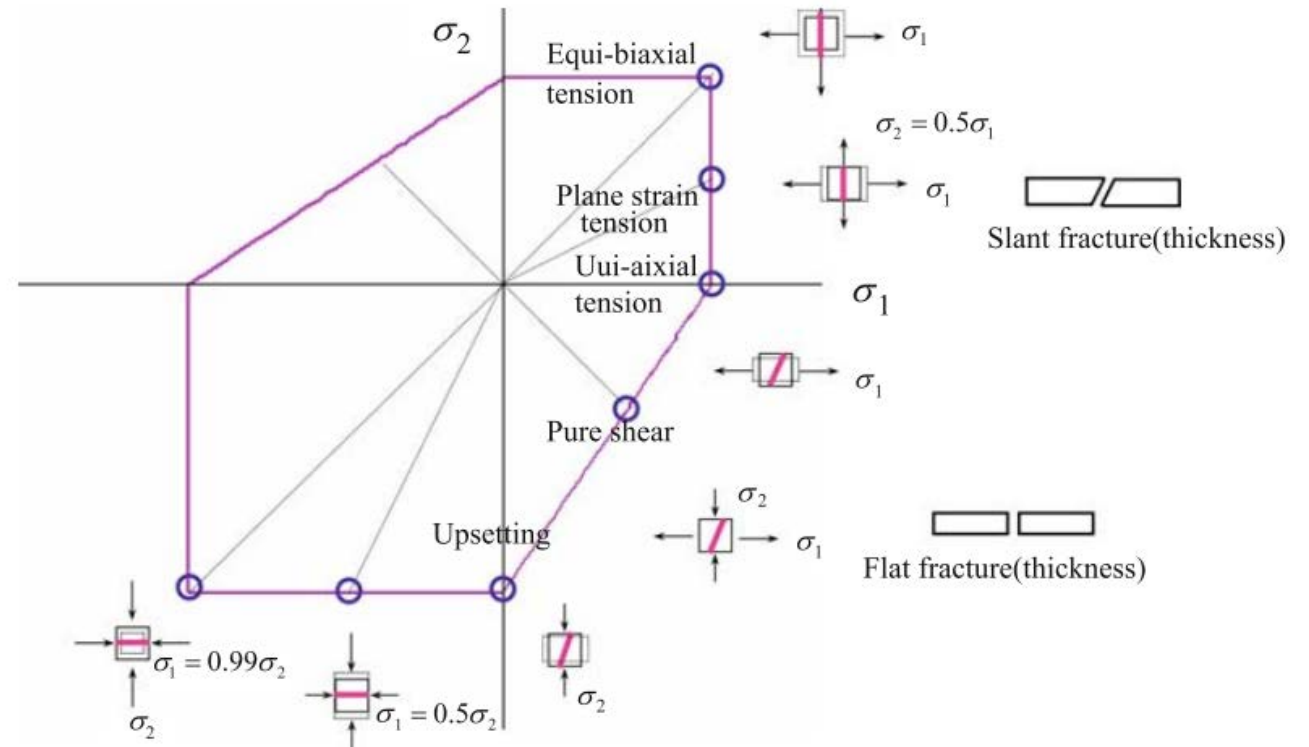
1. Location of onset of damage coincides with location of maximum equivalent plastic strain at the instant of onset of fracture.
2. The numerical simulation of tensile test with the best fit of piecewise linear extrapolation post necking.

# Mohr Coulomb model – (Bai-Wierzbicki 2010)

DOI: [10.1007/s10704-009-9422-8](https://doi.org/10.1007/s10704-009-9422-8)



**Fig. 22** 3D geometric representation of Mohr–Coulomb fracture locus for 2024-T351 aluminum alloy. ( $A = 740$  MPa,  $n = 0.15$ ,  $c_1 = 0.0345$ ,  $c_2 = 338.6$  MPa,  $c_\theta^s = c_\theta^c = 1.0$ )





## Stress state in terms of Triaxiality and Lode angle parameter

$$p = -\sigma_m = -\frac{1}{3}\text{tr}([\sigma]) = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$\eta = \frac{\sigma_m}{\bar{\sigma}}$$

$$q = \bar{\sigma} = \sqrt{\frac{3}{2}[S] : [S]}$$

$$\xi = \left(\frac{r}{q}\right)^3 = \cos(3\theta).$$

$$= \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

$$r = \left(\frac{9}{2}[S] \cdot [S] : [S]\right)^{1/3} = \left[\frac{27}{2}\det([S])\right]^{1/3}$$

$$= \left[\frac{27}{2}(\sigma_1 - \sigma_m)(\sigma_2 - \sigma_m)(\sigma_3 - \sigma_m)\right]^{1/3}$$

$$\bar{\theta} = 1 - \frac{6\theta}{\pi} = 1 - \frac{2}{\pi} \arccos \xi.$$

where  $[S]$  is the deviatoric stress tensor defined by,

$$[S] = [\sigma] + p[I],$$

# Mohr Coulomb in terms of $\varepsilon, \eta, \bar{\theta}$

$$\left(\sqrt{1+c_1^2}+c_1\right)\sigma_1-\left(\sqrt{1+c_1^2}-c_1\right)\sigma_3=2c_2, \quad \sigma_1 \geq \sigma_2 \geq \sigma_3.$$

$$\bar{\sigma}=c_2\left[\sqrt{\frac{1+c_1^2}{3}}\cos\left(\frac{\pi}{6}-\theta\right)+c_1\left(\eta+\frac{1}{3}\sin\left(\frac{\pi}{6}-\theta\right)\right)\right]^{-1}$$

$$\bar{\sigma}=A\bar{\varepsilon}^n\left[1-c_\eta(\eta-\eta_0)\right]\left[c_\theta^s+(c_\theta^{ax}-c_\theta^s)\gamma\right]$$

$$\gamma=\frac{\sqrt{3}}{2-\sqrt{3}}\left[\sec\left(\theta-\frac{\pi}{6}\right)-1\right]$$

$$\bar{\varepsilon}_f=\left\{\frac{A}{c_2}\left[c_\theta^s+\frac{\sqrt{3}}{2-\sqrt{3}}(c_\theta^{ax}-c_\theta^s)\left(\sec\left(\frac{\bar{\theta}\pi}{6}\right)-1\right)\right]\left[\sqrt{\frac{1+c_1^2}{3}}\cos\left(\frac{\bar{\theta}\pi}{6}\right)+c_1\left(\eta+\frac{1}{3}\sin\left(\frac{\bar{\theta}\pi}{6}\right)\right)\right]\right\}^{-\frac{1}{n}},$$

# Hosford - Coulomb Criteria

2015: Dirk Mohr et al

Mohr-Coulomb criteria :

$$\max_{\mathbf{n}}[\tau + c_1 \sigma_n] = c_2, \longrightarrow (\sigma_I - \sigma_{III}) + c(\sigma_I + \sigma_{III}) = b, \quad c = \frac{c_1}{\sqrt{1 + c_1^2}} \quad \text{and} \quad b = \frac{2c_2}{\sqrt{1 + c_1^2}},$$

an extension of the MC criterion is proposed by substituting the Tresca equivalent stress in by the Hosford (1972) equivalent stress

$$\bar{\sigma}_{HF} + c(\sigma_I + \sigma_{III}) = b, \quad c = \frac{c_1}{\sqrt{1 + c_1^2}} \quad \text{and} \quad b = \frac{2c_2}{\sqrt{1 + c_1^2}},$$

$$\bar{\sigma}_{HF} = \left\{ \frac{1}{2} \left( (\sigma_I - \sigma_{II})^a + (\sigma_{II} - \sigma_{III})^a + (\sigma_I - \sigma_{III})^a \right) \right\}^{\frac{1}{a}}, \quad \{0 < a < 2\}$$

# Hosford - Coulomb Criteria

$$\bar{\sigma}_{HF} + c(\sigma_I + \sigma_{III}) = b, \quad c = \frac{c_1}{\sqrt{1 + c_1^2}} \quad \text{and} \quad b = \frac{2c_2}{\sqrt{1 + c_1^2}},$$

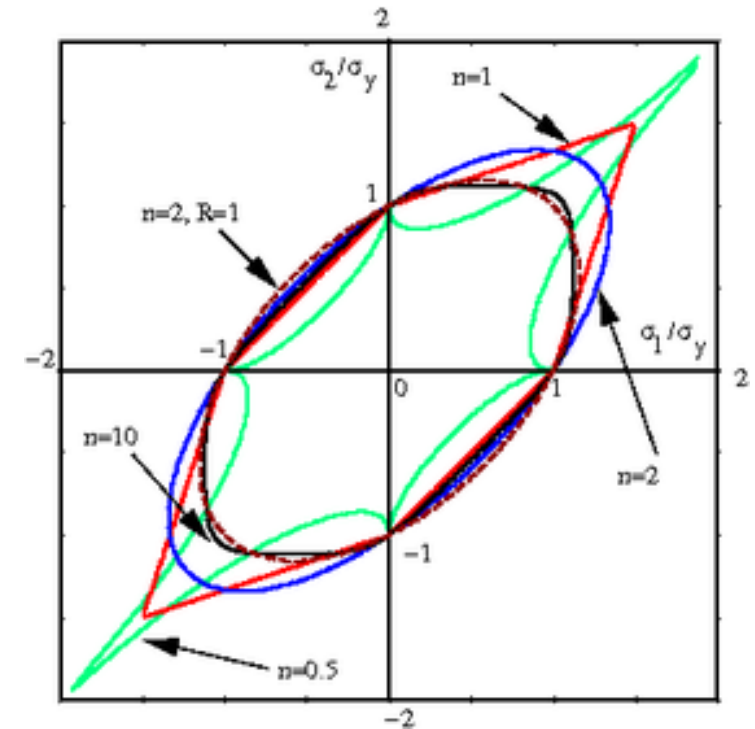
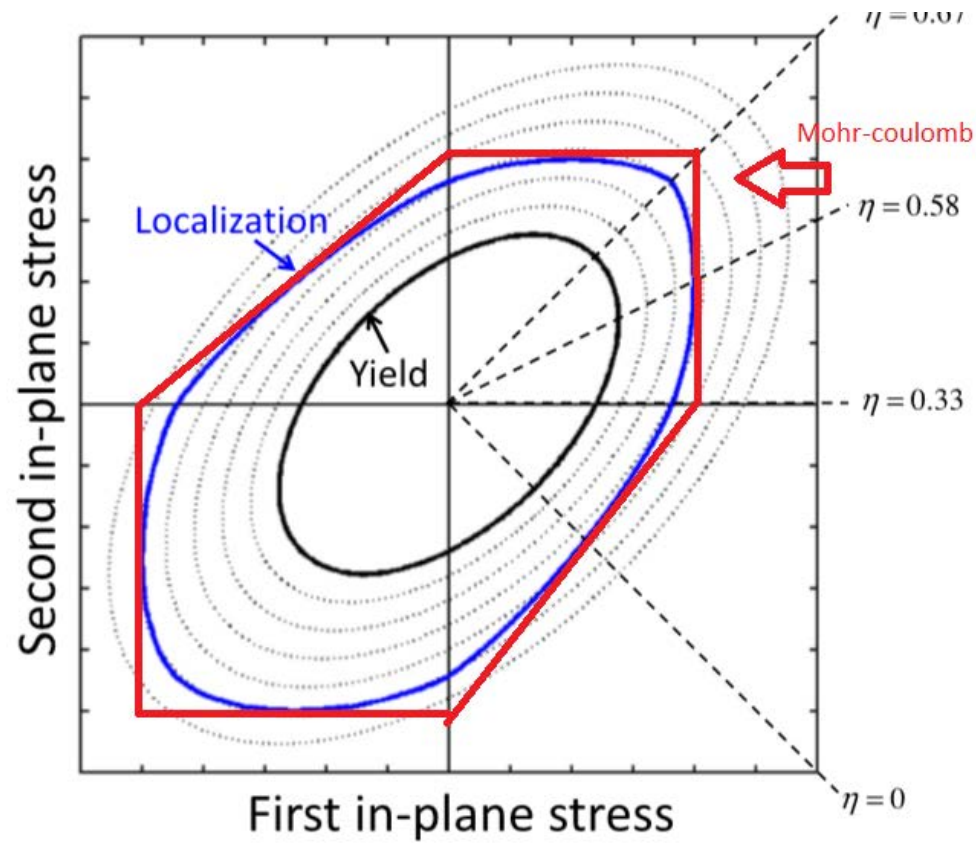
$$\bar{\sigma}_{HF} = \left\{ \frac{1}{2} \left( (\sigma_I - \sigma_{II})^a + (\sigma_{II} - \sigma_{III})^a + (\sigma_I - \sigma_{III})^a \right) \right\}^{\frac{1}{a}}, \quad \{0 < a < 2\}$$

For a=1 The above criteria becomes Mohr-Coulomb Criteria

Note: The Hosford criterion becomes non-convex for  $a < 1$ . This requires special care when using the Hosford function as yield surface, but there is no restriction with respect to convexity when it is used as localization criterion

# Hosford - Coulomb Criteria

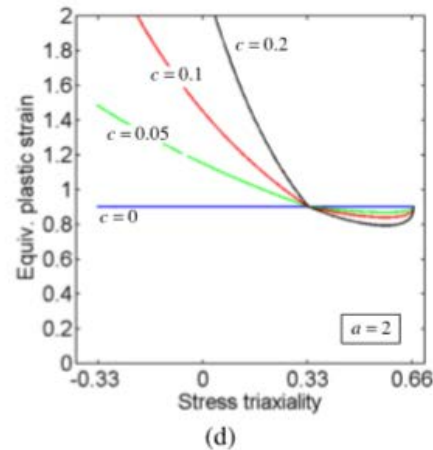
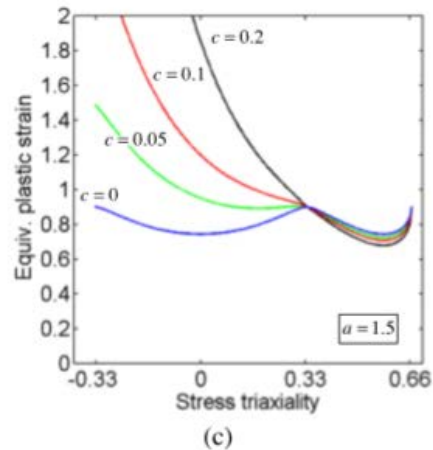
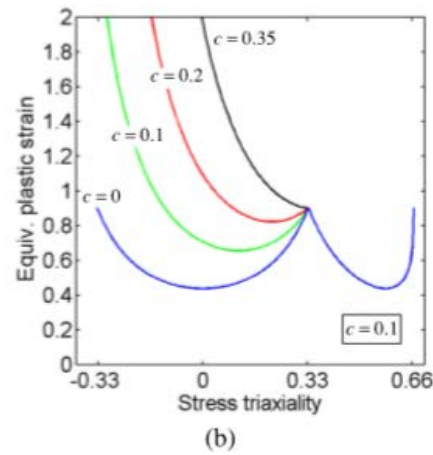
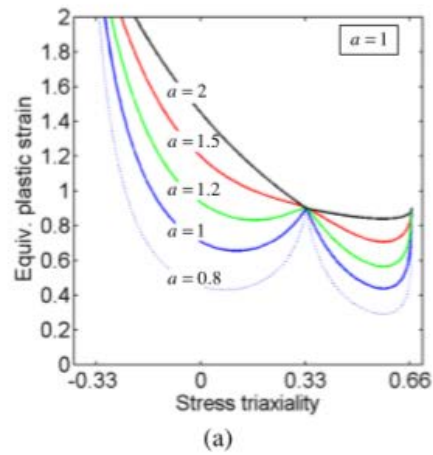
$$\bar{\sigma}_{HF} + c(\sigma_I + \sigma_{III}) = b, \quad \bar{\sigma}_{HF} = \left\{ \frac{1}{2} \left( (\sigma_I - \sigma_{II})^a + (\sigma_{II} - \sigma_{III})^a + (\sigma_I - \sigma_{III})^a \right) \right\}^{\frac{1}{a}},$$



# Hosford - Coulomb Criteria

$$\bar{\sigma}_{HF} + c(\sigma_I + \sigma_{III}) = b, \quad \bar{\sigma}_{HF} = \left\{ \frac{1}{2} \left( (\sigma_I - \sigma_{II})^a + (\sigma_{II} - \sigma_{III})^a + (\sigma_I - \sigma_{III})^a \right) \right\}^{\frac{1}{a}},$$

*D. Mohr, S.J. Marcadet/International Journal of Solids and Structures 67-68 (2015) 40-55*



. Effect of the parameters of the Hosford–Coulomb (HC) model on the fracture envelope for plane stress loading.

# Hosford - Coulomb Criteria

$$\bar{\sigma}_{HF} + c(\sigma_I + \sigma_{III}) = b,$$

$$\bar{\sigma}_{HF} = \left\{ \frac{1}{2} ((\sigma_I - \sigma_{II})^a + (\sigma_{II} - \sigma_{III})^a + (\sigma_I - \sigma_{III})^a) \right\}^{\frac{1}{a}},$$

$$\bar{\sigma} = \bar{\sigma}_f[\eta, \bar{\theta}] = \frac{b}{\left\{ \frac{1}{2} ((f_1 - f_2)^a + (f_2 - f_3)^a + (f_1 - f_3)^a) \right\}^{\frac{1}{a}} + c(2\eta + f_1 + f_3)}.$$

$$\bar{\epsilon}_f^{pr} = k^{-1} [\bar{\sigma}_f[\eta, \bar{\theta}]].$$

$$\int_0^{\bar{\epsilon}_f} \frac{d\bar{\epsilon}_p}{\bar{\epsilon}_f^{pr}[\eta, \bar{\theta}]} = 1,$$

$$\sigma_I = \bar{\sigma}(\eta + f_1),$$

$$\sigma_{II} = \bar{\sigma}(\eta + f_2),$$

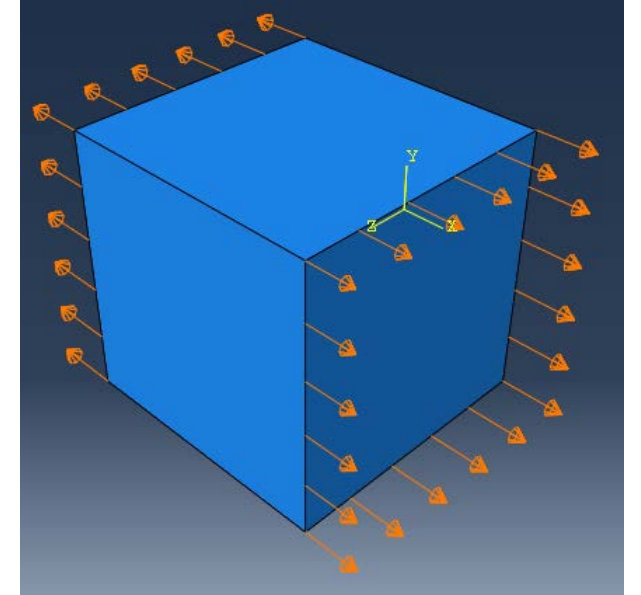
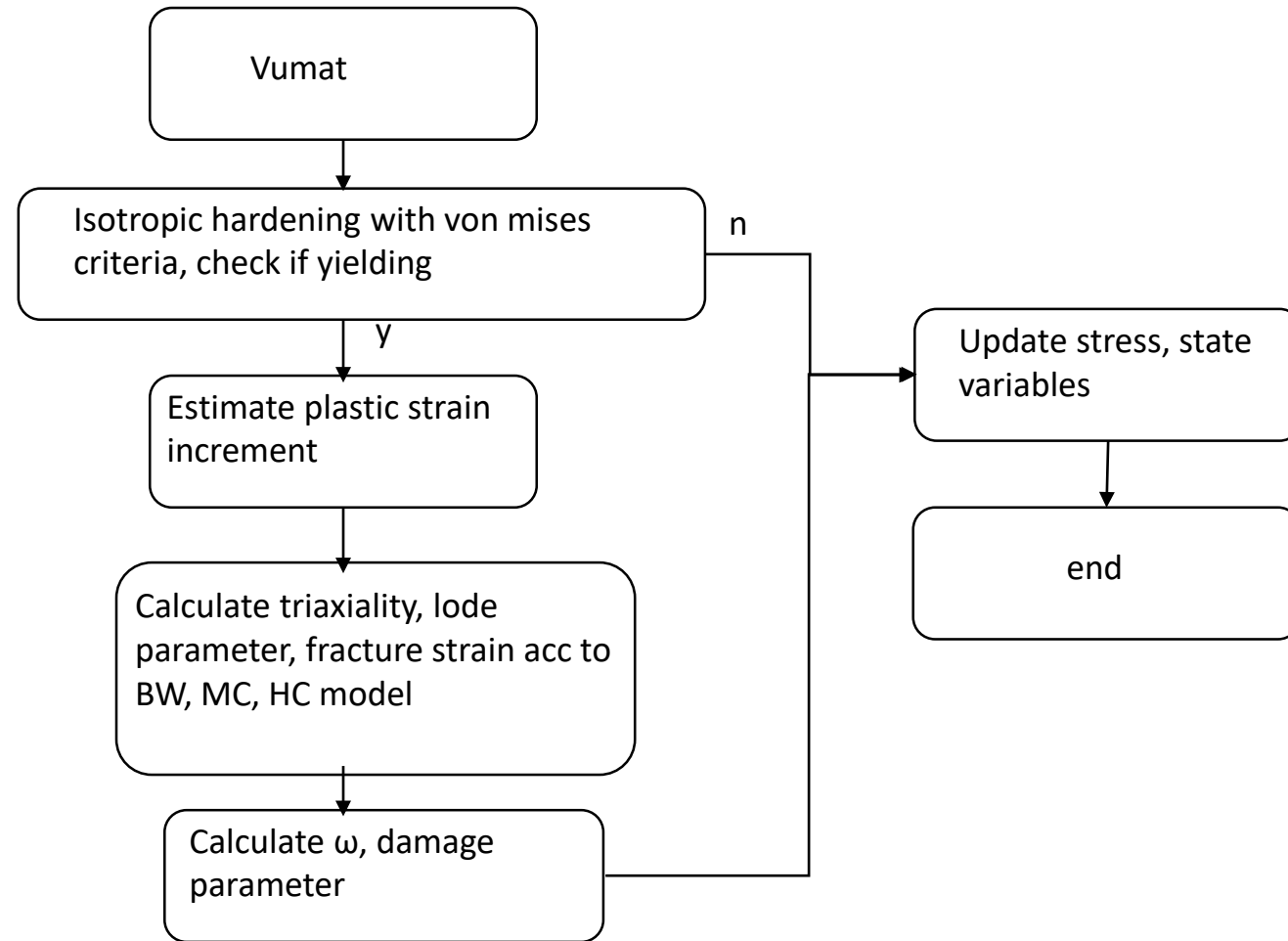
$$\sigma_{III} = \bar{\sigma}(\eta + f_3),$$

$$f_1[\bar{\theta}] = \frac{2}{3} \cos \left[ \frac{\pi}{6} (1 - \bar{\theta}) \right],$$

$$f_2[\bar{\theta}] = \frac{2}{3} \cos \left[ \frac{\pi}{6} (3 + \bar{\theta}) \right],$$

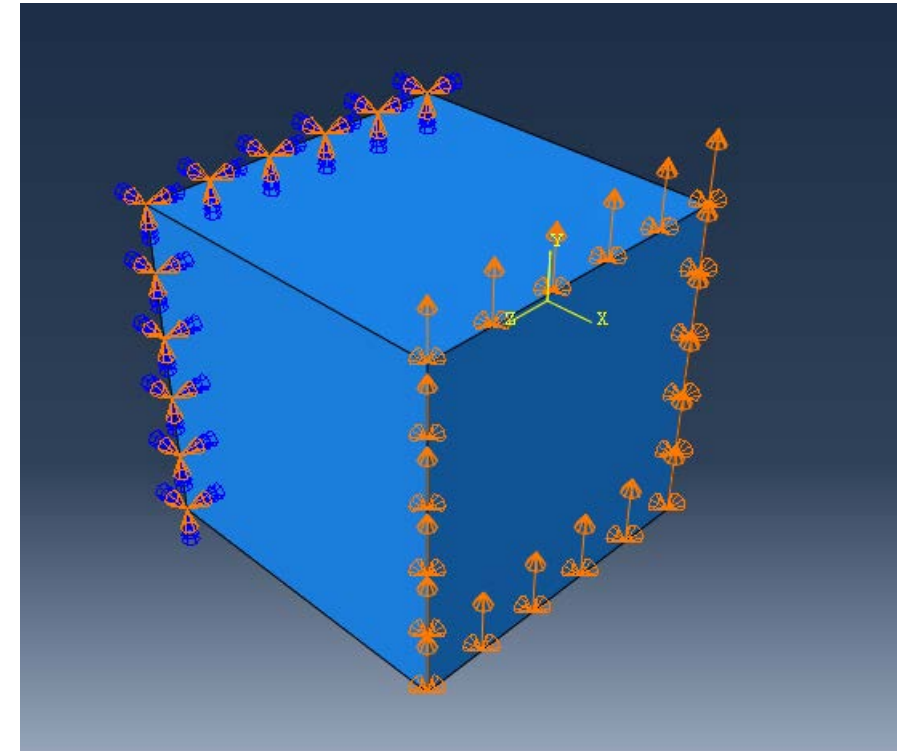
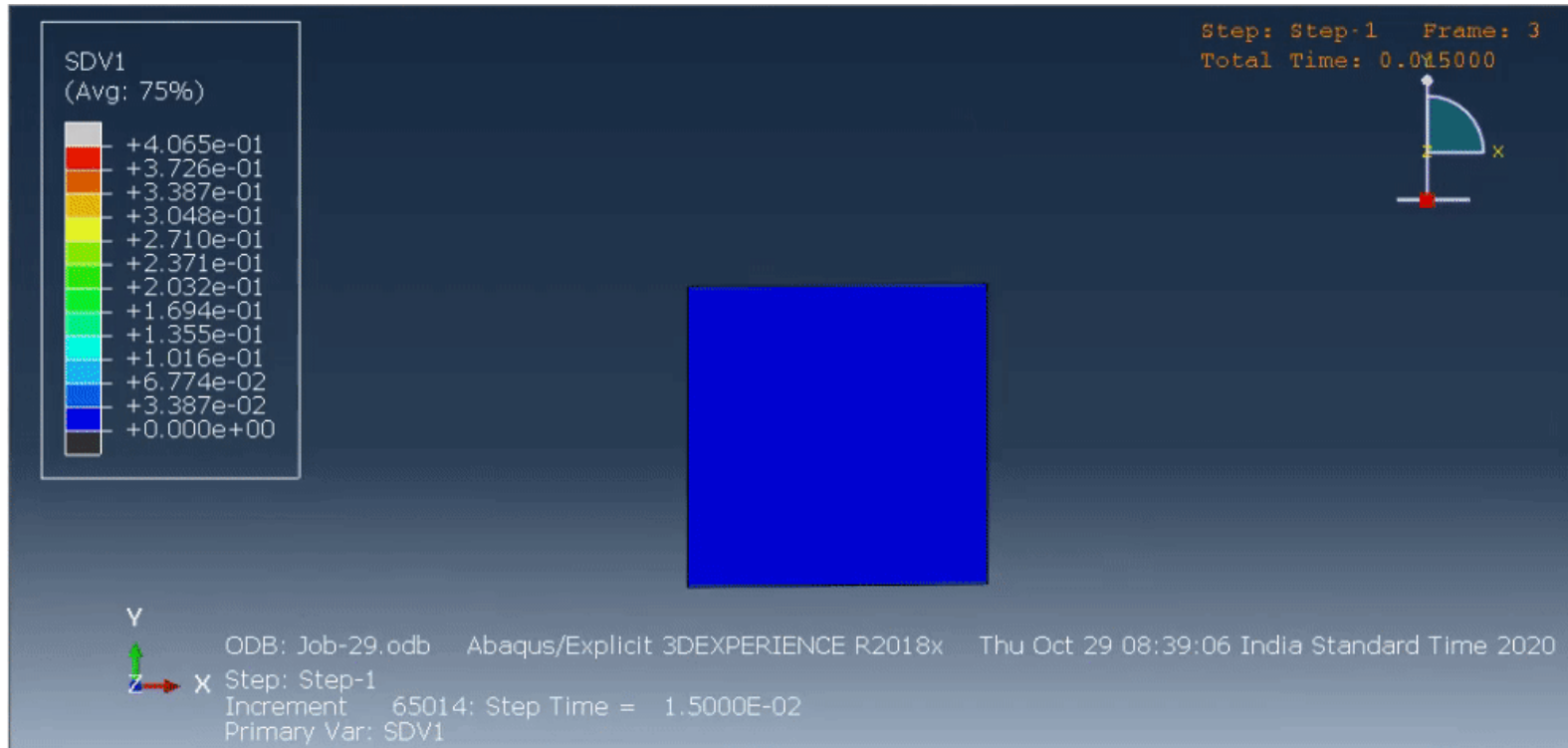
$$f_3[\bar{\theta}] = -\frac{2}{3} \cos \left[ \frac{\pi}{6} (1 + \bar{\theta}) \right].$$

# Vumat model and single element test





# single element test validation (pure shear case) || Mohr Coulomb Criteria



## single element test validation (pure shear case)

	Mechanical Constants
1	71659
2	0.33
3	5
4	120
5	120
6	1
7	1
8	1
9	101.9138
10	0
11	103.5364971
12	0.0005
13	104.7055913

C User needs **to** input  
 C props (1) - Young's modulus, e  
 C props (2) - Poisson's ratio, nu  
 C props (3) - Cone  
 C props (4) - Ctwo  
 C props (5) - power law constant  
 C props (6) - strain hardening expon  
 C props (7) - hardening law const cs  
 C props (8) - hardening law const cc  
 C props (9..) - syield and hardening **data**

E	nu	C1	C2	A	n	Cc	Cs
71659	0.33	5	120	120	1	1	1

# single element test validation (Pure shear case)

For uniaxial case,

Lode angle parameter ( $\bar{\theta}$ ) = 0, triaxiality= 0, putting below parameters in MC equation,

E	nu	C1	C2	A	n	Cc	Cs
71659	0.33	5	120	120	1	1	1

$$\bar{\epsilon}_f = \left\{ \frac{A}{c_2} \left[ c_\theta^s + \frac{\sqrt{3}}{2 - \sqrt{3}} (c_\theta^{ax} - c_\theta^s) \left( \sec\left(\frac{\bar{\theta}\pi}{6}\right) - 1 \right) \right] \right. \\ \left. \left[ \sqrt{\frac{1 + c_1^2}{3}} \cos\left(\frac{\bar{\theta}\pi}{6}\right) + c_1 \left( \eta + \frac{1}{3} \sin\left(\frac{\bar{\theta}\pi}{6}\right) \right) \right] \right\}^{-\frac{1}{n}},$$

fracture strain = 0.339683

# single element test validation (Pure shear case)

Name: SDV2 Pl: PART-1-1 E: 1 IP: 1

	X	Y
152	0.755	0.953788
153	0.76	0.960038
154	0.765	0.966288
155	0.77	0.972538
156	0.775	0.978788
157	0.78	0.985038
158	0.785	0.991288
159	0.79	0.997539
160	0.795	1.00303
161	0.8	1.00803
162	0.805	1.01303
163	0.81	1.01803
164	0.815	1.02303
165	0.82	1.02804
166	0.825	1.03304

Damage parameter

Name: SDV1 Pl: PART-1-1 E: 1 IP: 1

	X	Y
152	0.755	0.309076
153	0.76	0.310951
154	0.765	0.312827
155	0.77	0.314702
156	0.775	0.316577
157	0.78	0.318452
158	0.785	0.320328
159	0.79	0.322203
160	0.795	0.324078
161	0.8	0.325953
162	0.805	0.327829
163	0.81	0.329704
164	0.815	0.331579
165	0.82	0.333454
166	0.825	0.335329

Eq plastic strain

Name: SDV4 Pl: PART-1-1 E: 1 IP: 1

	X	Y
34	0.165	0.339683
35	0.17	0.339683
36	0.175	0.339683
37	0.18	0.339683
38	0.185	0.339683
39	0.19	0.339683
40	0.195	0.339683
41	0.2	0.339683
42	0.205	0.339683
43	0.21	0.339683
44	0.215	0.339683
45	0.22	0.339683
46	0.225	0.339683
47	0.23	0.339683
48	0.235	0.339683

Fracture strain EPLO

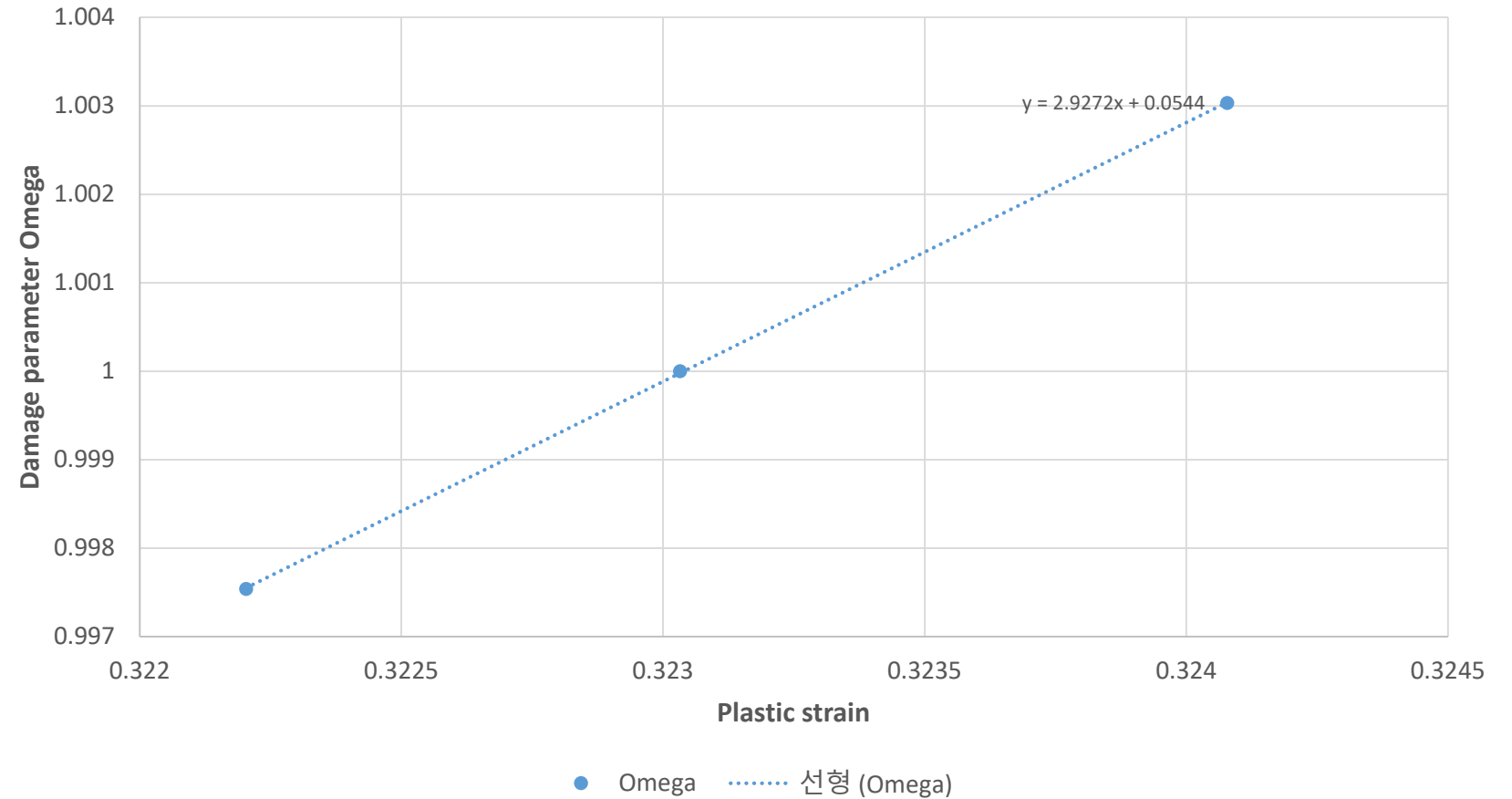
Also, checked through state variables lode angle parameter comes nearly zero (in the order of E-6) and triaxiality fluctuates near zero (in the order of E-9)

# single element test validation (Pure shear case)

Omega	Plastic strain
0.997539	0.322203
1.00303	0.324078
1	0.32303227

fracture strain = 0.339683

## Linear interpolation



# Model calibration and simulation results



# Hardening Rule

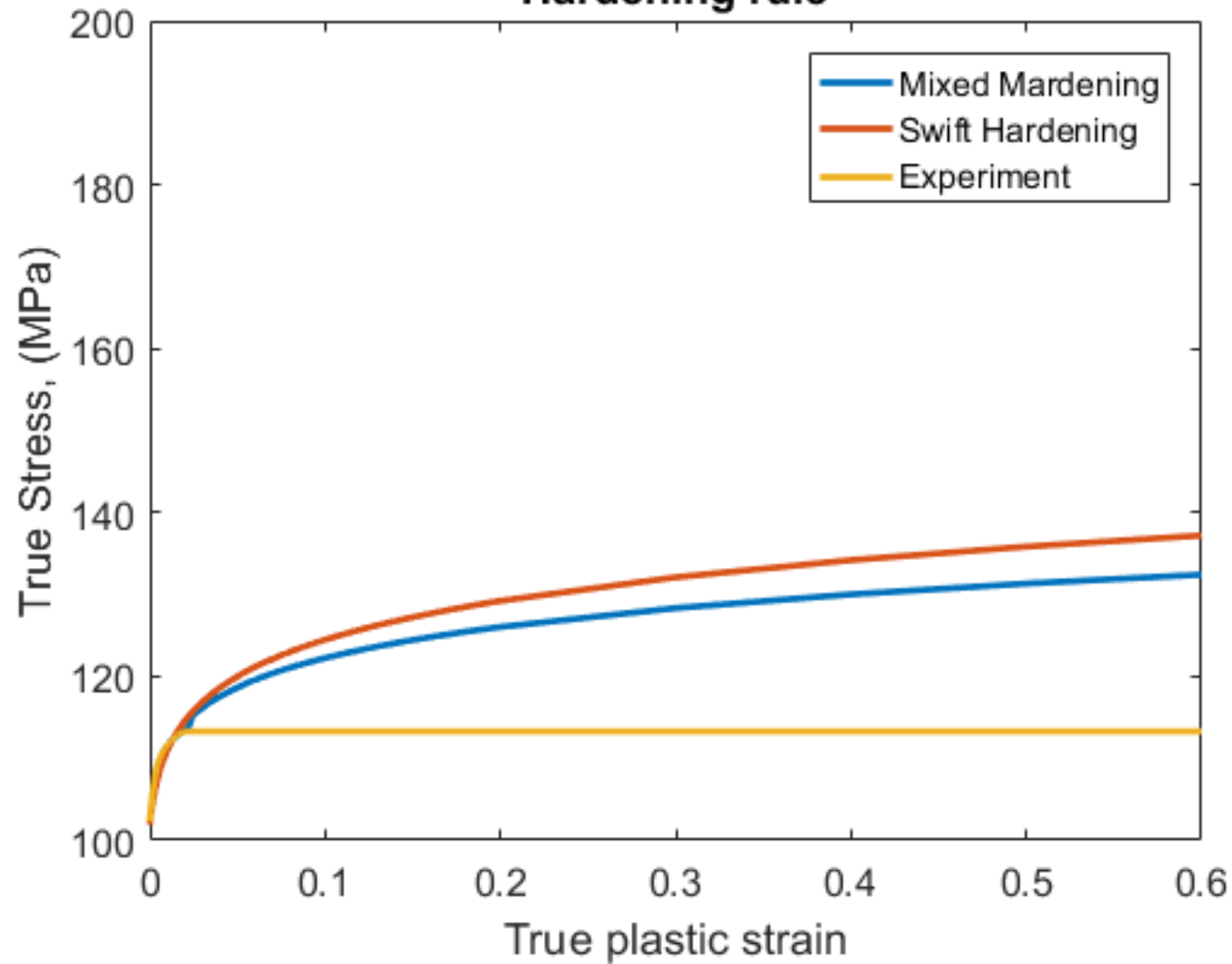
- Like the approach utilized by Mohr and Marcadet (2015), in the present work, the hardening behaviour till necking point is supposed to be described using the Swift hardening law and after necking is expressed as a linear combination of the Swift equation and no hardening behaviour as follows:

$$\bar{\sigma} = \begin{cases} K(\varepsilon_0 + \bar{\varepsilon}^p)^n & \bar{\varepsilon}^p \leq \bar{\varepsilon}^p_{necking} \\ Q [K(\varepsilon_0 + \bar{\varepsilon}^p)^n] + (1 - Q) [\bar{\sigma}_{UTS}] & \bar{\varepsilon}^p > \bar{\varepsilon}^p_{necking} \end{cases}$$

K	eo	Sigma yield	Sigma UTS	n	E	ep_neck	Q
141.0735	0.002707	101.9138	113.2987	0.055	71659	0.01962	0.8



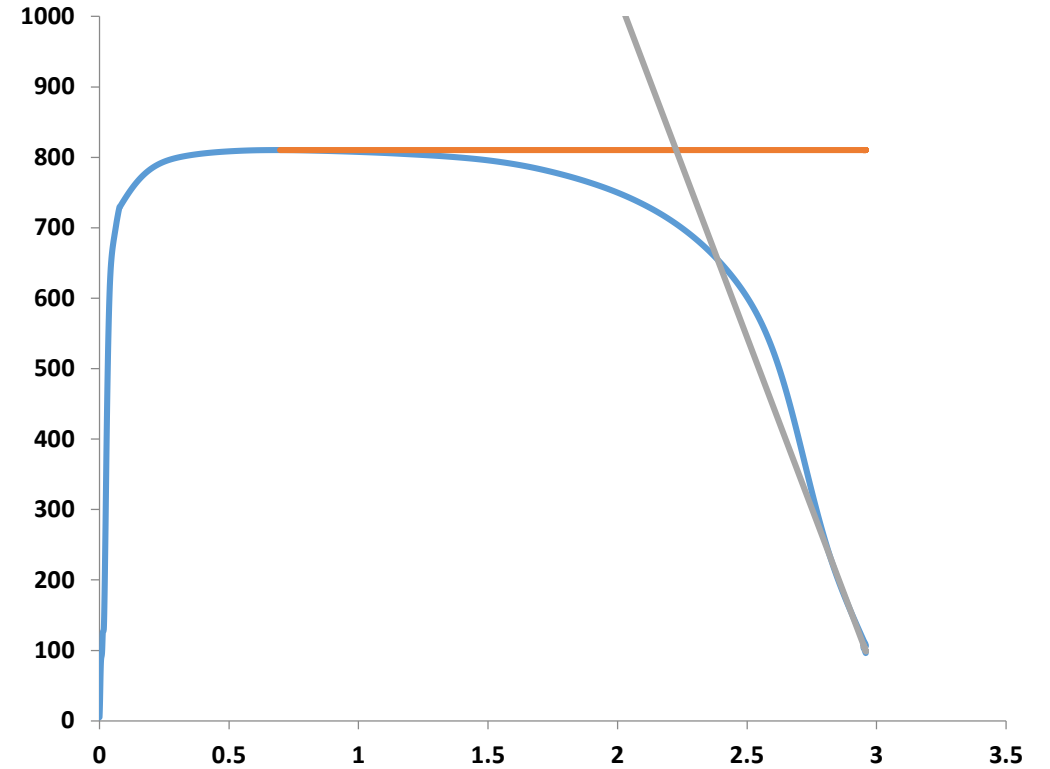
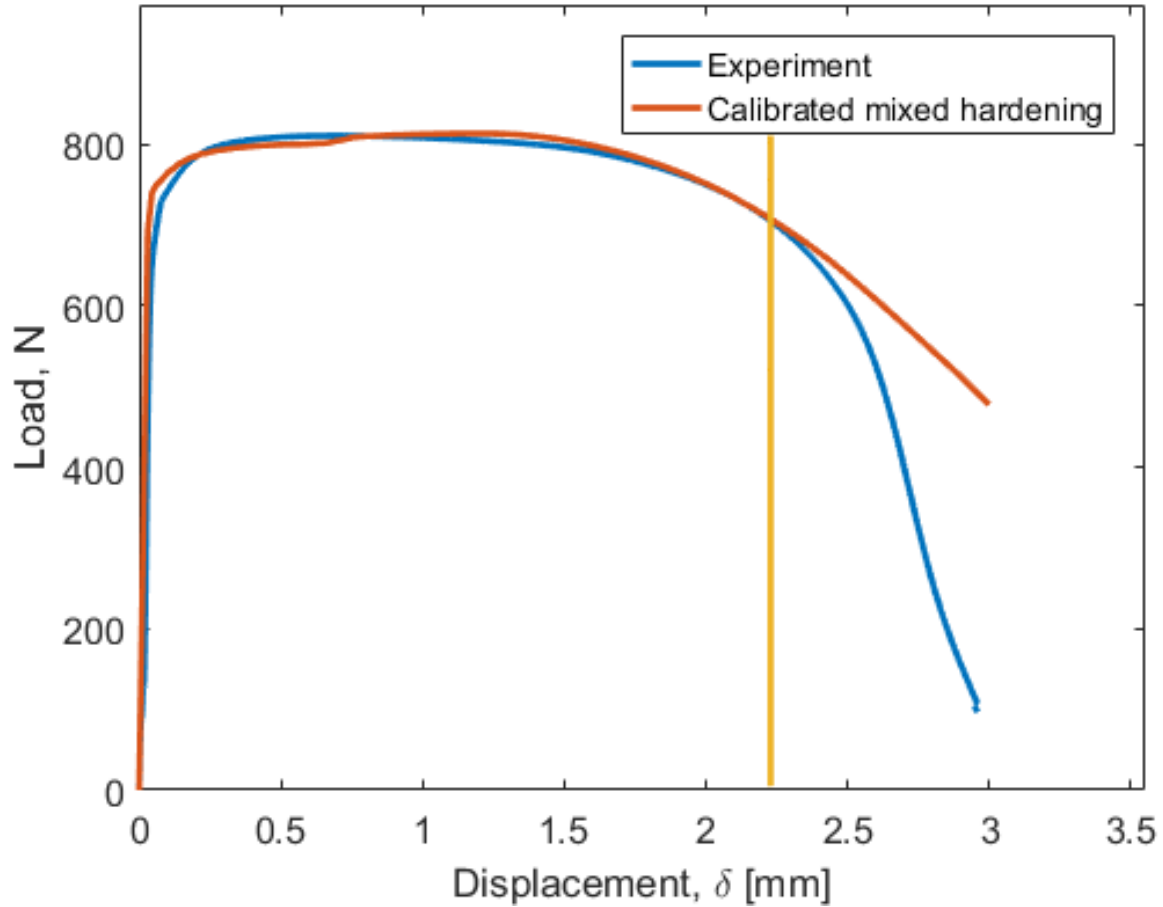
### Hardening rule



$$\sigma_i = \begin{cases} K(\varepsilon_0 + \bar{\varepsilon}^p)^n & \bar{\varepsilon}^p \leq \bar{\varepsilon}^p_{necking} \\ Q [K(\varepsilon_0 + \bar{\varepsilon}^p)^n] + (1 - Q)[\bar{\sigma}_{UTS}] & \bar{\varepsilon}^p > \bar{\varepsilon}^p_{necking} \end{cases}$$

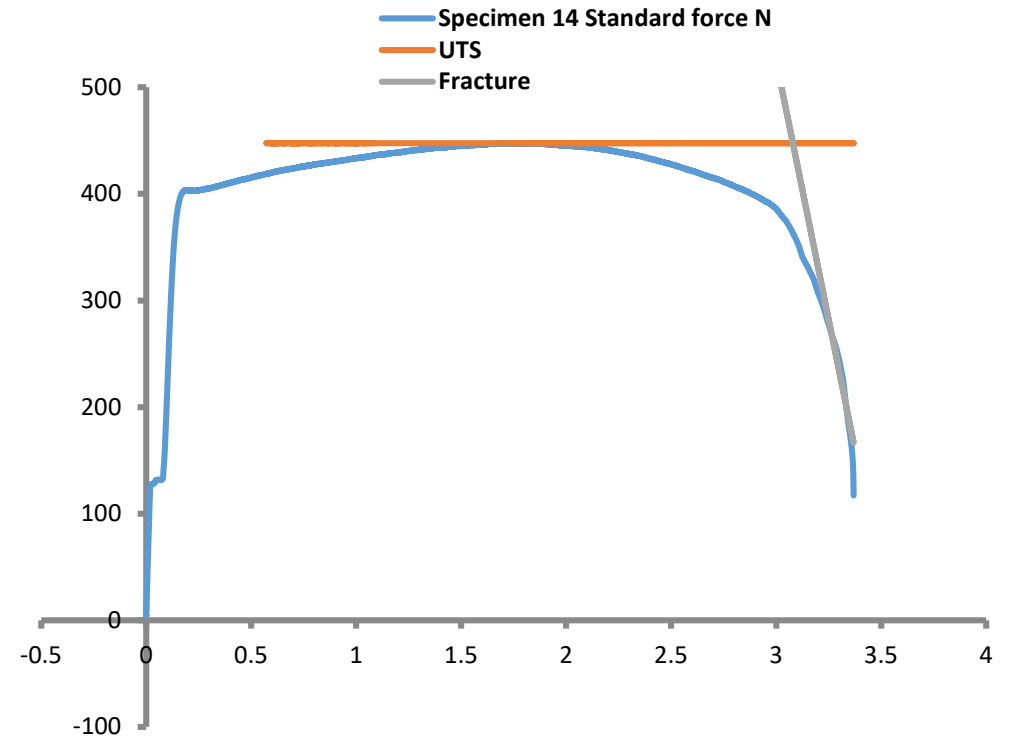
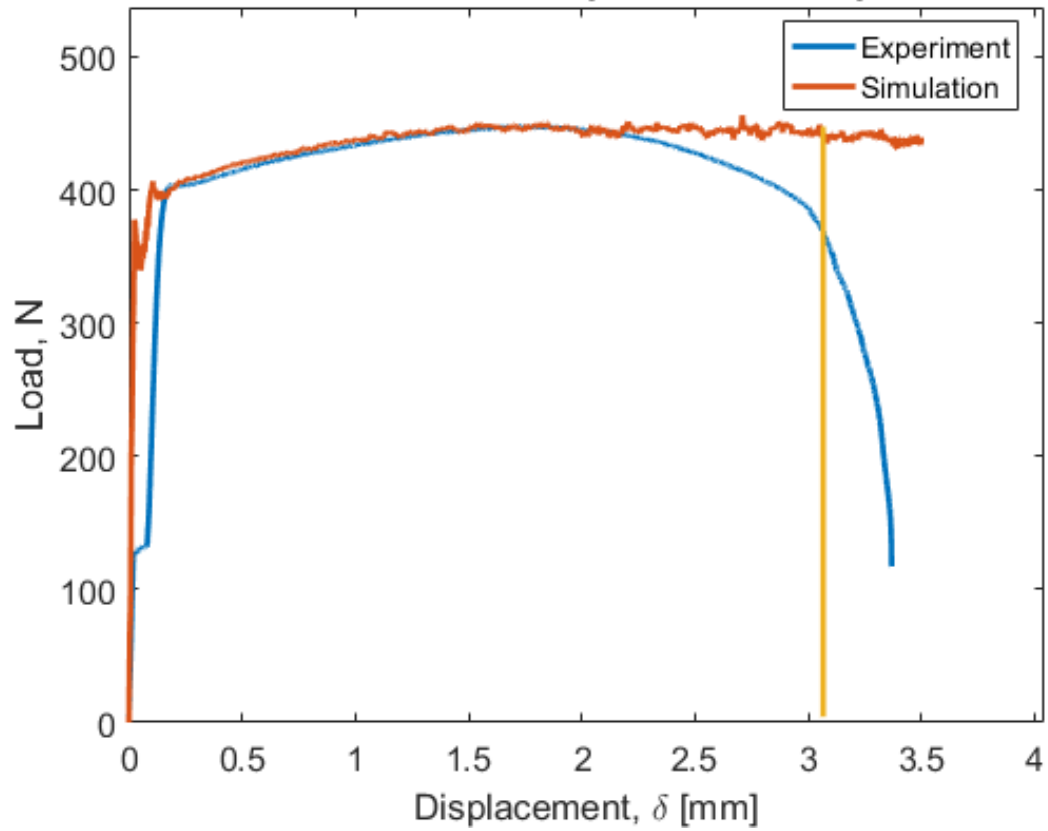
# Estimation of displacement to fracture ( Literature)

## Simple Tensile test -Force Displacement Response



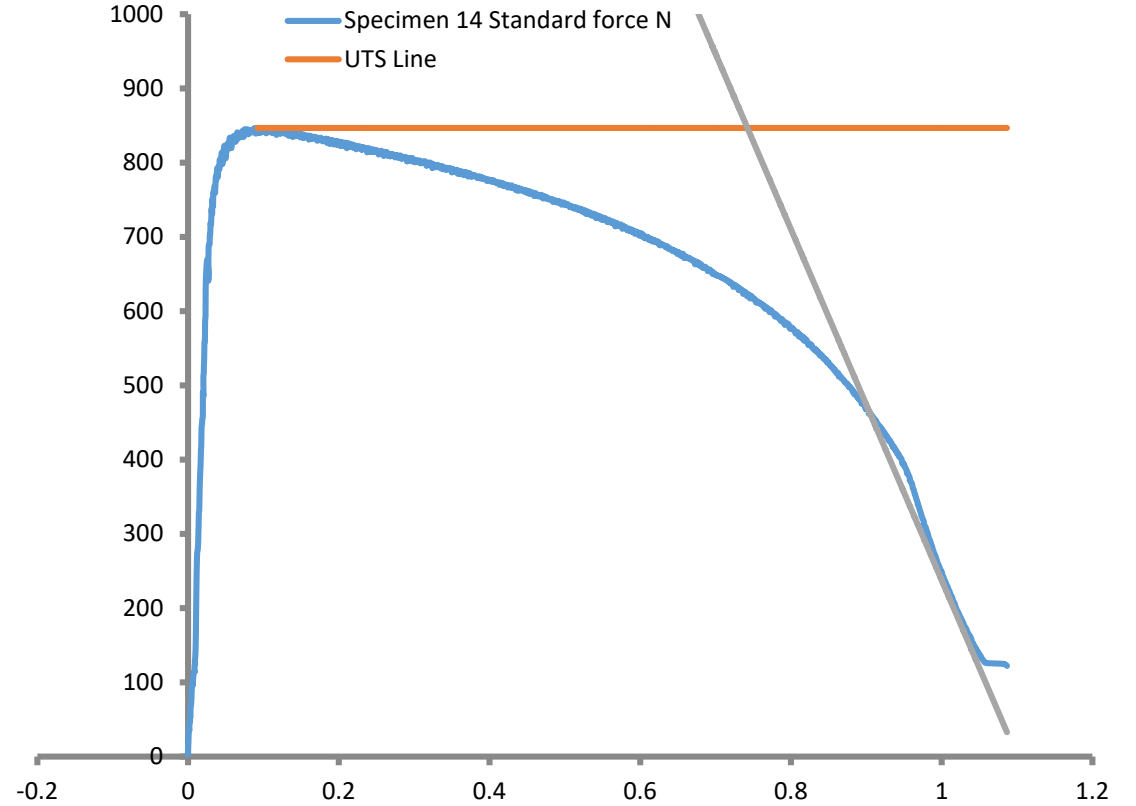
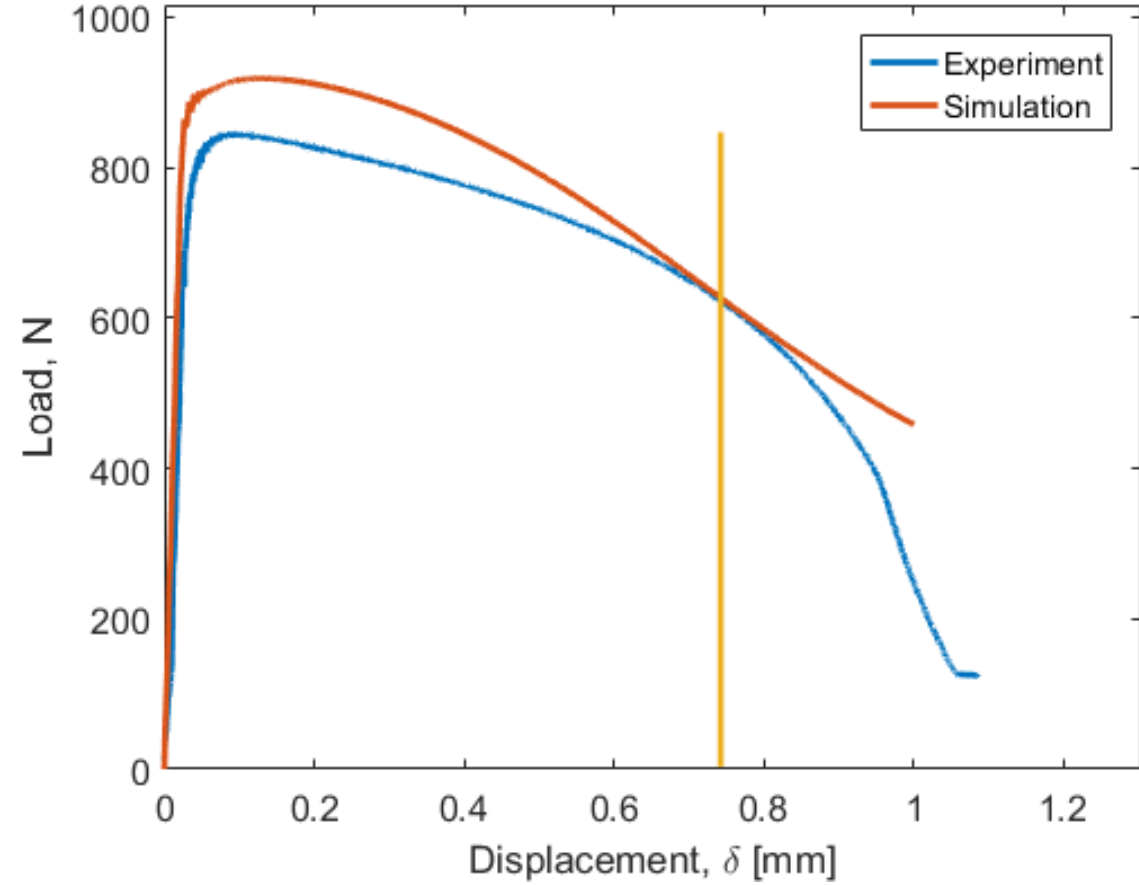
S. No	Displacement to fracture (mm)	% load drop
1	2.228	13.01 %

Shear test - Force Displacement Response



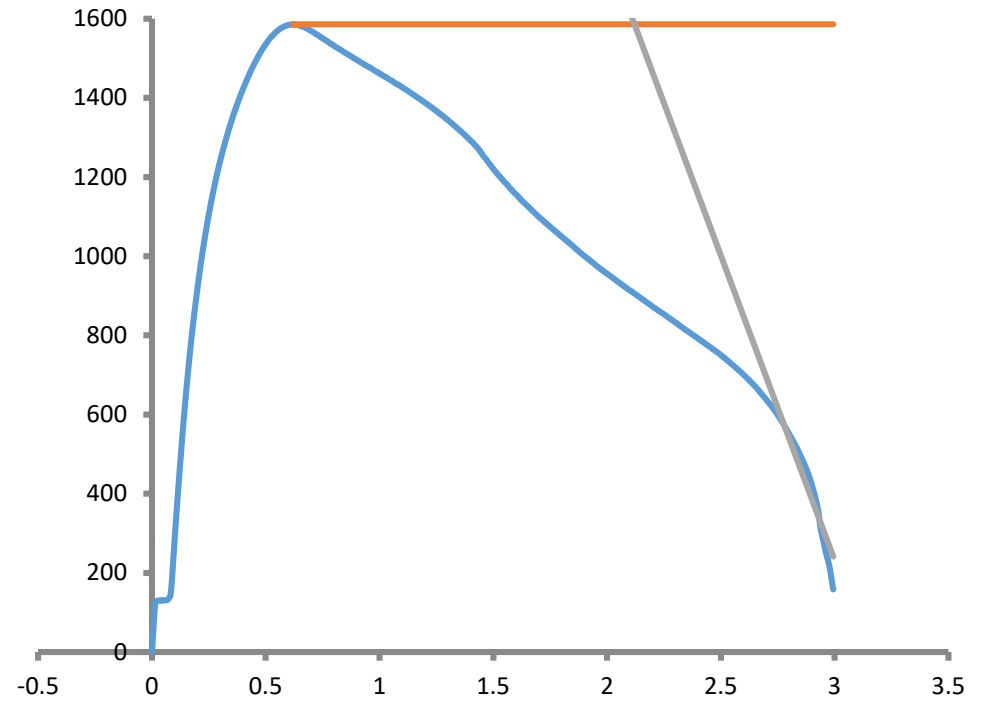
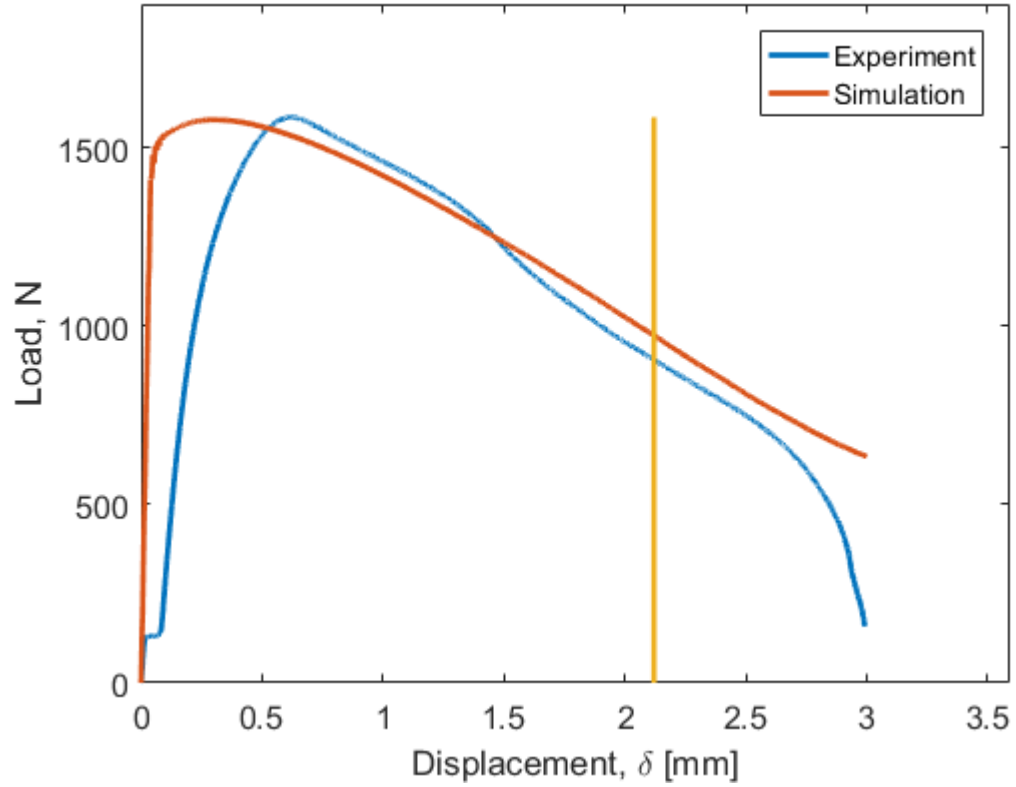
S.No	Displacement to fracture (mm)	% load drop
1	3.067	17.75%

### Notch Test - Force Displacement Response



S.No	Displacement to fracture (mm)	% load drop
1	0.742199361	26.466 %

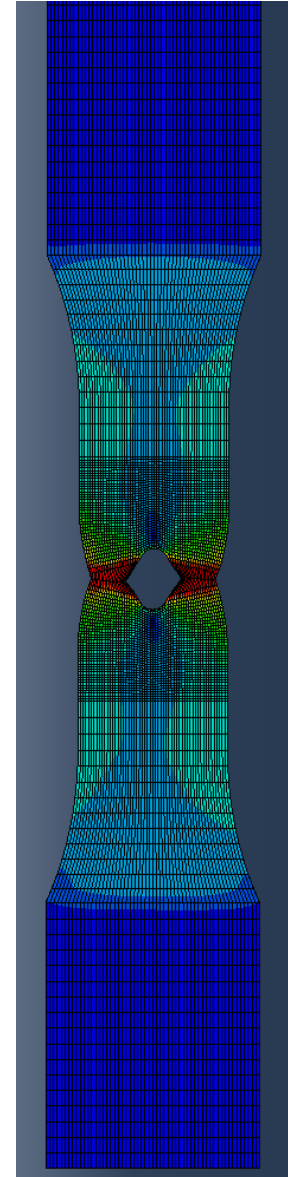
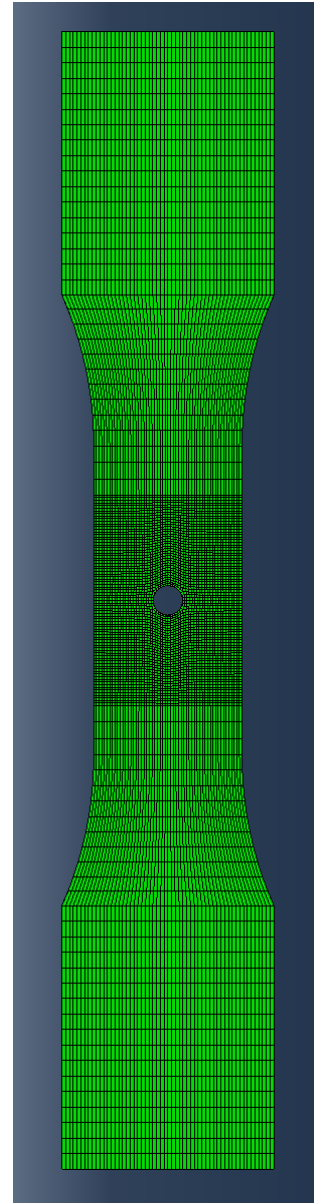
Center Hole Specimen- Force Displacement Response

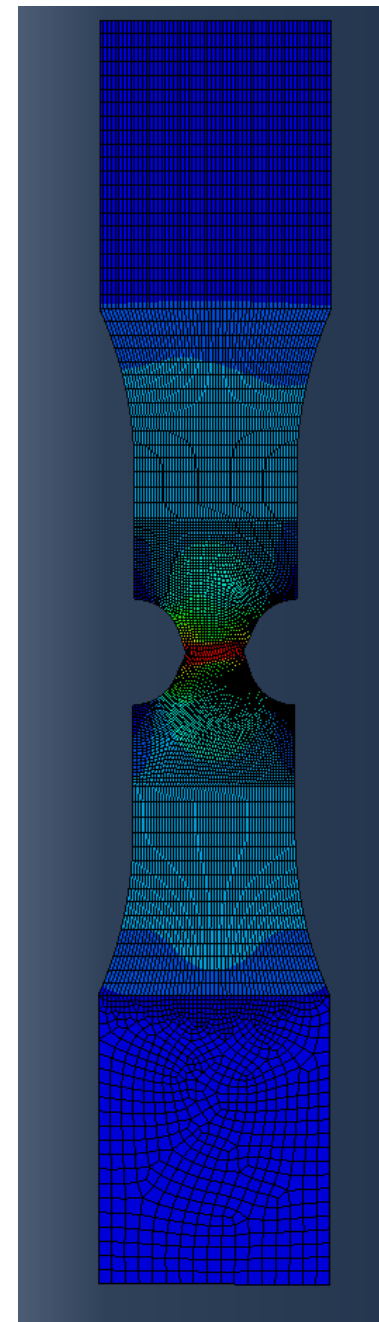
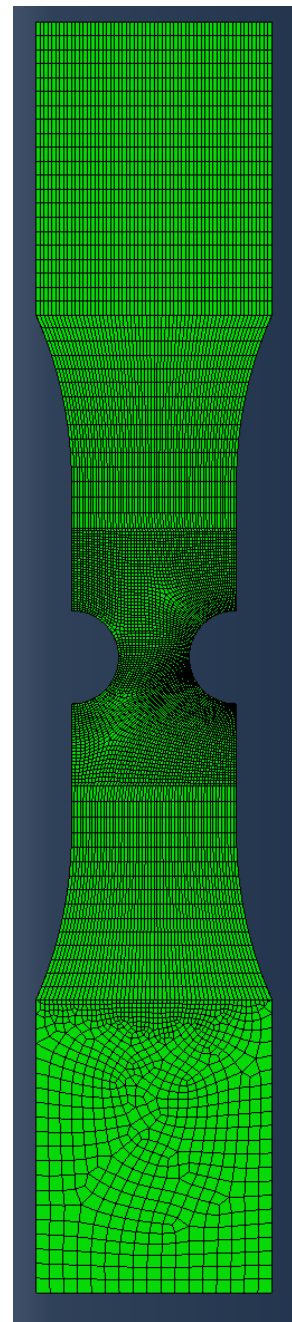
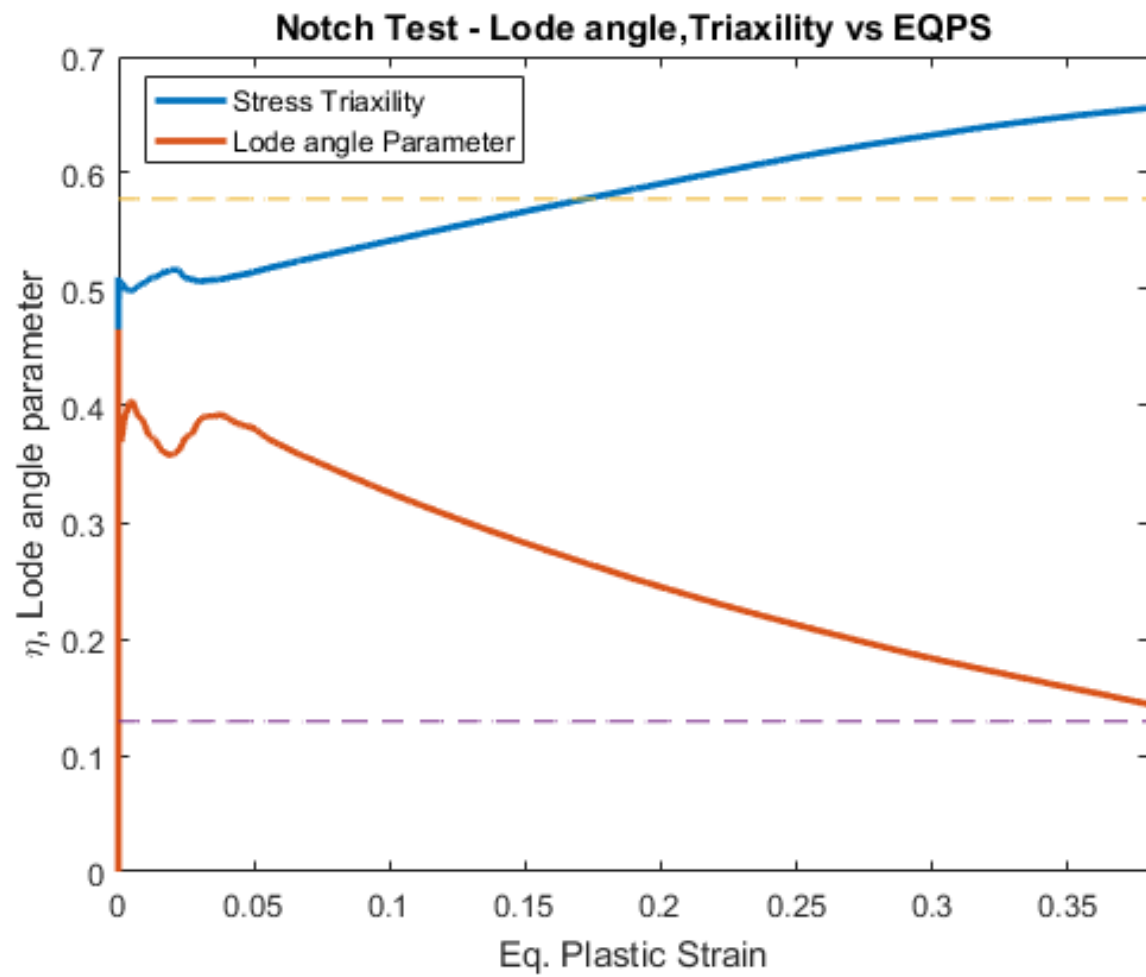


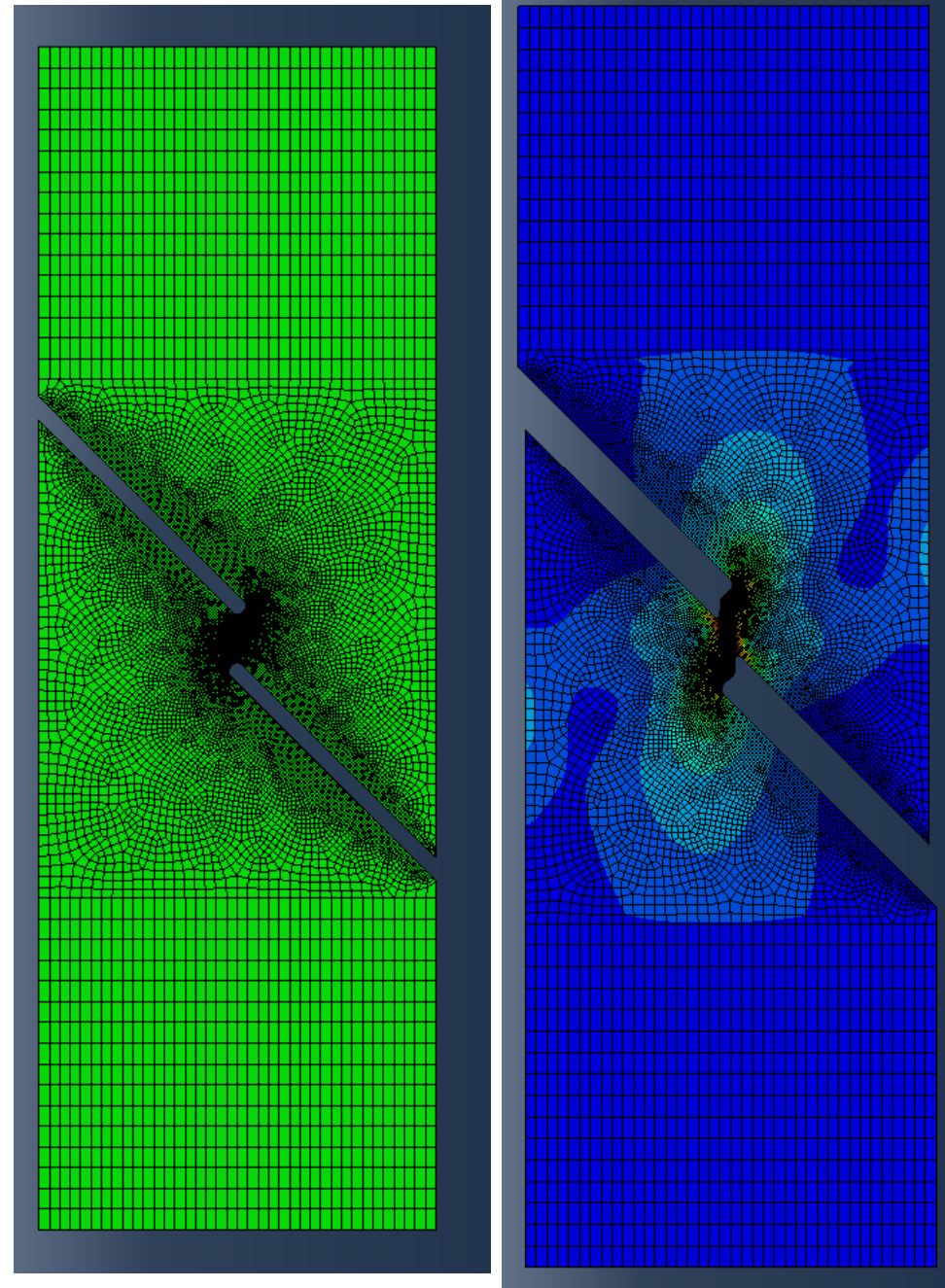
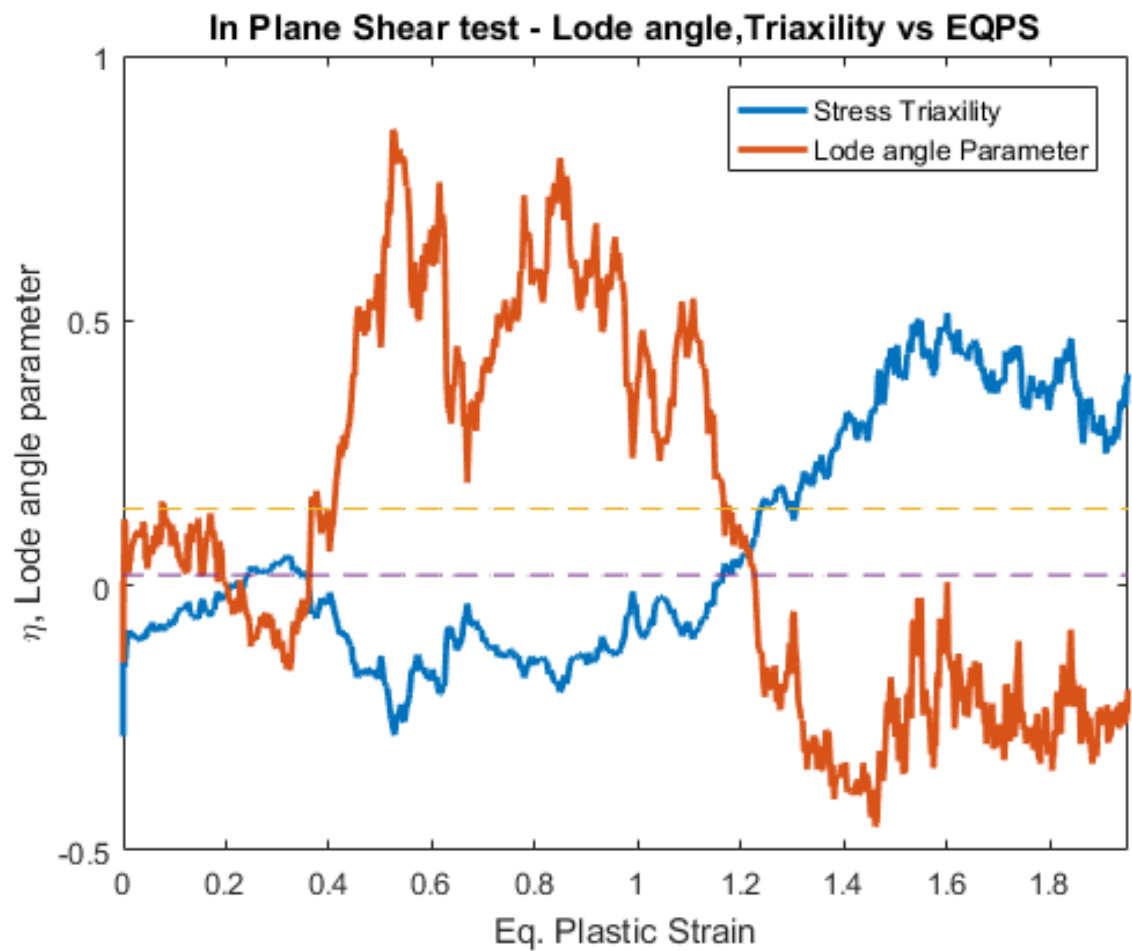
S.No	Displacement to fracture (mm)	% load drop
1	2.120648146	42.9032754 %

**Comment:**

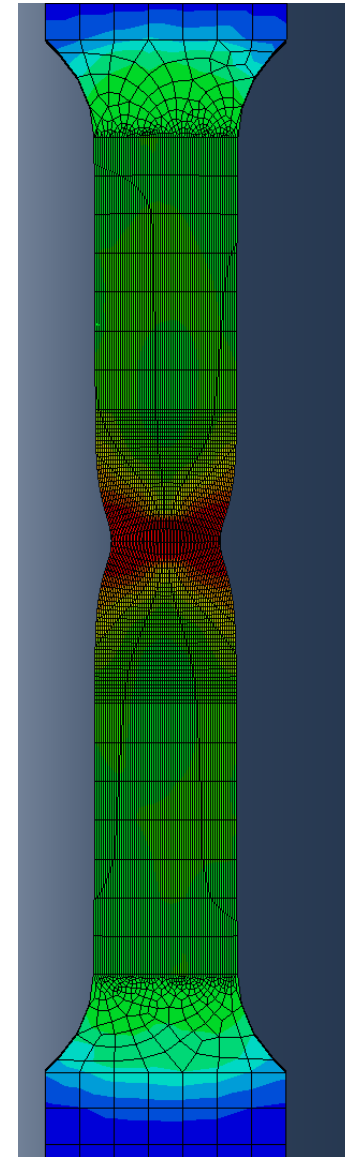
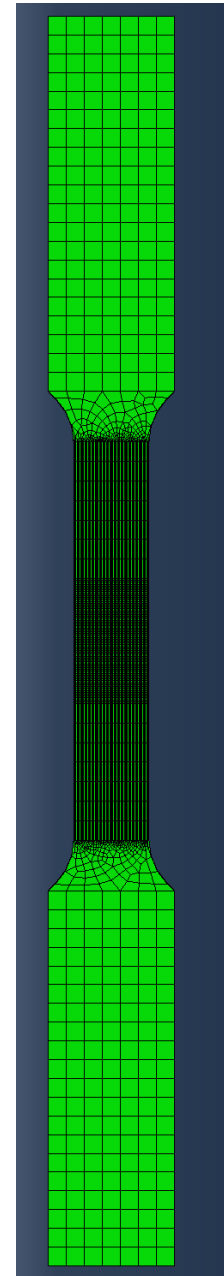
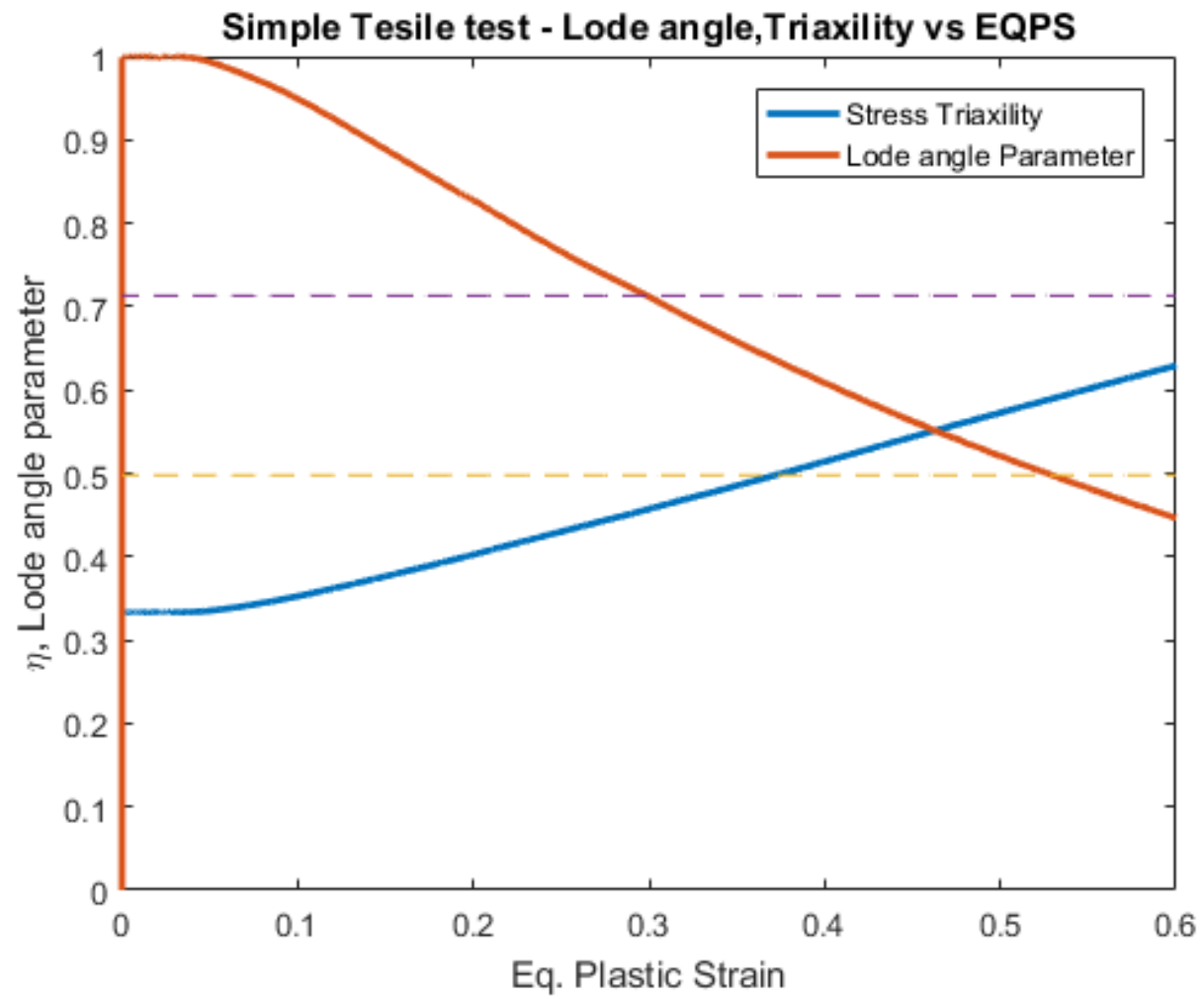
Here, load drop is very high

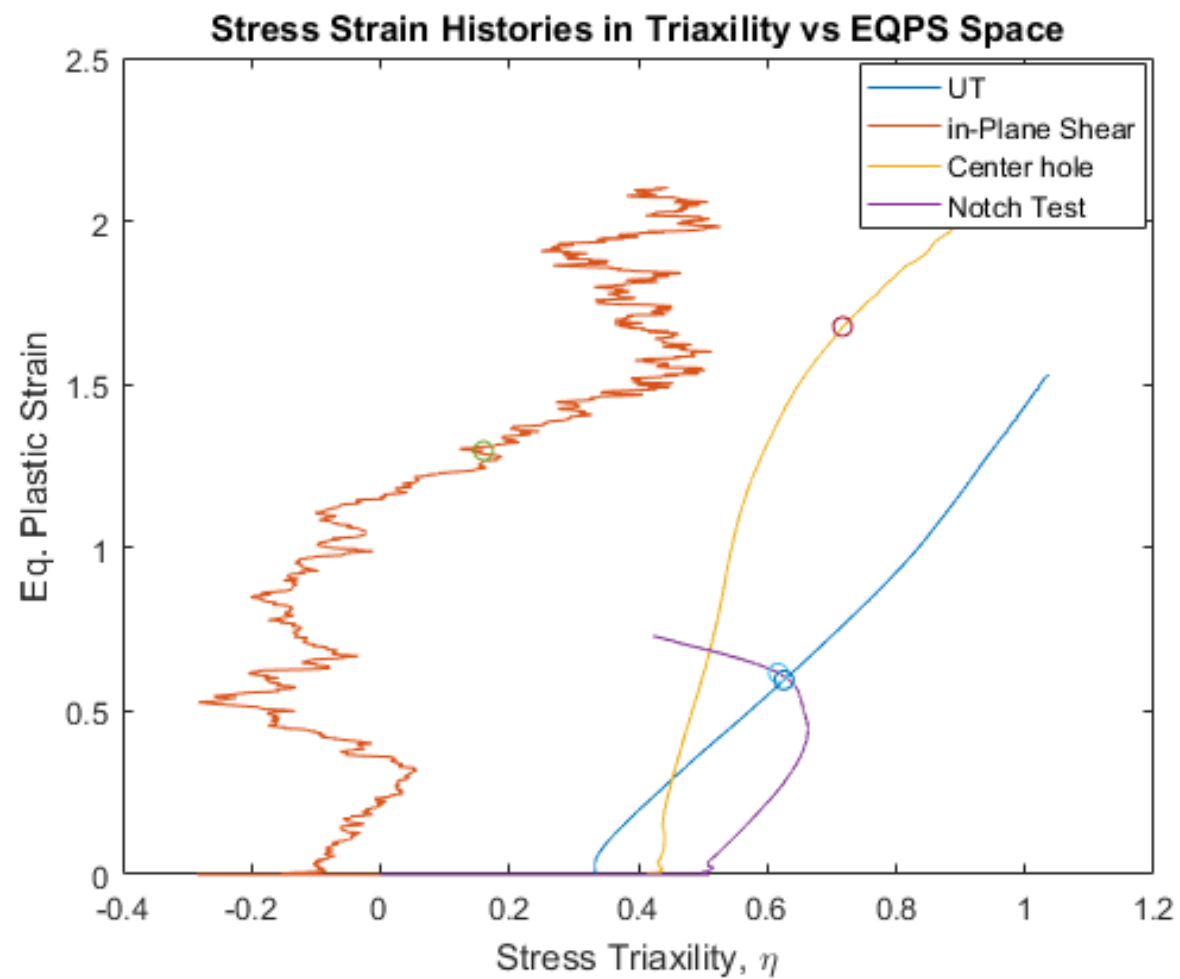
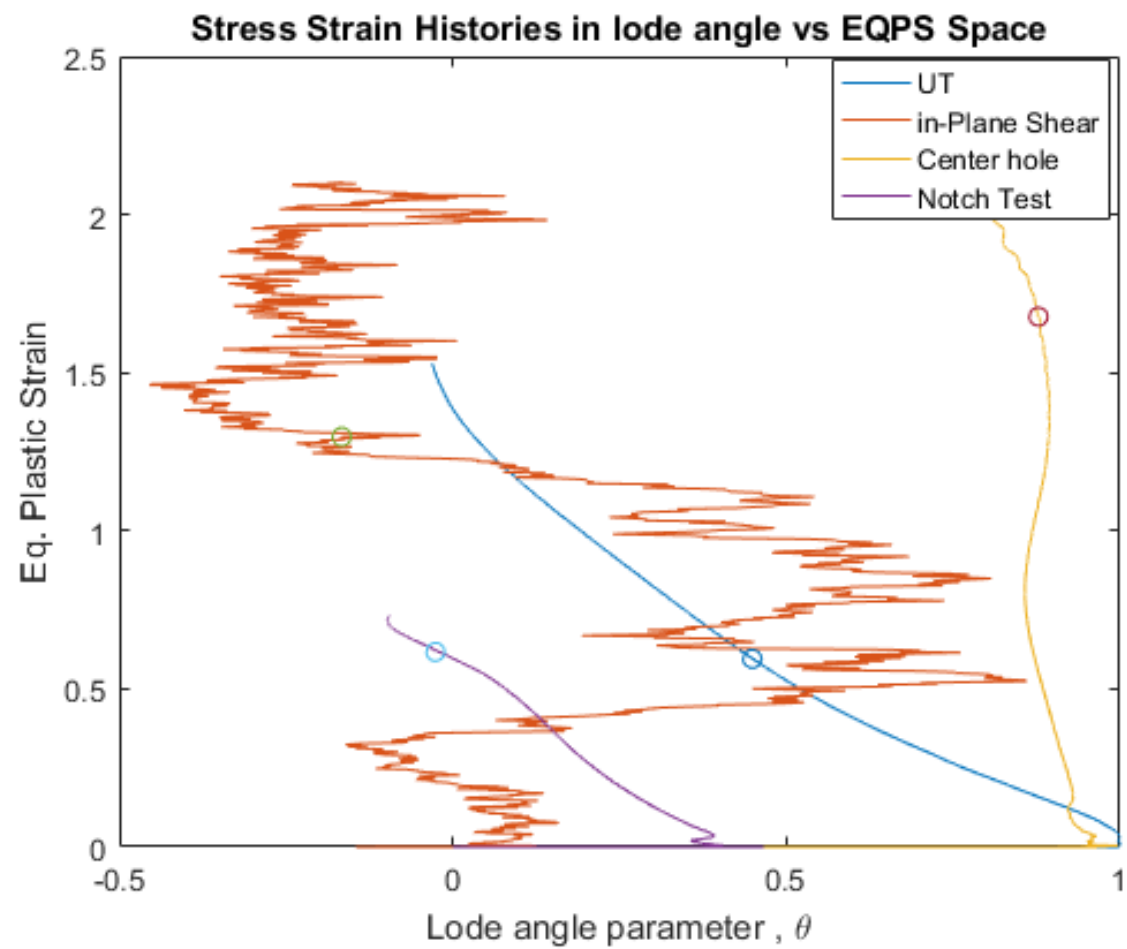


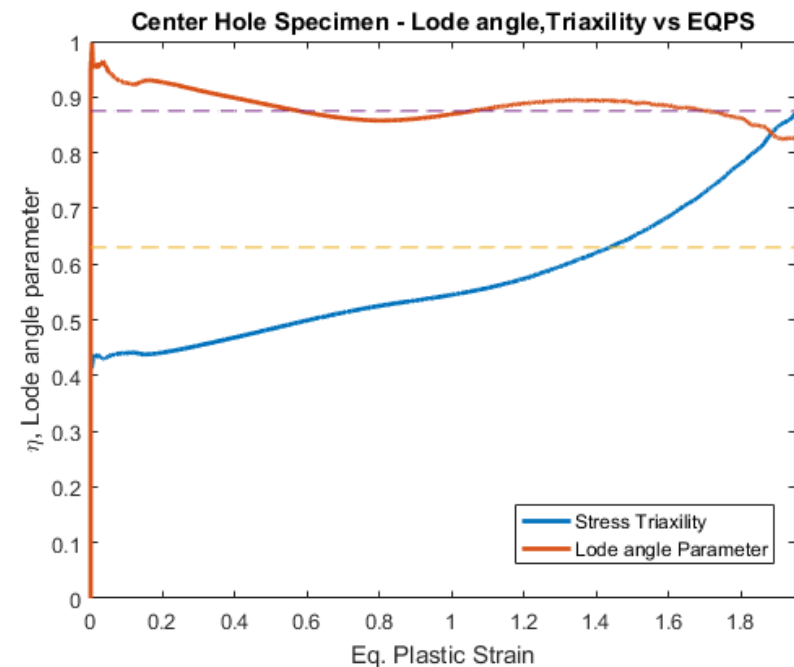
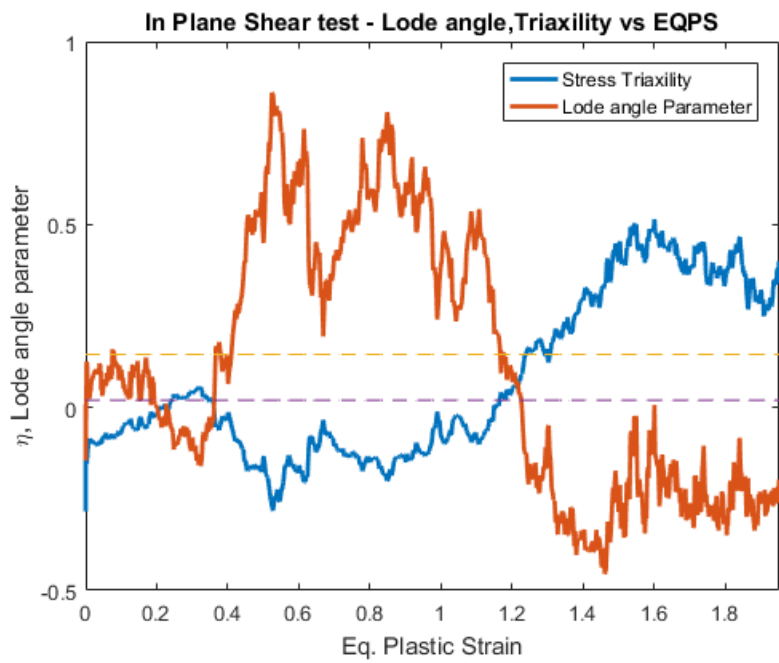
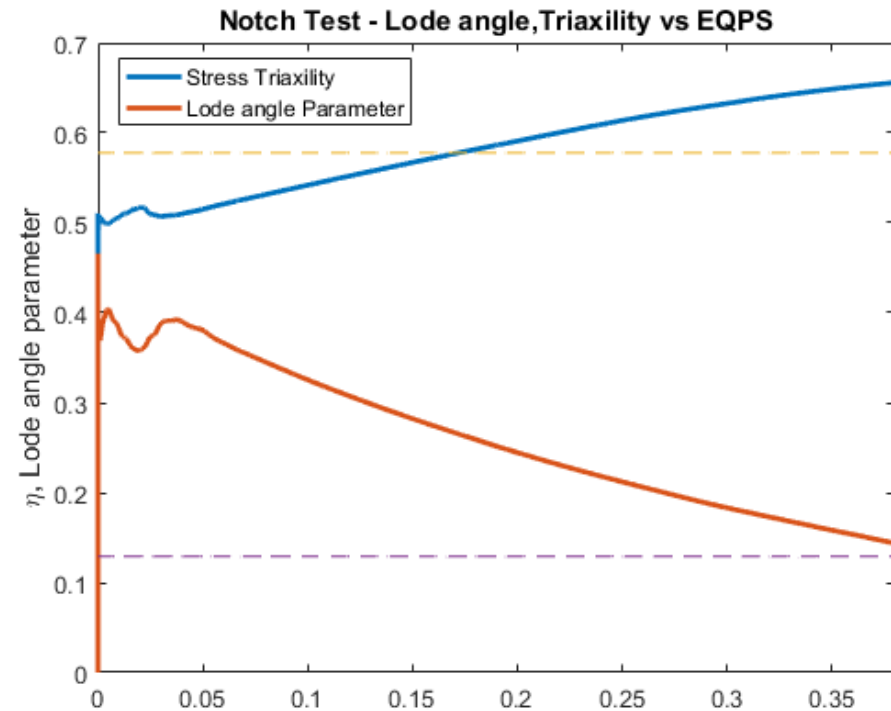
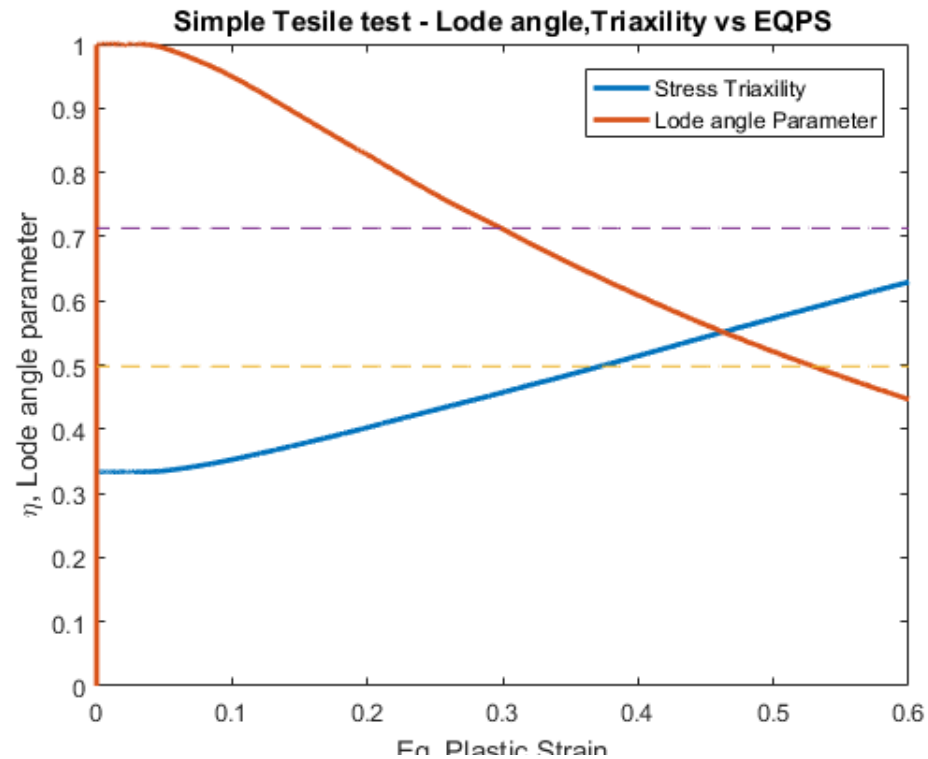












# **Damage models**

## Calibration for damage models:

S. No	Damage model	Tests to calibrate
1	B W model	Two tests; 1) Uniaxial Tension(UT) 2) In Plane Shear (ST)
2	M C Model	Three Tests: 1) ST 2) Notch Test (NT4) ( 4 mm radius) 3) Centre Hole Test (CH) (2.66 mm dia)
3	HC Model	Three Tests: 1) ST 2) Notch Test (NT4) ( 4 mm radius) 3) Centre Hole Test (CH) (2.66 mm dia)

# BW Criteria

```

% ef      tri      Lode      time      Avg Tri
C=[ 1.1354   0.049035 0.223   0.53   0.02616 ;      % Shear
    0.6167   0.617   -0.026   0.737   0.59867 ;      % NT4
    1.677    0.717    0.878    0.702   0.55284 ;      % CH
    0.594774 0.6261   0.449    0.742   0.3777 ];      % UT

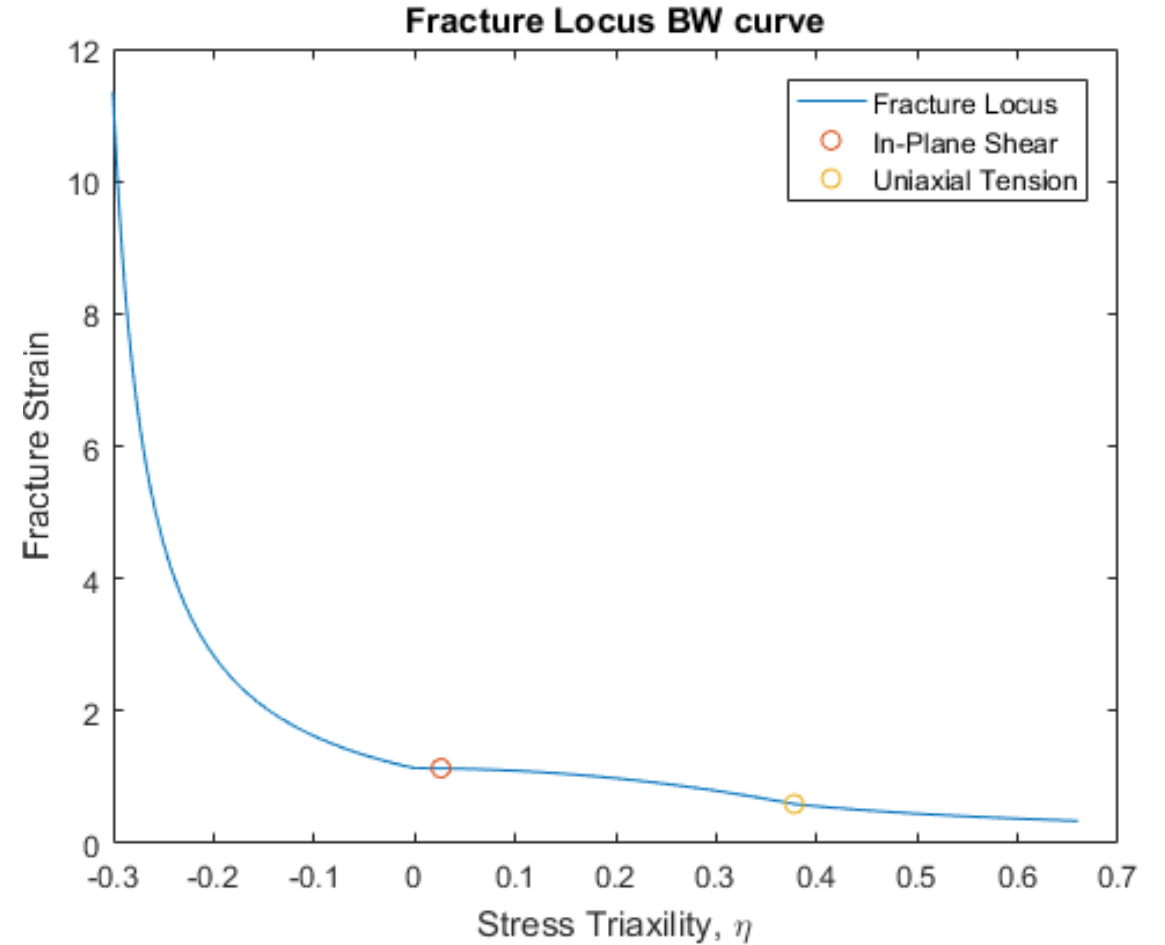
% constants matrix [A]
% c1      c2      BW
% a      b      c      HC
% c1      c2      c3      MMC
A = [C(1,1)  C(4,1)  0;      % BW
     1.31    149.7   0.05; % HC
     0.067   80.85   0.93]; % MMC
    
```

$$\bar{\varepsilon}_0^p = \frac{c_1}{(1+3\eta)} \quad \left\{ -\frac{1}{3} \leq \eta \leq 0 \right\}$$

$$\bar{\varepsilon}_0^p = C_1 + (C_2 - C_1)(\eta/\eta_0)^2 \quad \{0 \leq \eta \leq \eta_0\}$$

$$\bar{\varepsilon}_0^p = C_2(\eta_0/\eta) \quad \{\eta \geq \eta_0\}$$

$$\omega = \int \frac{d\varepsilon^p}{\bar{\varepsilon}_0^p}$$



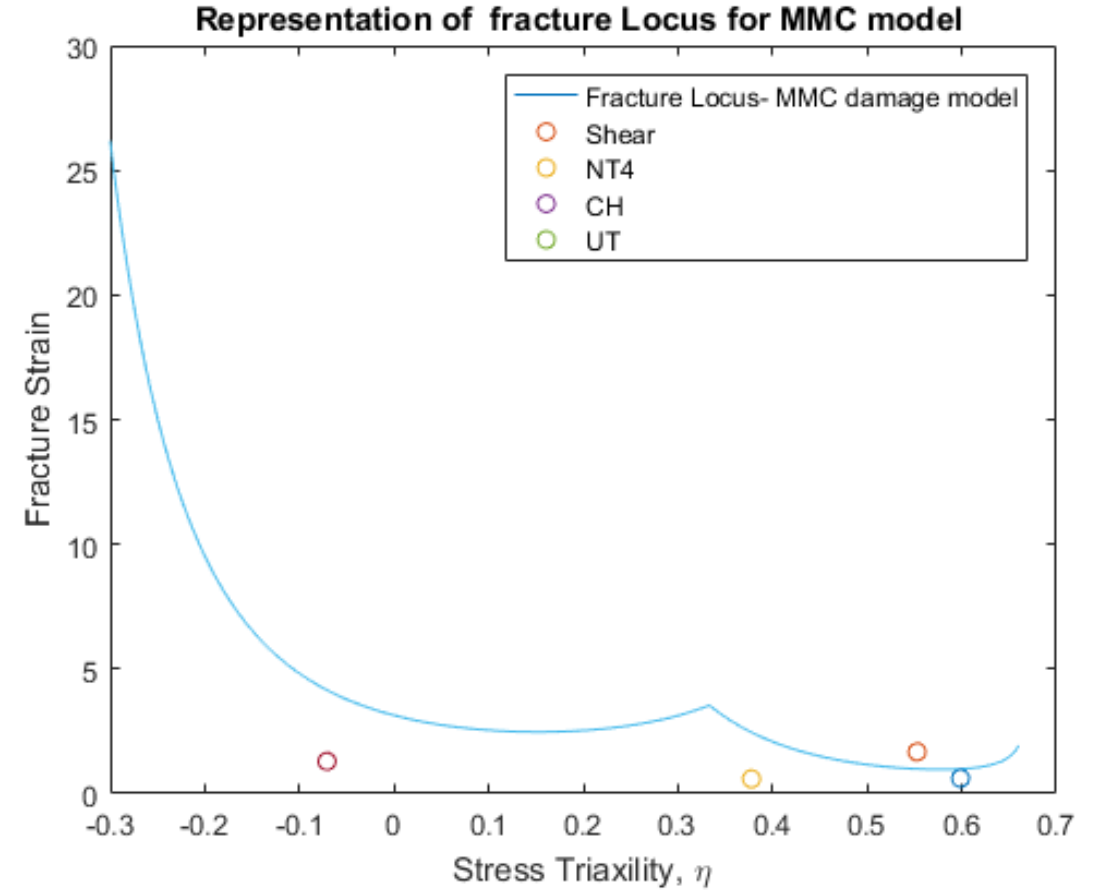
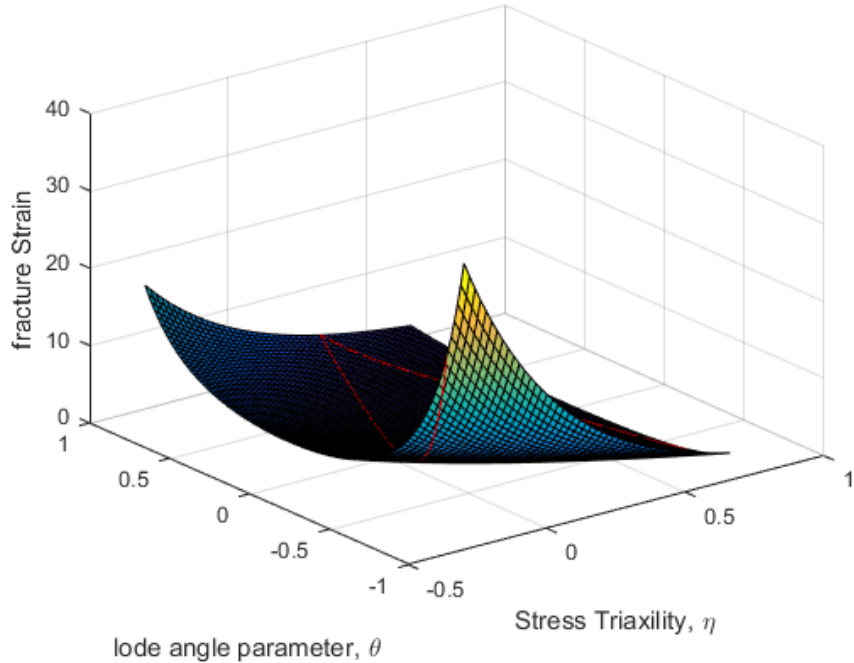
# Mohrs - Coulomb Criteria

```

% ef      tri      Lode      time      Avg Tri
C=[ 1.1354    0.049035 0.223    0.53    0.02616 ;
   0.6167    0.617    -0.026    0.737    0.59867 ;
   1.677     0.717     0.878     0.702    0.55284 ;
   0.594774 0.6261    0.449     0.742    0.3777 ] ;
% Shear
% NT4
% CH
% UT

% constants matrix [A]
% c1      c2      BW
% a      b      c      HC
% c1      c2      c3      MMC
A = [C(1,1)  C(4,1)  0;      % BW
     1.31    149.7    0.05;  % HC
     0.067   80.85   0.93]; % MMC
    
```

Representation of fracture strain in MC model in Haigh-Westergaard space



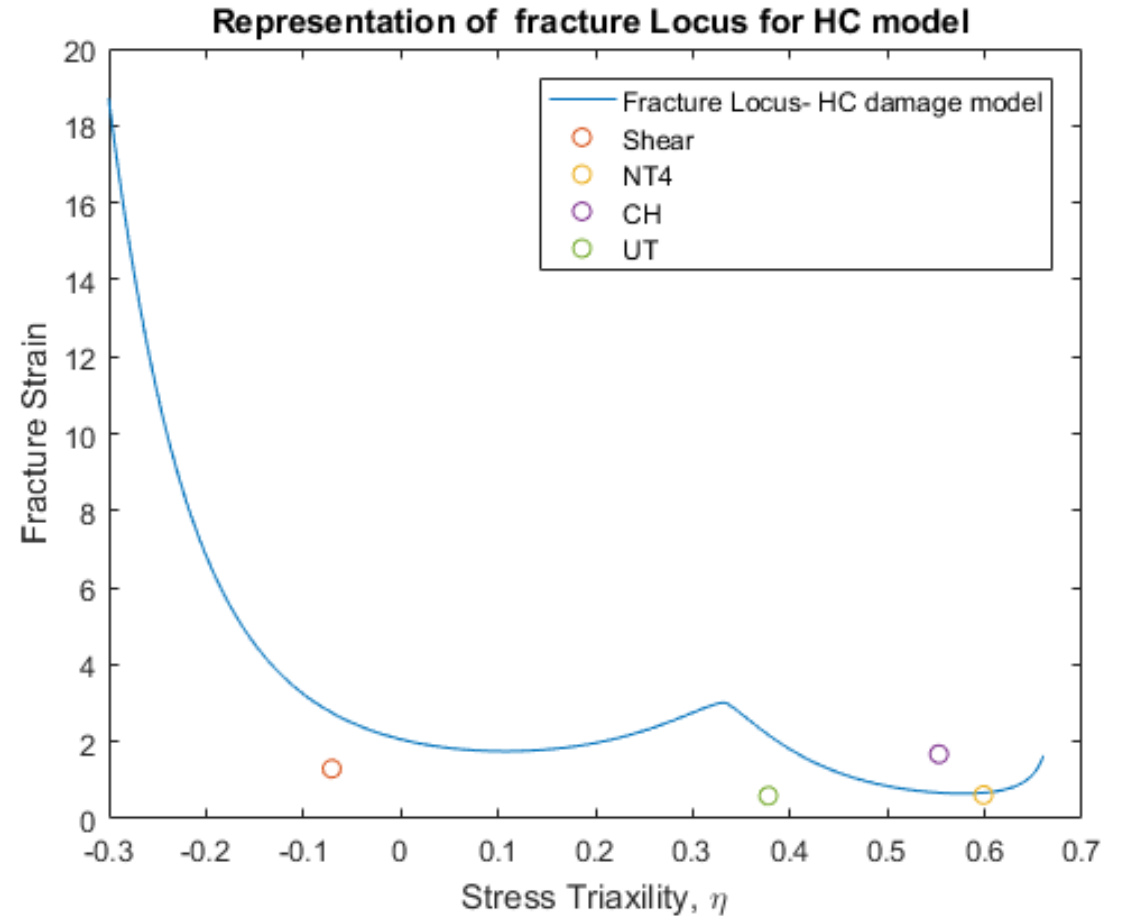
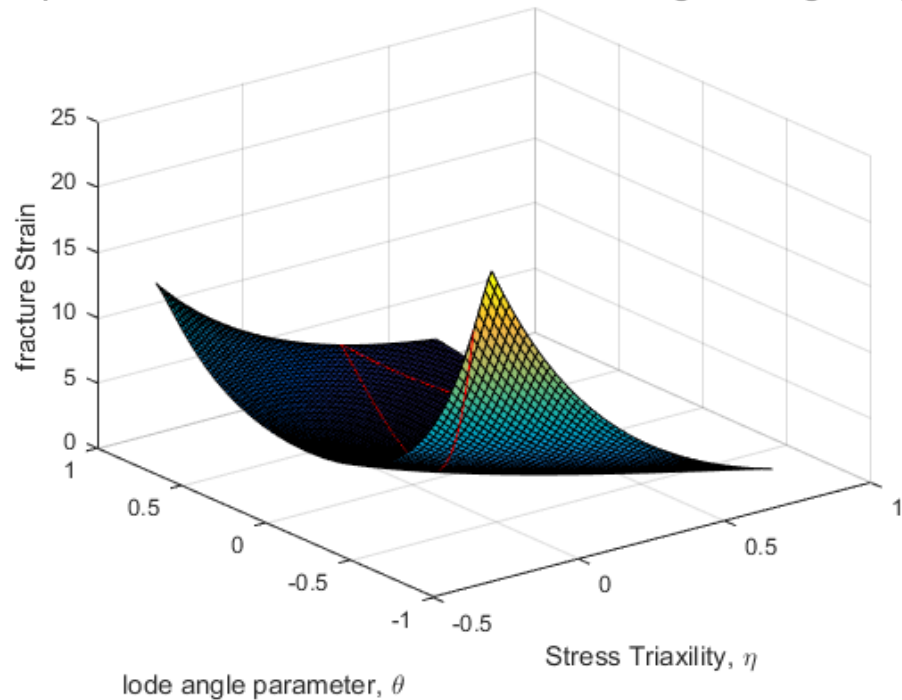
# Hosford - Coulomb Criteria

```

% ef      tri      Lode      time      Avg Tri
C=[ 1.1354    0.049035  0.223    0.53    0.02616 ;      % Shear
    0.6167    0.617    -0.026    0.737    0.59867 ;      % NT4
    1.677    0.717    0.878    0.702    0.55284 ;      % CH
    0.594774  0.6261    0.449    0.742    0.3777 ] ;      % UT

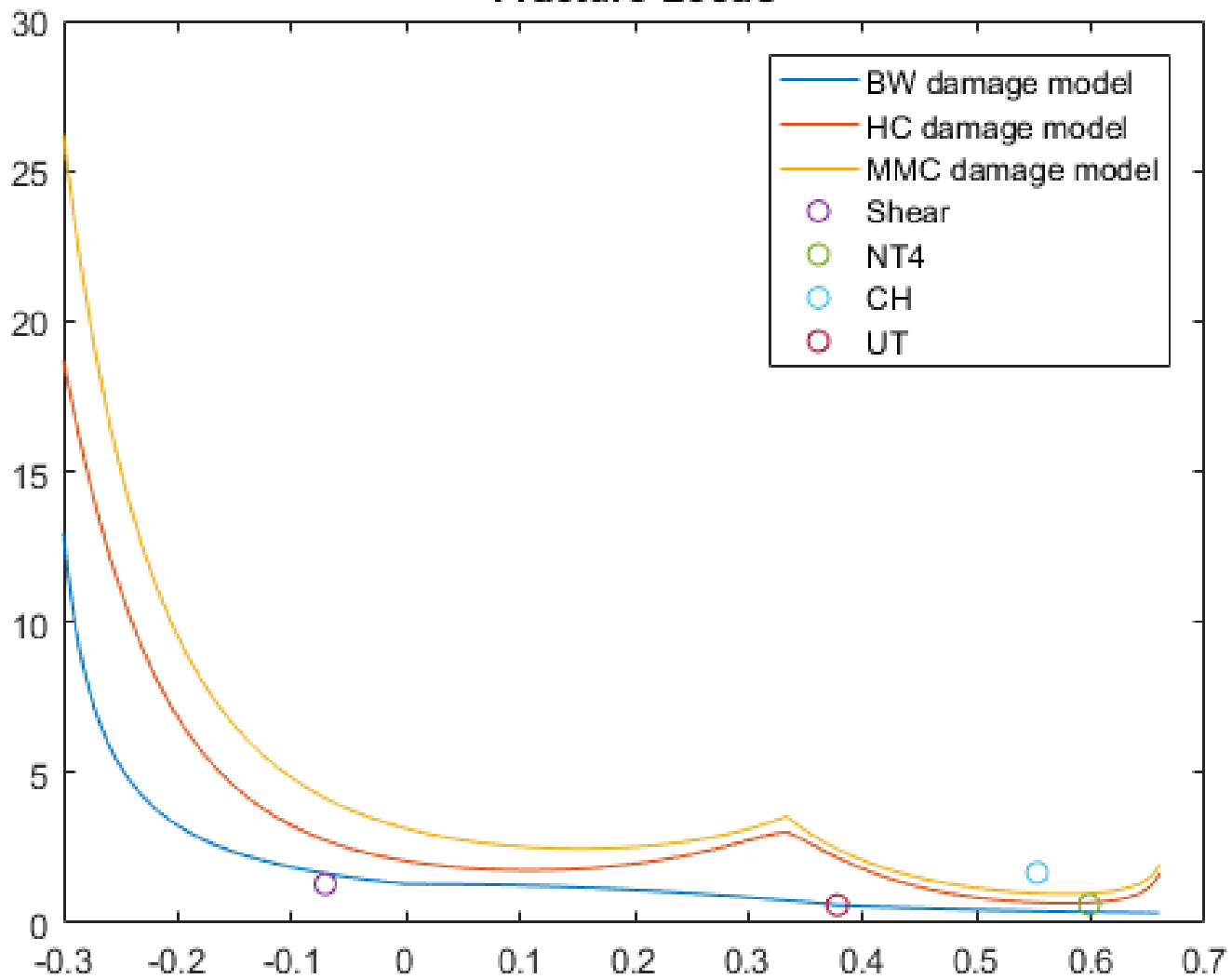
% constants matrix [A]
%      c1      c2      BW
%      a      b      c      HC
%      c1      c2      c3      MMC
A = [C(1,1)    C(4,1)    0;      % BW
     1.31      149.7    0.05;    % HC
     0.067     80.85    0.93];    % MMC
    
```

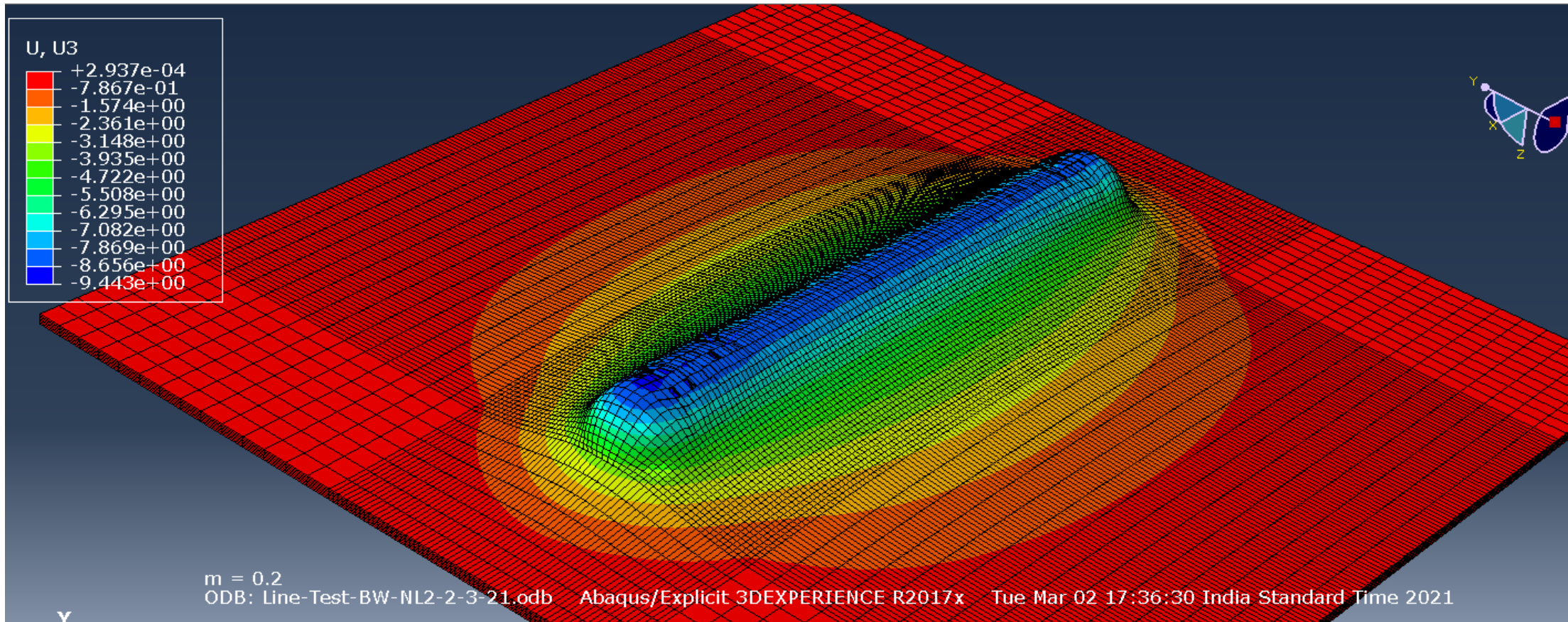
Representation of fracture strain in HC model in Haigh-Westergaard space



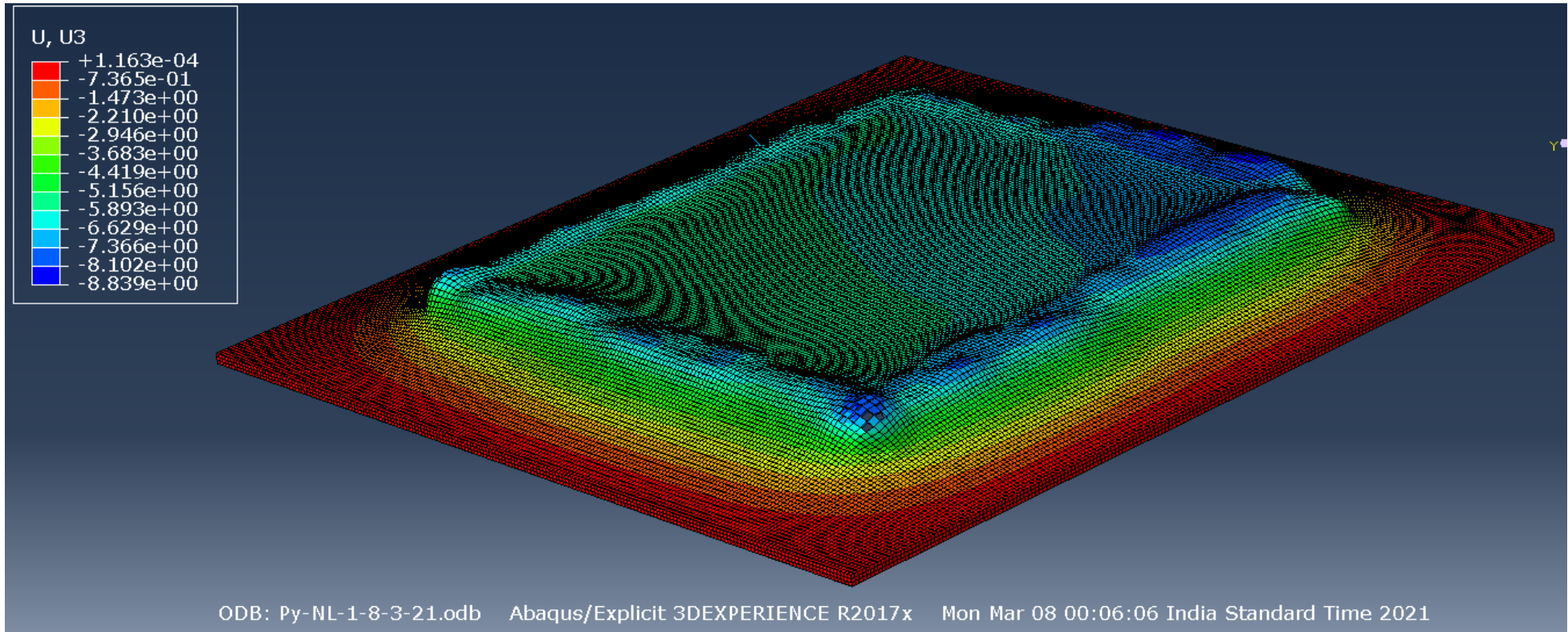


### Fracture Locus

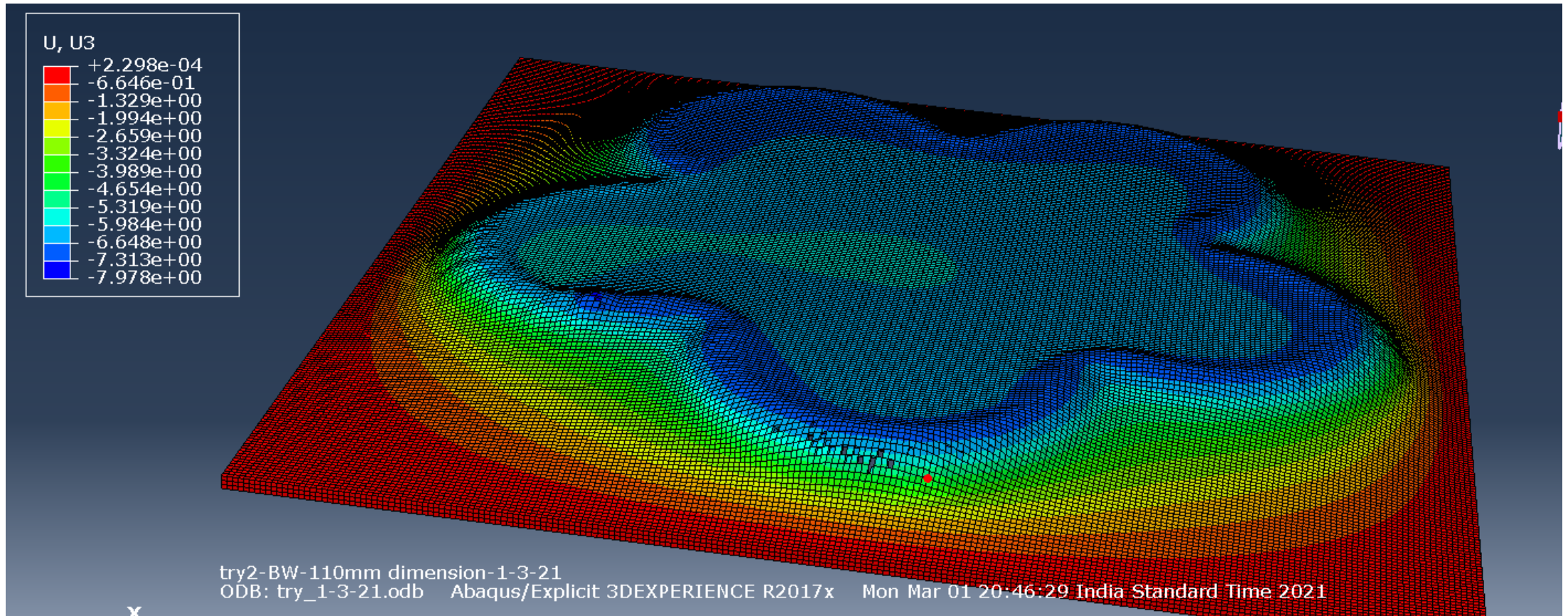




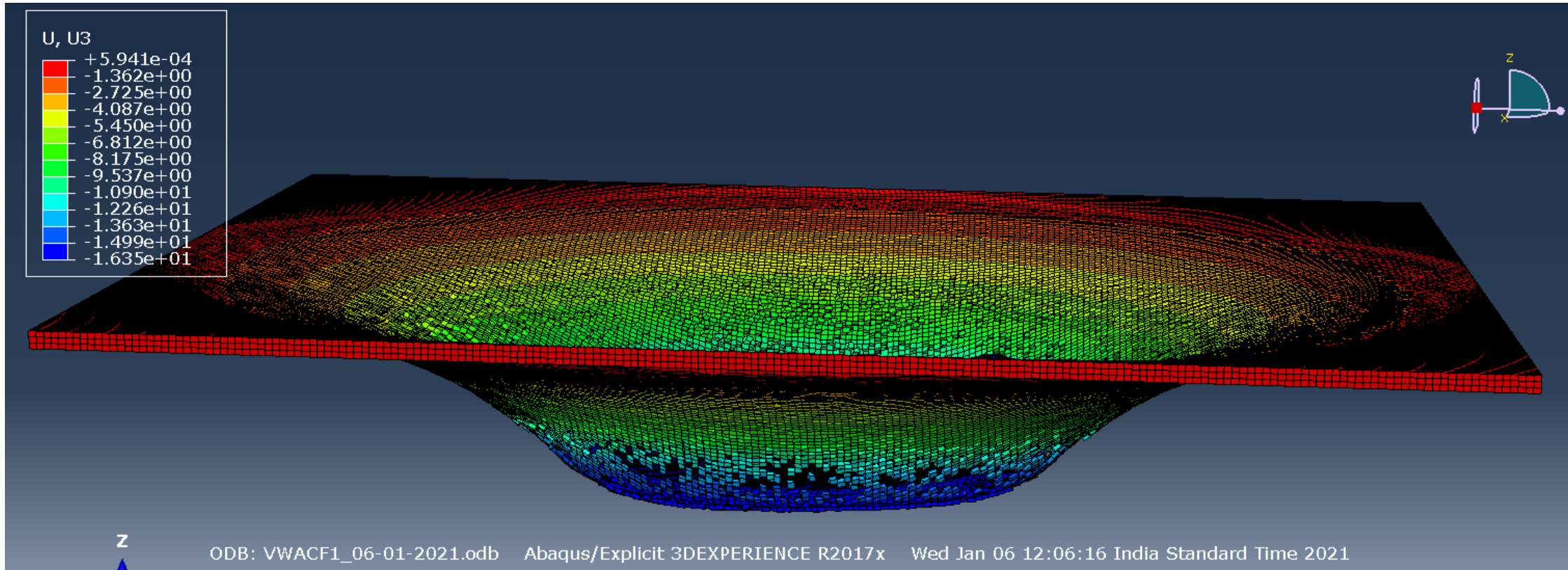
S.No	Exp. Fracture Depth	Predicted
1	12 mm	10.5 mm



S.No	Exp. Fracture Depth	Predicted
1	16.82 mm	12.38

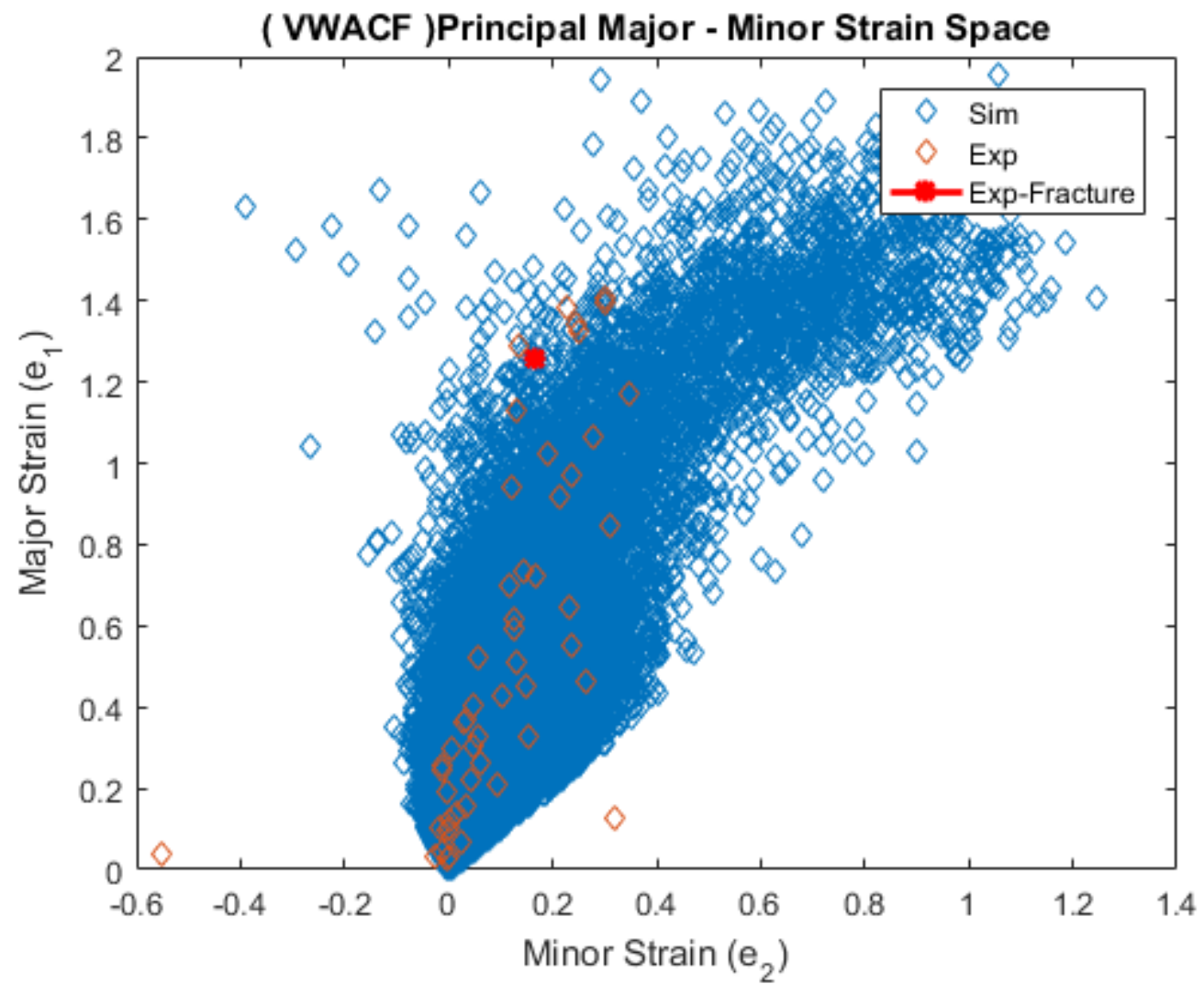


S.No	Exp. Fracture Depth	Predicted
1	12 mm	9.52 mm

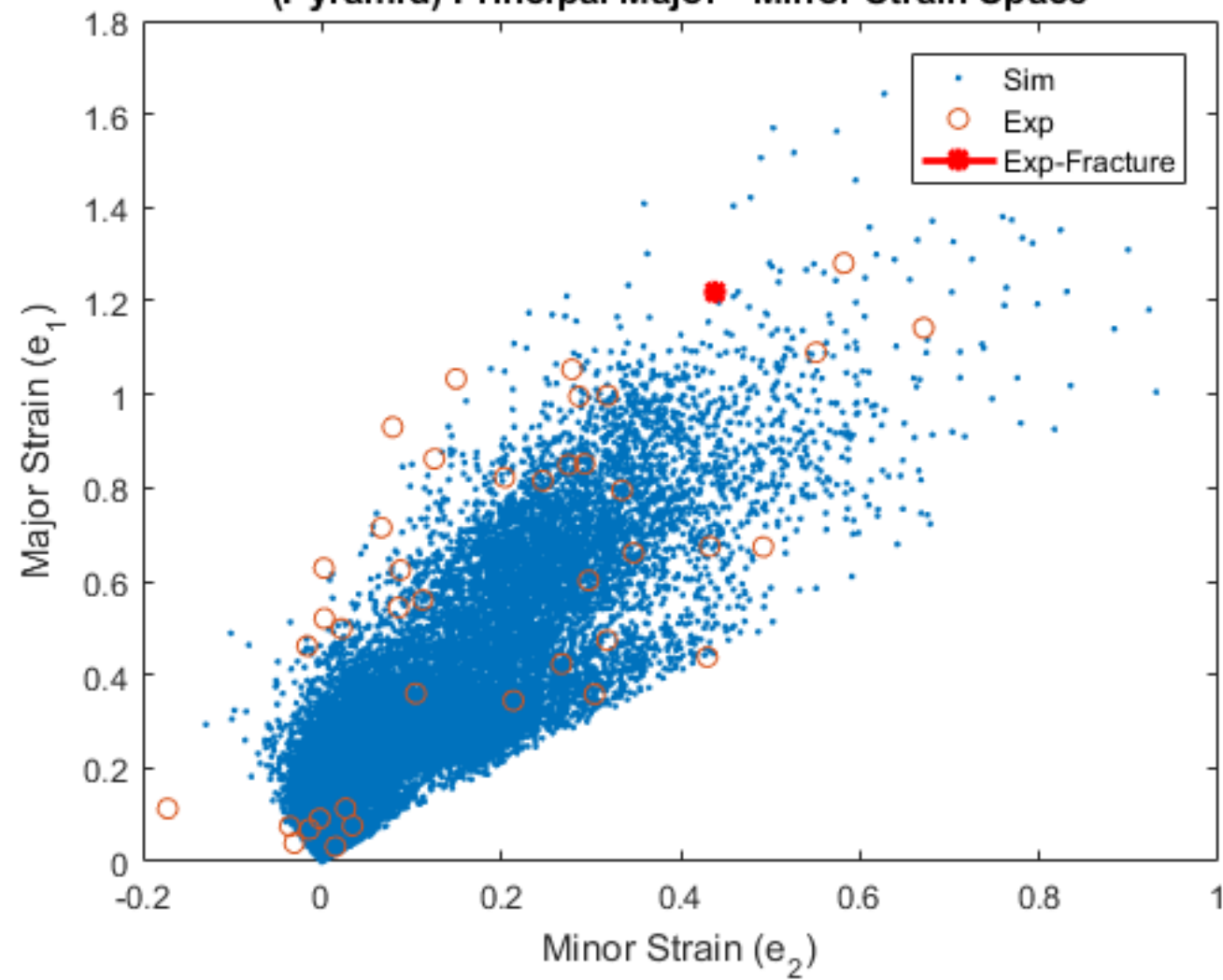


S.No	Exp. Fracture Depth	Predicted
1	40 mm	18 mm

**FFLD**

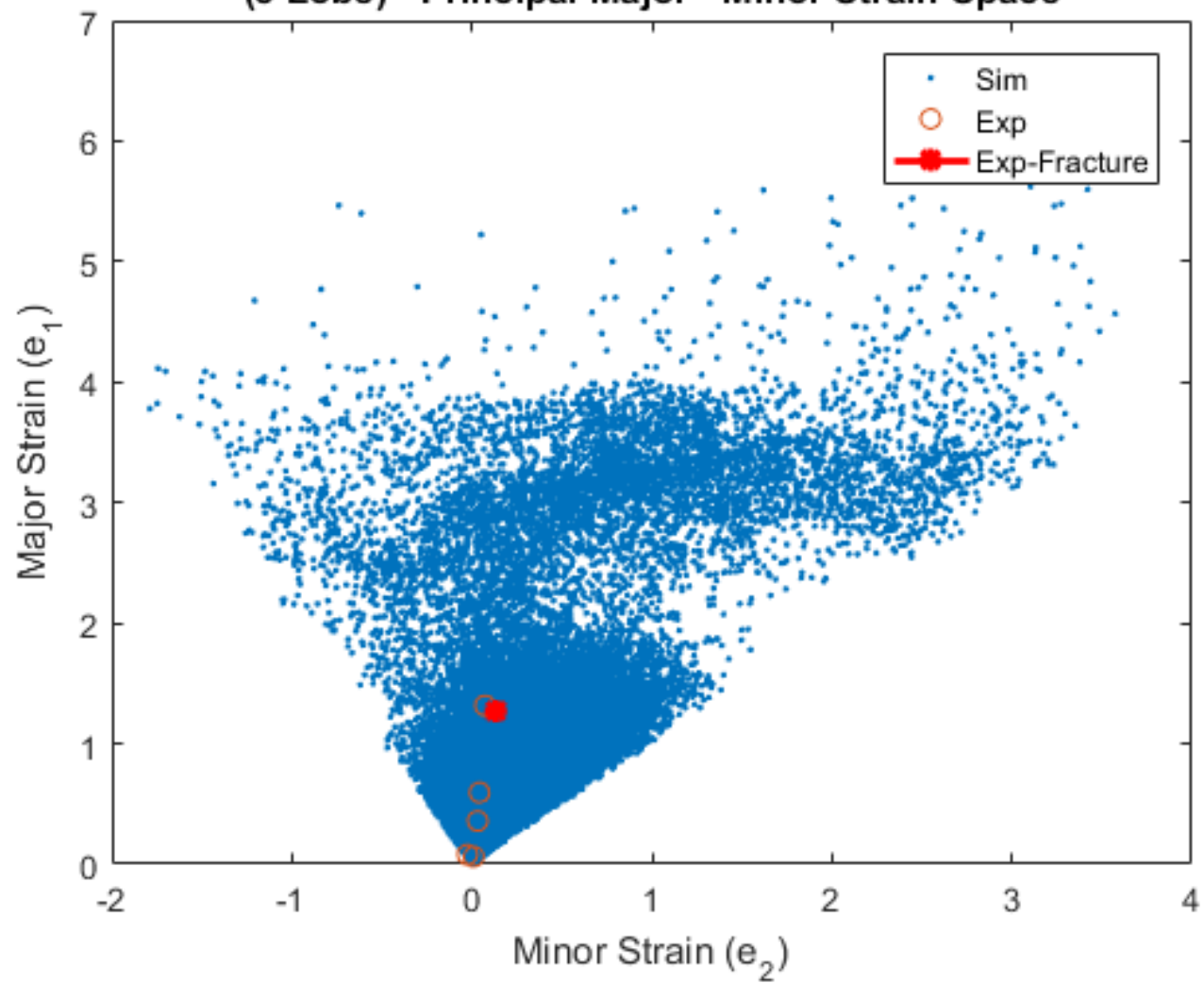


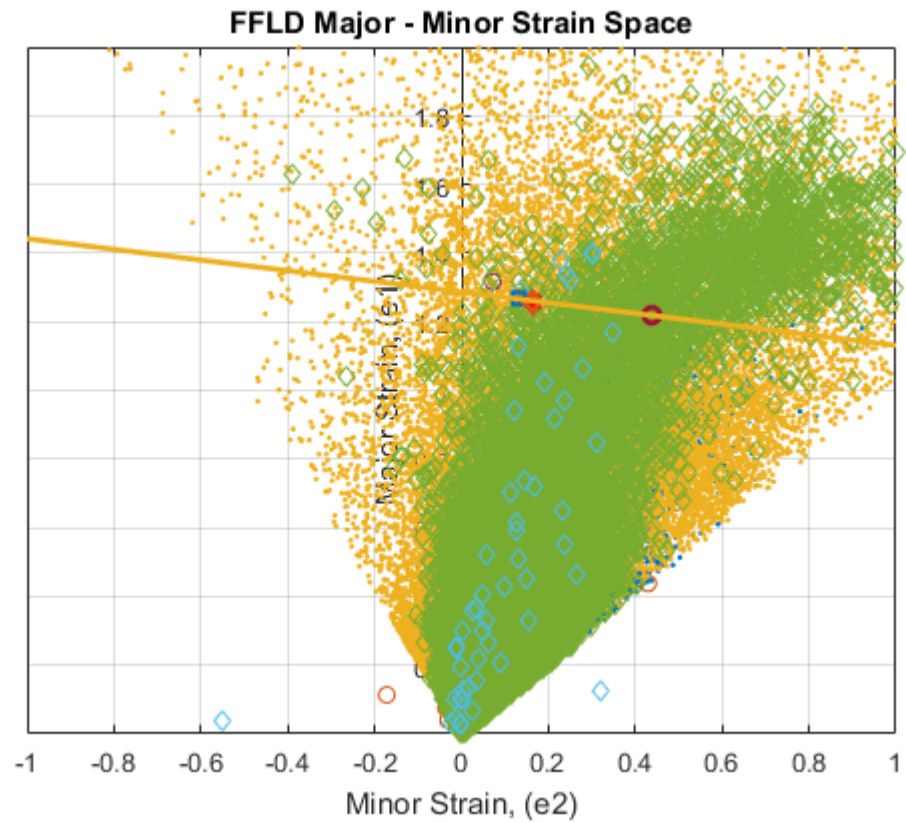
(Pyramid) Principal Major - Minor Strain Space



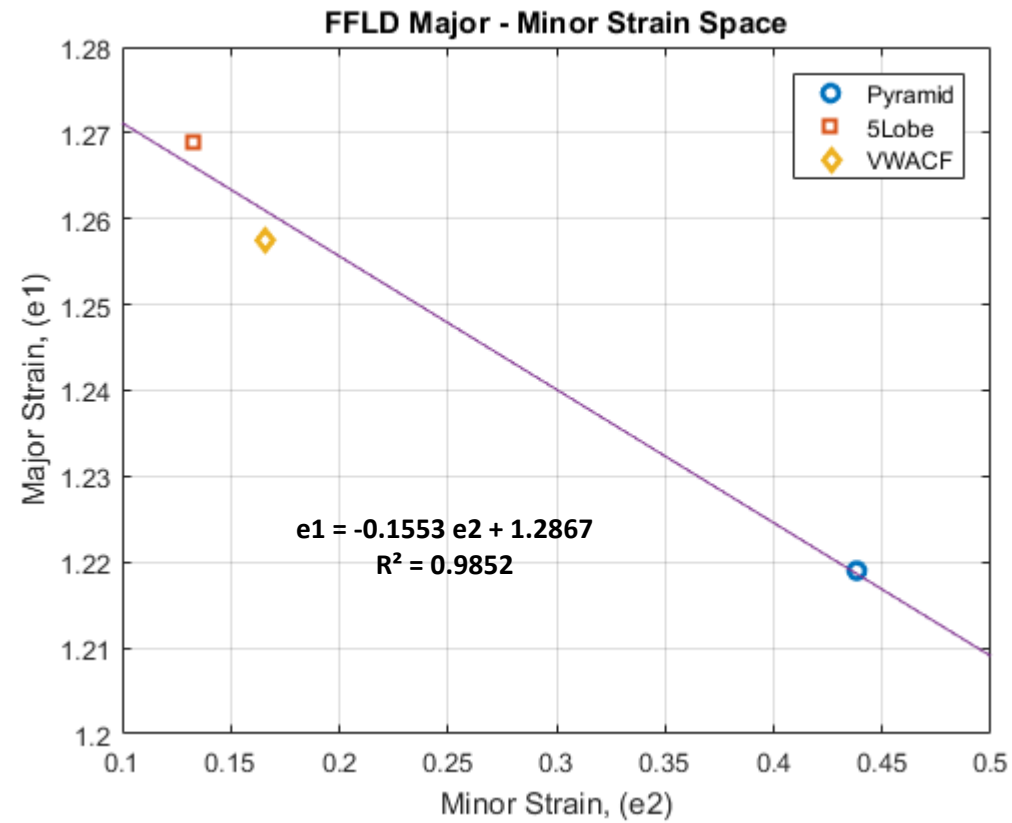


(5 Lobe) - Principal Major - Minor Strain Space





Combined



Experimental Fracture Strain-  
FFLD

# Summary

- Objective: to predict fracture for AL1050 in single point incremental sheet metal forming process (SPIF).
- Use of three uncoupled damage models i.e. BW (Bao-wierzbicki), MC (Mohr Coulomb), HC (Hosford Coulomb)
- Model the three model with help of damage parameter Omega which indicates fracture when it becomes unity.
- Material Model developed in ABAQUS subroutine VUMAT and UMAT for the three fracture models and validated by single element tests.
- Shear test, uniaxial tests, Notch test and central hole tests done to calibrate model and find model coefficients for all three models
- Finite element simulations are run with the developed material model (on ABAQUS with VUMAT) to predict fracture for various SPIF shapes like Line test, Pyramid, Five lobe, Variable wall angle conical frustum.
- Comparisons of models based on fracture predictions in SPIF simulations with respect to experimental observations.

Thank You.

Aishwary Gupta  
2021-31075