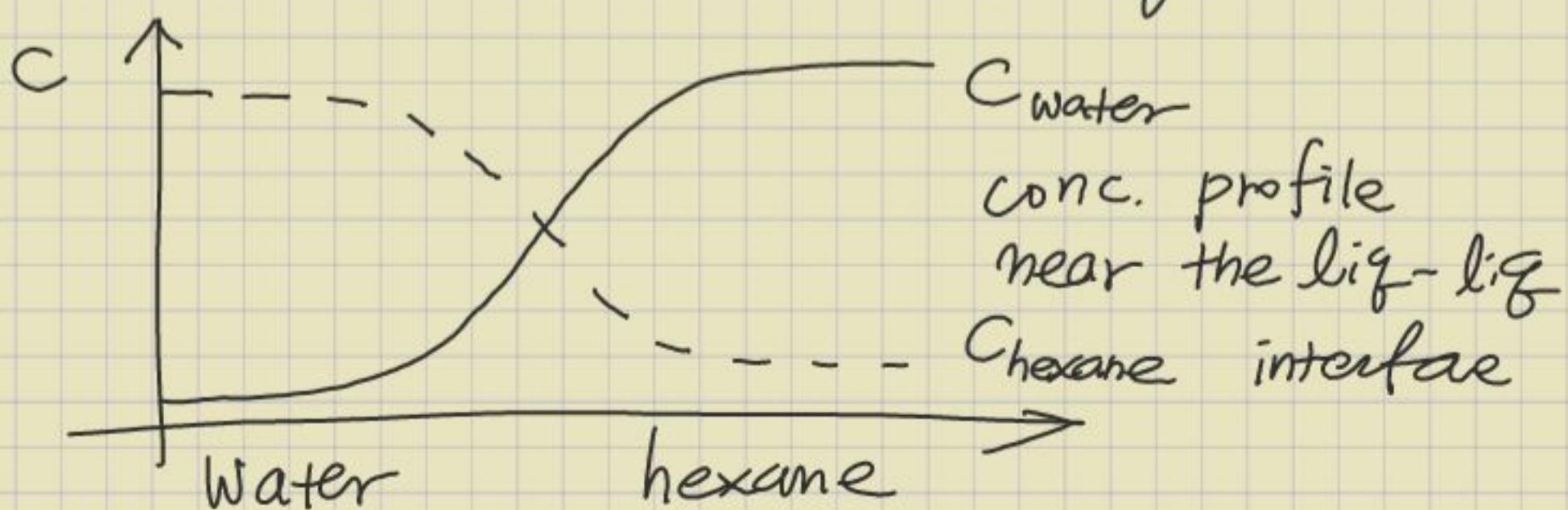
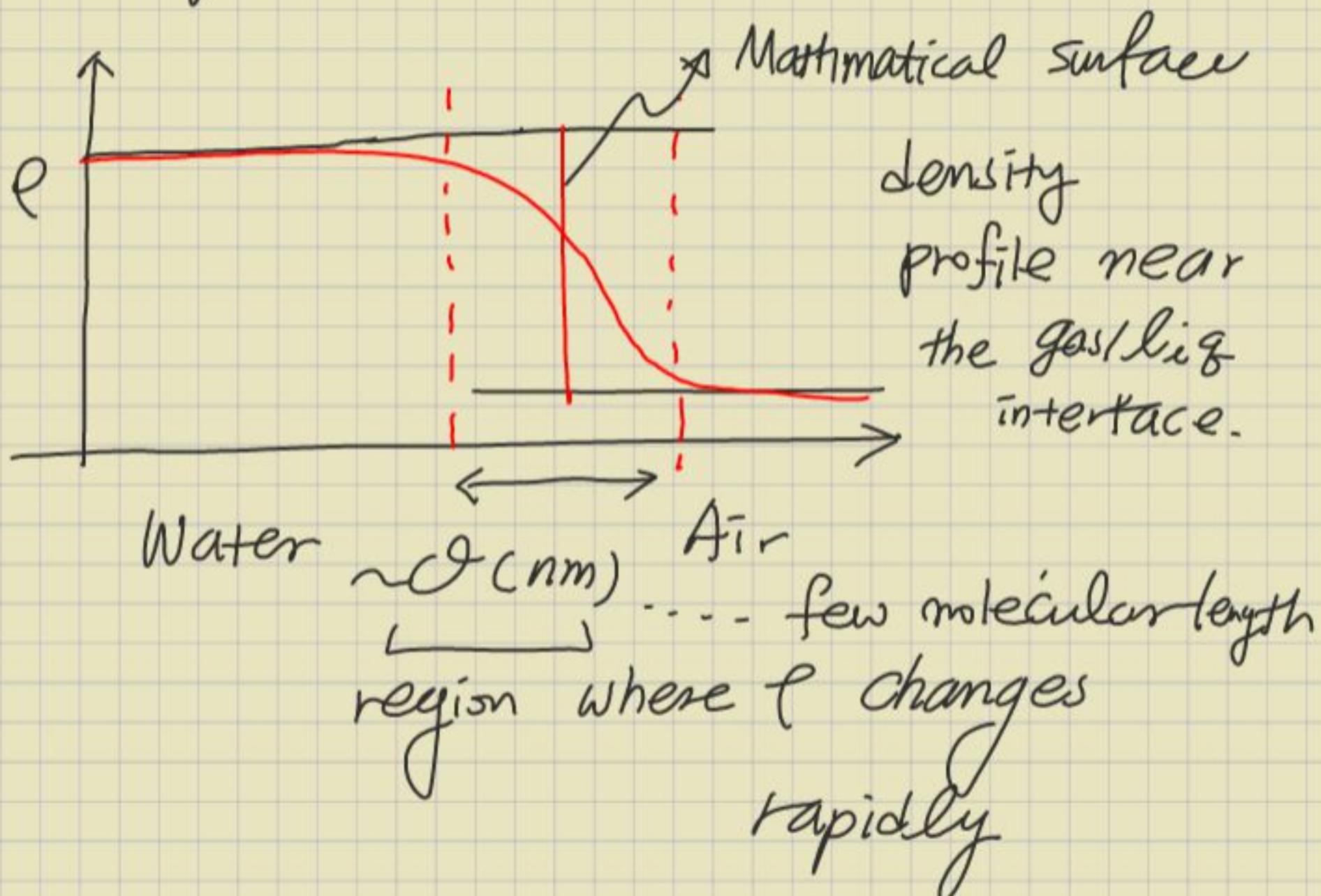


# Interfacial tension

# Lecture 3

So far free surface is treated as a mathematical surface.

In reality, it has finite thickness



Both interfaces can be treated as mathematical "sharp" surface.

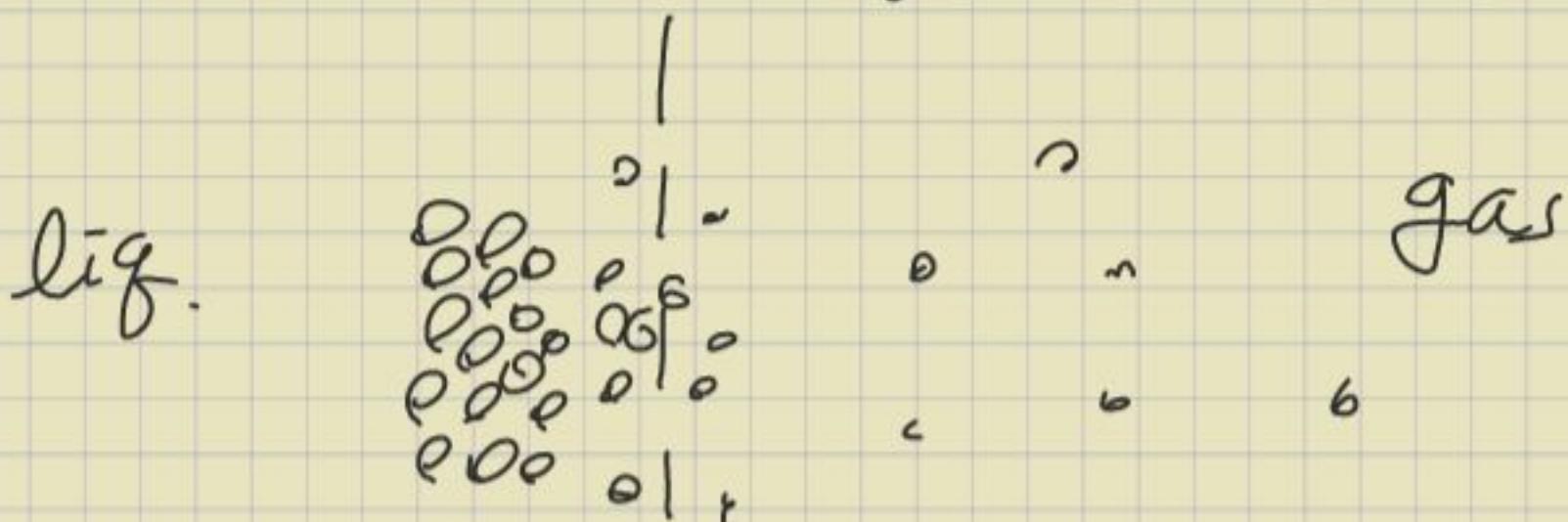
for the sake of simplicity.

⇒ By treating the sharp surface, you have to introduce some anisotropy in the state of stress at the surface.

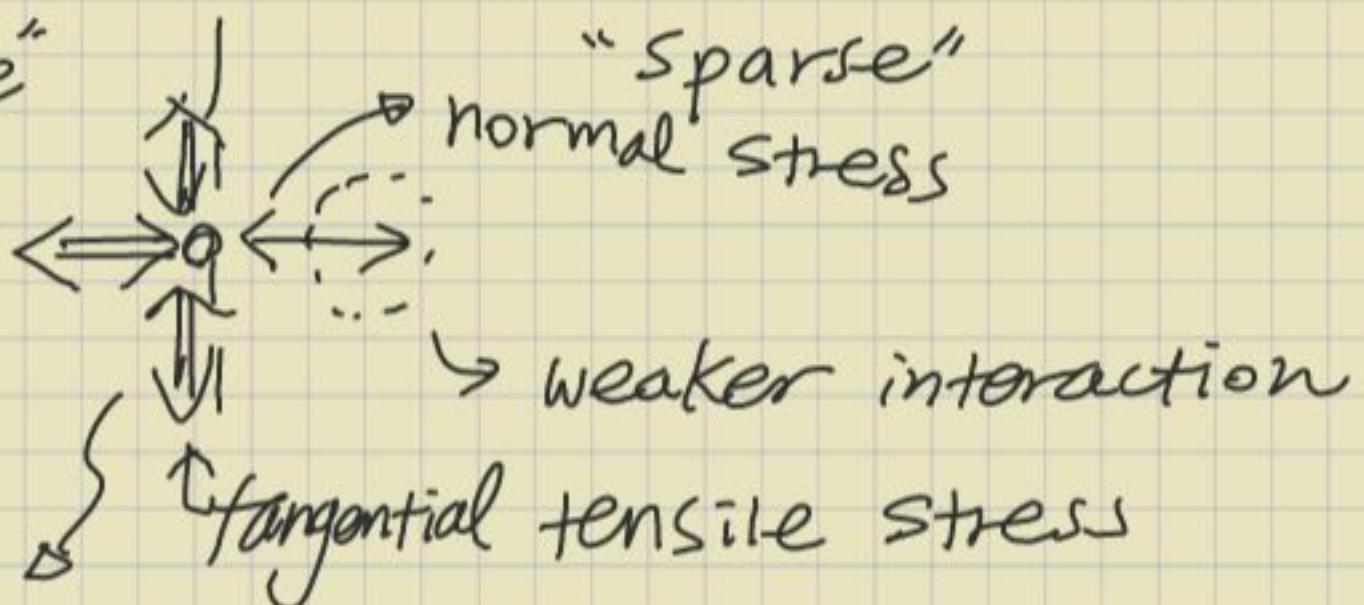
- └ Intenfacial tension → liq/liq
- └ Surface tension → gas/liq

# Origin of surface tension

In mechanical equilibrium



Very "dense"



Stress is not isotropic anymore

- Contributions to Stress in fluid.

(1) Kinetic pressure (due to collision)

Isotropic  $\sim \rho kT \sim n kT$

(2) Tensile stress e.g. VdW eqn

due to intermolecular attraction/  
repulsion

→ Anisotropic in interfacial region

lateral stress more tensile than normal stress

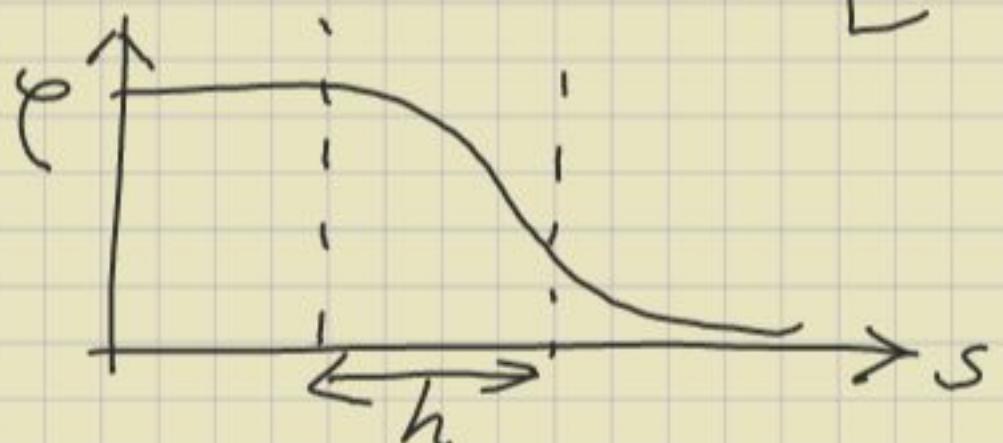
Extra stress acting on  
a given surface  
along tangential direction  
Should be related to

$$(2) = (1).$$

Precisely,

$$= \frac{[\text{Force}]}{L^2}$$

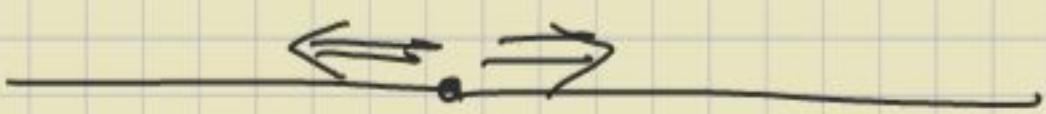
$$\sigma = \int_h (T_T - T_N) ds \quad \left[ \frac{\text{force}}{L} \right]$$



Collection of  
extra stress

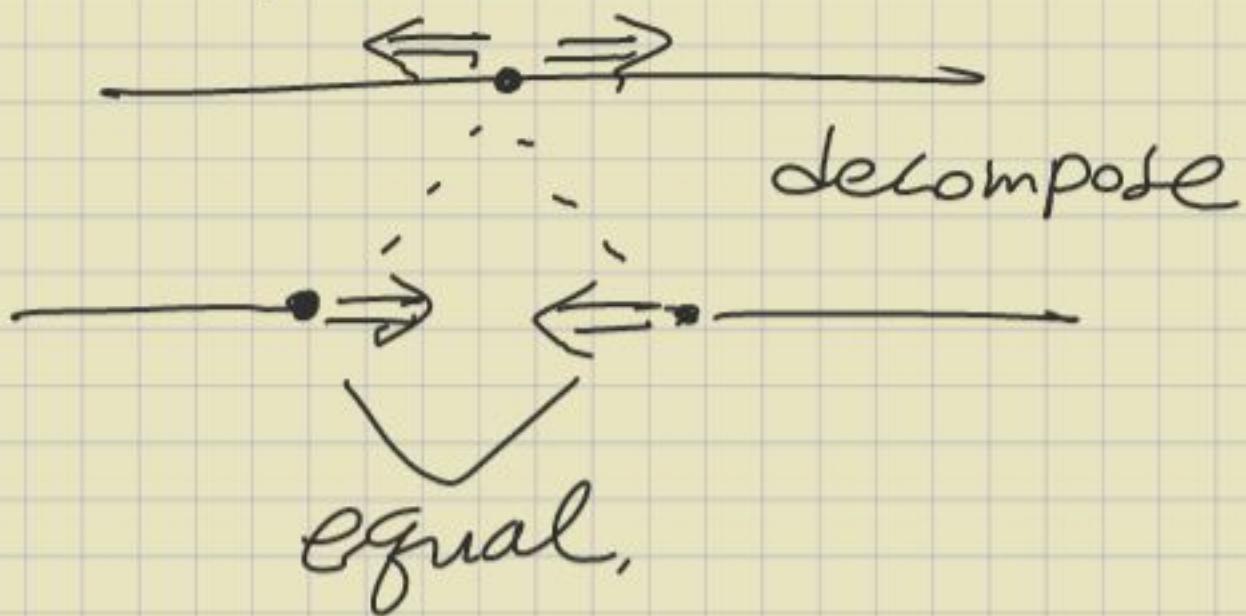
over the interfacial region.

Direction of Surface tension



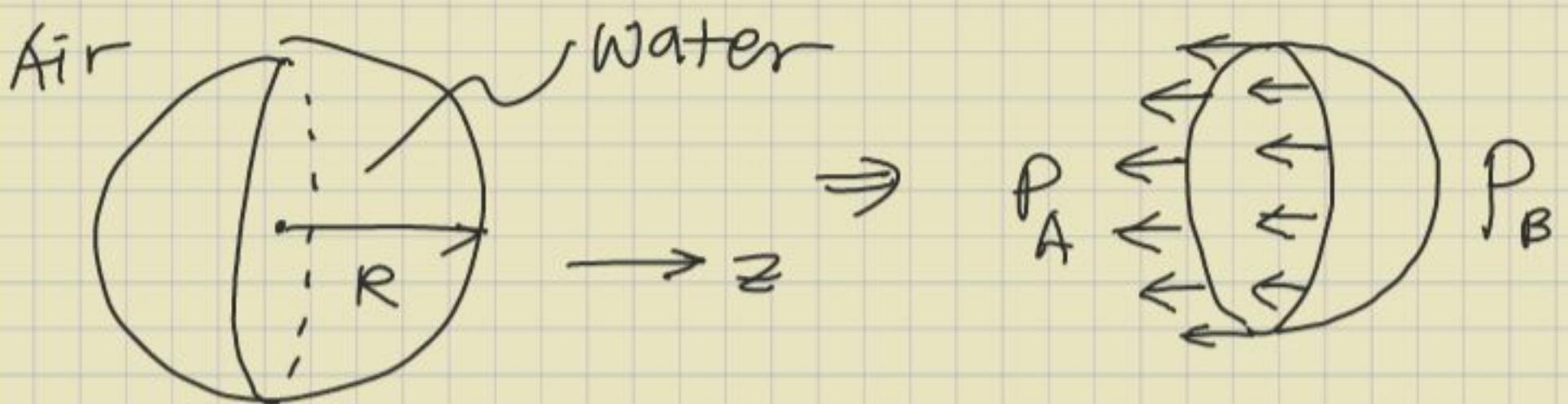
tangential direction

like pressure



Young - Laplace equation from

Force balance on Hemisphere.



Force balance in  $z$  direction

(act in  $-z$  dir)

(1) Surface tension force

$$\sigma \cdot 2\pi R \quad \cdots \square$$

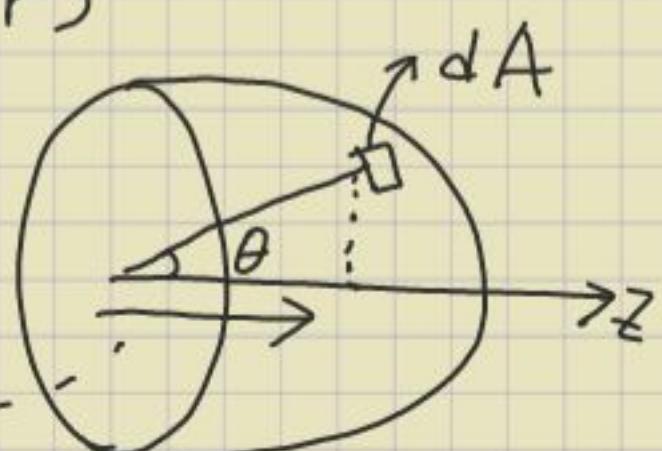
$$= \left[ \frac{N}{L} \right] \quad = [L]$$

Perimeter length  
 $= 2\pi R$

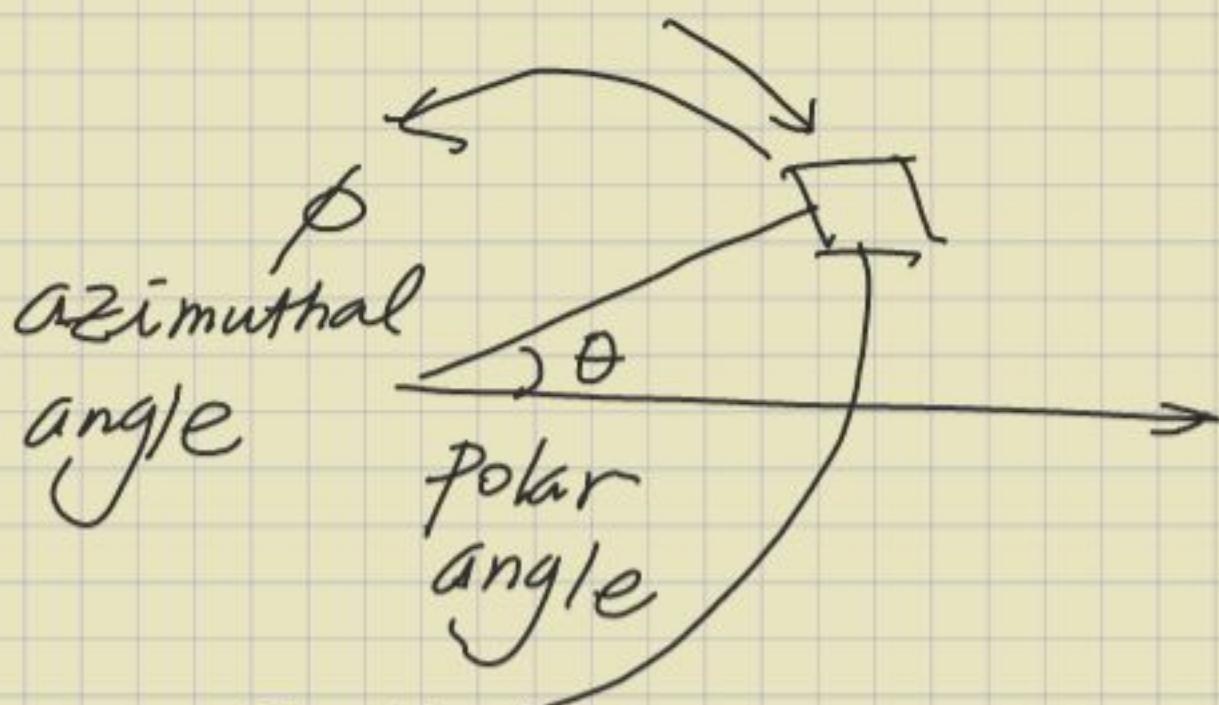
(2) Force due to pressure diff.

(act in  $+z$  dir)

$$\int_A (P_A - P_B) \cos \theta dA$$



$$\text{while } dA = R^2 \sin \theta d\theta d\phi$$



$$\therefore \int_0^{2\pi} \int_0^{\pi/2} (P_A - P_B) \cos \theta R^2 \sin \theta d\theta d\phi$$

$$= 2\pi R^2 (P_A - P_B) \cdot \frac{1}{2} \sin^2 \theta \Big|_0^{\pi/2}$$

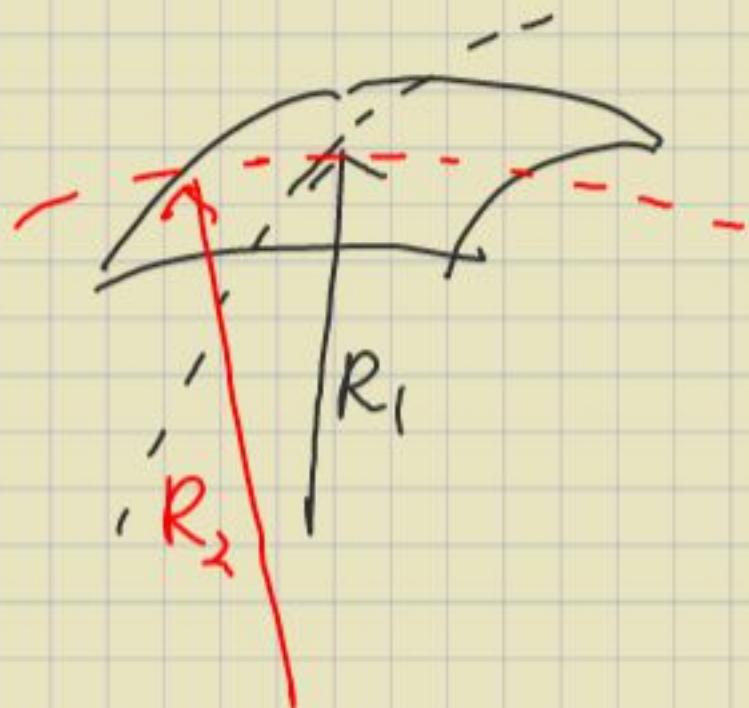
$$= \pi R^2 (P_A - P_B) \quad \square$$

Equating Ⓛ & Ⓜ, we have

$$P_A - P_B = \frac{2\sigma}{R} \quad (\Rightarrow P_A > P_B !)$$

For general surface

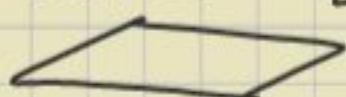
$$\Delta P = P_A - P_B = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$



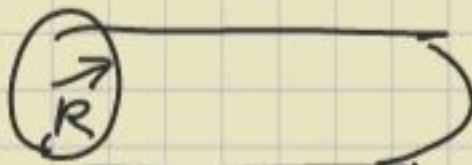
✓  
two principle  
radii of curvature  
in perpendicular  
direction along  
the surface.

example)

plane :  $R_1, R_2 \rightarrow \infty, \Delta P = 0$

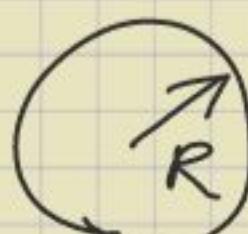


cylinder :  $R_1 = R, R_2 \rightarrow \infty, \Delta P = \frac{\sigma}{R}$



+

sphere :  $R_1 = R_2 = R, \Delta P = \frac{2\sigma}{R}$



(show, representative  
values slide)

Note that Y-L eqn can also be derived from thermodynamics  
 (See "Helmholtz - Capillary.pdf")

In a viewpoint from thermo.

$$\sigma = - \left( \frac{\partial F}{\partial A} \right)_{T, V, c}$$

area      temp      volume

Free energy  
 (Helmholtz)

or

$$= - \left( \frac{\partial G}{\partial A} \right)_{T, p, c}$$

Gibbs free E.

• Unit of  $\sigma$

$$\Delta P = \frac{2\sigma}{R} \Rightarrow \sigma = \frac{1}{2} \Delta PR$$

( $k$ )  $\uparrow$   
 case

$$= [\text{Pa} \cdot \text{m}]$$

$$= [N/m]$$

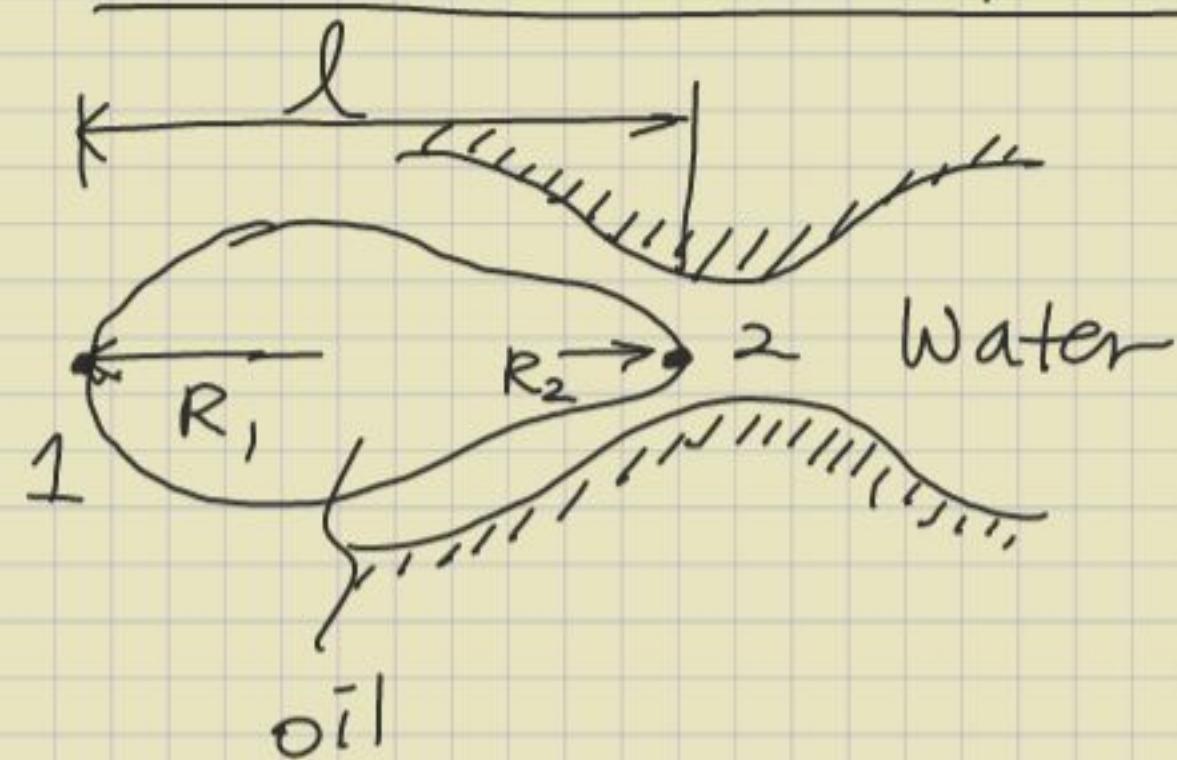
# Typical values of surface tension.

- Metal  $\sigma = 500 \sim 800 \text{ mN/m}$
- Water  $\sigma = 72 \text{ mN/m}$
- Organic liquid  $\sigma = 15 \sim 25 \text{ mN/m}$

Water is more tightly bounded than organic liquid

# Trapping of oil or organic lig.

Contaminant in porous media



$$\text{at point 1: } P_{W_1} - P_{\text{oil}} = \frac{2\sigma}{R_1}$$

$$\text{at point 2: } P_{W_2} - P_{\text{oil}} = \frac{2\sigma}{R_2}$$

∴ The pressure difference btw 1 & 2

$$P_{W_1} - P_{W_2} = 2\sigma \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

( $\Delta P$  inside oil)

In order to maintain drop in the position shown,  $P_{W_1} - P_{W_2}$  should be maintained inside the water.

For typical value of  $\sigma$   
( $10 \sim 50 \text{ mN/m}$ ) &  
typical flow rates in oil reservoirs  
or ground water acquifer.

( $\sim 1 \text{ ft/day}$ ),  
the pressure gradient  $\frac{P_w - P_{w_2}}{l}$ ,

↳ an underground  
layer of water-  
bearing rock

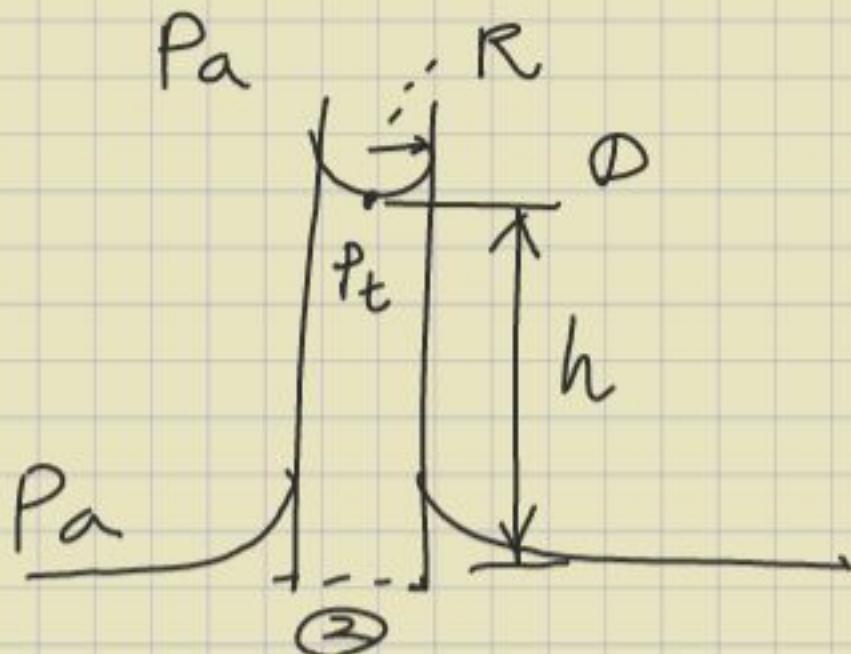
is not sufficient to force  
a drop through most pores  
and the oil drop will be trapped.

However, the drop can pass  
through pores if  $\sigma$  is reduced

to  $10^{-3} \sim 10^{-2} \text{ mN/m}$ .

Low interfacial tension (interfacial free  $\sigma$ )  
make drop easily deformable.

Capillary rise in a tube of radius  $R$ .



$$\textcircled{1} \quad P_t = P_a - \frac{2\sigma}{R} \quad ] \rightarrow \frac{2\sigma}{R} = \rho g h$$

$$\textcircled{2} \quad P_a = P_t + \rho g h \quad \Rightarrow h = \frac{2\sigma}{\rho g R}$$

$$\rho = 10^3 \text{ kg/m}^3$$

$$g = 10 \text{ kg/m s}^2$$

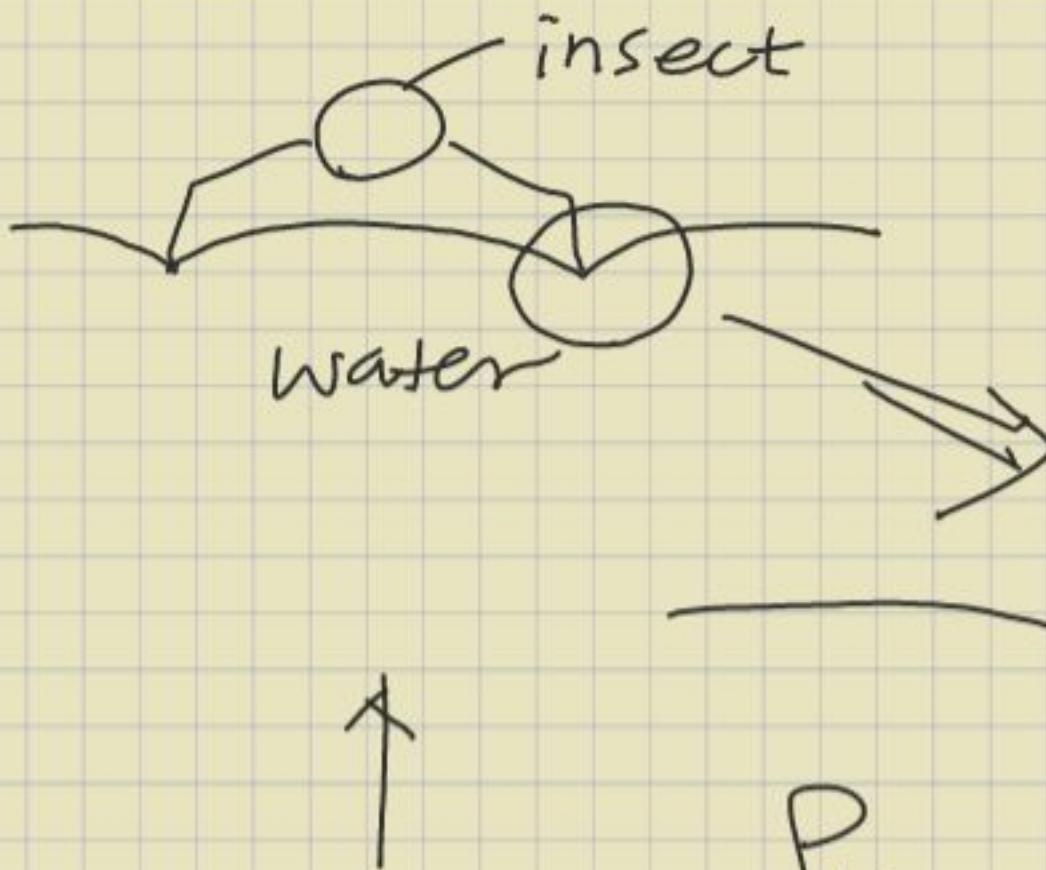
$$\sigma = 70 \text{ mN/m}$$

- When  $R \sim 1 \text{ m} \Rightarrow h \sim \mathcal{O}(\mu\text{m})$

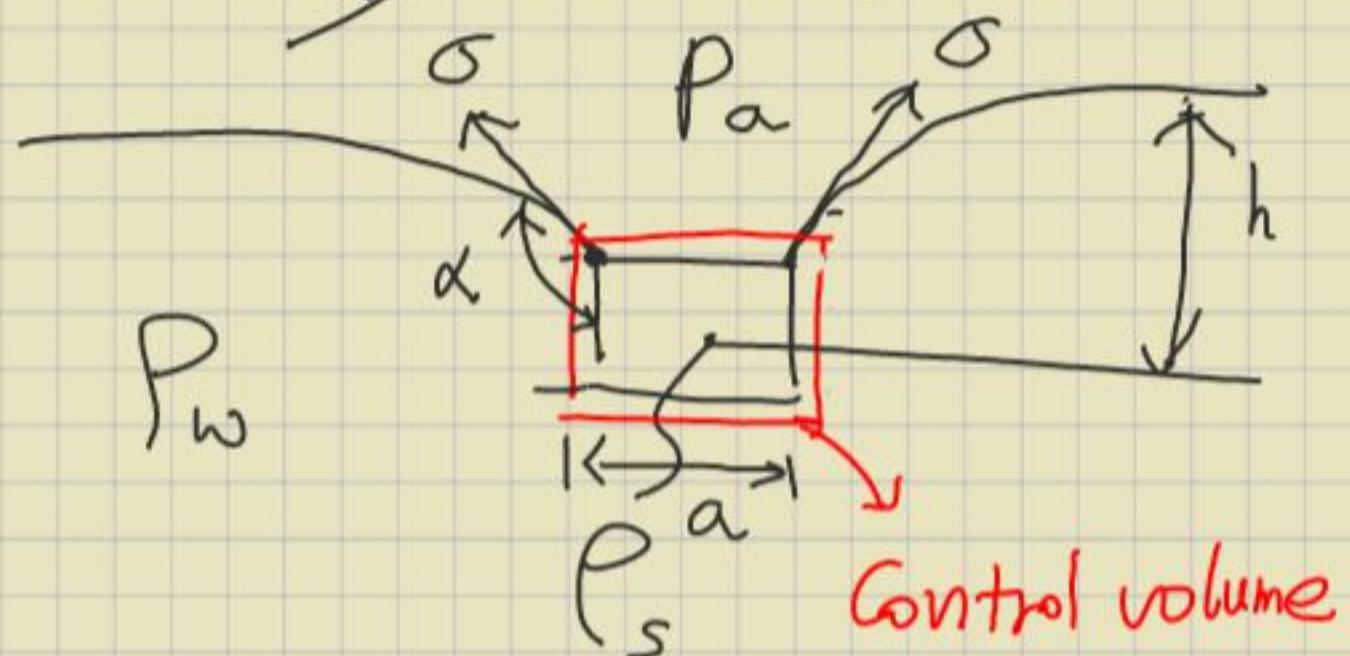
$R \sim 1 \text{ mm} \Rightarrow h \sim \mathcal{O}(10 \text{ mm})$

You can see the height rise.

# Insect that walking on water



Simplify



Consider vertical ( $z$ -dir) force balance

- weight :  $\rho_s g a^2$  (base line =  $z$ -dir)
- Surface tension force :  $-2\sigma \cos \alpha$
- pressure force on top & bottom surface

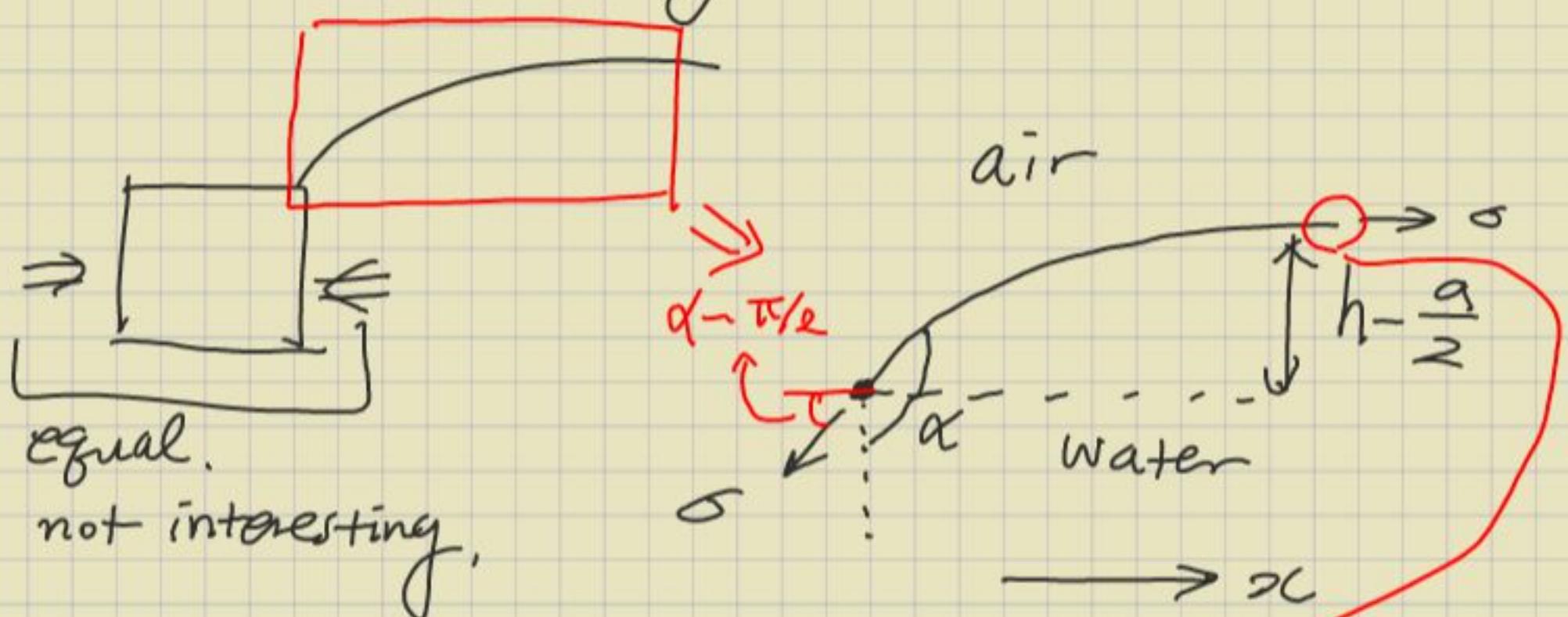
$$\underbrace{P_a \cdot a}_{\text{top}} - \underbrace{\left[ P_a + \rho_w g \left( h + \frac{a}{2} \right) \right] a}_{\text{bottom}}$$

$$\rho_s g a^2 = -2\sigma \cos \alpha + \rho_w g a \left( h + \frac{a}{2} \right)$$

(Vol) (Surface)

$$\left[ \left( \rho_s - \frac{\rho_w}{2} \right) g \right] a^2 - (\rho_w g h) a + 2 \sigma \cos \alpha = 0 \quad \dots \textcircled{D}$$

(b) horizontal force balance  
on unit length of fluid interface



- Surface tension force

$$-\sigma \cos(\alpha - \frac{\pi}{2}) + \sigma \quad \therefore \quad \sigma(1 - \sin \alpha)$$

$\alpha - \frac{\pi}{2}$

- Pressure force due to liquid

$$-(P_a + \rho_w g \frac{h - \frac{a}{2}}{2})(h - \frac{a}{2}) \int_0^{h - \frac{a}{2}} P(z) dz$$

Water

- pressure force due to air



$$\rho_a (h - \frac{a}{2})$$

Sum to get eq. Condition ✓

$$0 = \sigma(1 - \sin\alpha) - \frac{\rho_w g}{2} (h - \frac{a}{2})^2 \quad \dots \textcircled{2}$$

- unknown  $\alpha$  &  $h$
- equation  $\textcircled{1}$  &  $\textcircled{2}$
- Using software Matlab or Mathematica to solve.

It turns out that

the solution depends on 'a'  
( scale of )

- { small  $a \rightarrow$  float (there is a solution)
- large  $a \rightarrow$  sink (No solution)