446.631A

# 소성재료역학
# (Metal Plasticity)

## Chapter 3: Instability in simple tension test

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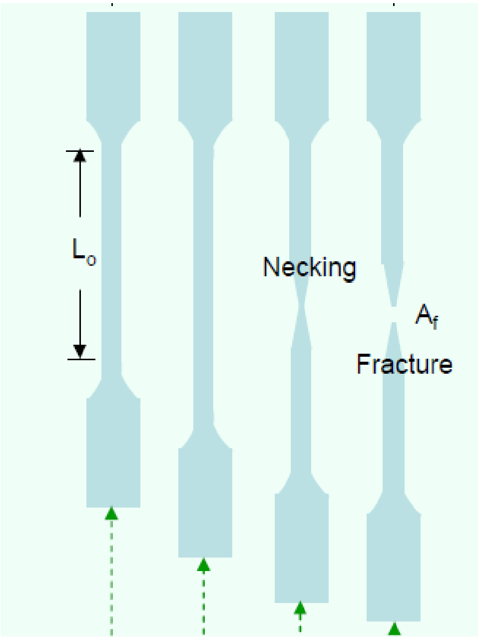
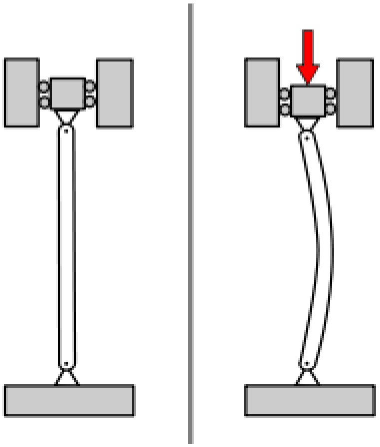
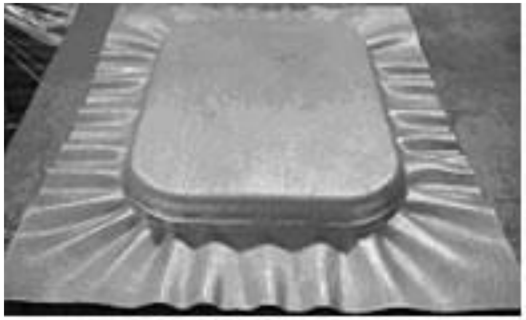
# Instability

- Necking
  - ✓ sheet and bulk under tension
  - ✓ localized deformation
- Barreling
  - ✓ Bulk under compression
- Buckling
  - ✓ sheet under compression

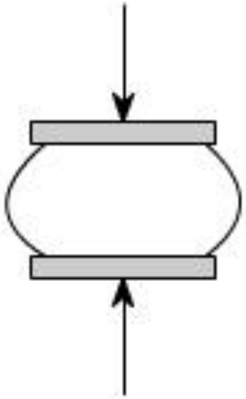




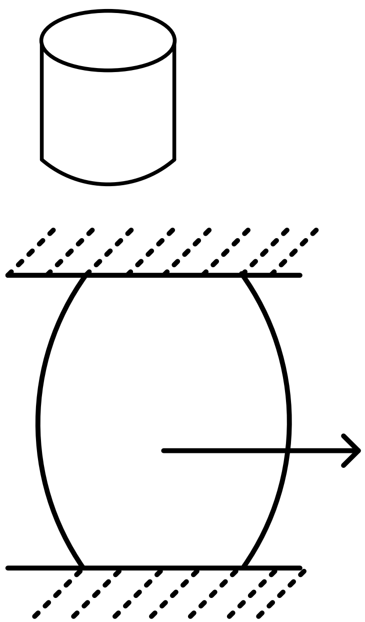
(a)

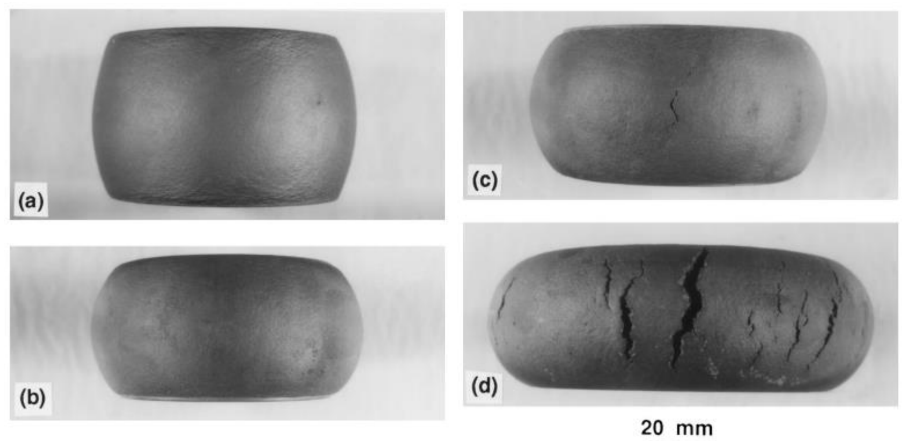
(b)



# Compression - barreling

Typical specimen geometries in compressive tests



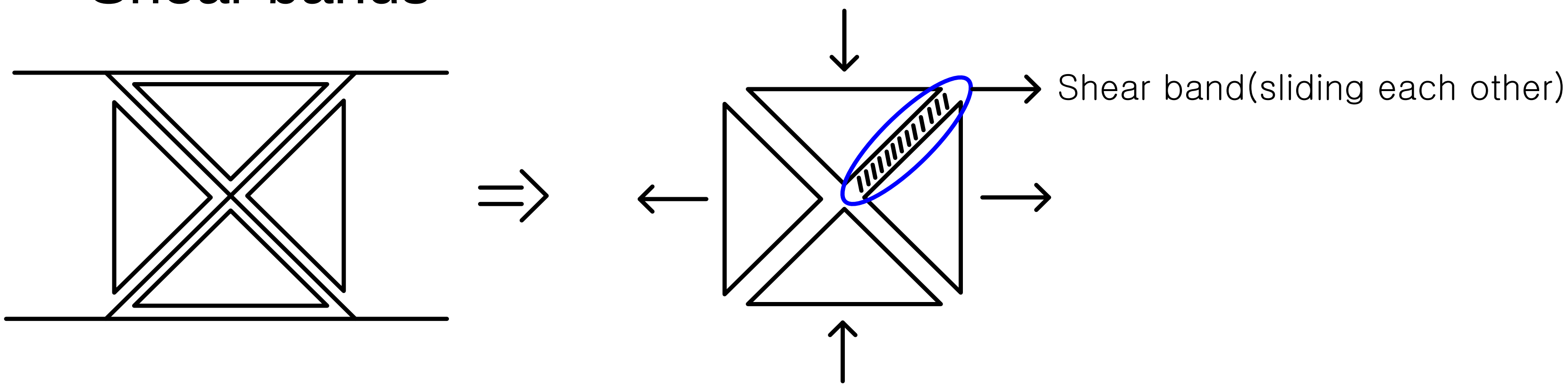


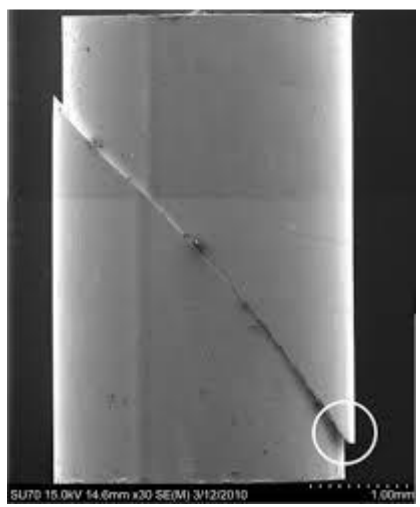

Non-uniform deformation(Near-simple compression)

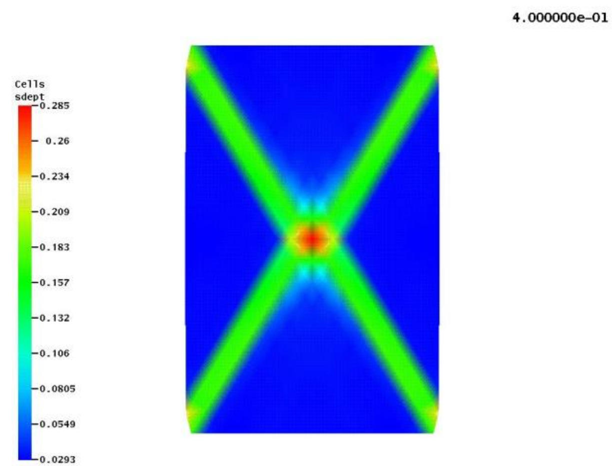
Needs good lubrication at the contact surfaces

# Compression – shear band

Shear bands

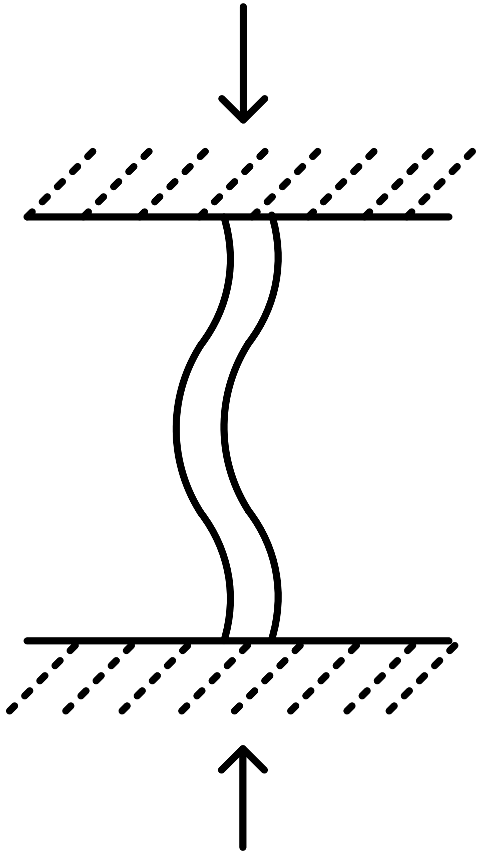


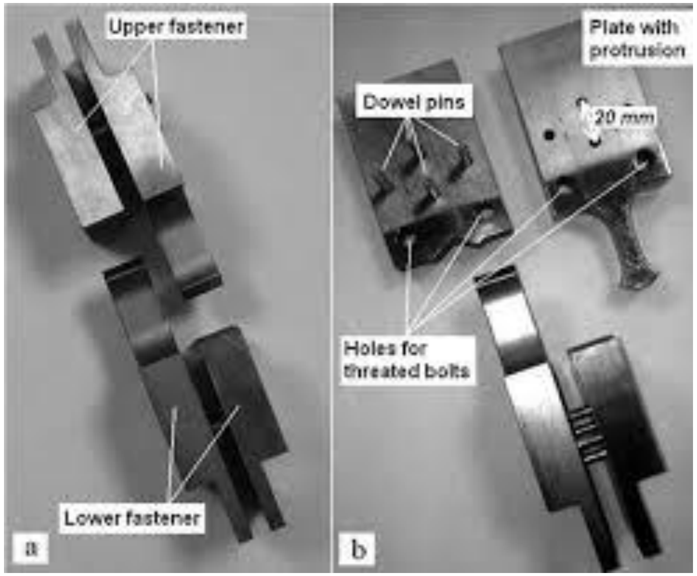



# Compression - buckling

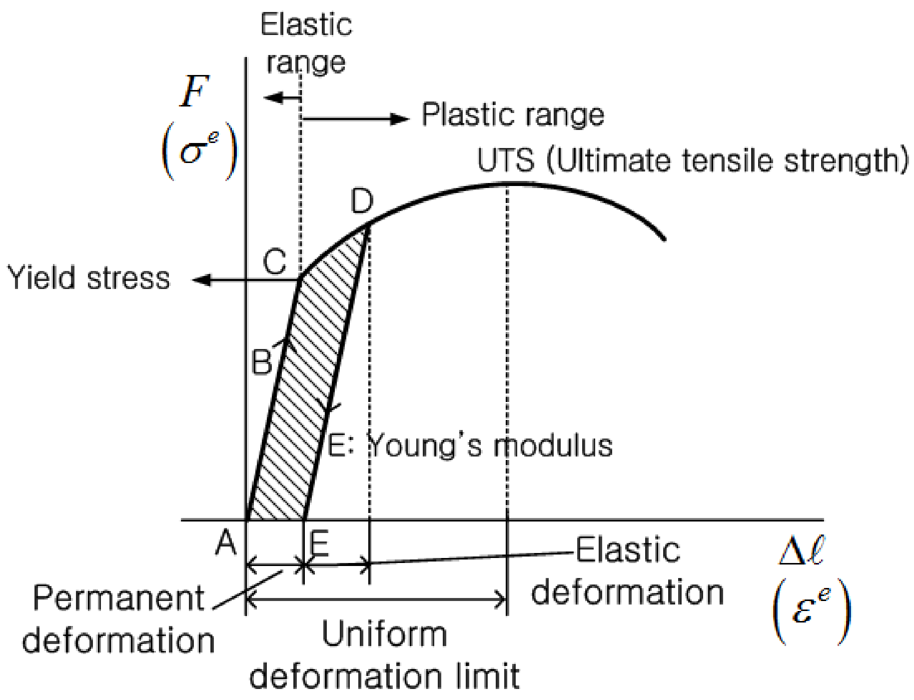
Buckling for sheet and thin rod

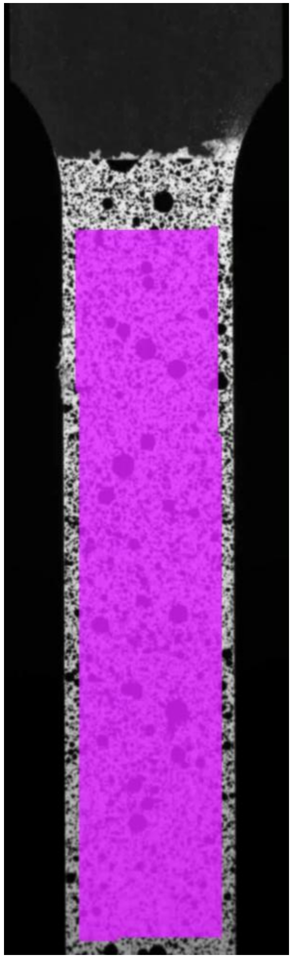





Anti-buckling device

# Necking for metals

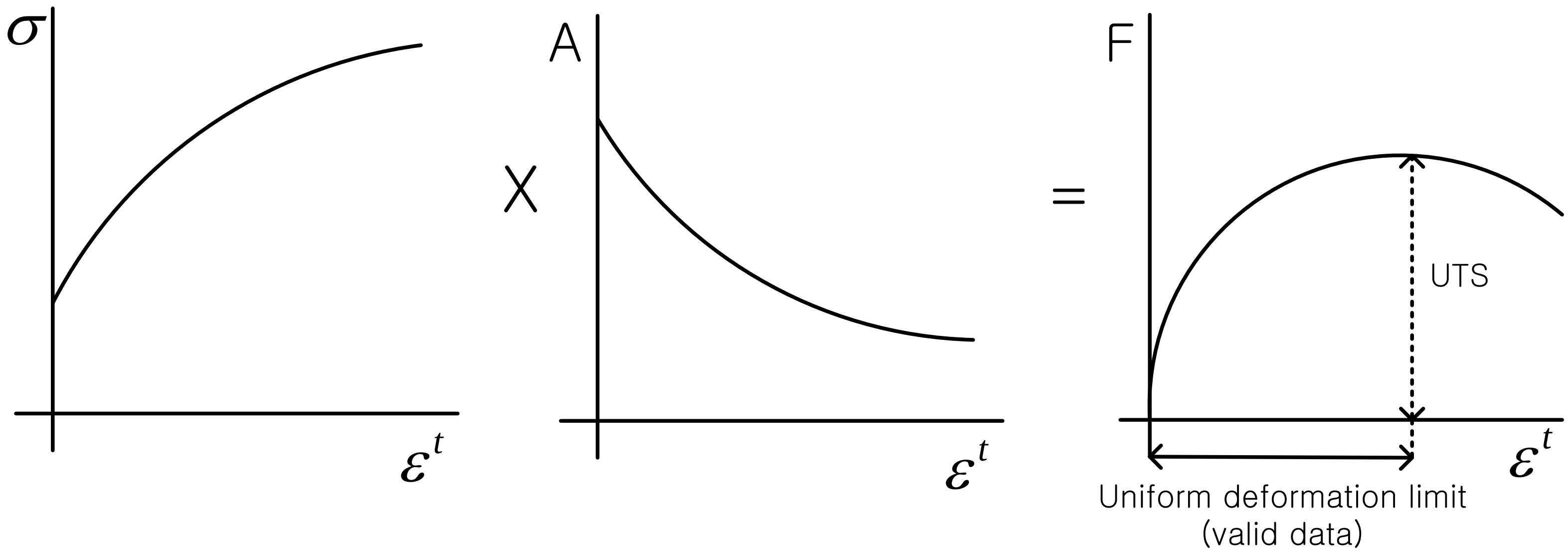


# Necking for metals

Uniform deformation limit

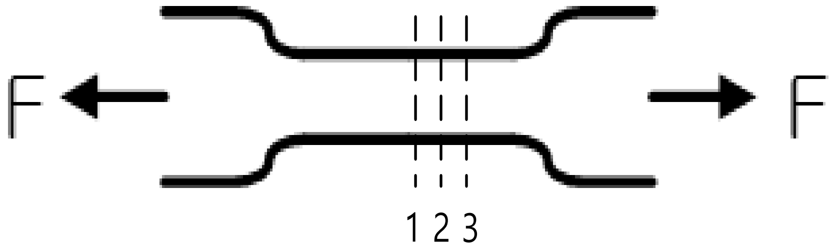
True stress　　Area　　Force




$$F\left(\varepsilon^{t}\right)=\sigma\left(\varepsilon^{t}\right)\times A\left(\varepsilon^{t}\right)$$

# Necking for metals

Uniform deformation limit



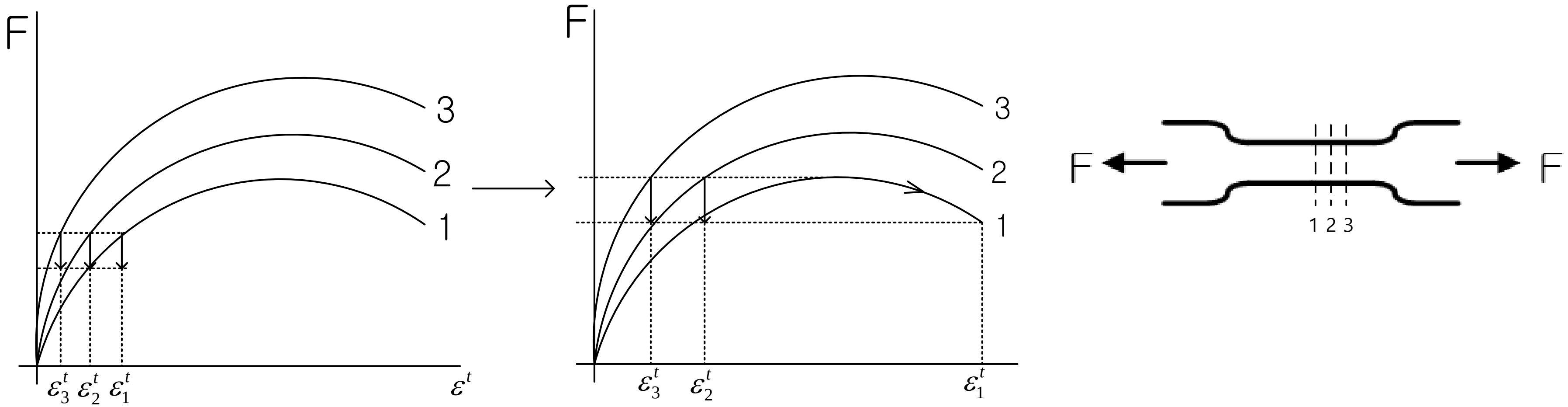

$A_o^1 < A_o^2 < A_o^3$ (either by real area difference or boundary constraint effect)

$F = F^1 = F^2 = F^3$

If Incompressible rigid-plasticity

$$F = \sigma A = \sigma A_o \frac{l_o}{l} = \sigma A_o e^{-\varepsilon^t}$$

# Diffuse necking condition for metals




Considère criterion (or limit of uniform deformation)

$$dF = 0$$

$$F = \sigma A$$

$$dF = d\sigma A + \sigma dA = 0$$

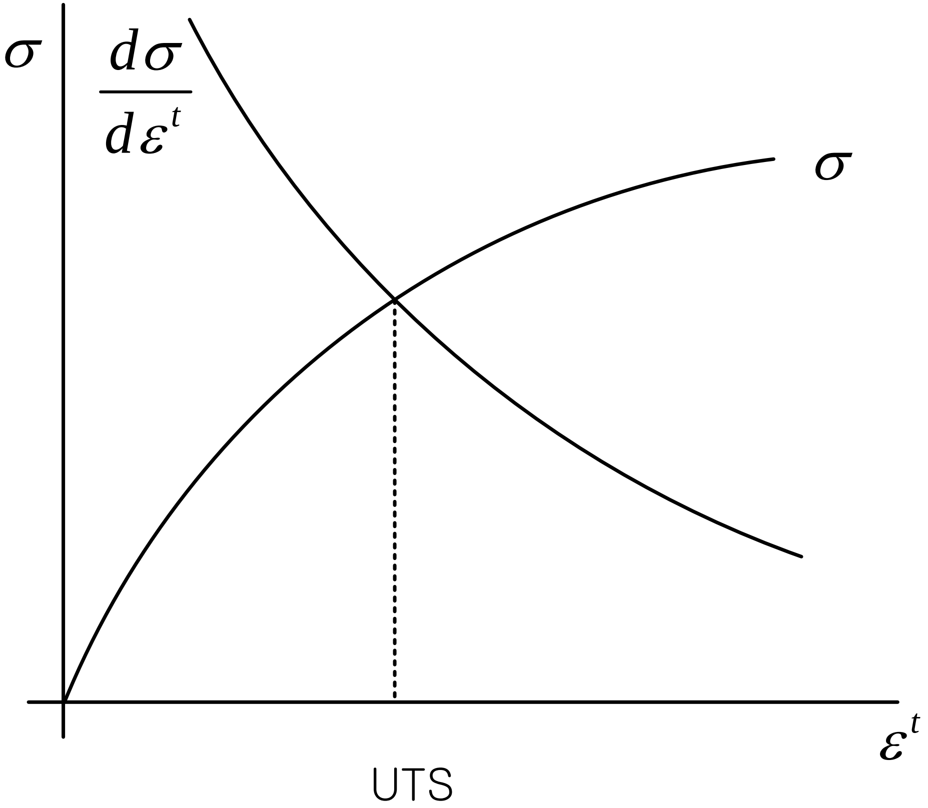
$$\frac{d\sigma}{\sigma} = -\frac{dA}{A} = \frac{dl}{l} = d\varepsilon^t$$

$$\left(\text{volume constant } \therefore Al = \text{Constant} \rightarrow \text{dA}l + Adl = 0\right)$$

$$\therefore \frac{d\sigma}{d\varepsilon^t} = \sigma \left(\text{Considère criterion}\right)$$

# Considère condition

$$\frac{d\sigma}{d\varepsilon^t} = \sigma \left(\text{Considère criterion}\right)$$



# Considère condition

Hardening laws

$$\bar{\sigma} = K\bar{\varepsilon}^{n}$$ Hollomon

$$\bar{\sigma} = \bar{\sigma}_y + K\bar{\varepsilon}^{n}$$ Ludwick

$$\bar{\sigma} = K\left(\varepsilon_o + \bar{\varepsilon}\right)^{n}$$ Swift

$$\bar{\sigma} = \bar{\sigma}_o - e^{-n\left(\bar{\varepsilon} - \varepsilon_o\right)} \text{ or } \bar{\sigma} = A - Be^{-c\bar{\varepsilon}} \left(B = e^{n\varepsilon_o}\right)$$ Voce

# Considère condition

Hollomon hardening law $\bar{\sigma} = K\bar{\varepsilon}^{n}$

$$\frac{d\sigma}{d\varepsilon} = \sigma \Rightarrow k \cdot n \cdot \varepsilon^{n-1} = k \cdot \varepsilon^{n}$$

$$\therefore \varepsilon = n$$

Swift hardening law $\bar{\sigma} = K\left(\varepsilon_o + \bar{\varepsilon}\right)^{n}$

$$\frac{d\sigma}{d\varepsilon} = \sigma \Rightarrow k \cdot n \cdot \left(\varepsilon_o + \varepsilon\right)^{n-1} = k \cdot \left(\varepsilon_o + \varepsilon\right)^{n}$$

$$\therefore \varepsilon_o + \varepsilon = n$$

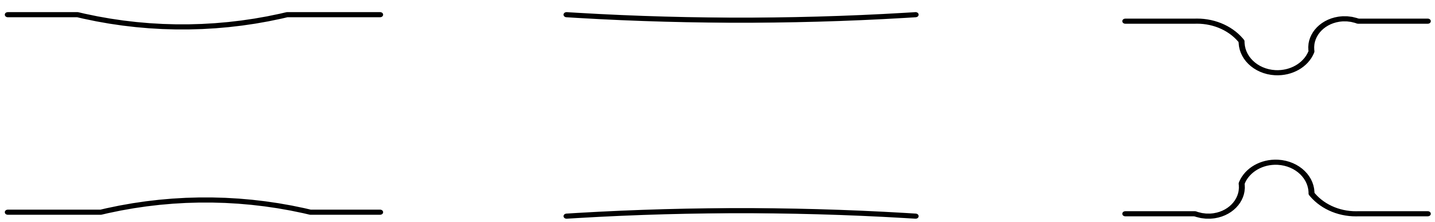
# Ductility measurement & necking

- Elongation: elongation is influenced by uniform deformation, strain hardening capacity

- Reduction of area
  - measure of deformation to produce failure and contribution from necking (or localization)
  - Due to the complicated stress state around necked region the value is dependent on the specimen geometry: not true material properties
  - RA is a structure-sensitive ductility parameter

# Necking
# : effect of strain rate sensitivity

$$\bar{\sigma} = f\left(\bar{\varepsilon}\right) \cdot \left(\frac{\dot{\bar{\varepsilon}}}{\dot{\bar{\varepsilon}}_o}\right)^m$$

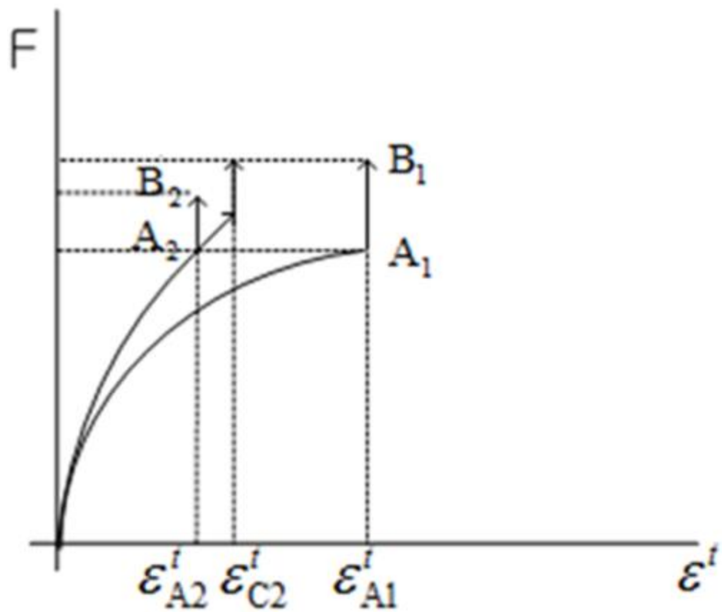


m=0 m>0 m<0

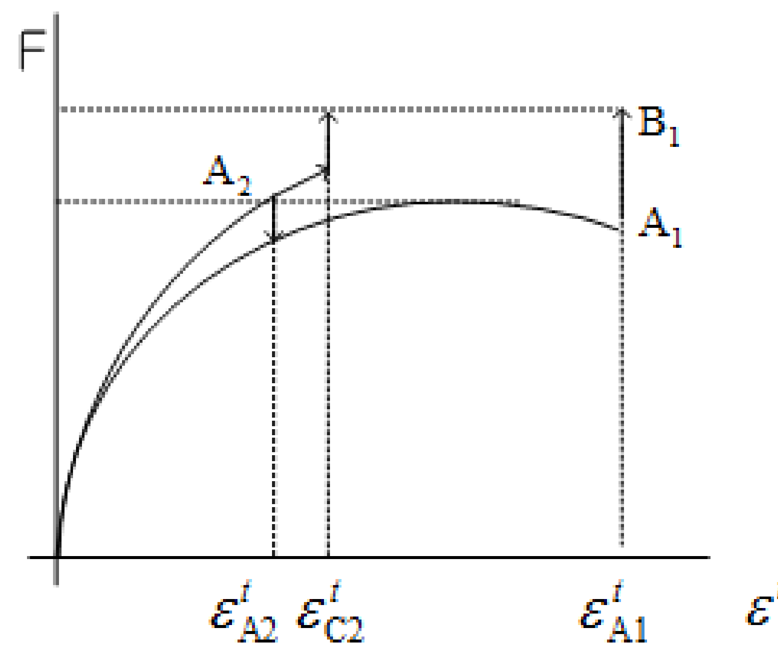
# Necking
# : effect of strain rate sensitivity

Positive strain rate sensitivity

⟶ Formation of diffuse necking is delayed

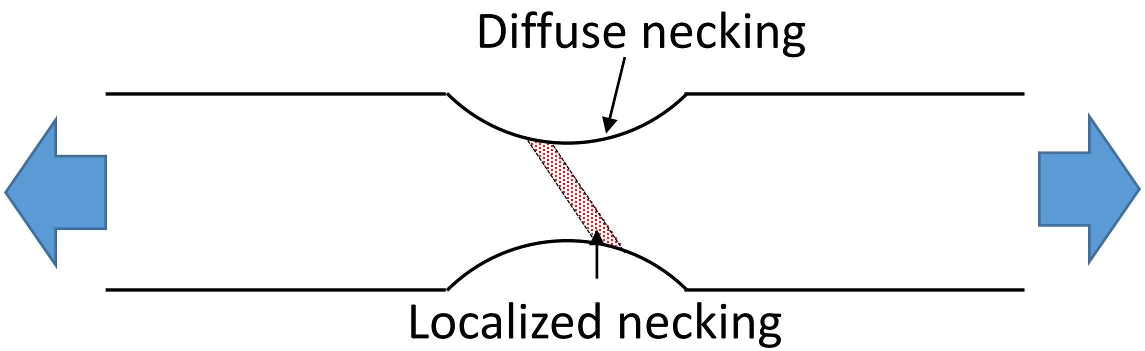



Before UTS




After UTS

# Diffuse vs localized necking




- Diffuse necking
  - Provide a large extent of necking on the tensile specimen similar to necking from a cylindrical specimen
  - Diffuse necking might terminate in fracture but normally followed by localized necking
- Localized necking
  - Localized necking is a narrow band with its size ~ specimen thickness, and a certain inclined angle ~55 degree
  - Give no change in width through the localized necking : plane strain deformation

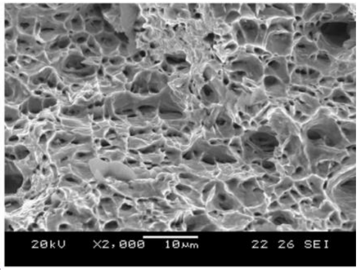
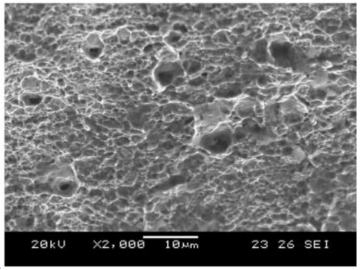
# Necking & failure

## n- value and failure

### Specimen shape after fracture

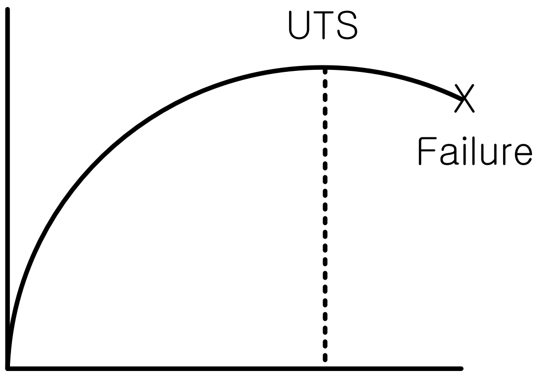
| Material | 340R* | DP980 | TWIP940* |
|---|---|---|---|
| Fracture | Fracture with strain localization | Fracture with strain localization | Fracture without strain localization |
| Top view |  |  |  |
| Side view |  |  |  |

### Magnified fracture surface

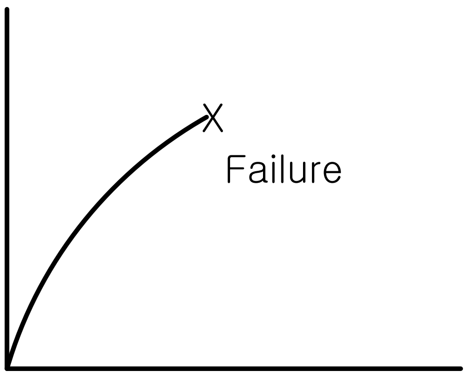
| Material | 340R* | TWIP940* |
|---|---|---|
| Fractured surfaces |  |  |

# Necking & failure

n- value and failure




Failure after necking

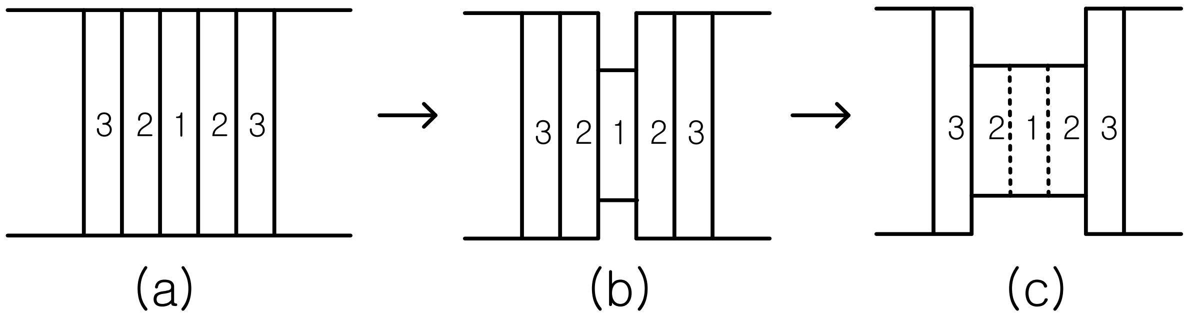


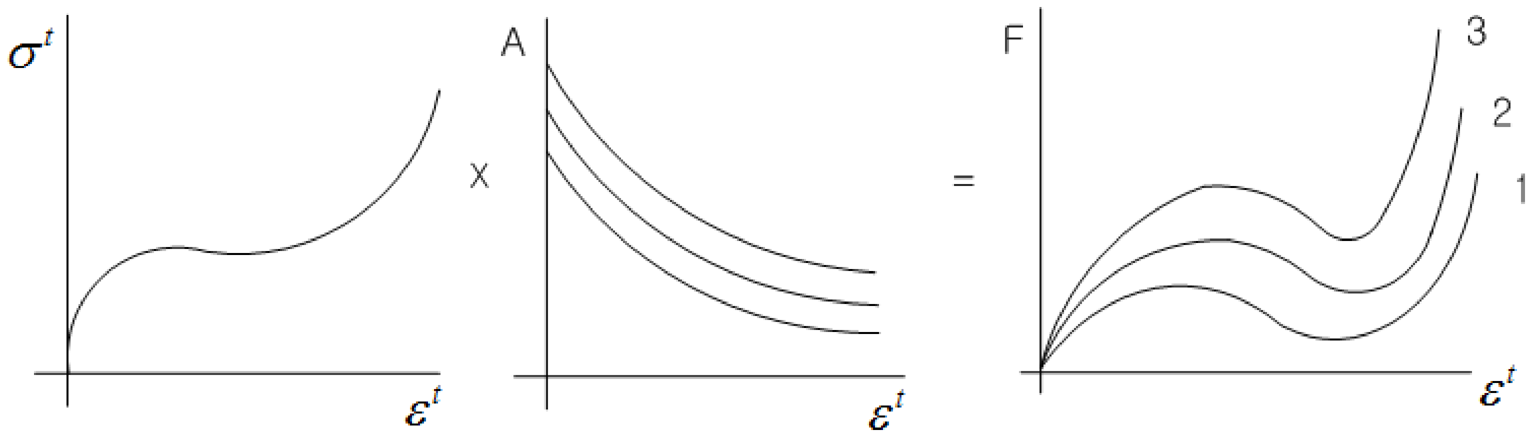

Failure before necking

- Failure after necking: the UTS point is the uniform deformation limit, after which all deformations are frozen (elastic unloading) except at the middle, leading to strain localization, diffused necking and micro-void growth till failure.

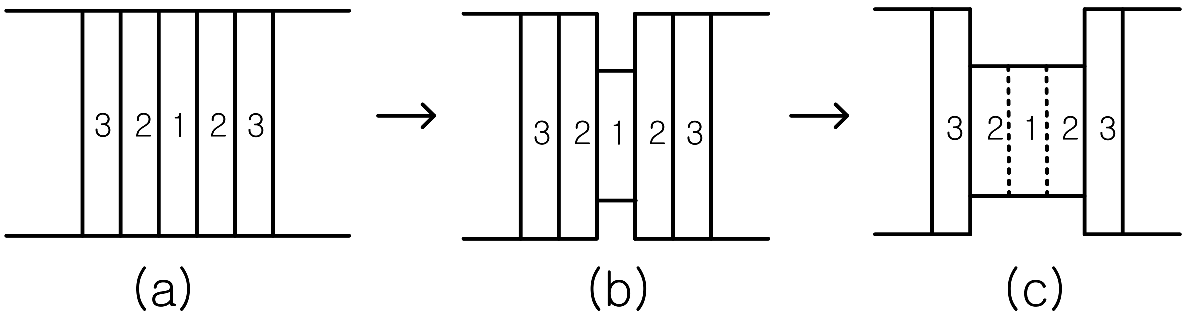
- Failure before necking: Failure does not accompany strain localization.

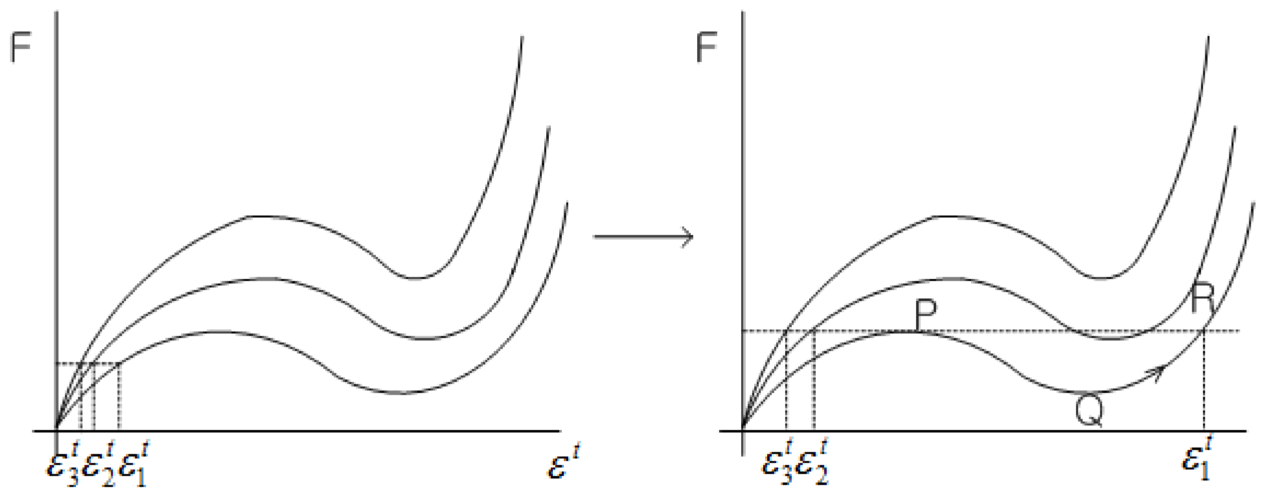
# Necking of semi-crystalline polymers



# Necking of semi-crystalline polymers

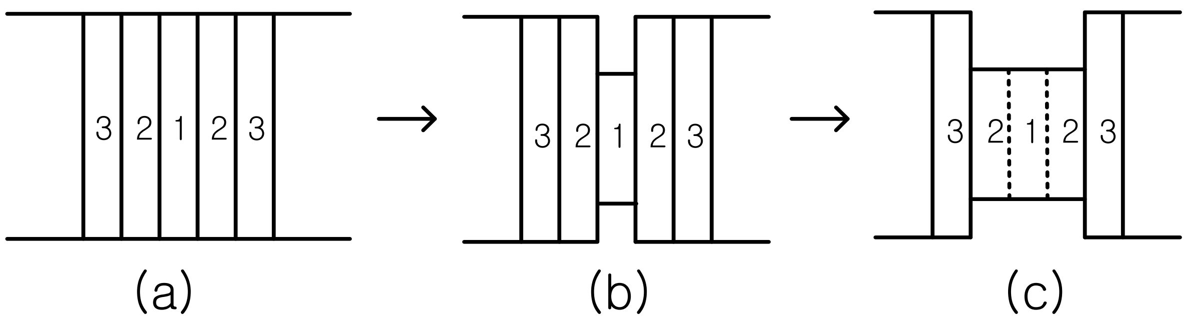


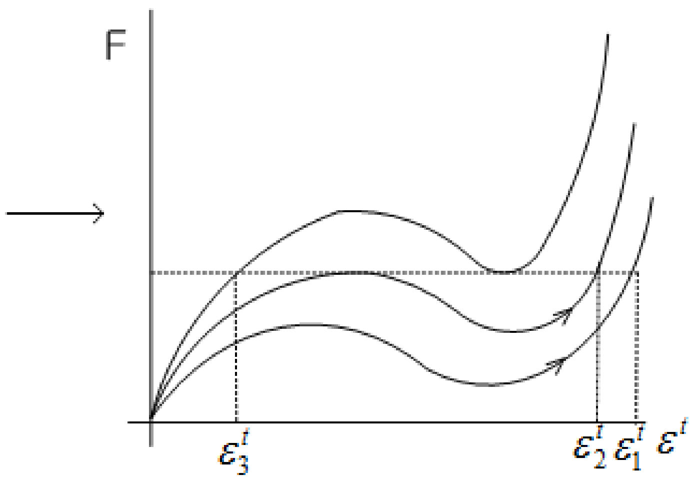




Uniform deformation range

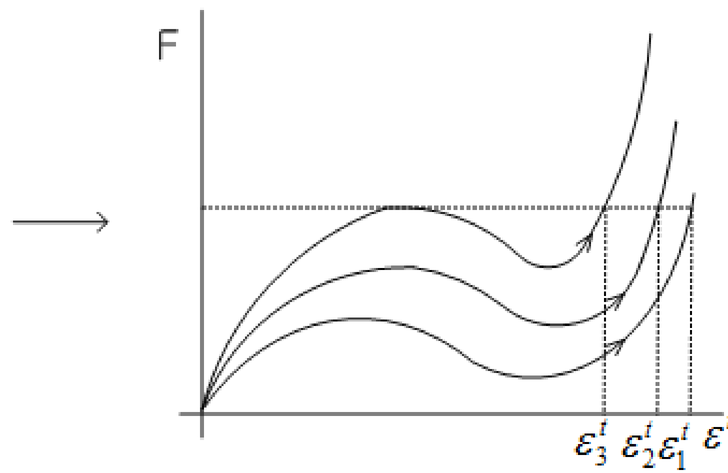
Localization at the center(b)

# Necking of semi-crystalline polymers




Further localization
from element 1 to 2(c)




Extension of localization
From element 2 to 3 (after c)