NUCLEAR SYSTEMS ENGINEERING

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NuTHEL
Nuclear Thermal Hydraulic Engineering Lab.

Course description

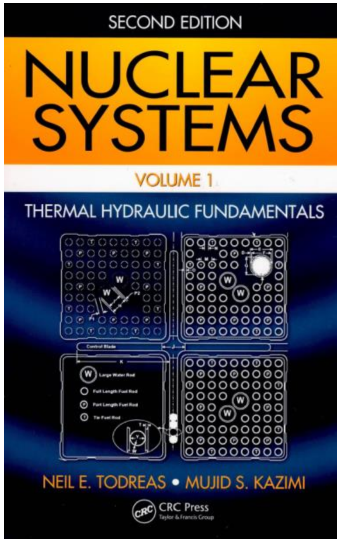
- Contents
  - Chapter 1 Principal Characteristics of Power Reactors
    - Will be replaced by the lecture note
    - Introduction to Nuclear Systems

Nuclear system

  - Chapter 4 Transport Equations for Single-Phase Flow (up to energy equation)
  - Chapter 6 Thermodynamics of Nuclear Energy Conversion Systems: Nonflow and Steady Flow : First- and Second-Law Applications
  - Chapter 7 Thermodynamics of Nuclear Energy Conversion Systems : Nonsteady Flow First Law Analysis

Thermodynamics

  - Chapter 3 Reactor Energy Distribution
  - Chapter 8 Thermal Analysis of Fuel Elements



Heat transport
Conduction
heat transfer

6. Thermodynamics of Nuclear Energy Conversion Systems

Nonflow and steady flow: First- and second-law applications

Contents

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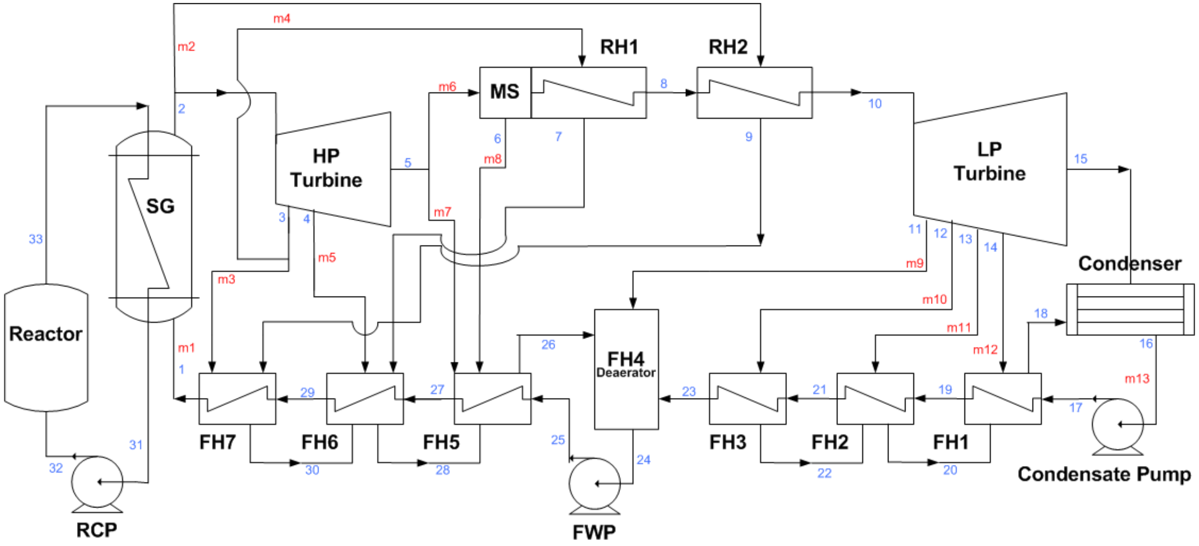
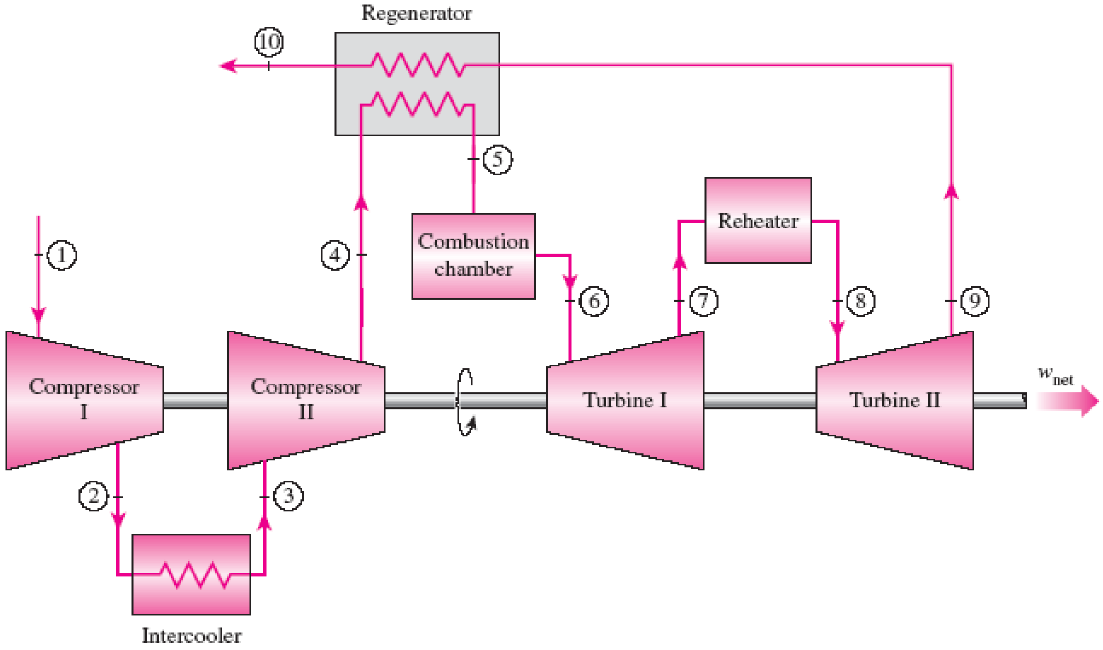
Complex Rankine cycle analysis



Complex Brayton cycle analysis



Energy Equation

- Entropy equation in Chapter 4: control mass approach
  - Definition of entropy

$$ds = \boxed{\left(\frac{\delta Q}{T}\right)_{\text{int},rev}}$$

Entropy is an extensive property of a system.

a measure of a system's thermal energy
per unit temperature
that is unavailable for doing useful work.

  - The second law of thermodynamics
    - Entropy exchanges with the surroundings are associated with heat interaction with the surroundings in a control mass system.

$$\frac{\mathrm{D}S}{\mathrm{D}t} = \left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\text{c.m.}} \geq \frac{(\mathrm{d}Q/\mathrm{d}t)_{\text{c.m.}}}{T_s}$$

system

where T_s = temperature at the ~~location where the energy (Q) is supplied as heat~~

$$\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\text{c.m.}} = \boxed{\dot{S}_{\text{gen}} + \frac{(\mathrm{d}Q/\mathrm{d}t)_{\text{c.m.}}}{T_s}} \qquad (4.25\text{b})$$

$\dot{S}_{\text{gen}}$ = rate of entropy generation due to irreversibilities

Energy Equation

- Entropy equation in Chapter 4: control mass approach
  - The second law of thermodynamics

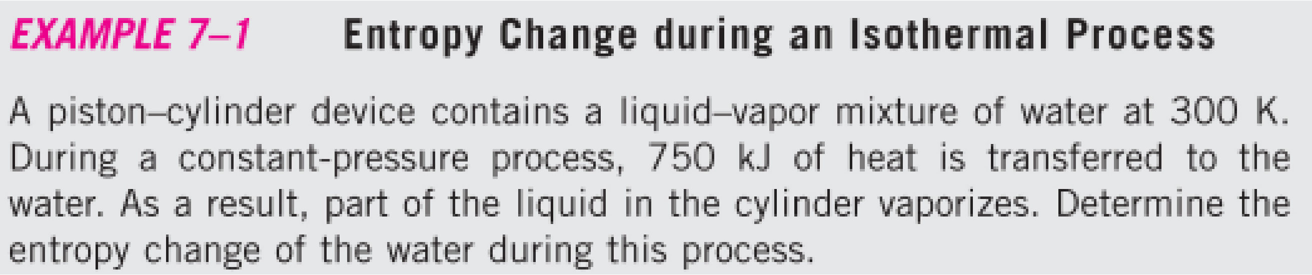
$$\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\text{c.m.}} = \dot{S}_{\text{gen}} + \frac{(\mathrm{d}Q/\mathrm{d}t)_{\text{c.m.}}}{T_{\text{s}}} \tag{4.25b}$$

    - Reversible adiabatic process

$$\boxed{\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\text{c.m.}} = 0} \quad \text{reversible adiabatic process}$$

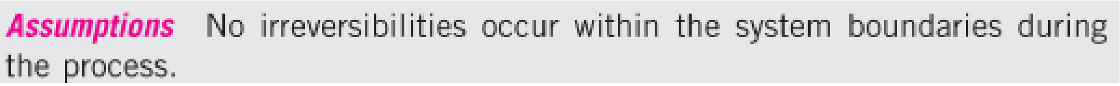
Energy Equation

- Entropy equation in Chapter 4: control mass approach
  - The second law of thermodynamics

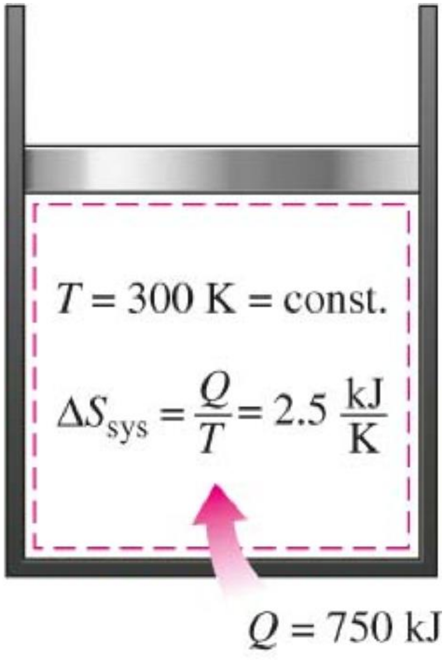


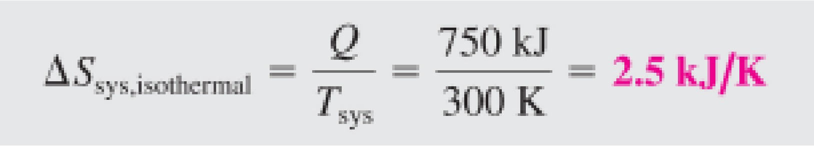
EXAMPLE 7–1 **Entropy Change during an Isothermal Process**

A piston–cylinder device contains a liquid–vapor mixture of water at 300 K. During a constant-pressure process, 750 kJ of heat is transferred to the water. As a result, part of the liquid in the cylinder vaporizes. Determine the entropy change of the water during this process.

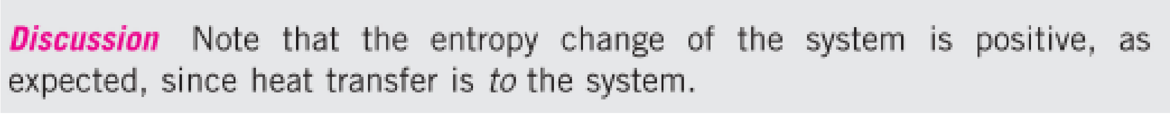


Assumptions No irreversibilities occur within the system boundaries during the process.





$$\Delta S_{\text{sys,isothermal}} = \frac{Q}{T_{\text{sys}}} = \frac{750\text{ kJ}}{300\text{ K}} = \mathbf{2.5\ kJ/K}$$



Discussion Note that the entropy change of the system is positive, as expected, since heat transfer is *to* the system.

- Entropy equation in Chapter 4: control mass approach
  - **EXAMPLE 7–2** **Entropy Generation during Heat Transfer Processes**

A heat source at 800 K loses 2000 kJ of heat to a sink at (*a*) 500 K and (*b*) 750 K. Determine which heat transfer process is more irreversible.

(*a*) For the heat transfer process to a sink at 500 K:

$$\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = \frac{-2000\ \text{kJ}}{800\ \text{K}} = -2.5\ \text{kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{Q_{\text{sink}}}{T_{\text{sink}}} = \frac{2000\ \text{kJ}}{500\ \text{K}} = +4.0\ \text{kJ/K}$$

and

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{source}} + \Delta S_{\text{sink}} = (-2.5 + 4.0)\ \text{kJ/K} = \mathbf{1.5\ kJ/K}$$

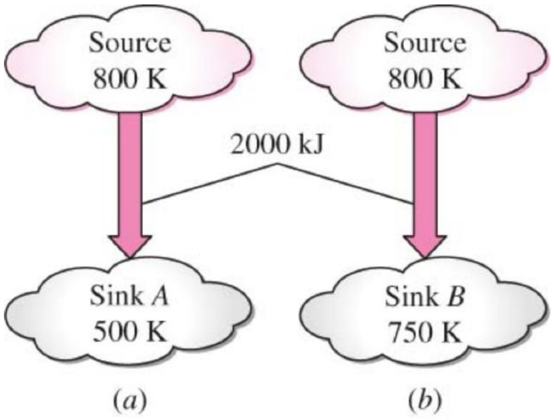
(*b*) Repeating the calculations in part (*a*) for a sink temperature of 750 K, we obtain

$$\Delta S_{\text{source}} = -2.5\ \text{kJ/k}$$

$$\Delta S_{\text{sink}} = +2.7\ \text{kJ/K}$$

and

$$S_{\text{gen}} = \Delta S_{\text{total}} = (-2.5 + 2.7)\ \text{kJ/K} = \mathbf{0.2\ kJ/K}$$



Introduction

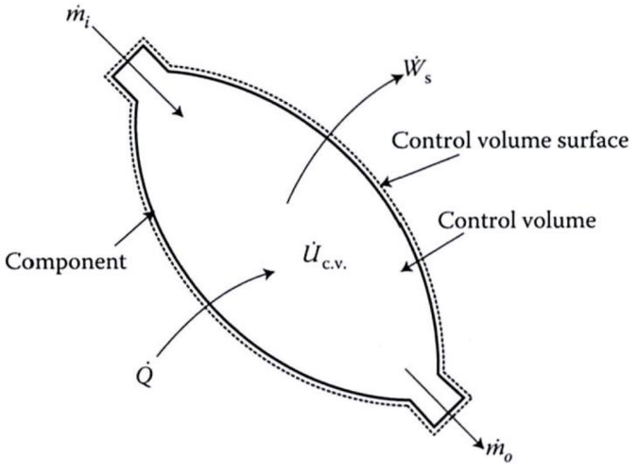
- Summary of the working forms of the first and second laws
  - First law for control volume

$$\dot{E}_{\text{c.v.}} = \sum_{i=1}^{I} \dot{m}_i (h_i^{\text{o}} + gz_i) + \dot{Q} + \dot{Q}_{\text{gen}} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{normal}} - \dot{W}_{\text{shear}}$$

$$\dot{U}_{\text{c.v.}} = \sum_{i=1}^{I} \dot{m}_i h_i + \dot{Q} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{normal}}$$

  - Second law

$$\dot{S}_{\text{c.v.}} = \sum_{i=1}^{I} \dot{m}_i s_i + \dot{s}_{\text{gen}} + \frac{\dot{Q}}{T_{\text{s}}}$$



Thermodynamic Analysis of Nuclear Power Plants

Analysis of NPPs

- Provides the relation between the mixed mean outlet coolant temperatures (or enthalpy for BWR) of the core through the primary and secondary systems to the generation of electricity at the turbine.
- Cycles used for the various reactor types and the methods of thermodynamic analysis of these cycles are described.
  - Rankine cycle for steam-driven electric turbines(PWR, BWR)
  - Brayton cycle can be considered for gas cooled reactor systems
  - Both cycles are constant-pressure heat addition and rejection cycles for steady-flow operation

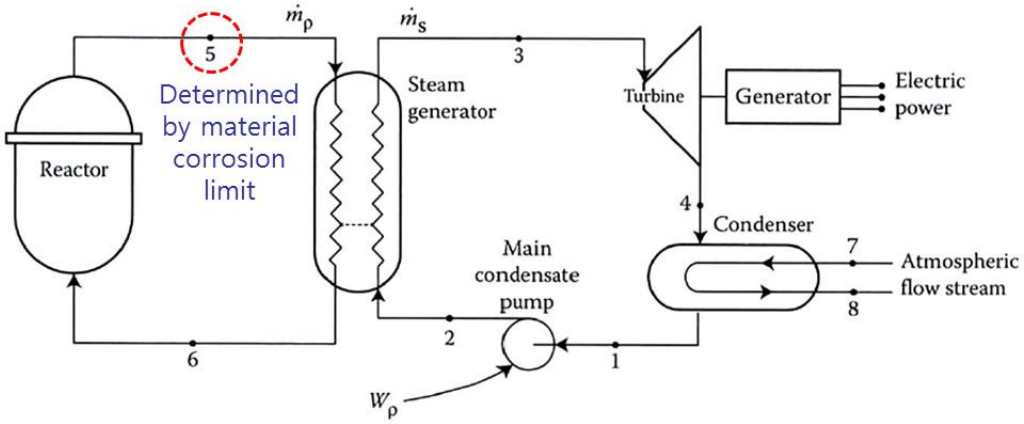
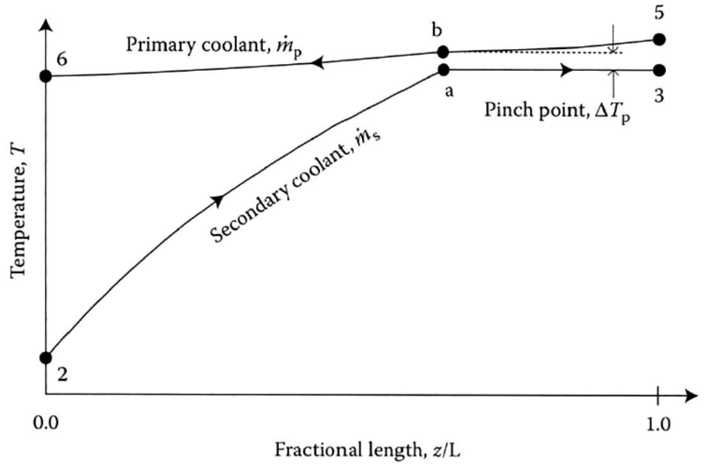


FIGURE 6.6 Simplified PWR plant.



Thermodynamic Analysis of Nuclear Power Plants

- Analysis of NPPs
  - Simple Brayton cycle

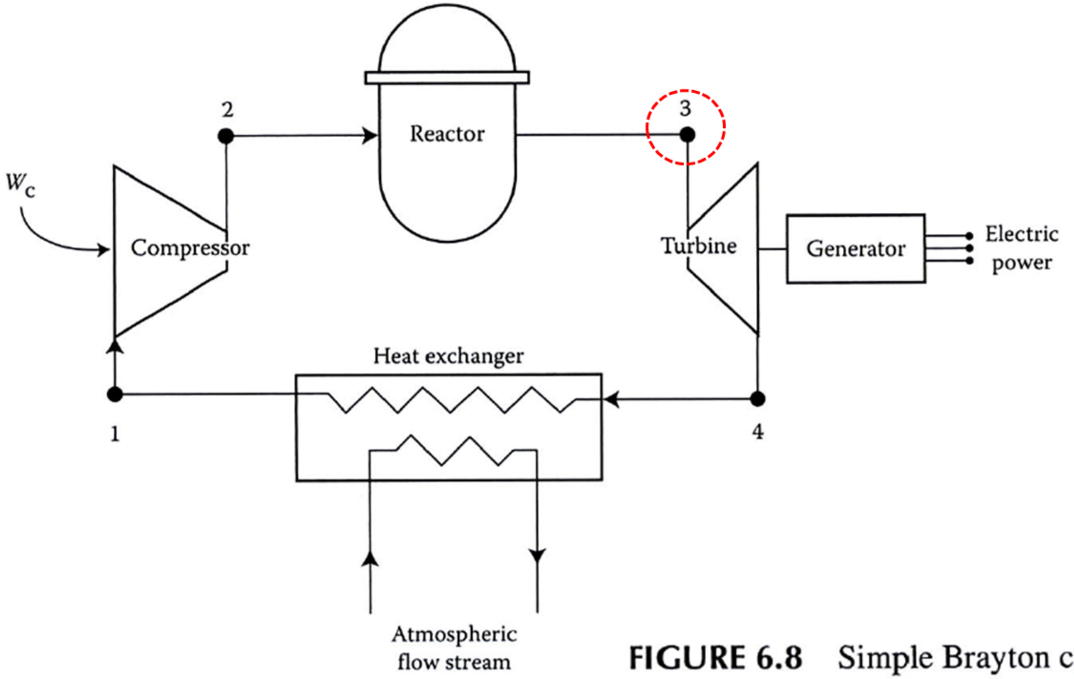


FIGURE 6.8 Simple Brayton cycle.

Maximum temperature of the Brayton cycle (T_3) is set by turbine blade and gas cooled reactor core material limits far higher than those for the Rankine cycle, which is set by liquid-cooled reactor core materials limits .

Thermodynamic Analysis of Nuclear Power Plants

- Efficiency to assess the thermodynamic performance of components and cycles
  - Thermodynamic efficiency (or effectiveness): ζ
    - The ratio between the actual useful work and the maximum useful work
  - Isentropic efficiency: η_s
    - Special case of thermodynamics efficiency
    - Adiabatic case
    - Important for devices such as pump, turbine, compressor
  - Thermal efficiency: η_{th}
    - The fraction of the heat input that is converted to net work output
    - The cycle efficiency that we are interested in.

Thermodynamic Analysis of Nuclear Power Plants

- Thermodynamic efficiency (or effectiveness): ζ

$$\zeta \equiv \frac{\dot{W}_{\mathrm{u,actual}}}{\dot{W}_{\mathrm{u,max}}}$$

  - Maximum useful work
    - No entropy generation

$$\left(\frac{\mathrm{d}W}{\mathrm{d}t}\right)_{\mathrm{u,max}} = -\left[\frac{\partial(E+p_{\mathrm{o}}V-T_{\mathrm{o}}S)}{\partial t}\right]_{\mathrm{c.v.}} + \sum_{i=1}^{I}\dot{m}_i(h^{\mathrm{o}}-T_{\mathrm{o}}s+gz)_i + \left(1-\frac{T_{\mathrm{o}}}{T_{\mathrm{s}}}\right)\frac{\mathrm{d}Q}{\mathrm{d}t} + \left(\frac{\mathrm{d}Q}{\mathrm{d}t}\right)_{\mathrm{gen}} - \left(\frac{\mathrm{d}W}{\mathrm{d}t}\right)_{\mathrm{shear}} \qquad (4.46)$$

    - Fixed, non-deformable control volume with zero shear work
    - Negligible kinetic and potential energy differences

$$\dot{W}_{\mathrm{u,max}} = -\left[\frac{\partial(U-T_{\mathrm{o}}S)}{\partial t}\right] + \sum_{i=1}^{I}\dot{m}_i(h-T_{\mathrm{o}}s)_i + \left(1-\frac{T_{\mathrm{o}}}{T_{\mathrm{s}}}\right)\dot{Q} \qquad (6.32)$$

where T_{s} = temperature at which heat is supplied and $(\mathrm{d}Q/\mathrm{d}t)_{\mathrm{gen}}$ is treated as part of $\mathrm{d}U/\mathrm{d}t$.

Thermodynamic Analysis of Nuclear Power Plants

- Isentropic efficiency: η_s

$$\eta_s = \left(\frac{\dot{W}_{\text{u,actual}}}{\dot{W}_{\text{u,max}}} \right)_{\dot{Q}=0} \qquad \zeta = \eta_s \quad \text{for an adiabatic control volume}$$

- Useful actual work for an adiabatic control volume with zero shear and negligible kinetic and potential energy differences

$$\left(\frac{\mathrm{d}W}{\mathrm{d}t} \right)_{\text{u,actual}} = -\left[\frac{\partial (E + p_o V - T_o S)}{\partial t} \right]_{\text{c.v.}} + \sum_{i=1}^{I} \dot{m}_i (h^{\circ} - T_o s + gz)_i$$

$$+ \left(1 - \frac{T_o}{T_s} \right) \frac{\mathrm{d}Q}{\mathrm{d}t} + \left(\frac{\mathrm{d}Q}{\mathrm{d}t} \right)_{\text{gen}} - \left(\frac{\mathrm{d}W}{\mathrm{d}t} \right)_{\text{shear}} - T_o \dot{S}_{\text{gen}}$$

$$\left(\frac{\partial S}{\partial t} \right)_{\text{c.v.}} = \sum_{i=1}^{I} \dot{m}_i s_i + \dot{S}_{\text{gen}} + \frac{(\mathrm{d}Q/\mathrm{d}t)}{T_s}$$

  - For steady state

$$\dot{W}_{\text{u,actual}} = \sum_{i=1}^{I} \dot{m}_i h_i$$

$$\boxed{\eta_s = \frac{\sum_{i=1}^{I} \dot{m}_i h_i}{\sum_{i=1}^{I} \dot{m}_i h_{is}}}$$

Thermodynamic Analysis of Nuclear Power Plants

- Thermal efficiency: η_{th}
  - Cycle efficiency

$$\eta_{\mathrm{th}} = \frac{\dot{W}_{\mathrm{u,actual}}}{\dot{Q}_{\mathrm{in}}}$$

    - For adiabatic systems, it is not useful.

Thermodynamic Analysis of Nuclear Power Plants

Example problem (a)

- Thermal efficiency: η_{th}

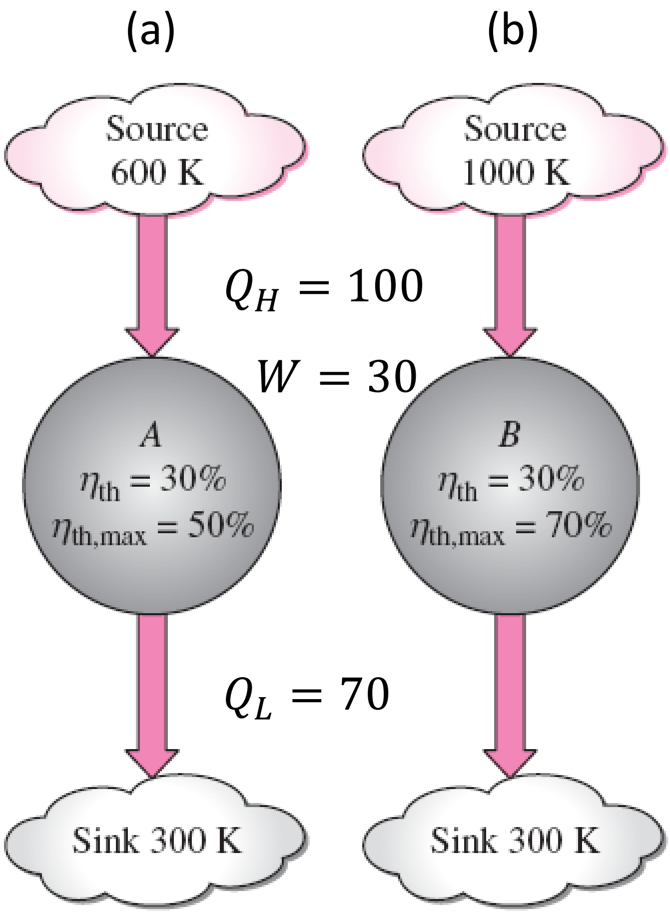
$$W = 30$$

- Thermodynamic efficiency (or effectiveness): ζ

$$\zeta \equiv \frac{\dot{W}_{\text{u,actual}}}{\dot{W}_{\text{u,max}}} \quad = \frac{30}{50} = 60\% \qquad = \frac{30}{70} = 42.9\%$$

$$\dot{W}_{u,max} = \left(1 - \frac{T_L}{T_H}\right) Q_H = \left(1 - \frac{300}{600}\right) \cdot 100 = 50$$

$$\dot{W}_{u,max} = \left(1 - \frac{T_L}{T_H}\right) Q_H = \left(1 - \frac{300}{1000}\right) \cdot 100 = 70$$



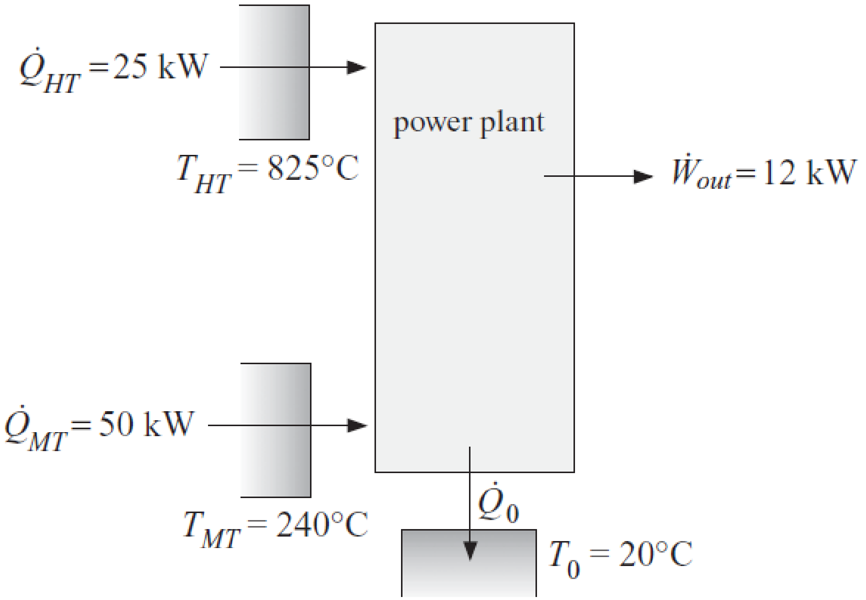
Thermodynamic Analysis of Nuclear Power Plants

- Example problem (b)

The plant receives $\dot{Q}_{HT} = 25$ kW at temperature $T_{HT} = 825^{\circ}$C. The plant also receives $\dot{Q}_{MT} = 50$ kW at temperature $T_{MT} = 240^{\circ}$C. The plant rejects heat to the environment at $T_0 = 20^{\circ}$C and produces power $\dot{W}_{out} = 12$ kW.

a) Determine the Second Law efficiency of the power plant.

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{u,actual}}}{\dot{Q}_{\text{in}}} \qquad \zeta \equiv \frac{\dot{W}_{\text{u,actual}}}{\dot{W}_{\text{u,max}}}$$



$$\dot{Q}_{HT} + \dot{Q}_{MT} = \dot{Q}_{0,rev} + \dot{W}_{out,rev}$$

Maximum useful work ⇒ no entropy generation
Entropy in = entropy out

$$\frac{\dot{Q}_{HT}}{T_{HT}} + \frac{\dot{Q}_{MT}}{T_{MT}} = \frac{\dot{Q}_{0,rev}}{T_0}$$

Entropy decrease in hot reservoirs
= entropy increase in cold reservoir

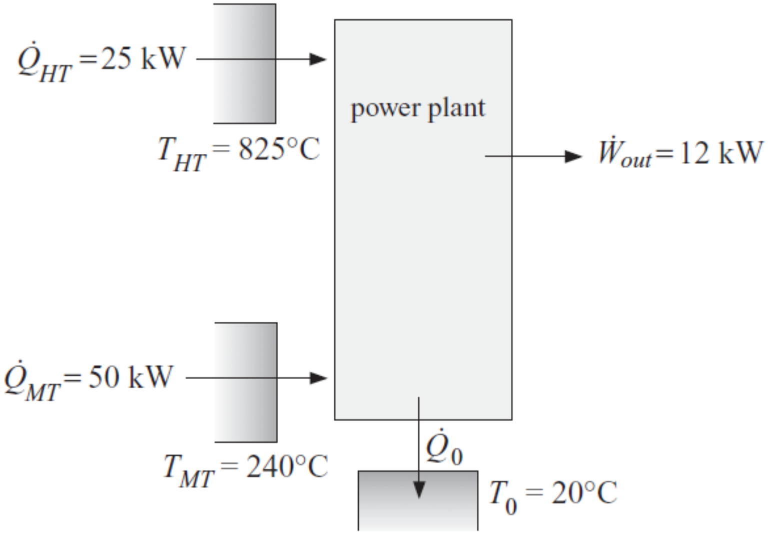
Thermodynamic Analysis of Nuclear Power Plants

- Example problem (b)

The plant receives $\dot{Q}_{HT} = 25$ kW at temperature $T_{HT} = 825^{\circ}$C. The plant also receives $\dot{Q}_{MT} = 50$ kW at temperature $T_{MT} = 240^{\circ}$C. The plant rejects heat to the environment at $T_0 = 20^{\circ}$C and produces power $\dot{W}_{out} = 12$ kW.

a) Determine the Second Law efficiency of the power plant.

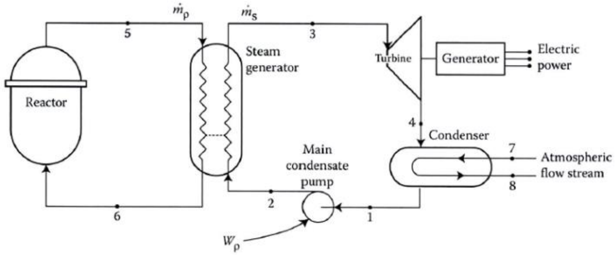
$$\eta_{th} = \frac{\dot{W}_{u,actual}}{\dot{Q}_{in}} = \frac{12}{75} = 16\%$$



$$\frac{\dot{Q}_{HT}}{T_{HT}} + \frac{\dot{Q}_{MT}}{T_{MT}} = \frac{\dot{Q}_{0,rev}}{T_0} \qquad \dot{Q}_{o,rev} = 35.2\ kW$$

$$\dot{Q}_{HT} + \dot{Q}_{MT} = \dot{Q}_{0,rev} + \dot{W}_{out,rev} \qquad \dot{W}_{out,rev} = 39.8\ kW$$

$$\zeta \equiv \frac{\dot{W}_{u,actual}}{\dot{W}_{u,max}} = \frac{12}{39.8} = 30.2\ \%$$



Contents

Thermodynamic Analysis of A Simplified PWR System

First Law Analysis of a Simplified PWR System

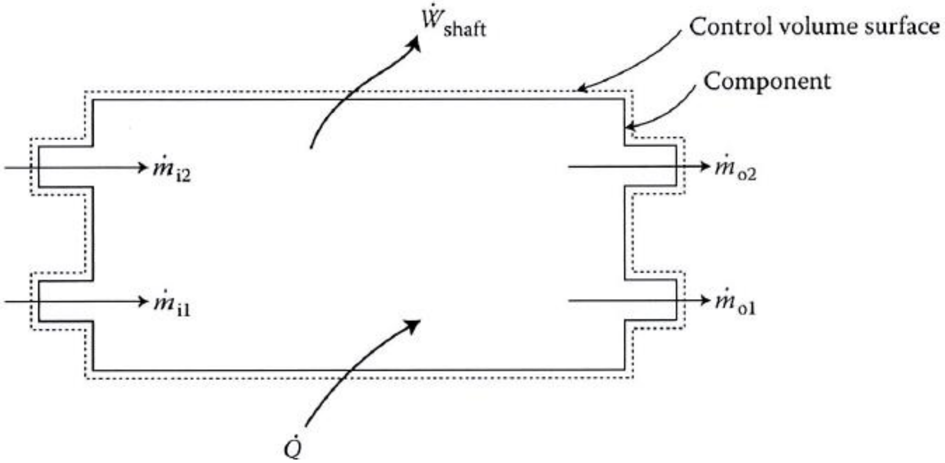
- One component with multiple inlet and outlet flow streams operating at steady state and surround it with a non-deformable, stationary control volume
- Energy equation

$$\sum_{k=1}^{I} (\dot{m}h)_{\mathrm{in,k}} - \sum_{k=1}^{I} (\dot{m}h)_{\mathrm{out,k}} = \dot{W}_{\mathrm{shaft}} - \dot{Q}$$

- Turbine
  - $\dot{Q} = 0, \dot{W}_{shaft} > 0$
  - Left side > 0
- Condenser
  - $\dot{Q} < 0, \dot{W}_{shaft} = 0$
  - Left side > 0
- Pump
  - $\dot{Q} = 0, \dot{W}_{shaft} < 0$
  - Left side < 0
- Steam generator
  - $\dot{Q} > 0, \dot{W}_{shaft} = 0$
  - Left side < 0

- Mass conservation equation

$$\dot{m} = \sum_{k=1}^{I} \dot{m}_{\mathrm{in},k} - \sum_{k=1}^{I} \dot{m}_{\mathrm{out},k}$$

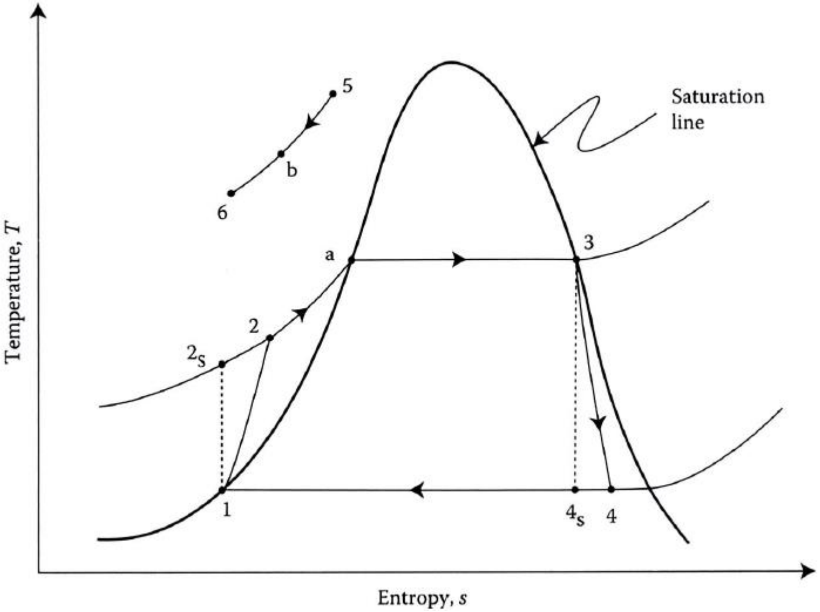


Thermodynamic Analysis of A Simplified PWR System

First Law Analysis of a Simplified PWR System

- Turbine
  - Shaft work, no heat addition ⇒ adiabatic turbine
  - Actual turbine work is related to the ideal work by the component isentropic efficiency.
  - Isentropic efficiency

$$\eta_T = \frac{h_{in} - h_{out}}{h_{in} - h_{out,s}} \qquad h_{out,s} = f[p(h_{out}), s_{in}]$$



- Pump
  - Shaft work, no heat addition ⇒ adiabatic pump

$$\eta_P = \frac{\text{Ideal work required}}{\text{Actual work required}} = \frac{h_{in} - h_{out,s}}{h_{in} - h_{out}} = \frac{h_{out,s} - h_{in}}{h_{out} - h_{in}}$$

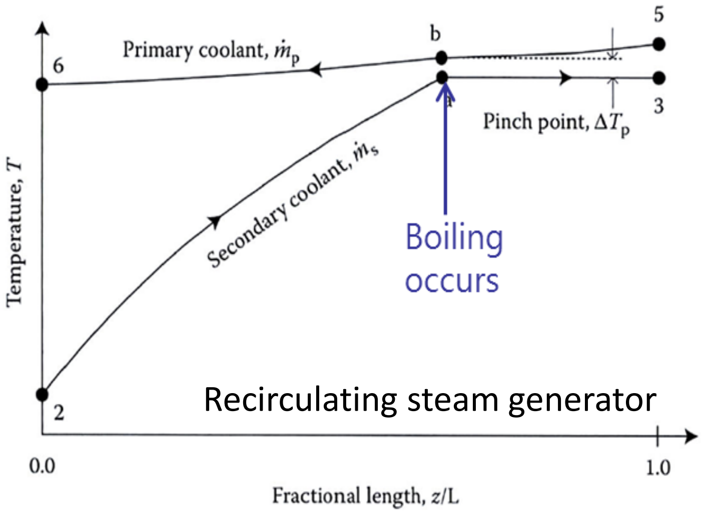
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Steam generator
    - Heat transfer between the primary and the secondary

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{(h_{out} - h_{in})_s}{(h_{in} - h_{out})_p}$$

$$\sum_{k=1}^{I} (\dot{m}h)_{in,k} - \sum_{k=1}^{I} (\dot{m}h)_{out,k} = \dot{W}_{shaft} - \dot{Q}$$

    - Pinch point temperature
      - Minimum temperature difference between the primary and the secondary
      - Inversely related to the heat transfer area
        - Small temperature difference: large heat transfer area
        - Large temperature difference: small heat transfer area
      - Directly related to the irreversibility of the SG
        - Large temperature difference: large irreversibility
      - Important design choice based on tradeoff between cost and irreversibility



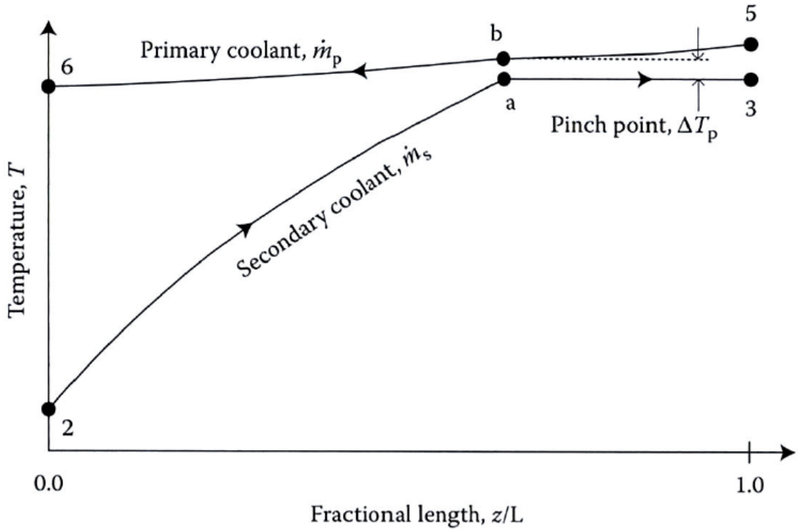
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Steam generator

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{(h_{out} - h_{in})_s}{(h_{in} - h_{out})_p}$$

$$dh = c_p dT$$

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{h_3 - h_a}{\bar{c}_p[T_5 - (T_a + \Delta T_p)]} = \frac{h_a - h_2}{\bar{c}_p[(T_a + \Delta T_p) - T_6]}$$



- Reactor plant
  - Actual work

$$\dot{W}_{u,actual} = \dot{W}_T + \dot{W}_P = [\dot{m}_s(h_{in} - h_{out})]_T + [\dot{m}_s(h_{in} - h_{out})]_P$$

$$= \underline{[\eta_T \dot{m}_s(h_{in} - h_{out,s})]_T} + \underline{\left[\frac{\dot{m}_s}{\eta_P}(h_{in} - h_{out,s})\right]_P}$$

$$\eta_T = \frac{h_{in} - h_{out}}{h_{in} - h_{out,s}}$$

$$\eta_P = \frac{h_{in} - h_{out,s}}{h_{in} - h_{out}}$$

Positive or negative?

Thermodynamic Analysis of A Simplified PWR System

First Law Analysis of a Simplified PWR System

- Reactor plant
  - Maximum useful work

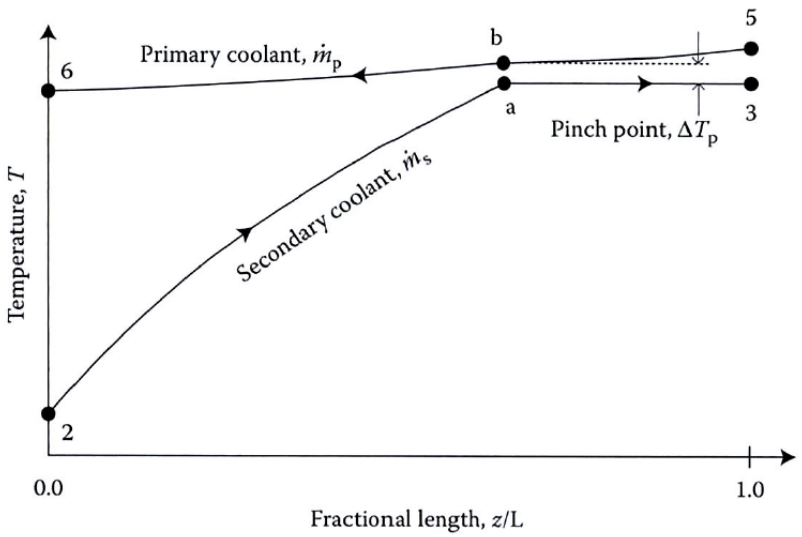
$$\dot{W}_{\mathrm{u,max}} = \dot{m}_{\mathrm{p}}(h_{\mathrm{out}} - h_{\mathrm{in}})_{\mathrm{R}} = \dot{m}_{\mathrm{p}}(h_5 - h_6)$$

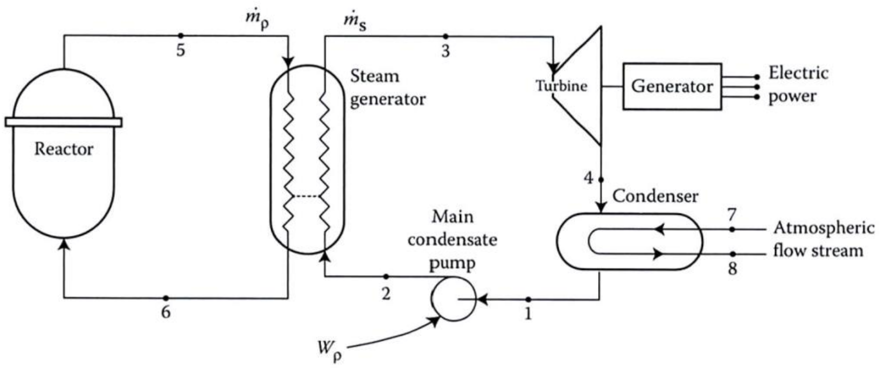
    - In section 6.4.2.3
  - Maximum useful work expressed with the secondary side properties

$$\dot{W}_{\mathrm{u,max}} = \dot{m}_{\mathrm{p}}(h_{\mathrm{out}} - h_{\mathrm{in}})_{\mathrm{R}} = [\dot{m}_{\mathrm{s}}(h_{\mathrm{out}} - h_{\mathrm{in}})]_{\mathrm{SGs}}$$

$$\dot{W}_{\mathrm{u,max}} = \dot{m}_{\mathrm{p}}(h_5 - h_6) = \dot{m}_{\mathrm{s}}(h_3 - h_2)$$

  - Thermodynamic efficiency or effectiveness: ζ





$$\zeta = \frac{[\dot{m}_{\mathrm{s}}(h_{\mathrm{in}} - h_{\mathrm{out}})]_{\mathrm{T}} + [\dot{m}_{\mathrm{s}}(h_{\mathrm{in}} - h_{\mathrm{out}})]_{\mathrm{P}}}{[\dot{m}_{\mathrm{s}}(h_{\mathrm{out}} - h_{\mathrm{in}})]_{\mathrm{SGs}}} \xrightarrow{\text{Single turbine case}} \zeta = \frac{h_3 - h_4 + h_1 - h_2}{h_3 - h_2}$$

$$\eta_{\mathrm{th}} = \frac{\dot{W}_{\mathrm{u,actual}}}{\dot{Q}_{\mathrm{in}}}$$

Thermodynamic Analysis of A Simplified PWR System

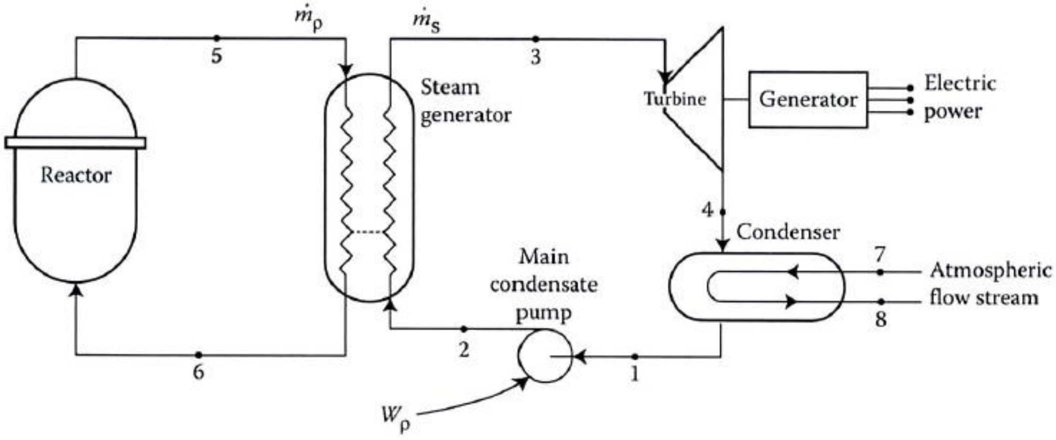
- First Law Analysis of a Simplified PWR System

Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant

PROBLEM

1. Draw the temperature–entropy (*T*–*s*) diagram for this cycle.
2. Compute the ratio of the primary to secondary flow rates.
3. Compute the nuclear plant thermodynamic efficiency.
4. Compute the cycle thermal efficiency.

- Assume that the turbine and pump have isentropic efficiencies of 85%.



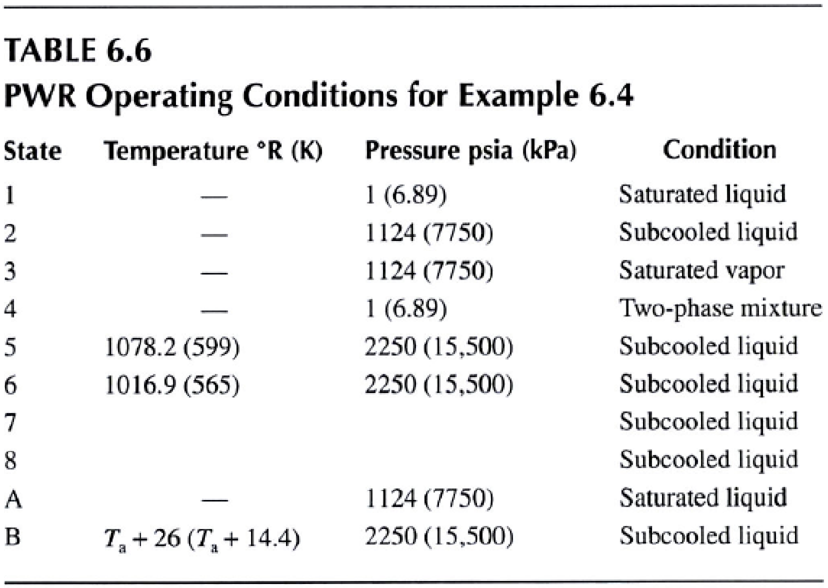
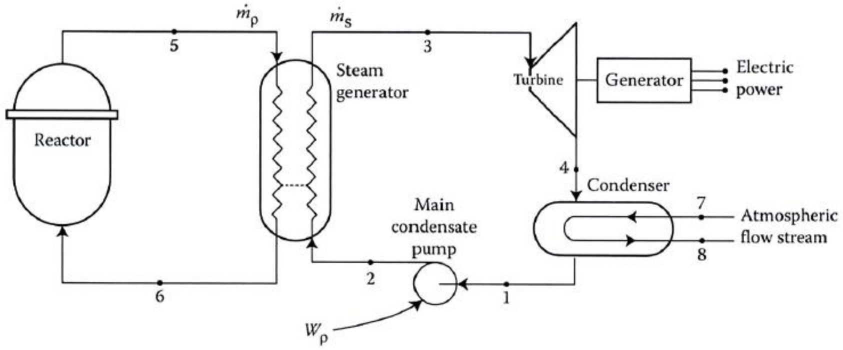


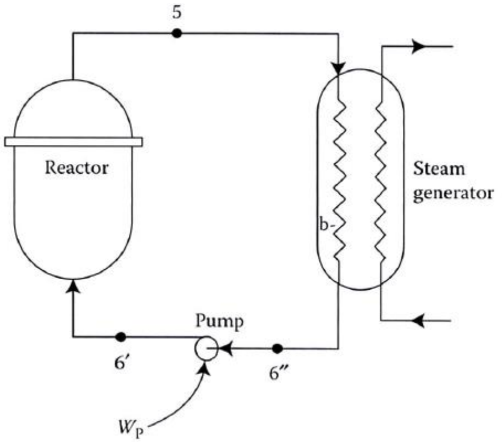
TABLE 6.6
PWR Operating Conditions for Example 6.4

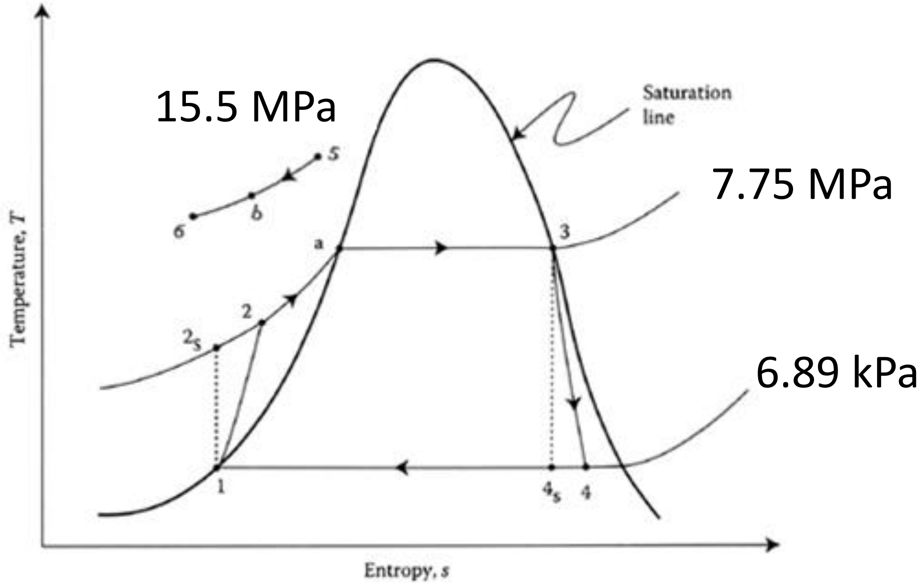
State	Temperature °R (K)	Pressure psia (kPa)	Condition
1	—	1 (6.89)	Saturated liquid
2	—	1124 (7750)	Subcooled liquid
3	—	1124 (7750)	Saturated vapor
4	—	1 (6.89)	Two-phase mixture
5	1078.2 (599)	2250 (15,500)	Subcooled liquid
6	1016.9 (565)	2250 (15,500)	Subcooled liquid
7			Subcooled liquid
8			Subcooled liquid
A	—	1124 (7750)	Saturated liquid
B	$T_a + 26$ ($T_a + 14.4$)	2250 (15,500)	Subcooled liquid

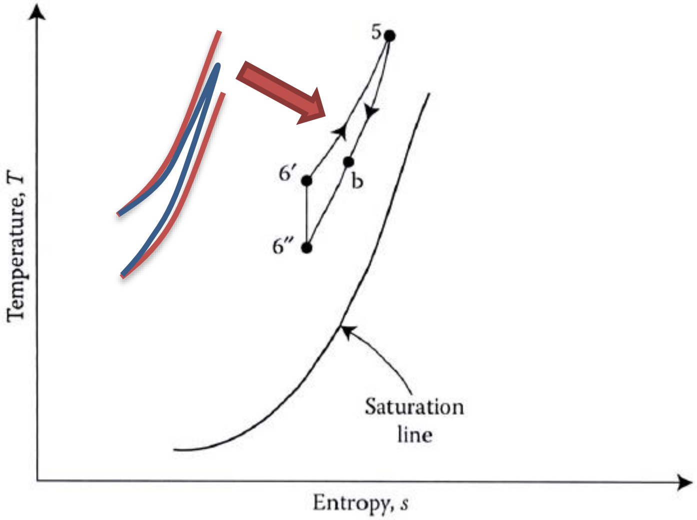
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - Draw the temperature-entropy *(T-s)* diagram for this cycle.









Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - Compute the ratio of the primary to secondary flow rates.

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{h_3 - h_a}{\bar{c}_p[T_5 - (T_a + \Delta T_p)]} = \frac{h_a - h_2}{\bar{c}_p[(T_a + \Delta T_p) - T_6]}$$

From steam tables:

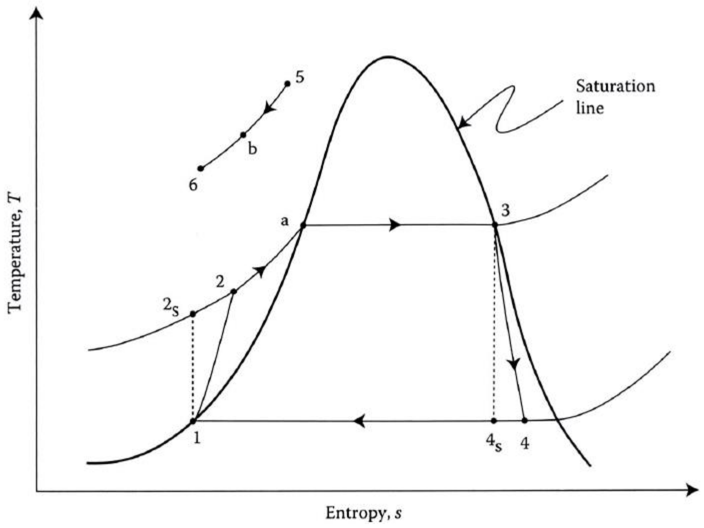
$h_3 = h_g$(sat. at 1124 psia) = 1187.29 Btu/lb (2.771 MJ/kg)

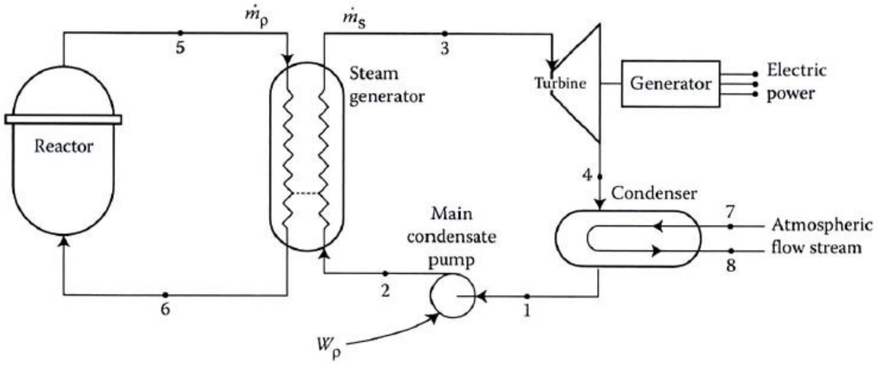
$h_a = h_f$(sat. at 1124 psia) = 560.86 Btu/lb (1.309 MJ/kg)

T_a = sat. liquid at 1124 psia = 1018.8 R° (566.0 K)

Hence

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{2.77 \times 10^6 - 1.309 \times 10^6}{5941\,[599 - (566 + 14.4)]} = 13.18$$





Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - Compute the nuclear plant thermodynamic efficiency.

$$\zeta = \frac{h_3 - h_4 + h_1 - h_2}{h_3 - h_2}$$

      - Calculate the enthalpies of each state!

    - State 1

From the steam tables:

$h_1 = h_f(\text{sat. at 1 psia}) = 69.74$ Btu/lb (0.163 MJ/kg)

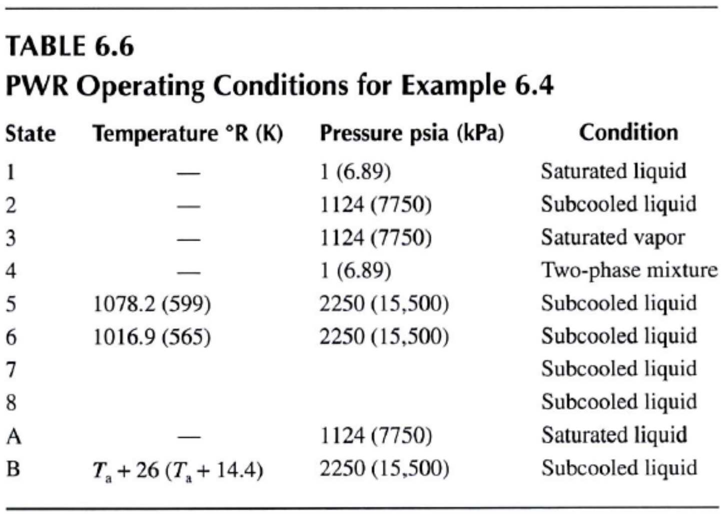


TABLE 6.6
PWR Operating Conditions for Example 6.4

State	Temperature °R (K)	Pressure psia (kPa)	Condition
1	—	1 (6.89)	Saturated liquid
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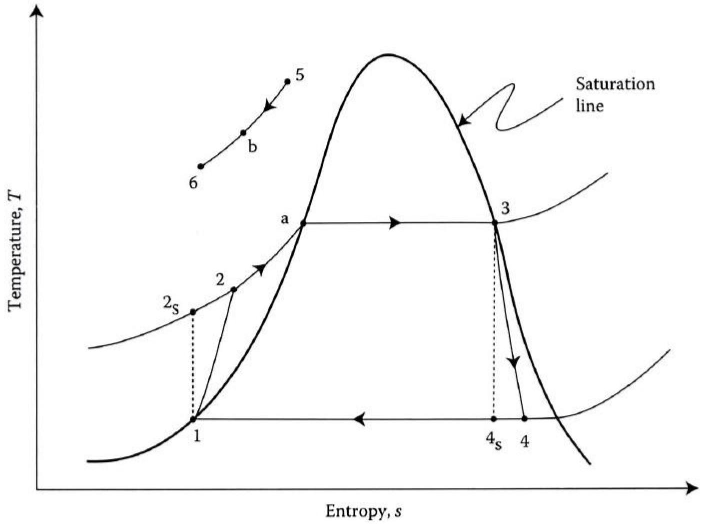


Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - State 2
      - Isentropic efficiencies of 85%.

$$\eta_P = \frac{h_{in} - h_{out,s}}{h_{in} - h_{out}}$$

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_P}$$

      - Isentropic process

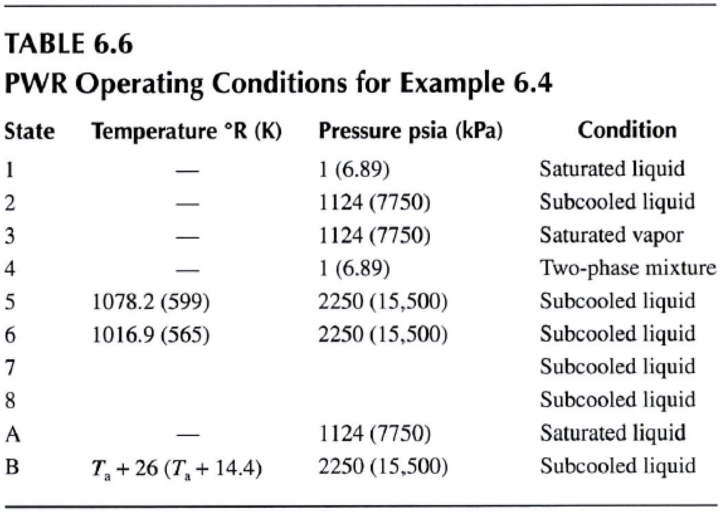
$$s_{2s} = s_1 = 0.13266 \text{ Btu/lb°R } (557 \text{ J/kg K})$$

$$p_{2s} = p_2 = 1124 \text{ psia } (7.75 \text{ MP}_a)$$

      - $h_{2s} = h(p_2, s_{2s})$

$$h_{2s} = 73.09 \text{ Btu/lb } (0.170 \text{ MJ/kg})$$

$$= 0.163 + \frac{0.170 - 0.163}{0.85} = 0.171 \text{MJ/kg}$$



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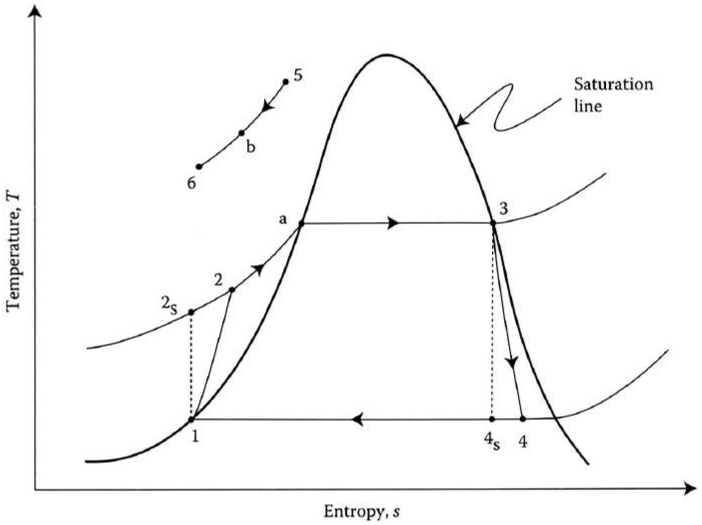


Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - State 3
      - $h_3 = h(p_3)$ From the steam tables:
    - State 4
      - Isentropic efficiency

$$\eta_T = \frac{h_{in} - h_{out}}{h_{in} - h_{out,s}} \qquad h_4 = h_3 - \eta_T (h_3 - h_{4s})$$

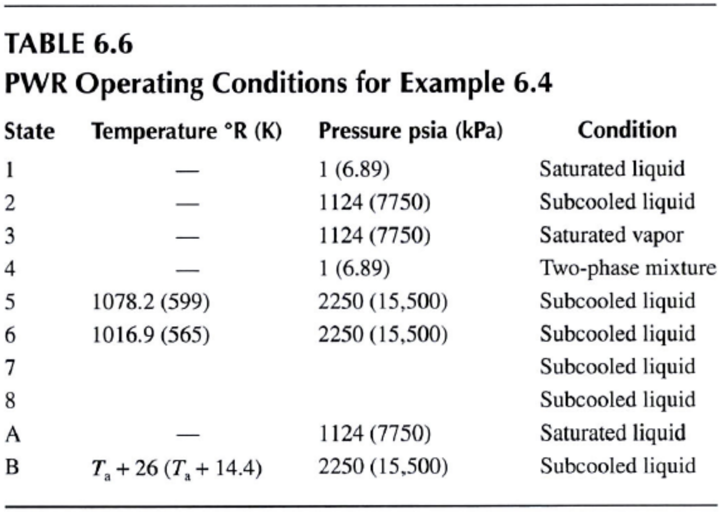
      - Isentropic process
        - $s_3 = s_{4s} = (1-x)\cdot s_f + x \cdot s_g$

$$x_{4s} = \frac{s_{4s} - s_f}{s_{fg}} = \frac{1.3759 - 0.13266}{1.8453} = 0.674$$

        - $h_{4s} = (1-x)\cdot h_f + x \cdot h_g$

$$h_{4s} = hf + x_{4s} h_{fg} = 69.74 + 0.674(1036.0)$$
$$= 768.00 \text{ Btu/lb } (1.79 \text{ MJ/kg})$$

$$h_4 = h_3 - \eta_T (h_3 - h_{4s}) = 1187.29 - 0.85(1187.29 - 768.00)$$
$$= 830.9 \text{ Btu/lb } (1.94 \text{ MJ/kg})$$



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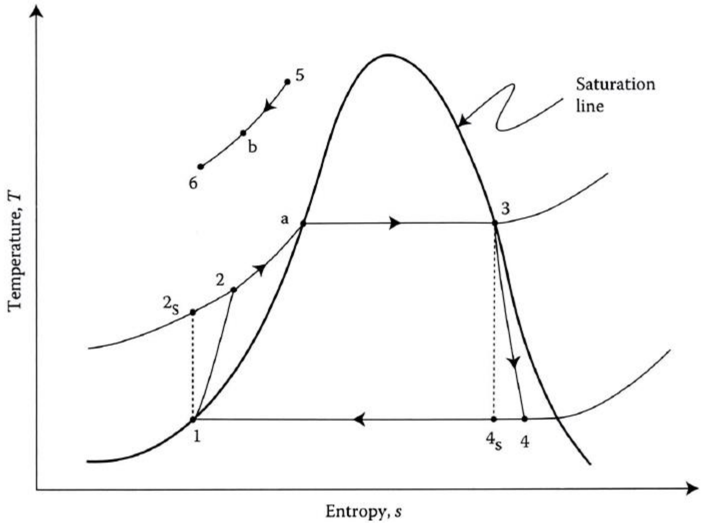


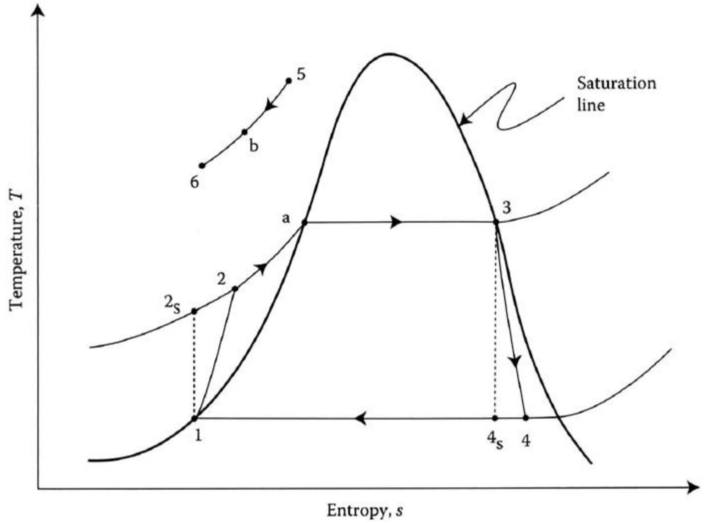
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - Compute the nuclear plant thermodynamic efficiency.

$$\zeta = \frac{h_3 - h_4 + h_1 - h_2}{h_3 - h_2}$$

$$\zeta = \frac{1187.29 - 830.9 + 69.74 - 73.69}{1187.29 - 73.69}\text{(English units)} = 0.317$$

$$= \frac{2.771 - 1.94 + 0.163 - 0.171}{2.771 - 0.171}\text{(SI units)} = 0.317$$



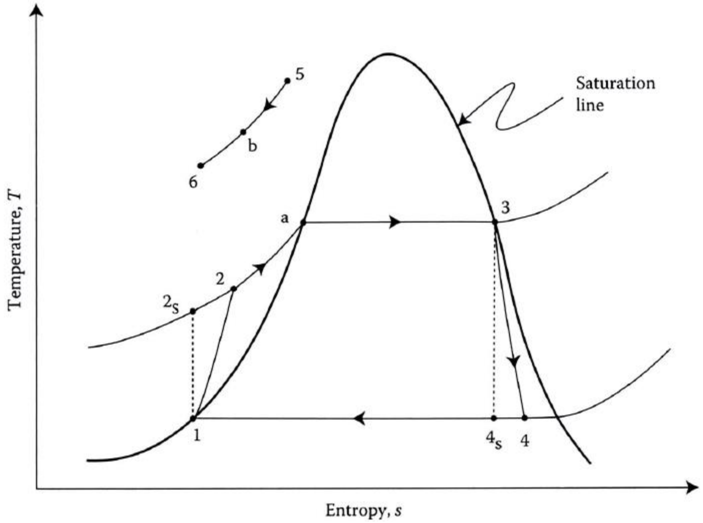
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - Compute the cycle thermal efficiency.

$$\eta_{th} = \frac{\dot{W}_{u,actual}}{\dot{Q}_{in}}$$

– $\dot{Q}_{in} = \dot{m}_s(h_3 - h_2)$

– $\dot{W}_{i,actual} = \dot{m}_s[(h_3 - h_4) - (h_2 - h_1)]$



    - Thermal efficiency (cycle thermal efficiency)

      = plant thermodynamic efficiency

$$\eta_{th} = \zeta$$

Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Improvements in thermal efficiency for this simple saturated cycle
    - Pressure
      - Increase the pump outlet pressure/decrease turbine outlet pressure

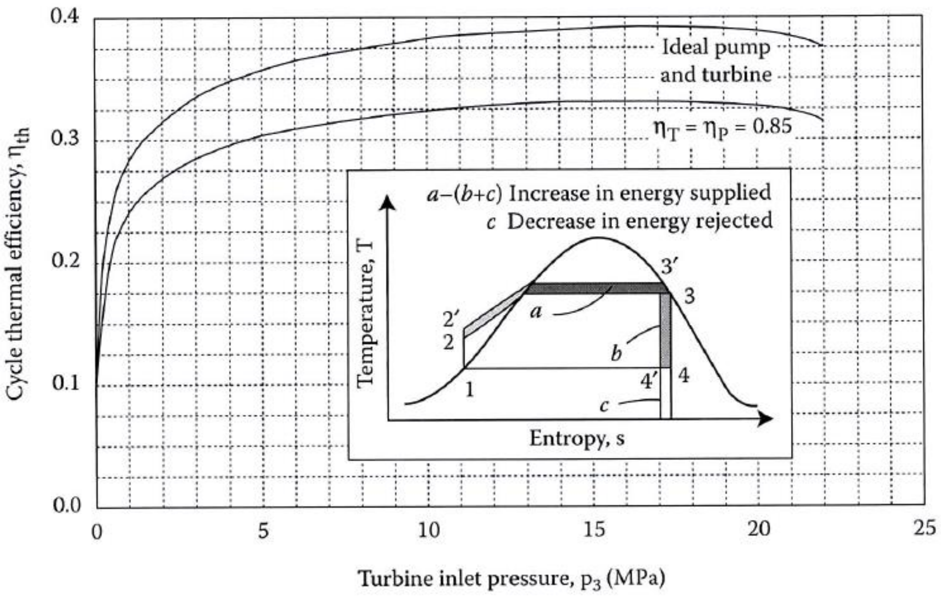


FIGURE 6.13 Thermal efficiency of Rankine cycle using saturated steam for varying turbine inlet pressure. Turbine inlet: saturated vapor. Exhaust pressure: 7 kPa.

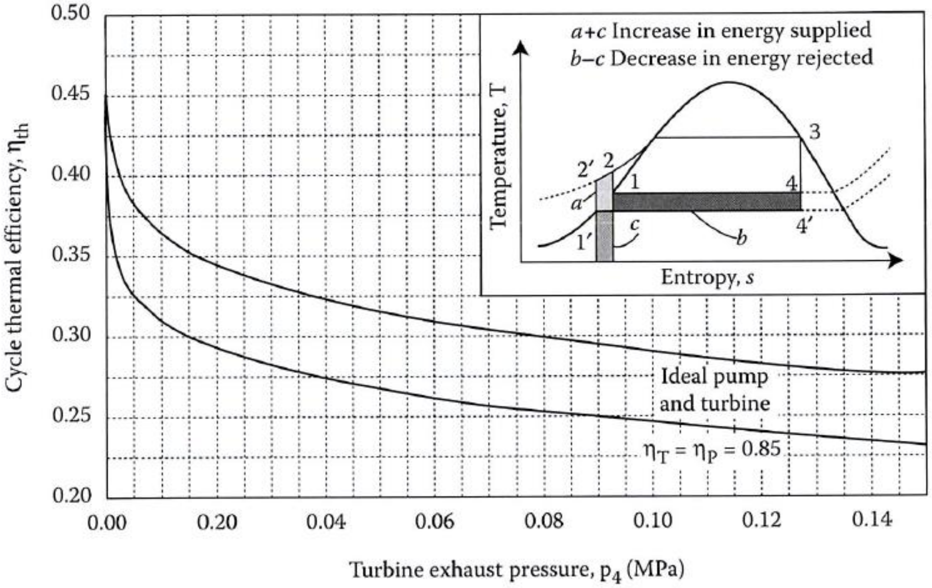
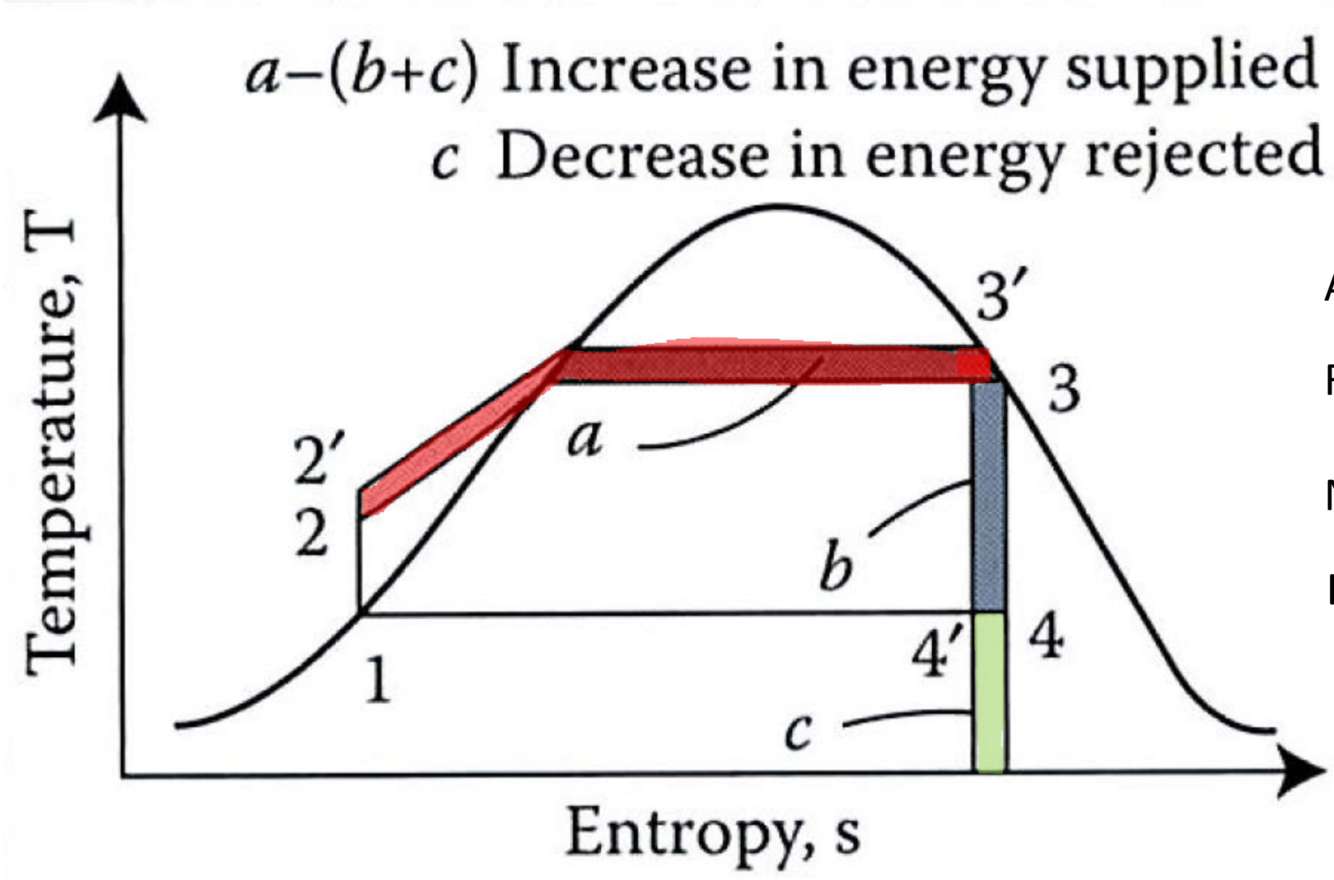


FIGURE 6.14 Thermal efficiency of Rankine cycle for a saturated turbine inlet state for varying turbine outlet pressure. Turbine inlet: 7.8 MPa saturated vapor.

Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Improvements in thermal efficiency for this simple saturated cycle
    - Pressure
      - Increase the pump outlet pressure/decrease turbine outlet pressure



Added heat ⥣

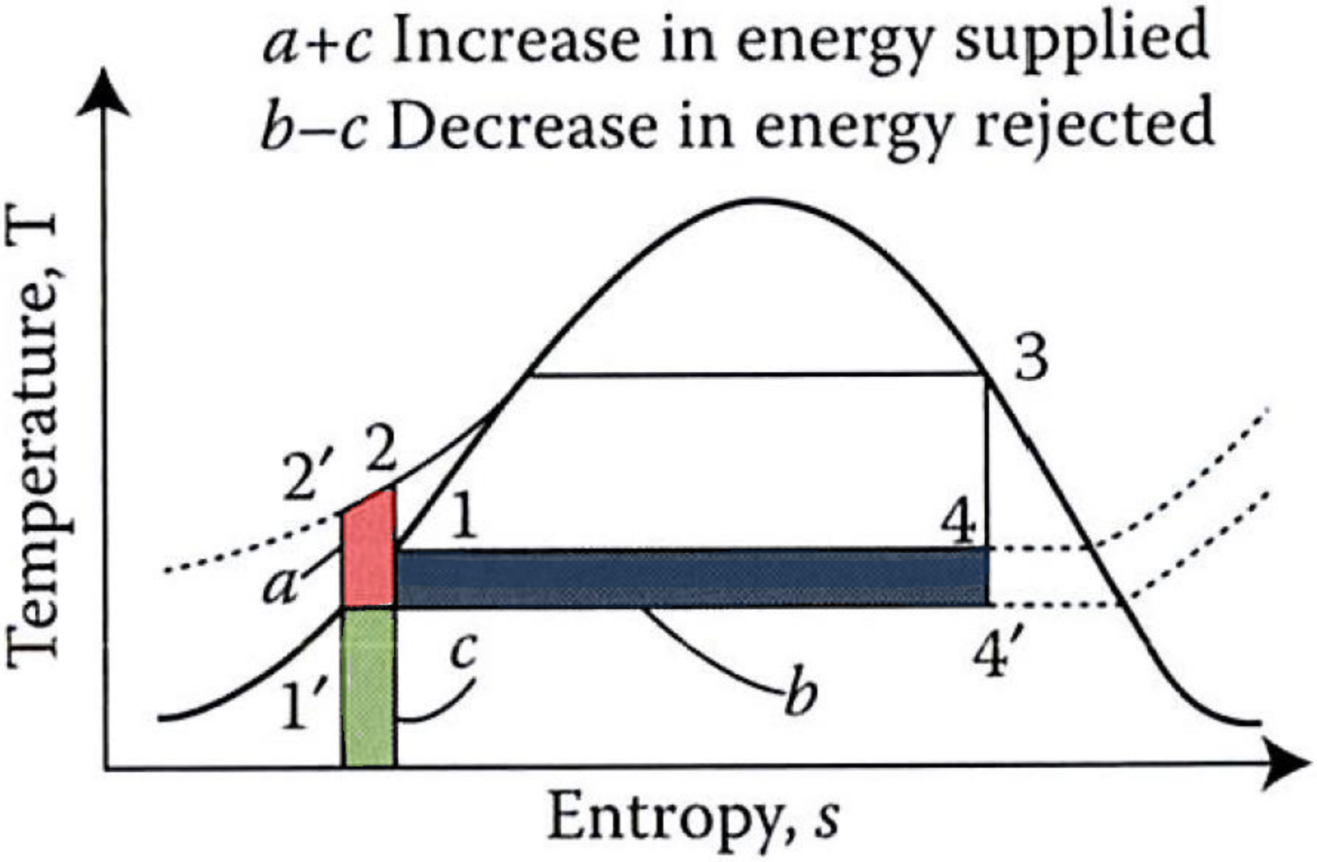
Rejected heat ⥥

Net work ⥣

Efficiency ⥣

Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Improvements in thermal efficiency for this simple saturated cycle
    - Pressure
      - Increase the pump outlet pressure/decrease turbine outlet pressure



Added heat ⥣

Rejected heat ⥥

Net work ⥣

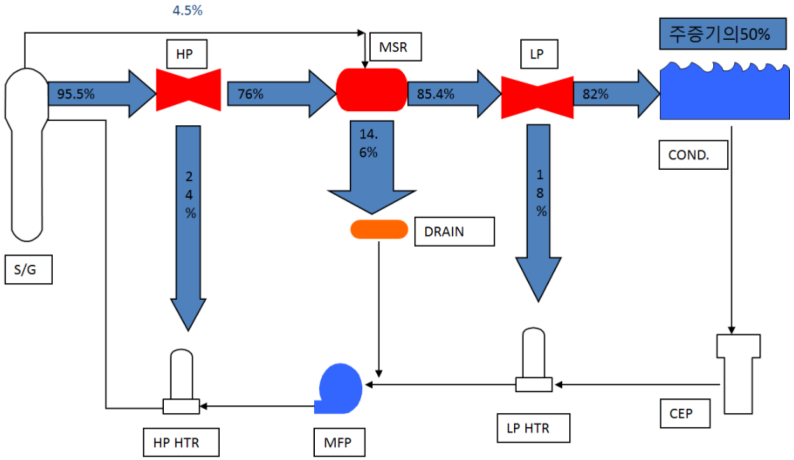
Efficiency ⥣

Contents

More Complex Rankine Cycles

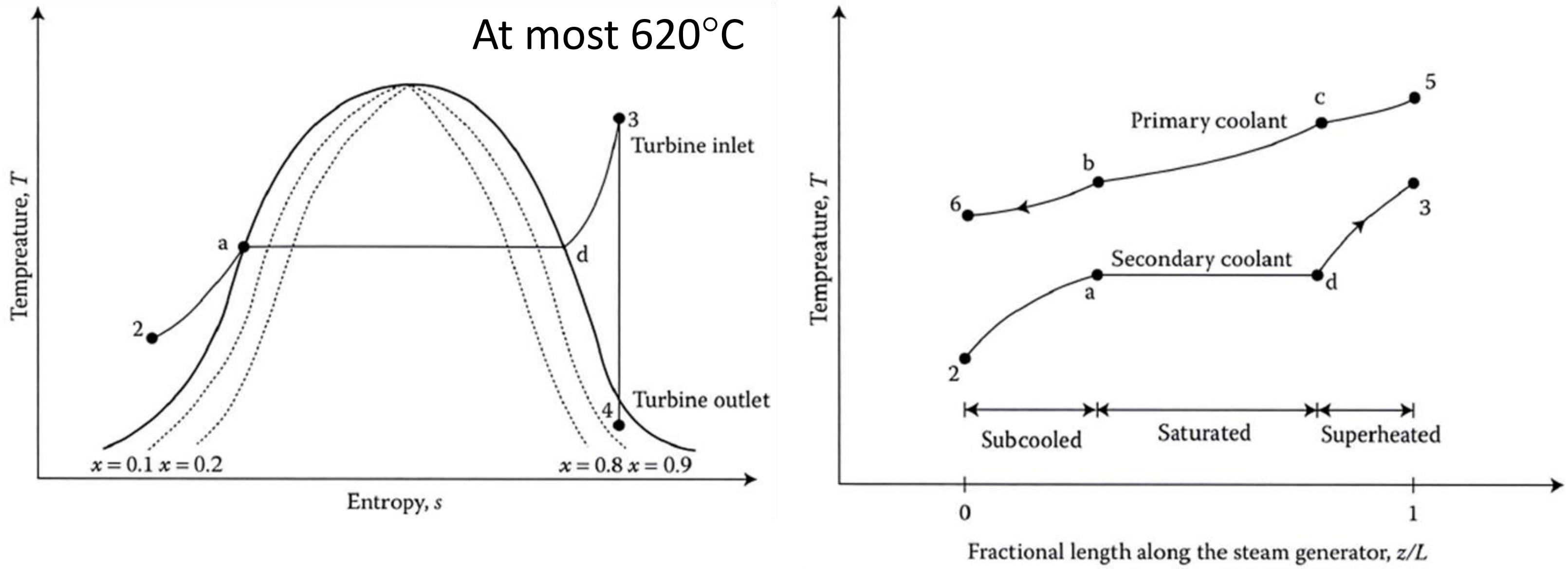
System Superheat, Reheat, Regeneration, Moisture Separation

- Improvements in overall cycle performance
  - Can be accomplished by superheat, reheat, and regeneration
    - Each approach leads to a higher average temperature at which heat is received by the working fluid
- Minimize erosion of the turbine blades
  - Multiple turbine stages with intermediate moisture separation
  - The superheat and reheat options also allow maintenance of higher exit steam quality, there by decreasing the liquid fraction in the turbine.



More Complex Rankine Cycles

- System Superheat, Reheat, Regeneration, Moisture Separation
  - Superheat: limited degree in PWR
    - Once-through SG: B&W
    - Recirculating SG: CE and Westinghouse

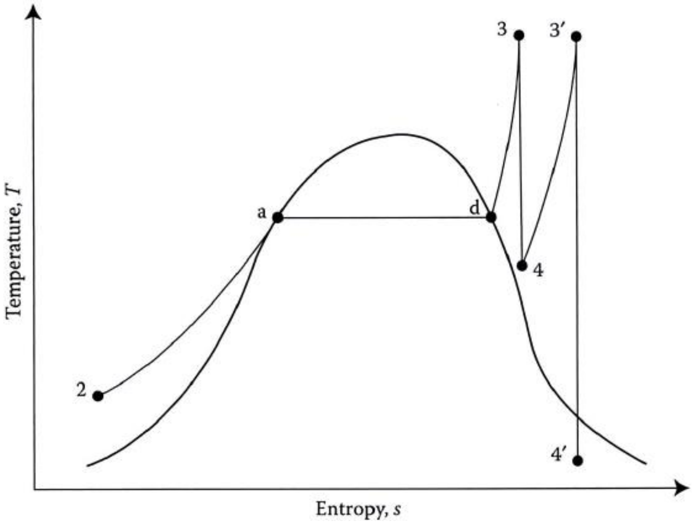


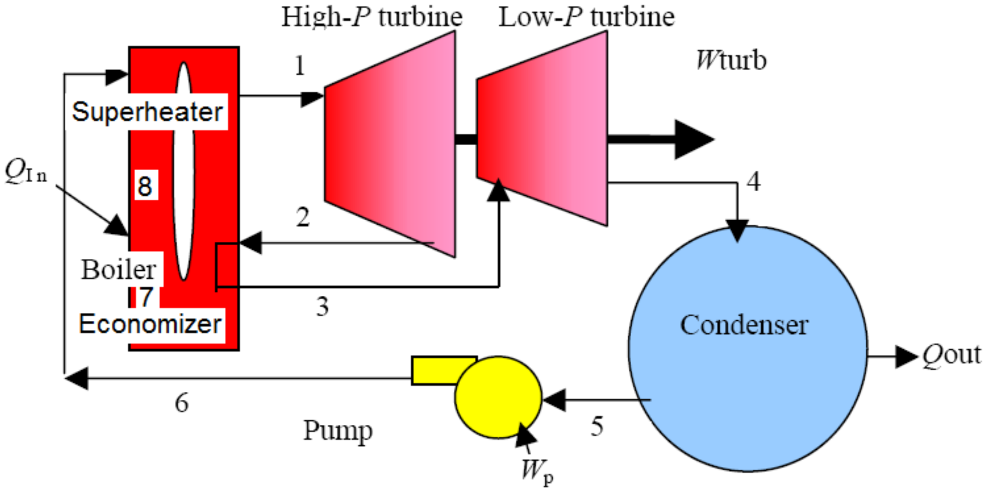
At most 620°C

FIGURE 6.16 Heat exchanger and turbine processes in a superheated power cycle.

More Complex Rankine Cycles

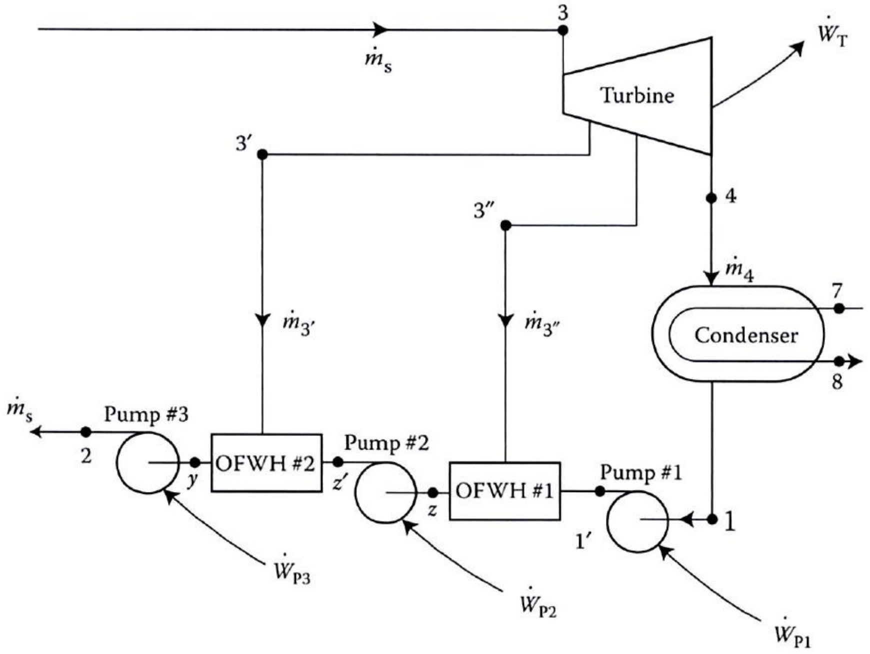
- System Superheat, Reheat, Regeneration, Moisture Separation
  - Reheat
    - The working fluid is returned to a heat exchanger after partial expansion in the turbine.
    - In PWR
      - Moisture separator and reheater

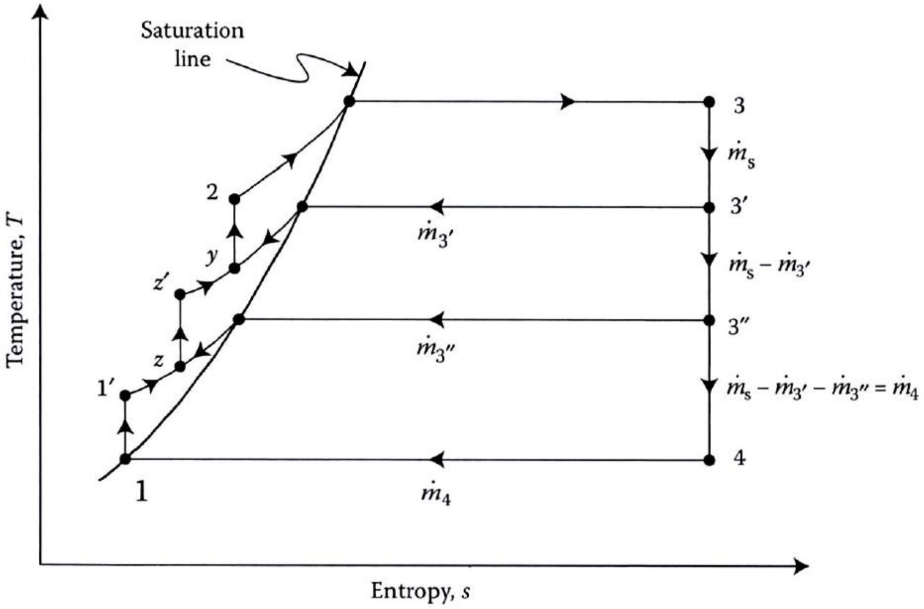




More Complex Rankine Cycles

- System Superheat, Reheat, Regeneration, Moisture Separation
  - Regeneration
    - Extracting steam from various turbine stages and directing it to a series of heaters where the condensate or feed water is preheated.
    - These feedwater heaters may be open (OFWH), in which the streams are directly mixed, or closed (CFWH) in which the heat transfer occurs through tube walls.





More Complex Rankine Cycles

- System Superheat, Reheat, Regeneration, Moisture Separation
  - Analysis of the regeneration with OFWH
    - Thermal efficiency

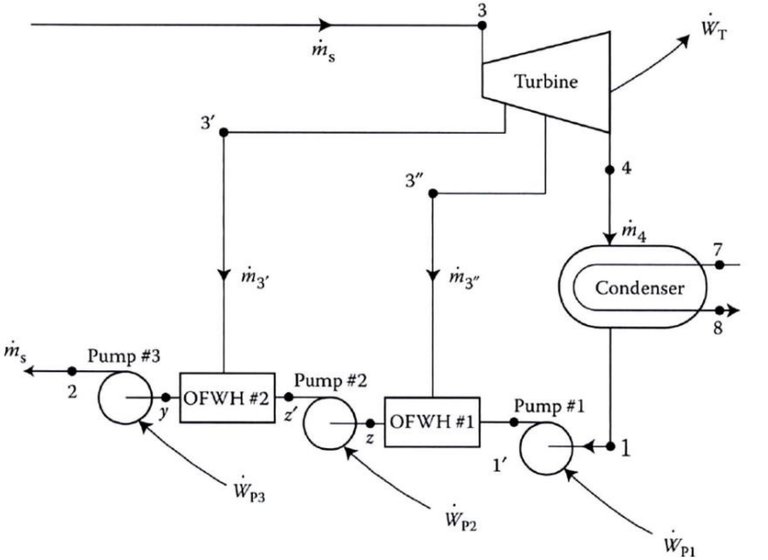
$$\eta_{\text{th}} = \frac{\dot{W}_{\text{u,actual}}}{\dot{Q}_{\text{in}}}$$

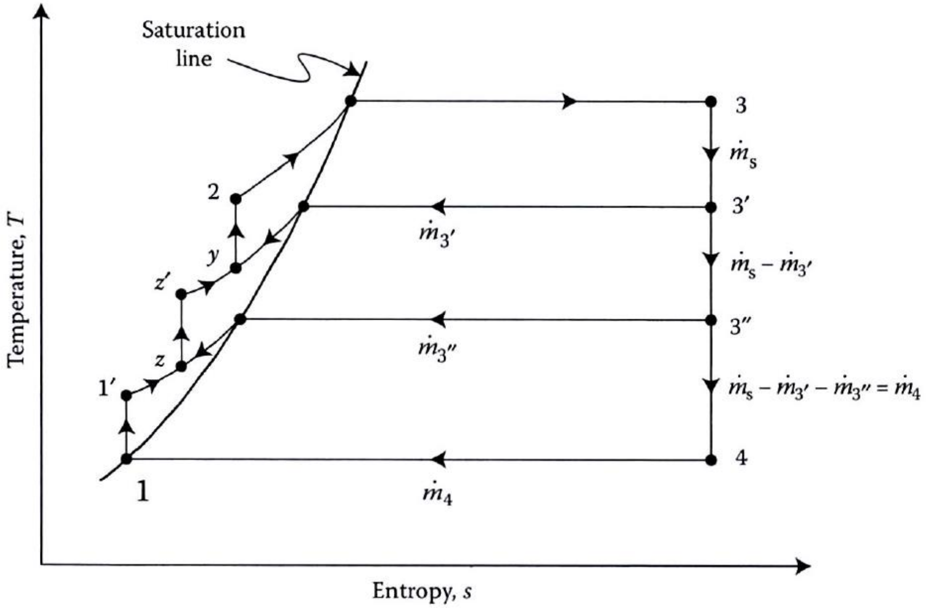
    - Actual useful work
      - Turbine work – pump work

$$\dot{W}_{\mathrm{T}} = \dot{m}_{\mathrm{s}}(h_3 - h_{3'}) + (\dot{m}_{\mathrm{s}} - \dot{m}_{3'})(h_{3'} - h_{3''}) + \dot{m}_4(h_{3''} - h_4)$$

$$\dot{W}_P = \dot{m}_4(h_{1'} - h_1) + (\dot{m}_4 + \dot{m}_{3''})(h_{z'} - h_z) + (\dot{m}_4 + \dot{m}_{3''} + \dot{m}_{3'})(h_2 - h_y)$$

$$\dot{Q}_{in} = \dot{m}_s(h_3 - h_2)$$





More Complex Rankine Cycles

- System Superheat, Reheat, Regeneration, Moisture Separation
  - Analysis of the regeneration with OFWH
    - First law & mass conservation
      - For OFWH No. 1

$$\sum_{k=1}^{I}(\dot{m}h)_{\mathrm{in,k}}-\sum_{k=1}^{I}(\dot{m}h)_{\mathrm{out,k}}=\cancel{\dot{W}_{\mathrm{shaft}}}-\cancel{\dot{Q}}$$

$$h_z\left(\dot{m}_{3''}+\dot{m}_4\right)=\dot{m}_{3''}h_{3''}+\dot{m}_4h_{1'}$$

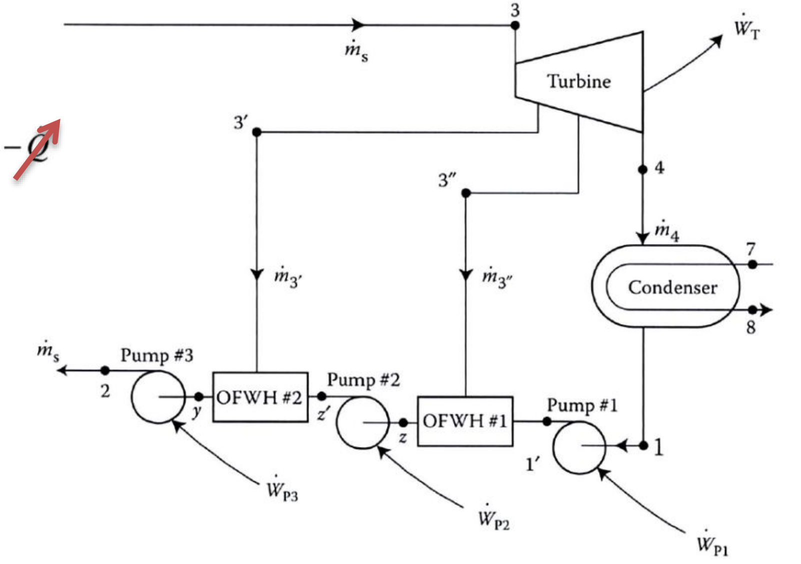
$$\left(\dot{m}_{3''}+\dot{m}_4\right)=\left(\dot{m}_s-\dot{m}_{3'}\right)$$

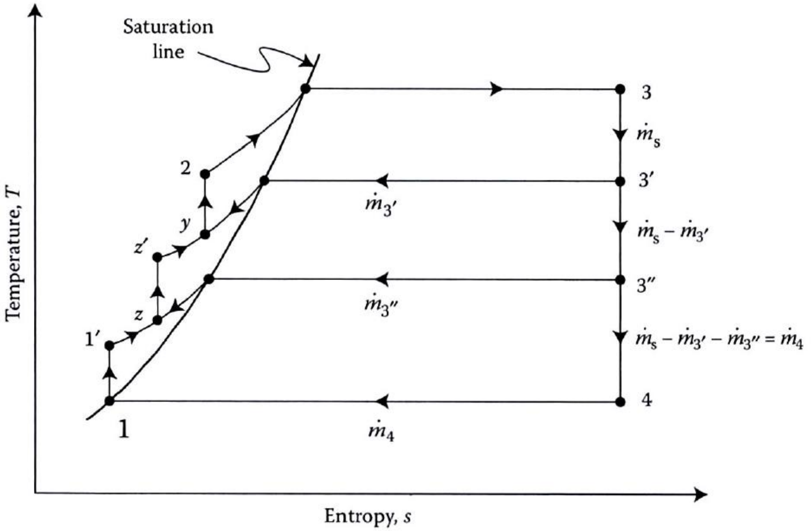
$$h_z=\frac{\dot{m}_{3''}h_{3''}+\dot{m}_4h_{1'}}{\dot{m}_{3''}+\dot{m}_4}=\frac{\dot{m}_{3''}h_{3''}+\dot{m}_4h_{1'}}{\dot{m}_s-\dot{m}_{3'}}\qquad(6.91)$$

      - For OFWH No. 2

$$h_y\dot{m}_s=\dot{m}_{3'}h_{3'}+\left(\dot{m}_s-\dot{m}_{3'}\right)h_{z'}$$

$$h_y=\frac{\dot{m}_{3'}h_{3'}+(\dot{m}_s-\dot{m}_{3'})h_{z'}}{\dot{m}_s}\qquad(6.92)$$

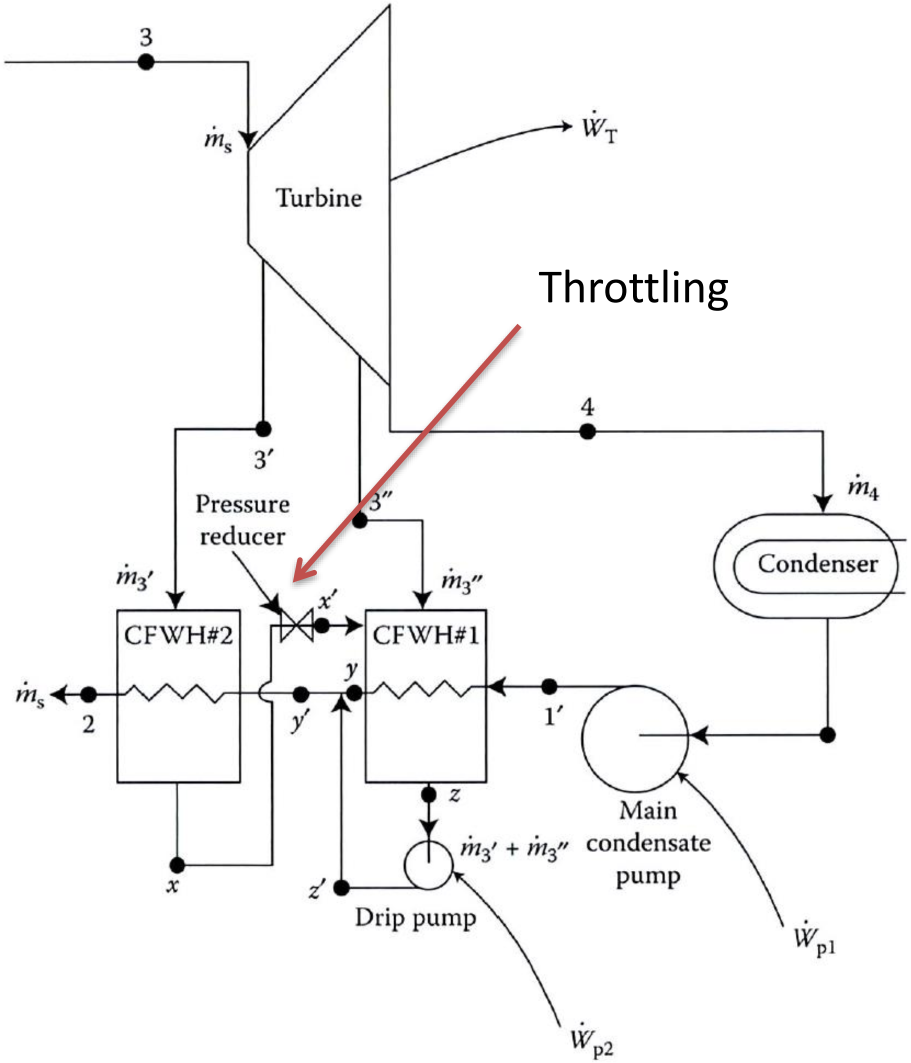




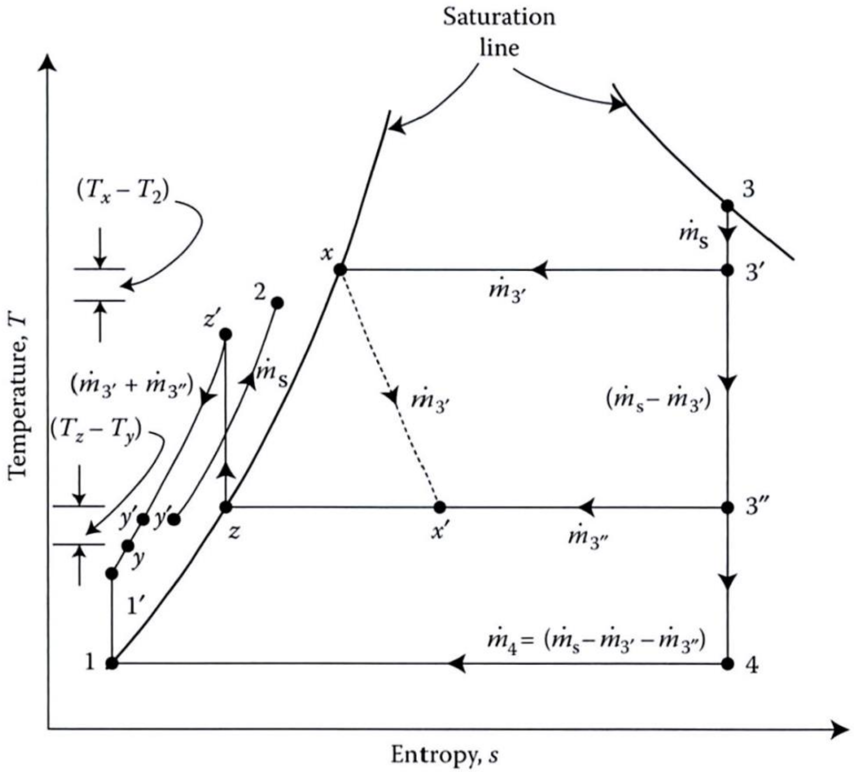
More Complex Rankine Cycles

- System Superheat, Reheat, Regeneration, Moisture Separation
  - Regeneration with CFWH

Closed feedwater heaters
- Drained backward
- Pumped forward



Throttling



More Complex Rankine Cycles

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Closed feedwater heaters
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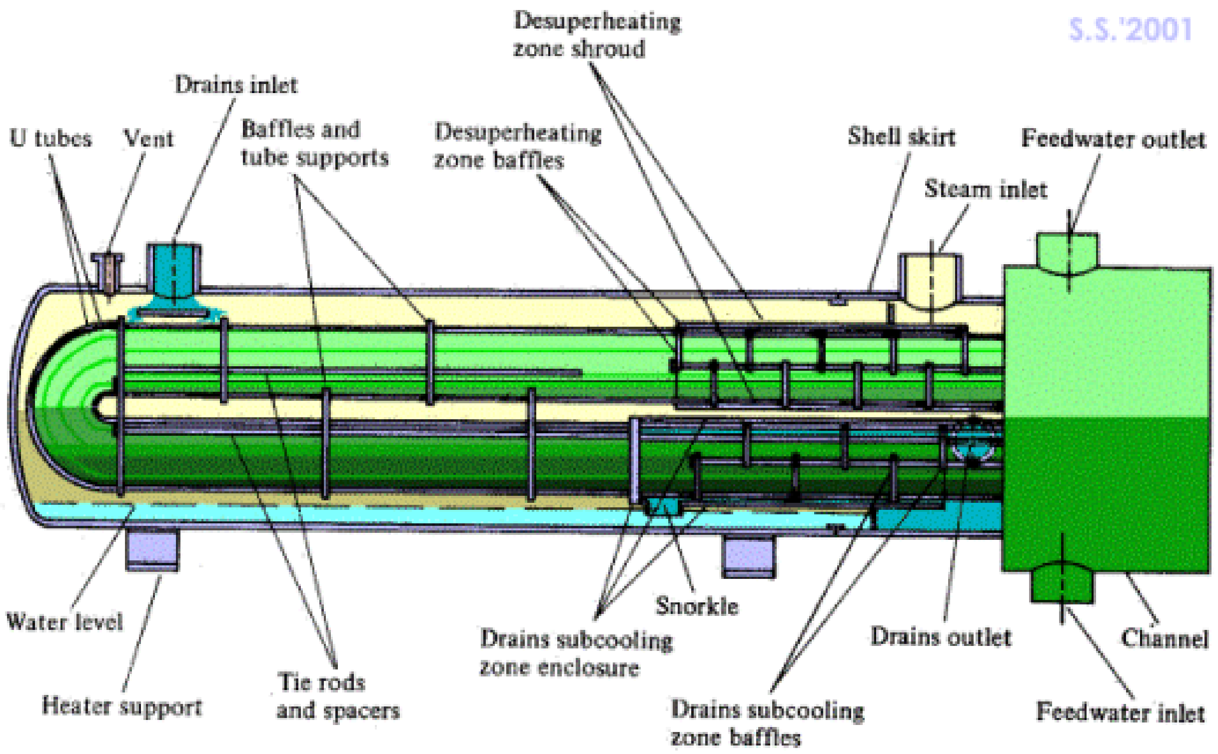
- Pumped forward



More Complex Rankine Cycles

- System Superheat, Reheat, Regeneration, Moisture Separation
  - Analysis of the regeneration with CFWH
    - First law & mass conservation
      - For CFWH No. 1

$$\dot{m}_4 h_y = \dot{m}_4 h_{1'} + \dot{m}_{3''} h_{3''} + \dot{m}_{3'} h_{x'} - \left(\dot{m}_{3'} + \dot{m}_{3''}\right) h_z$$

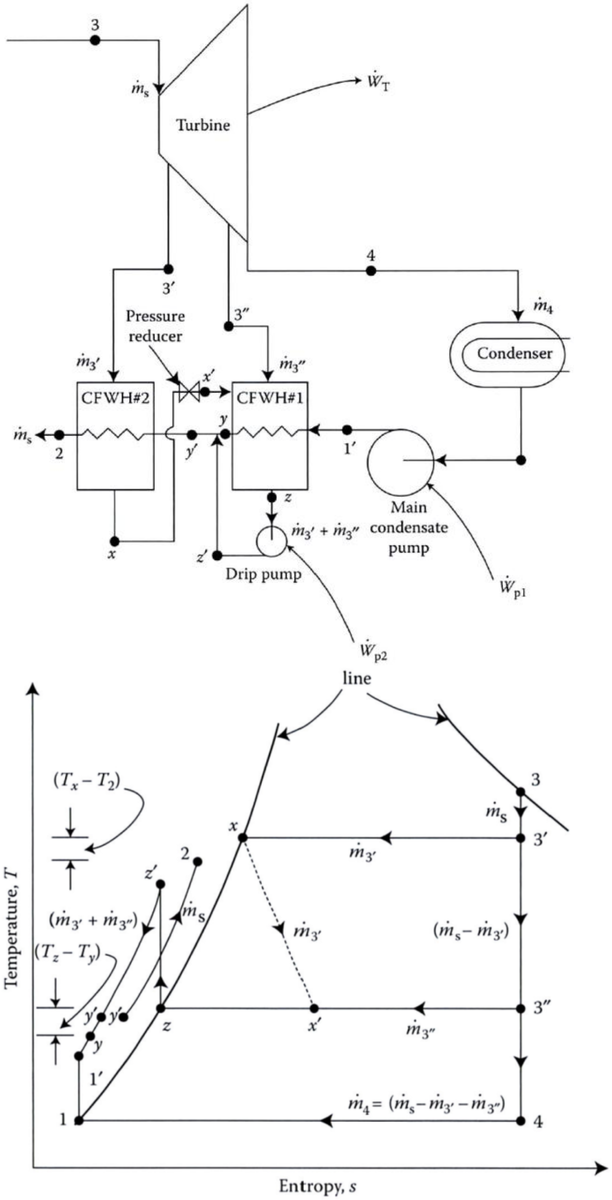
$$h_y = \frac{\dot{m}_{3''} h_{3''} + \dot{m}_4 h_{1'} + \dot{m}_{3'} h_{x'} - (\dot{m}_{3'} + \dot{m}_{3''}) h_z}{\dot{m}_4} \qquad (6.93)$$

- For the mixing tee, at position y’

$$\left(\dot{m}_4 + \dot{m}_{3'} + \dot{m}_{3''}\right) h_{y'} = \left(\dot{m}_{3'} + \dot{m}_{3''}\right) h_{z'} + \dot{m}_4 h_y$$

$$\dot{m}_s = \dot{m}_4 + \dot{m}_{3'} + \dot{m}_{3''}$$

$$h_{y'} = \frac{\dot{m}_4 h_y + (\dot{m}_{3'} + \dot{m}_{3''}) h_{z'}}{\dot{m}_s} \qquad (6.94)$$

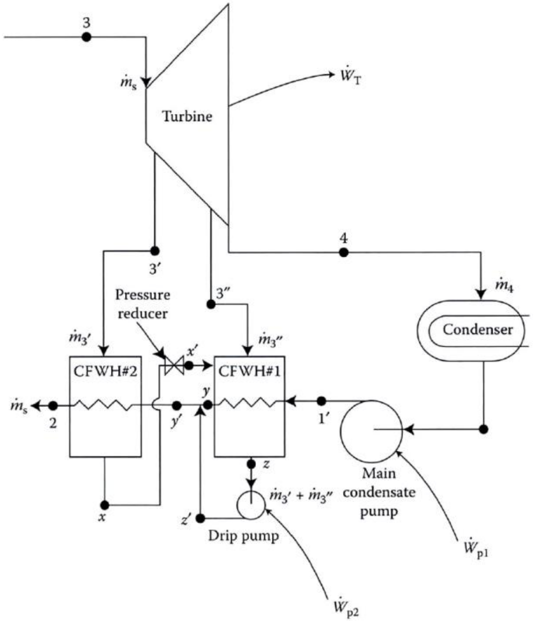


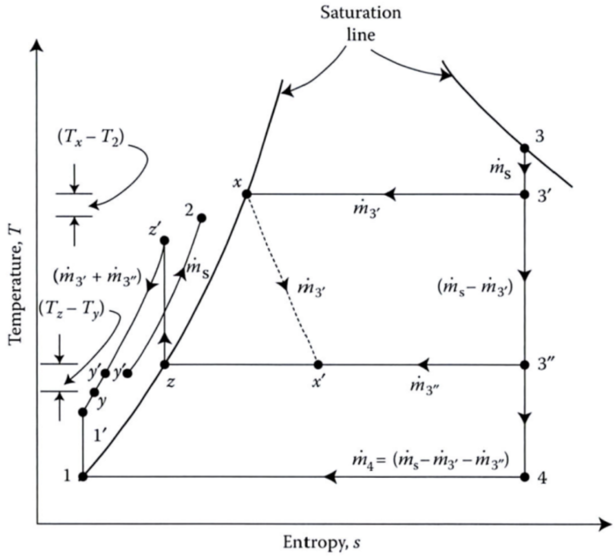
More Complex Rankine Cycles

- System Superheat, Reheat, Regeneration, Moisture Separation
  - Analysis of the regeneration with CFWH
    - First law & mass conservation
      - For CFWH No. 2

$$\dot{m}_s h_2 + \dot{m}_{3'} h_x = \dot{m}_{3'} h_{3'} + \dot{m}_s h_{y'}$$

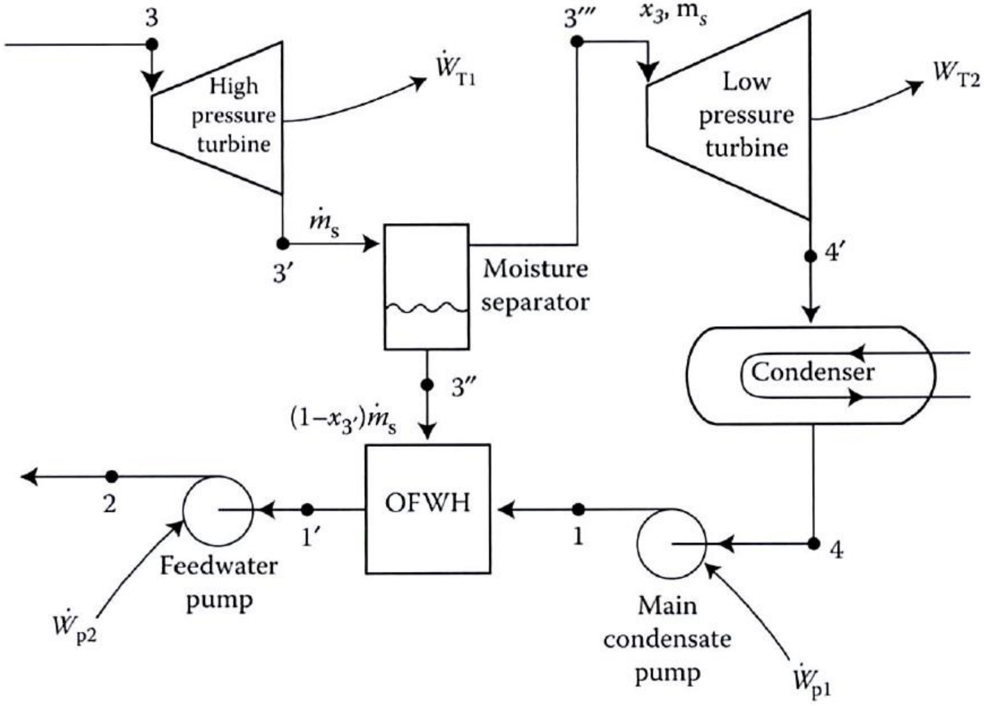
$$h_2 = \frac{\dot{m}_{3'}(h_{3'} - h_x) + \dot{m}_s h_{y'}}{\dot{m}_s} \qquad (6.95)$$

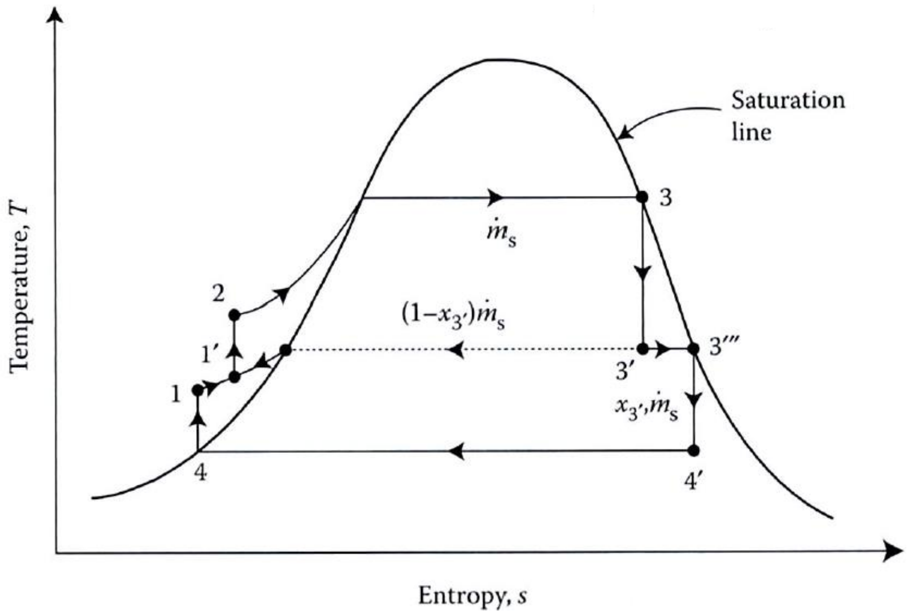




More Complex Rankine Cycles

- System Superheat, Reheat, Regeneration, Moisture Separation
  - Moisture separator
    - The steam from the high pressure turbine to a moisture separator
    - Separated liquid to a feedwater heater while vapor passes to a low-pressure turbine





More Complex Rankine Cycles

- System Superheat, Reheat, Regeneration, Moisture Separation
  - Moisture separator
    - First law & mass conservation
    - Saturated state of liquid and vapor
      - For the separated water (3”)

$$\dot{m}_{3''} = (1-x)\dot{m}_s \qquad h_{3''} = h_f(p_{3'})$$

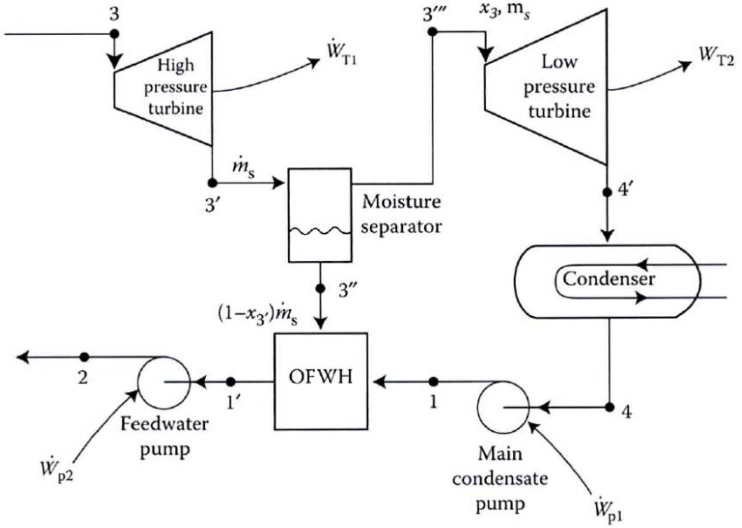
      - For the separated vapor (3’’’)

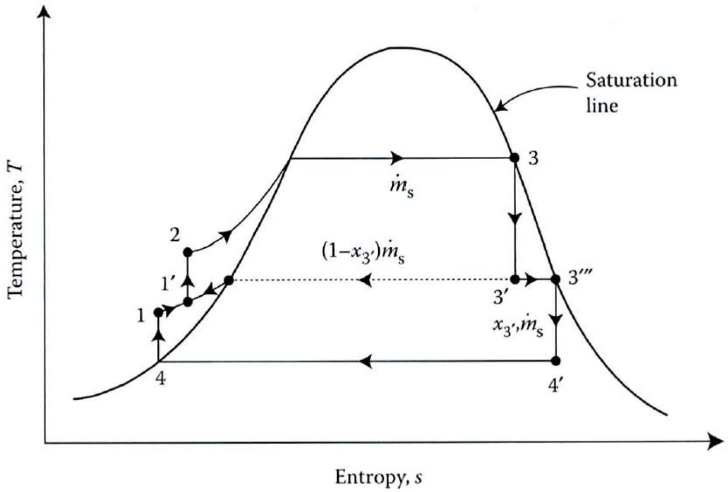
$$\dot{m}_{3'''} = x\dot{m}_s \qquad h_{3'''} = h_g(p_{3'})$$

      - Feedwater heater exit enthalpy (1’)

$$h_{1'}\dot{m}_s = h_{3''}(1-x_{3'})\dot{m}_s + h_1 x_{3'}\dot{m}_s$$

$$h_{1'} = \frac{h_{3''}(1-x_{3'})\dot{m}_s + h_1 x_{3'}\dot{m}_s}{\dot{m}_s} \qquad (6.96)$$

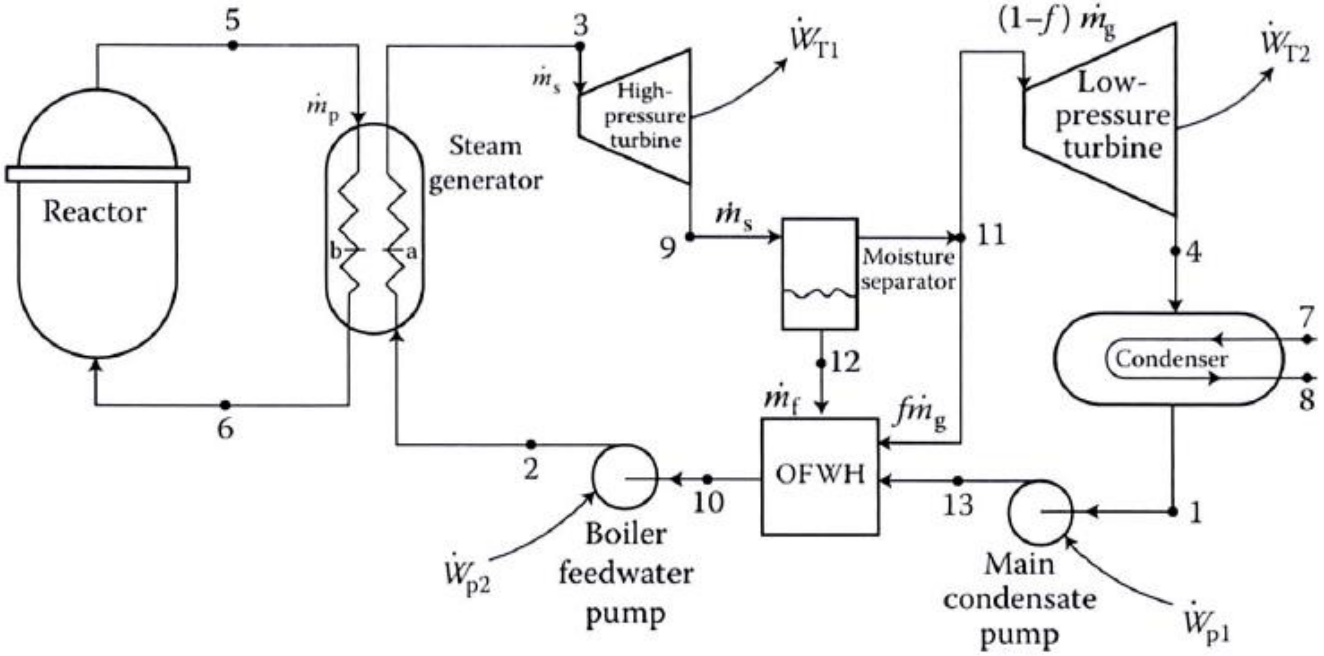




More Complex Rankine Cycles

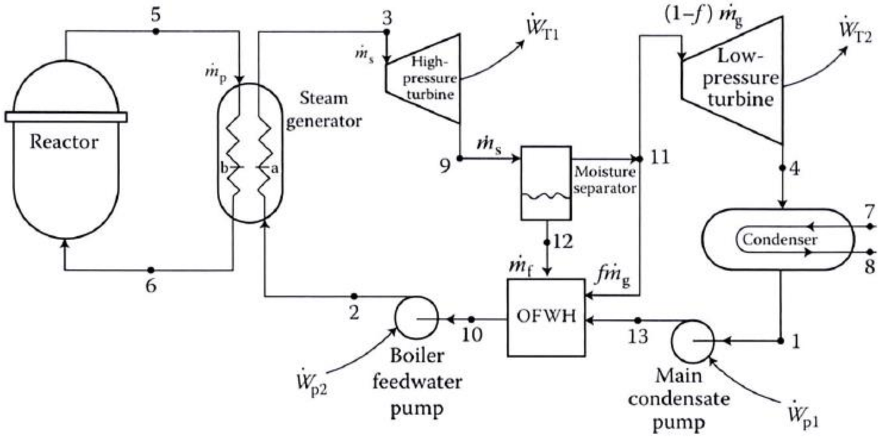
Example 6.6: Thermodynamic Analysis of a PWR Cycle with Moisture Separation and One Stage of Feedwater Heating

PROBLEM This example demonstrates the advantage in cycle performance gained by adding moisture separation and open feedwater heating to the cycle of Example 6.4. The cycle, as illustrated in Figure 6.22, operates under the conditions given in Table 6.8. All states are identical to those in Example 6.4 except state 2, as states 9 through 13 have been added. The flow rate at state 11, which is diverted to the open feedwater heater, is sufficient to preheat the condensate stream at state 13 to the conditions of state 10. Assume that all turbines and pumps have isentropic efficiencies of 85%.



More Complex Rankine Cycles

Example 6.6: Thermodynamic Analysis of a PWR Cycle with Moisture Separation and One Stage of Feedwater Heating

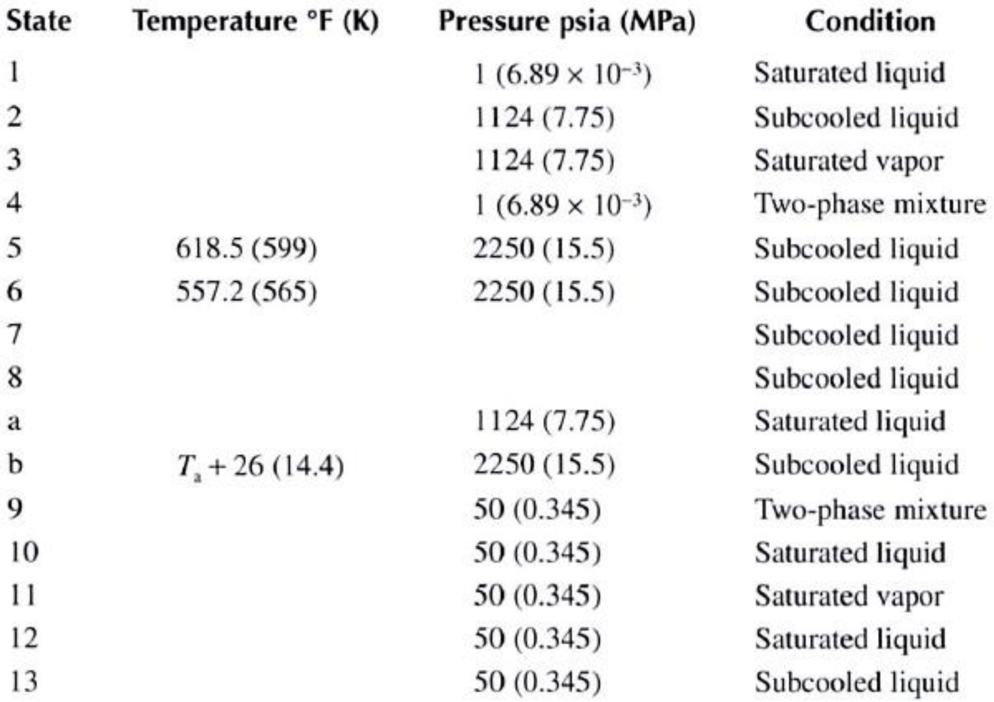


1. Draw the temperature-entropy *(T-s)* diagram for this cycle, as well as the temperature -fractional length *(T* vers us z/L) diagram for the steam generator.
2. Compute the ratio of primary to secondary flow rates.
3. Compute the cycle thermal efficiency.

f is not given!

TABLE 6.8

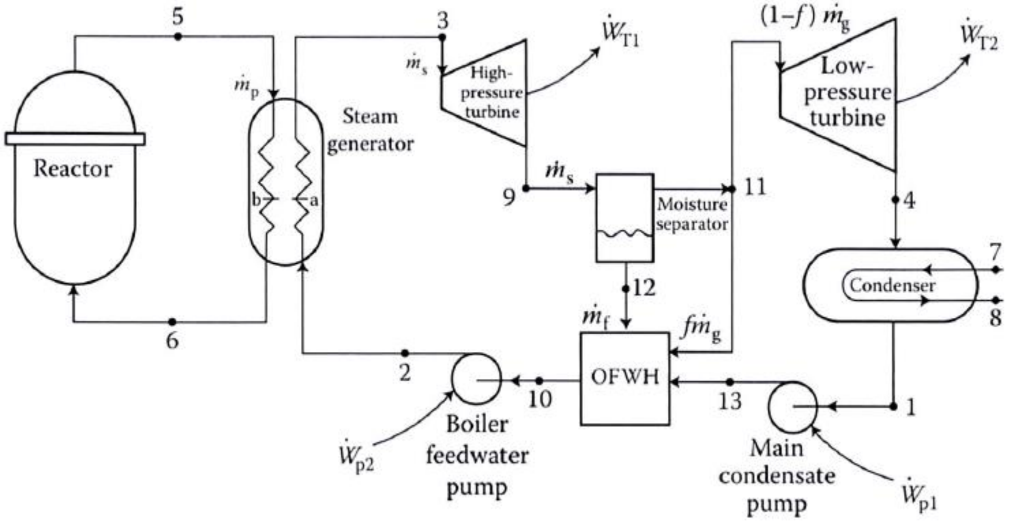
PWR Operating Conditions for Example 6.6



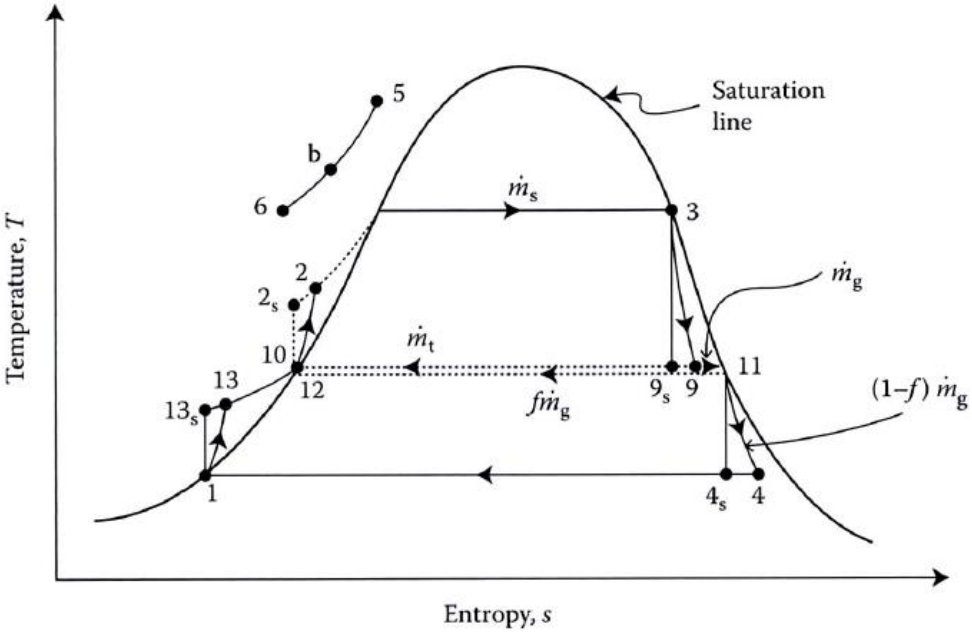
State	Temperature °F (K)	Pressure psia (MPa)	Condition
1		1 (6.89×10^{-3})	Saturated liquid
2		1124 (7.75)	Subcooled liquid
3		1124 (7.75)	Saturated vapor
4		1 (6.89×10^{-3})	Two-phase mixture
5	618.5 (599)	2250 (15.5)	Subcooled liquid
6	557.2 (565)	2250 (15.5)	Subcooled liquid
7			Subcooled liquid
8			Subcooled liquid
a		1124 (7.75)	Saturated liquid
b	$T_a + 26$ (14.4)	2250 (15.5)	Subcooled liquid
9		50 (0.345)	Two-phase mixture
10		50 (0.345)	Saturated liquid
11		50 (0.345)	Saturated vapor
12		50 (0.345)	Saturated liquid
13		50 (0.345)	Subcooled liquid

More Complex Rankine Cycles

Example 6.6: Thermodynamic Analysis of a PWR Cycle with Moisture Separation and One Stage of Feedwater Heating



T-s diagram



More Complex Rankine Cycles

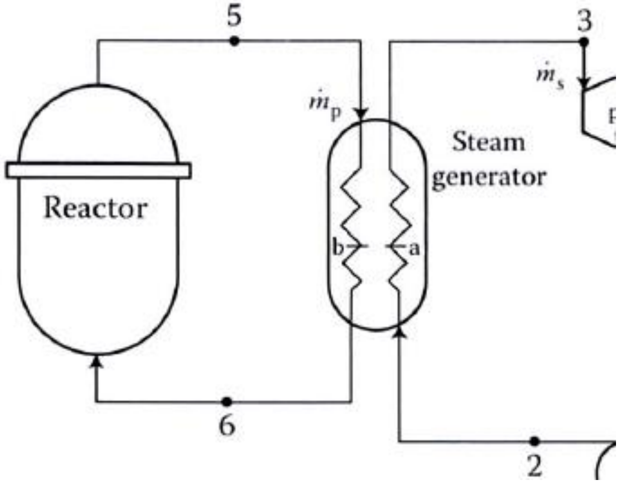
- Example 6.6
  - Compute the ratio of primary to secondary flow rates.
    - With given information only
    - Energy conservation

$$\dot{m}_p(h_5 - h_6) = \dot{m}_s(h_3 - h_2)$$

Information is not enough.

$$\dot{m}_p(h_5 - h_b) = \dot{m}_s(h_3 - h_a)$$

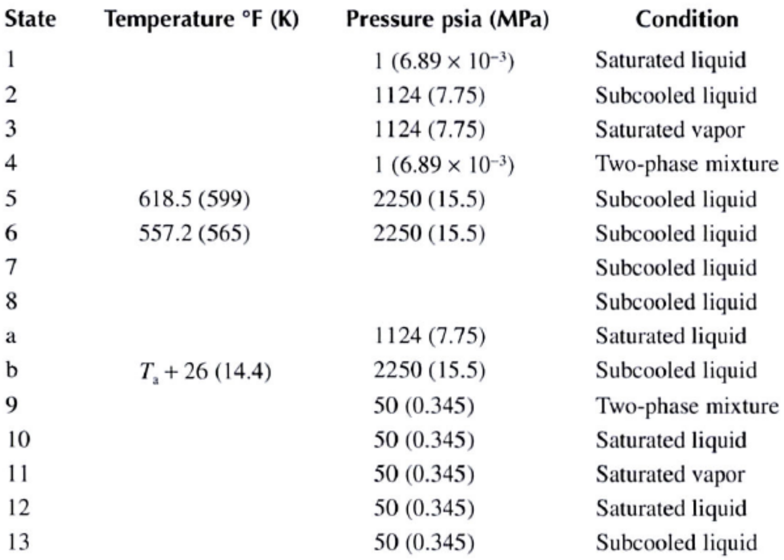
Information is enough.



$$(h_5 - h_b) = \bar{c}_p(T_5 - T_b) = \bar{c}_p(T_5 - T_a - 14.4)$$

$$\because T_b = T_a + 14.4$$

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{h_3 - h_a}{\bar{c}_p[T_5 - (T_a - \Delta T_p)]} = 13.18$$



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12		50 (0.345)	Saturated liquid
13		50 (0.345)	Subcooled liquid

More Complex Rankine Cycles

- Example 6.6
  - Compute the cycle thermal efficiency.
    - Neglecting pump work,

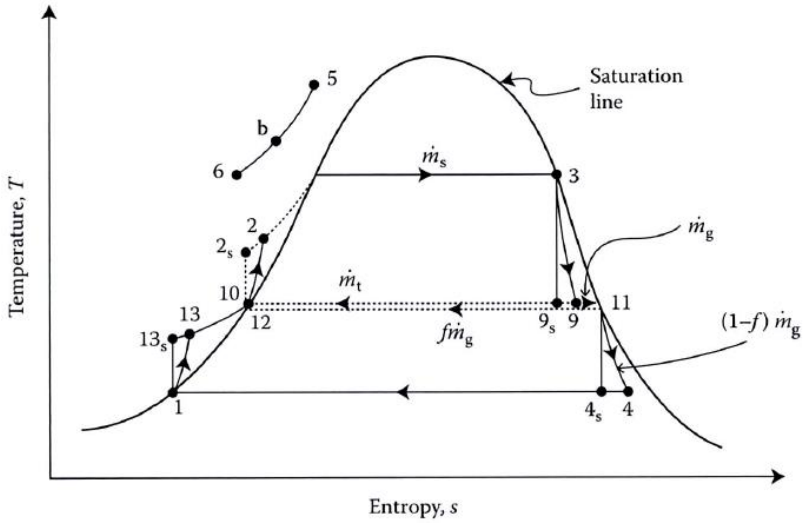
$$\eta_{th} = \frac{\dot{m}_s(h_3 - h_9) + (1-f)\dot{m}_g(h_{11} - h_4)}{\dot{m}_s(h_3 - h_2)}$$

    - $h_3 \Rightarrow$ by steam table
    - $h_{11} \Rightarrow$ by steam table
    - $h_{10} = h_{12} \Rightarrow$ by steam table
    - Unknowns: $\dot{m}_g, h_9, h_4, h_2, f$
    - h_9

$$s_3 = s_{9s} = s_f + x_{9s}(s_g - s_f) = s_f + xs_{gf}$$

$$h_{9s} = h_f + x_{9s}(h_g - h_f) = h_f + \frac{s_{9s} - s_f}{s_{gf}} h_{gf}$$

$$\eta_T = \frac{h_3 - h_9}{h_3 - h_{9,s}} \qquad h_9 = h_3 - \eta_T\left(h_3 - h_{9s}\right)$$



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More Complex Rankine Cycles

- Example 6.6
  - Compute the cycle thermal efficiency.
    - Unknowns: $\dot{m}_g, h_9, h_4, h_2, f$
    - $\dot{m}_g$, $\dot{m}_f$

$$h_9 = h_f + x_9(h_g - h_f)$$

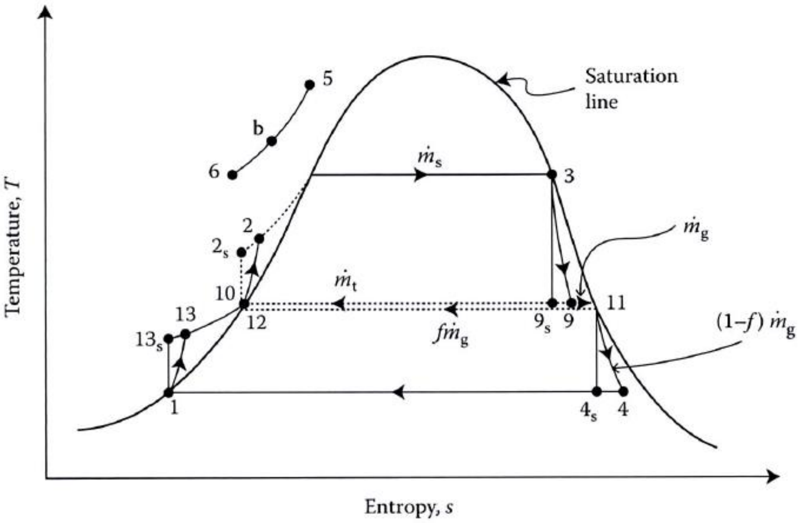
$$x_9 = \frac{h_9 - h_f}{h_g - h_f} \quad \Rightarrow \quad \dot{m}_g = x_9 \dot{m}_s \quad \dot{m}_f = (1 - x_9)\dot{m}_s$$

    - h_4

$$s_{11} = s_{4s} = s_f + x_{4s}(s_g - s_f) = s_f + x_{4s} s_{gf}$$

$$h_{4s} = h_f + x_{4s}(h_g - h_f) = h_f + \frac{s_{4s} - s_f}{s_{gf}} h_{gf}$$

$$\eta_T = \frac{h_{11} - h_4}{h_{11} - h_{4s}} \qquad h_4 = h_{11} - \eta_T\left(h_{11} - h_{4s}\right)$$



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More Complex Rankine Cycles

- Example 6.6
  - Compute the cycle thermal efficiency.
    - Unknowns: $\dot{m}_g, h_9, h_4, h_2, f$
    - h_2 and f

$$\left[(1-f)\dot{m}_g + \dot{m}_f + f\dot{m}_g\right] h_{10} = \dot{m}_s h_{10}$$

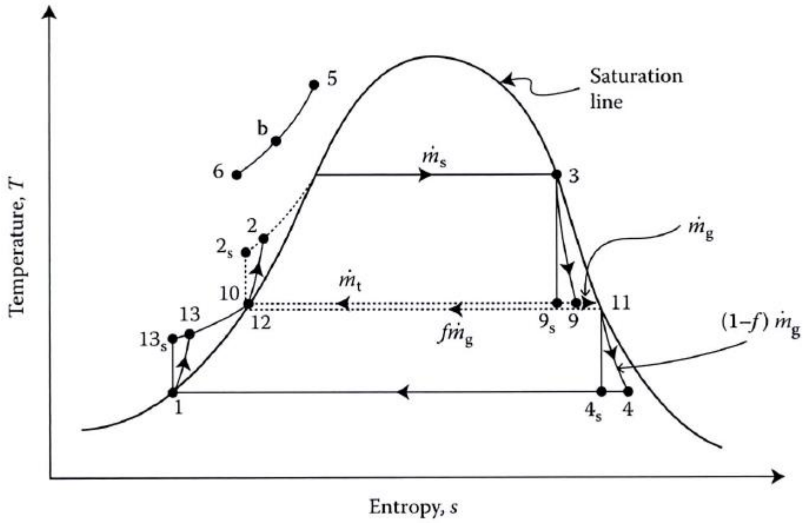
$$= (1-f)\dot{m}_g h_{13} + \dot{m}_f h_{12} + f\dot{m}_g h_{11}$$

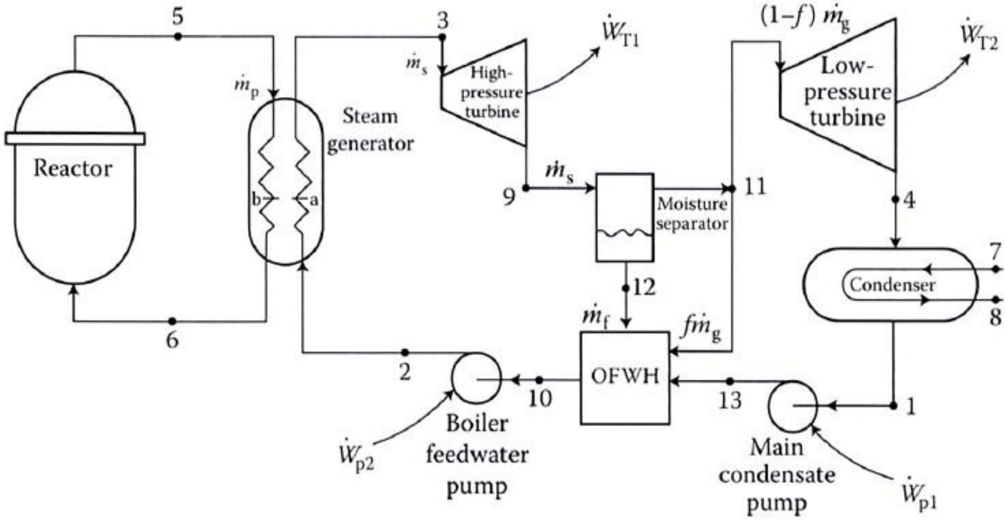
$$\dot{m}_s = \left[(1-f)\dot{m}_g + \dot{m}_f + f\dot{m}_g\right]$$

$$f = \frac{\dot{m}_s h_{10} - \dot{m}_g h_{13} - \dot{m}_f h_{12}}{\dot{m}_g\left(h_{11} - h_{13}\right)}$$

$$s_{10} = s_{2s}, h_{2s} = f(P_2, s_{2s})$$

$$\eta_p = \frac{h_{2s} - h_{10}}{h_2 - h_{10}} \quad h_2 = h_{10} + \frac{\left(h_{2s} - h_{10}\right)}{\eta_p}$$



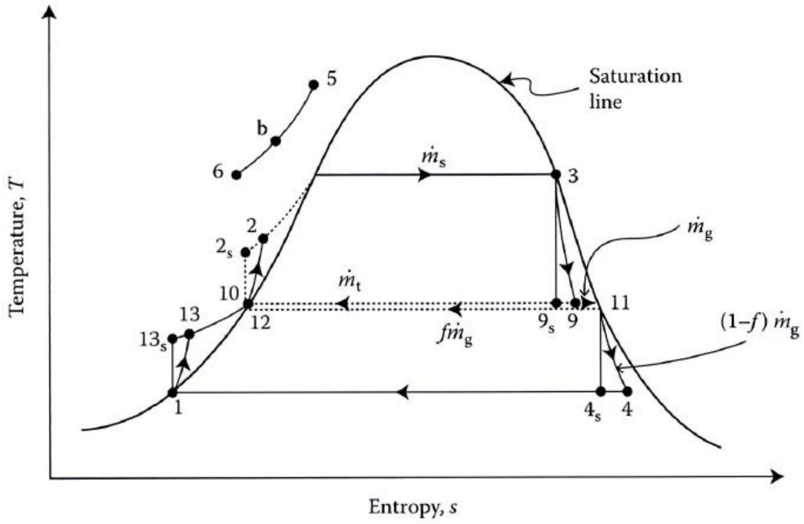


More Complex Rankine Cycles

- Example 6.6
  - Compute the cycle thermal efficiency.

$$\eta_{th} = \frac{\dot{m}_s(h_3 - h_9) + (1 - f)\dot{m}_g(h_{11} - h_4)}{\dot{m}_s(h_3 - h_2)}$$

Try by yourself!



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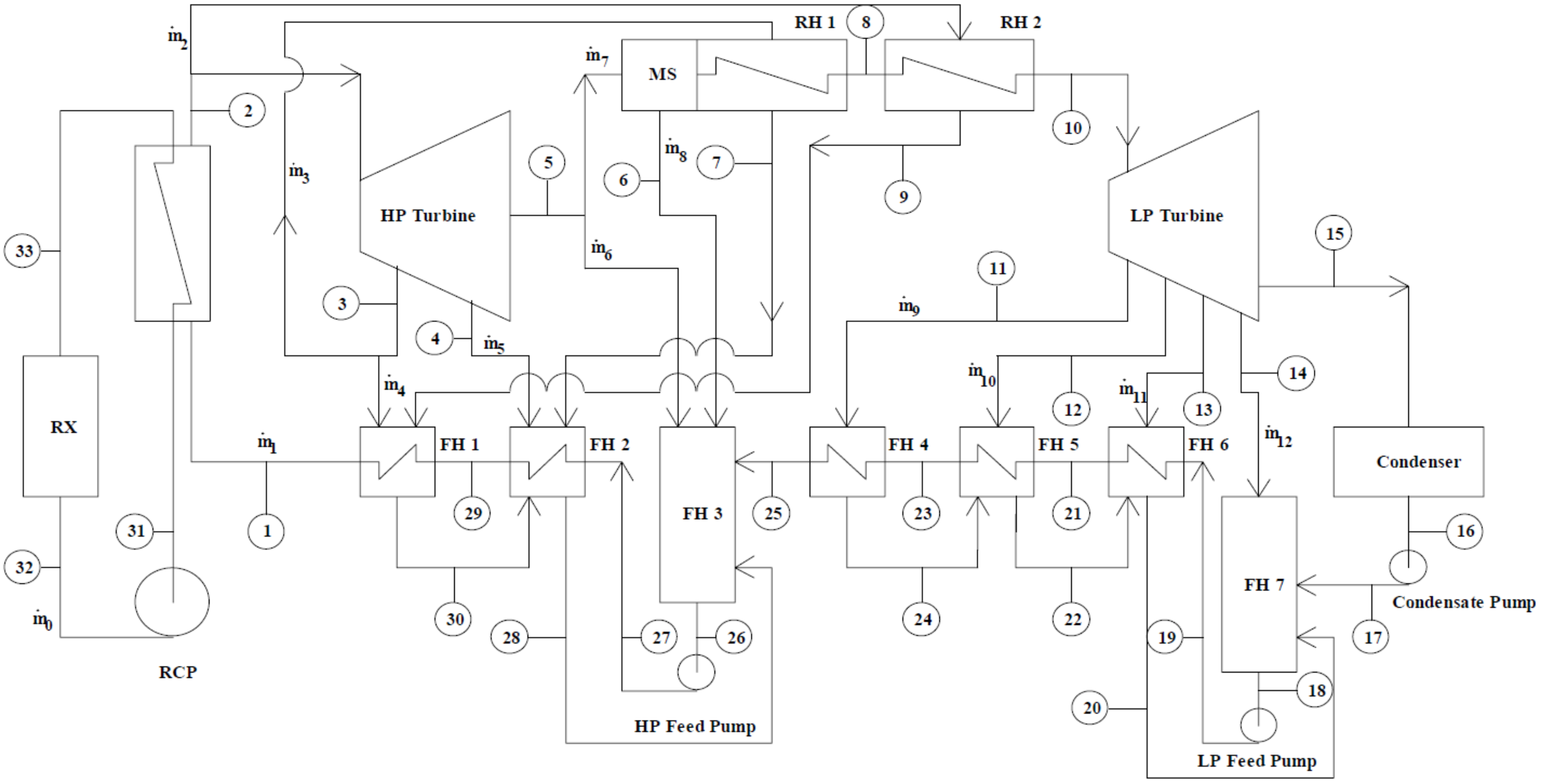
More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - 4 loop WH PWR
  - Efficiency of the turbines
  - Cycle efficiency
  - Pump efficiency: 75 %



More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - 4 loop WH PWR
  - Efficiency of the turbines
  - Cycle efficiency
  - Pump efficiency: 75 %

	State Point	Temperature (oC)	Quality	Pressure (MPa)	Enthalpy (kJ/kg)
1		226.7			
2			1	6.895	
3				2.827	
4				1.896	
5				1.206	2530.7
6					
7					
8		215.6			
9					
10		268.3			
11				0.483	
12				0.310	
13				0.110	
14				0.046	
15				0.00517	2330.7
16					
17					
18					
19					
20					
21		98.9			
22					
23		127.8			
24					
25		146.7			
26					
27					
28					
29		205.6			
30					
31				14.651	
32		293.3		15.513	
33		326.7			

More Complex Rankine Cycles

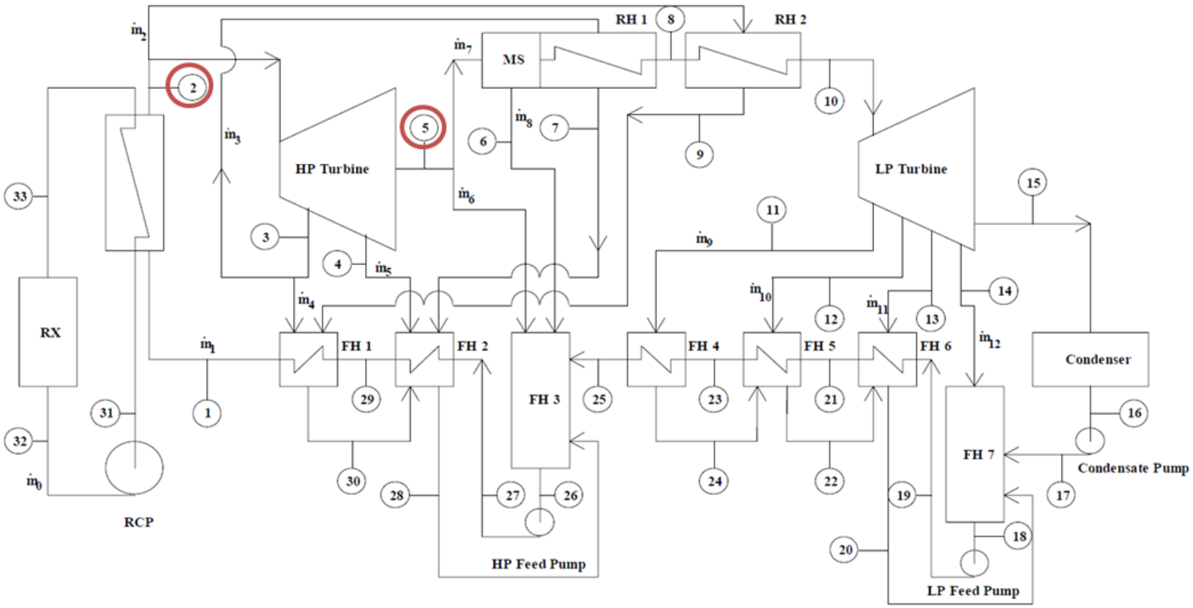
- More complex Rankine cycle analysis
  - Solution procedure
    - Step 1: evaluate the efficiencies of the high pressure turbine and low pressure turbine

– $\eta_{HPT} = \frac{h_2 - h_5}{h_2 - h_{5,s}}$

$\eta_{LPT} = \frac{h_{10} - h_{15}}{h_{10} - h_{15,s}}$

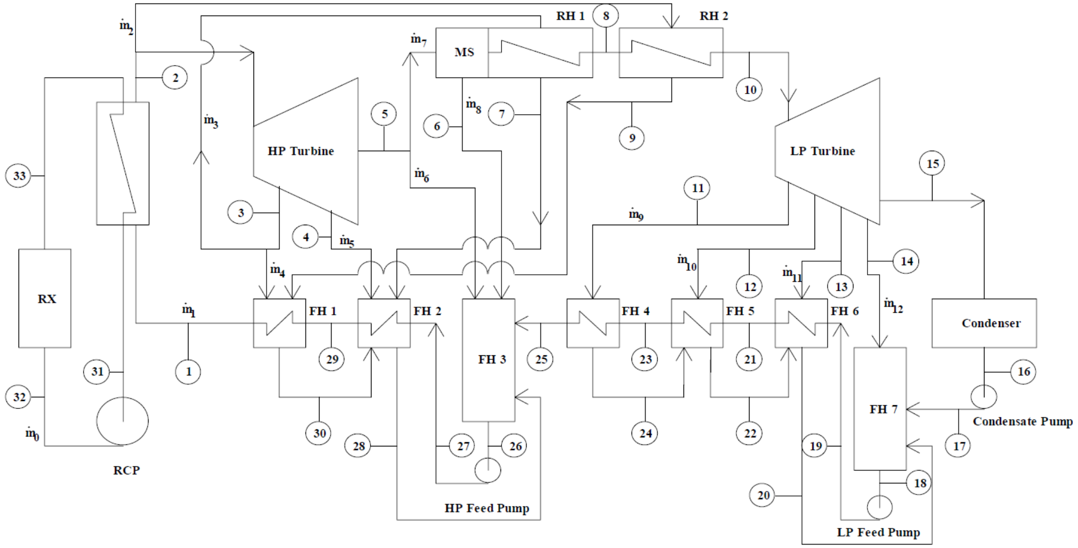
- $h_2 = h_g(P)$: saturated steam
- $h_5 = 2530.7$: two-phase mixture
- $h_{5,s} = h(P, s_2)$

$$\boxed{\eta_{HPT} \approx 0.779 \quad \eta_{LPT} \approx 0.745}$$



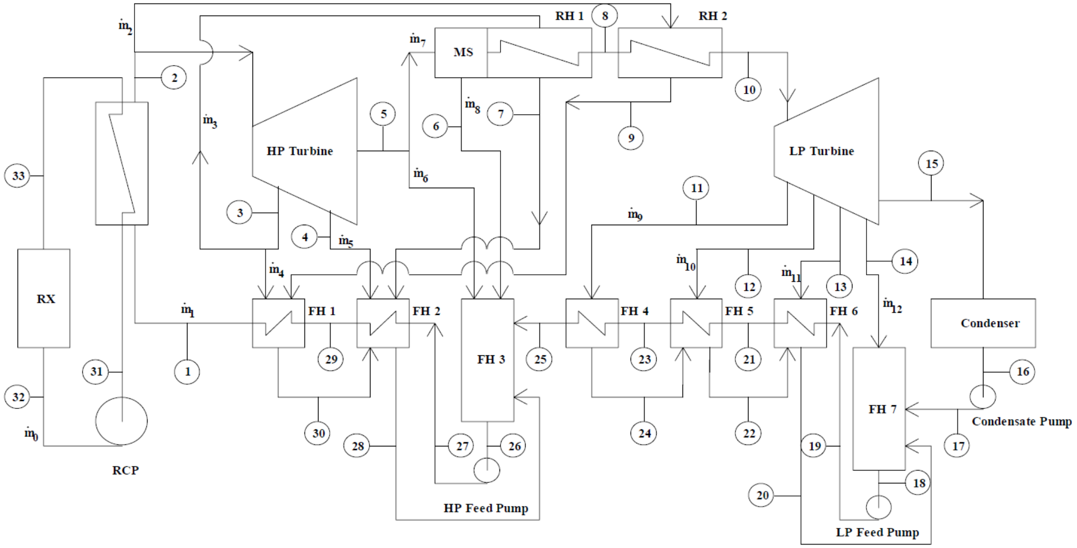
More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 2: evaluate the enthalpies of each state
    - $h_1 = h(P, T)$: subcooled liquid, $T_1 \rightarrow$ given
    - $h_2 = h_g(P)$: saturated steam
    - $h_{3,s} = h(P, s_2)$
    - $h_3 = f(h_2, h_{3,s}, \eta) \longleftarrow \eta_{HPT} = \frac{h_2 - h_x}{h_2 - h_{x,s}}$
    - h_5 : given
    - $h_6 = h_f(P)$: saturated liquid
    - $h_7 = h_f(P)$: saturated liquid
    - $h_8 = h(P, T)$: $T_8 \rightarrow$ given
    - $h_9 = h_f(P)$: saturated liquid
    - $h_{10} = h(P, T)$: $T_{10} \rightarrow$ given



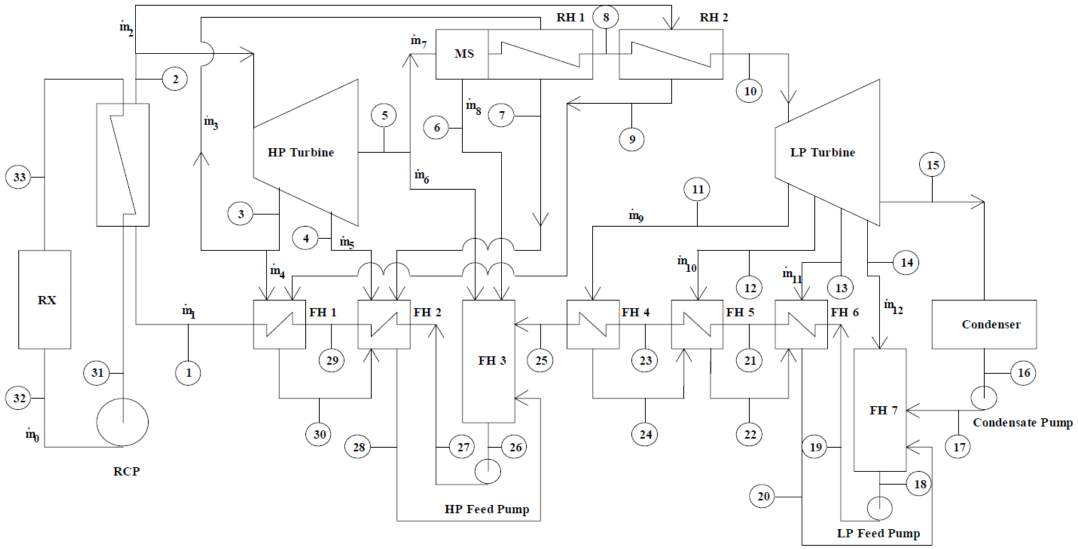
More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 2: evaluate the enthalpies of each state
    - – $h_{11,s} = h(P, s_{10})$
    - – $h_{11} \sim h_{14} = f(h_{10}, h_{11,s}, \eta)$ ⟵ $\eta_{LPT} = \dfrac{h_{10} - h_x}{h_{10} - h_{x,s}}$
    - – h_{15} : given
    - – $h_{16} = h_f(P)$: saturated liquid
    - – $h_{17,s} = h(P, s_{16})$
    - – $h_{17} = f(h_{16}, h_{17,s}, \eta)$: subcooled liquid, $\eta_{pump} \rightarrow$ given
    - – $h_{18} = h_f(P)$: saturated liquid
    - – $h_{19,s} = h(P, s_{18})$: saturated liquid
    - – $h_{19} = f(h_{18}, h_{19,s}, \eta)$
    - – $h_{20} = h_f(P)$: saturated liquid



More Complex Rankine Cycles

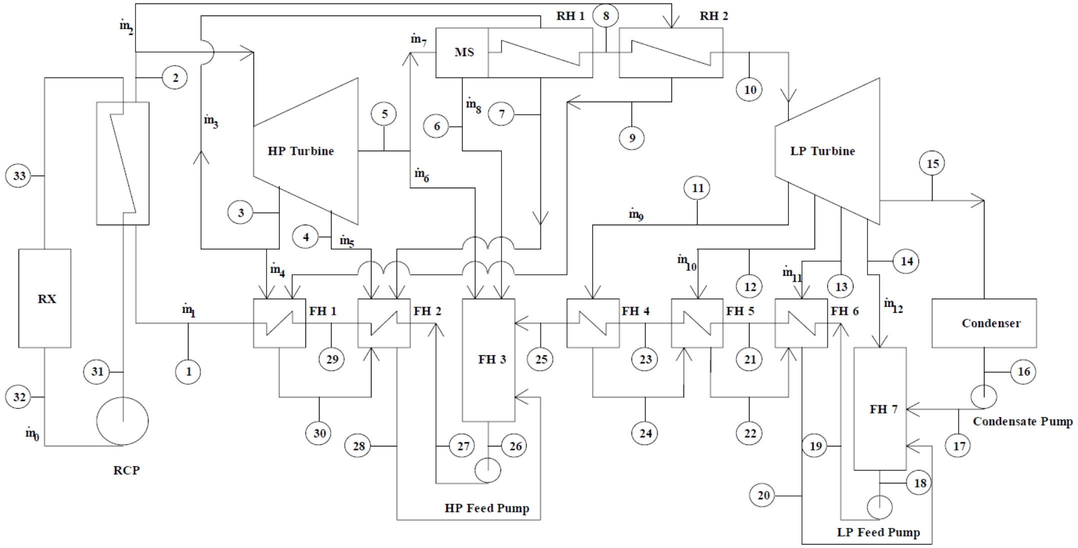
- More complex Rankine cycle analysis
  - Step 2: evaluate the enthalpies of each state
    - $h_{21} = h(P,T)$: subcooled liquid, $T_{21} \rightarrow$ given
    - $h_{22} = h_f(P)$: saturated liquid
    - $h_{23} = h(P,T)$: subcooled liquid, $T_{23} \rightarrow$ given
    - $h_{24} = h_f(P)$: saturated liquid
    - $h_{25} = h(P,T)$: subcooled liquid, $T_{25} \rightarrow$ given
    - $h_{26} = h_f(P)$: saturated liquid
    - $h_{27,s} = h(P,s_{26})$
    - $h_{27} = f(h_{26},h_{27,s},\eta)$: subcooled liquid, $\eta_{pump} \rightarrow$ given



More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 2: evaluate the enthalpies of each state
    - $h_{28} = h_f(P)$: saturated liquid
    - $h_{29} = h(P, T)$: subcooled liquid, $T_{29} \rightarrow$ given
    - $h_{30} = h_f(P)$: saturated liquid

    - $h_{32} = h(P, T)$: subcooled liquid, $T_{32} \rightarrow$ given
    - $h_{33} = h(P, T)$: subcooled liquid, $T_{33} \rightarrow$ given, $P_{33} = (P_{31} + P_{32})/2$
    - $w_{RCP,s} = v(P_{32} - P_{31})$
    - $w_{RCP} = w_{RCP,s}/\eta$
    - $h_{31} = h_{32} - w_{RCP}$



More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 2: evaluate the enthalpies of each state

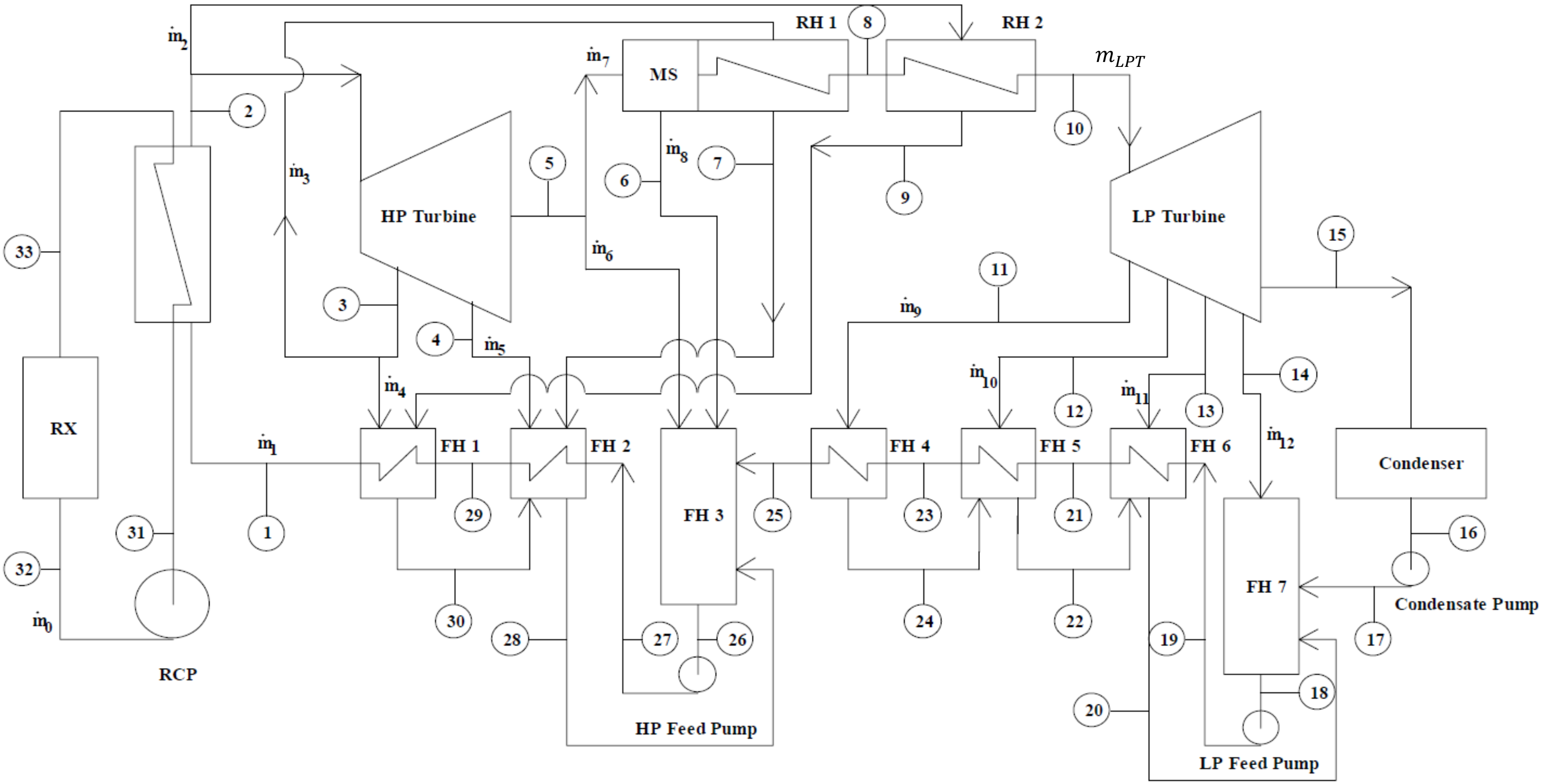
	h
1	975740.4
2	2773765.5
3	2644676.6
4	2590034.0
5	2530700.0
6	799286.7
7	992814.1
8	2854763.3
9	1262030.7
10	2976593.1

	h
11	2828965.9
12	2767621.9
13	2637452.8
14	2539523.6
15	2330700.0
16	140246.9
17	140300.9
18	331879.5
19	333473.2
20	428835.3

	h
21	415294.2
22	566217.1
23	537648.7
24	634466.5
25	618500.1
26	799286.7
27	807899.2
28	896155.4
29	879667.6
30	992814.1
5g	2783890.2

More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 3: evaluate the mass flow rates using mass and energy conservation equations



More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 3: evaluate the mass flow rates using mass and energy conservation equations
    - Feed Heater 1

$$\dot{m}_1 h_{29} + \dot{m}_2 h_9 + \dot{m}_4 h_3 = \dot{m}_1 h_1 + (\dot{m}_2 + \dot{m}_4) h_{30}$$

$$\tilde{\dot{m}}_2 (h_9 - h_{30}) + \tilde{\dot{m}}_4 (h_3 - h_{30}) = h_1 - h_{29} \qquad (1)$$

$$\text{where } \tilde{\dot{m}}_n \equiv \frac{\dot{m}_n}{\dot{m}_1}$$

- Feed Heater 2

$$\dot{m}_1 h_{27} + \dot{m}_3 h_7 + \dot{m}_5 h_4 + (\dot{m}_2 + \dot{m}_4) h_{30} = \dot{m}_1 h_{29} + (\dot{m}_2 + \dot{m}_3 + \dot{m}_4 + \dot{m}_5) h_{28}$$

$$\tilde{\dot{m}}_2 (h_{30} - h_{28}) + \tilde{\dot{m}}_3 (h_7 - h_{28}) + \tilde{\dot{m}}_4 (h_{30} - h_{28}) + \tilde{\dot{m}}_5 (h_4 - h_{28}) = h_{29} - h_{27} \qquad (2)$$

- Moisture Separator

$$\tilde{\dot{m}}_7 x_5 - \tilde{\dot{m}}_{LPT} = 0 \qquad \tilde{\dot{m}}_7 (1 - x_5) - \tilde{\dot{m}}_8 = 0$$

(3) (4)

More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 3: evaluate the mass flow rates using mass and energy conservation equations
    - Feed Heater 3

$$\dot{m}_{LPT}h_{25}+\dot{m}_8h_6+\dot{m}_6h_5+(\dot{m}_2+\dot{m}_3+\dot{m}_4+\dot{m}_5)h_{28}=\dot{m}_1h_{26}$$

$$\tilde{\dot{m}}_2h_{28}+\tilde{\dot{m}}_3h_{28}+\tilde{\dot{m}}_4h_{28}+\tilde{\dot{m}}_5h_{28}+\tilde{\dot{m}}_6h_5+\tilde{\dot{m}}_8h_6+\tilde{\dot{m}}_{LPT}h_{25}=h_{26} \qquad (5)$$

    - Reheater 1

$$\dot{m}_{LPT}h_{g_5}+\dot{m}_3h_3=\dot{m}_{LPT}h_8+\dot{m}_3h_7$$

$$\tilde{\dot{m}}_3(h_3-h_7)+\tilde{\dot{m}}_{LPT}(h_{g_5}-h_8)=0 \qquad (6)$$

    - Reheater 2

$$\dot{m}_{LPT}h_8+\dot{m}_2h_2=\dot{m}_{LPT}h_{10}+\dot{m}_2h_9$$

$$\tilde{\dot{m}}_2(h_2-h_9)+\tilde{\dot{m}}_{LPT}(h_8-h_{10})=0 \qquad (7)$$

More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 3: evaluate the mass flow rates using mass and energy conservation equations
    - Feed Heater 4

$$\dot{m}_{LPT}h_{23} + \dot{m}_9 h_{11} = \dot{m}_{LPT}h_{25} + \dot{m}_9 h_{24}$$

$$\tilde{\dot{m}}_{LPT}(h_{23} - h_{25}) + \tilde{\dot{m}}_9(h_{11} - h_{24}) = 0 \qquad (8)$$

    - Feed Heater 5

$$\dot{m}_{LPT}h_{21} + \dot{m}_{10}h_{12} + \dot{m}_9 h_{24} = \dot{m}_{LPT}h_{23} + (\dot{m}_9 + \dot{m}_{10})h_{22}$$

$$\tilde{\dot{m}}_{LPT}(h_{21} - h_{23}) + \tilde{\dot{m}}_9(h_{24} - h_{22}) + \tilde{\dot{m}}_{10}(h_{12} - h_{22}) = 0 \qquad (9)$$

    - Feed Heater 6

$$\dot{m}_{LPT}h_{19} + \dot{m}_{11}h_{13} + (\dot{m}_9 + \dot{m}_{10})h_{22} = \dot{m}_{LPT}h_{21} + (\dot{m}_9 + \dot{m}_{10} + \dot{m}_{11})h_{20}$$

$$\tilde{\dot{m}}_{LPT}(h_{19} - h_{21}) + \tilde{\dot{m}}_9(h_{22} - h_{20}) + \tilde{\dot{m}}_{10}(h_{22} - h_{20}) + \tilde{\dot{m}}_{11}(h_{13} - h_{20}) = 0 \qquad (10)$$

More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 3: evaluate the mass flow rates using mass and energy conservation equations
    - Feed Heater 7

$$(\dot{m}_{LPT} - \dot{m}_9 - \dot{m}_{10} - \dot{m}_{11} - \dot{m}_{12})h_{17} + \dot{m}_{12}h_{14} + (\dot{m}_9 + \dot{m}_{10} + \dot{m}_{11})h_{20} = \dot{m}_{LPT}h_{18}$$

$$\tilde{\dot{m}}_{LPT}(h_{17} - h_{18}) + \tilde{\dot{m}}_9(h_{20} - h_{17}) + \tilde{\dot{m}}_{10}(h_{20} - h_{17}) + \tilde{\dot{m}}_{11}(h_{20} - h_{17}) + \tilde{\dot{m}}_{12}(h_{14} - h_{17}) = 0 \qquad (11)$$

    - Mass balance on Feed Heater 3

$$\dot{m}_{LPT} + \dot{m}_2 + \dot{m}_3 + \dot{m}_4 + \dot{m}_5 + \dot{m}_6 + \dot{m}_8 = \dot{m}_1$$

$$\text{Since } \dot{m}_{LPT} = \dot{m}_7 x_5 \text{ and } \dot{m}_8 = \dot{m}_7(1 - x_5)$$

$$\dot{m}_2 + \dot{m}_3 + \dot{m}_4 + \dot{m}_5 + \dot{m}_6 + \dot{m}_7 = \dot{m}_1$$

$$\tilde{\dot{m}}_2 + \tilde{\dot{m}}_3 + \tilde{\dot{m}}_4 + \tilde{\dot{m}}_5 + \tilde{\dot{m}}_6 + \tilde{\dot{m}}_7 = 1 \qquad (12)$$

More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 3: evaluate the mass flow rates using mass and energy conservation equations
    - A linear system of 12 equations in 12 unknowns

m2	0.054
m3	0.029
m4	0.049
m5	0.035
m6	0.061
m7	0.772
m8	0.098
m9	0.025
m10	0.037
m11	0.021
m12	0.044
mlpt	0.673

More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 4: evaluate the turbine works and pump works
    - High pressure turbine

$$\dot{W}_{thp} = (\dot{m}_1 - \dot{m}_2)h_2 - (\dot{m}_3 + \dot{m}_4)h_3 - \dot{m}_5 h_4 - (\dot{m}_6 + \dot{m}_7)h_5$$

$$\dot{W}_{thp} / \dot{m}_1 = (1 - \tilde{\dot{m}}_2)h_2 - (\tilde{\dot{m}}_3 + \tilde{\dot{m}}_4)h_3 - \tilde{\dot{m}}_5 h_4 - (\tilde{\dot{m}}_6 + \tilde{\dot{m}}_7)h_5$$

    - Low pressure turbine

$$\dot{W}_{tlp} = \dot{m}_{LPT} h_{10} - \dot{m}_9 h_{11} - \dot{m}_{10} h_{12} - \dot{m}_{11} h_{13} - \dot{m}_{12} h_{14} - (\dot{m}_{LPT} - \dot{m}_9 - \dot{m}_{10} - \dot{m}_{11} - \dot{m}_{12})h_{15}$$

$$\dot{W}_{tlp} / \dot{m}_1 = \tilde{\dot{m}}_{LPT} h_{10} - \tilde{\dot{m}}_9 h_{11} - \tilde{\dot{m}}_{10} h_{12} - \tilde{\dot{m}}_{11} h_{13} - \tilde{\dot{m}}_{12} h_{14} - (\tilde{\dot{m}}_{LPT} - \tilde{\dot{m}}_9 - \tilde{\dot{m}}_{10} - \tilde{\dot{m}}_{11} - \tilde{\dot{m}}_{12})h_{15}$$

    - Condensate pump

$$\dot{W}_{cp} = (\dot{m}_{LPT} - \dot{m}_9 - \dot{m}_{10} - \dot{m}_{11} - \dot{m}_{12})w_{cp}$$

$$\dot{W}_{cp} / \dot{m}_1 = (\tilde{\dot{m}}_{LPT} - \tilde{\dot{m}}_9 - \tilde{\dot{m}}_{10} - \tilde{\dot{m}}_{11} - \tilde{\dot{m}}_{12})w_{cp}$$

More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 5: evaluate the turbine works and pump works
    - Low pressure feed pump

$$\dot{W}_{LPfp} = \dot{m}_{LPT} w_{LPfp}$$

$$\dot{W}_{LPfp} / \dot{m}_1 = \tilde{\dot{m}}_{LPT} w_{LPfp}$$

    - High pressure feed pump

$$\dot{W}_{HPfp} = \dot{m}_1 w_{HPfp}$$

$$\dot{W}_{HPfp} / \dot{m}_1 = w_{LPfp}$$

    - Reactor coolant pump

$$\dot{W}_{RCP} = \dot{m}_0 w_{RCP}$$

➢ Mass flow rate ratio between primary and secondary sides

$$\dot{m}_0 h_{33} + \dot{m}_1 h_1 = \dot{m}_0 h_{31} + \dot{m}_1 h_2$$

$$\frac{\dot{m}_0}{\dot{m}_1} = \frac{h_2 - h_1}{h_{33} - h_{31}}$$

More Complex Rankine Cycles

- More complex Rankine cycle analysis
  - Step 6: evaluate the cycle efficiency

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{reactor}} = \frac{\dot{W}_{thp} + \dot{W}_{tlp} + \dot{W}_{cp} + \dot{W}_{LPfp} + \dot{W}_{HPfp} + \dot{W}_{RCP}}{\dot{Q}_{reactor}}$$

$$= \frac{(\dot{W}_{thp} + \dot{W}_{tlp} + \dot{W}_{cp} + \dot{W}_{LPfp} + \dot{W}_{HPfp} + \dot{W}_{RCP}) / \dot{m}_1}{\dot{Q}_{reactor} / \dot{m}_1}$$

$$= 32.8\ \%$$