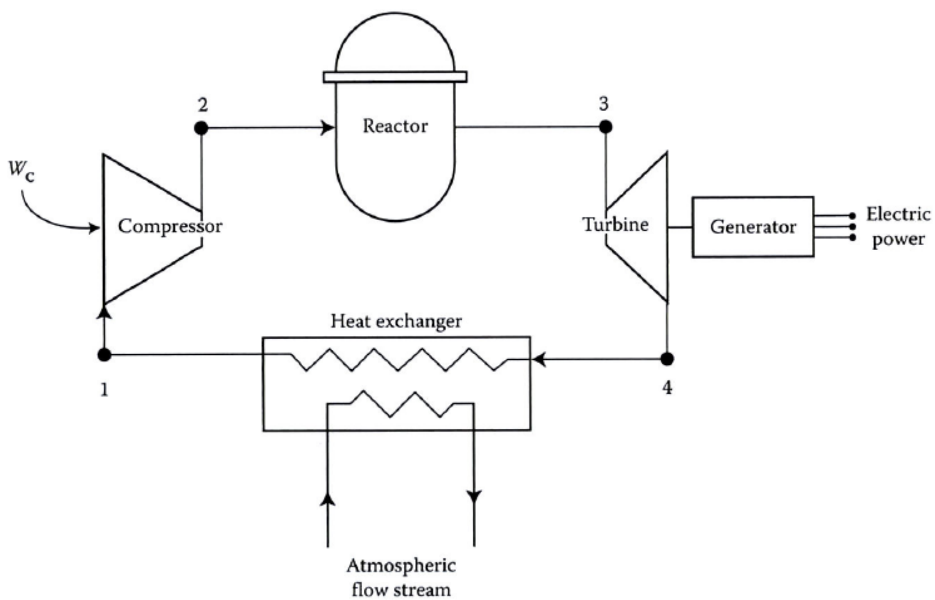
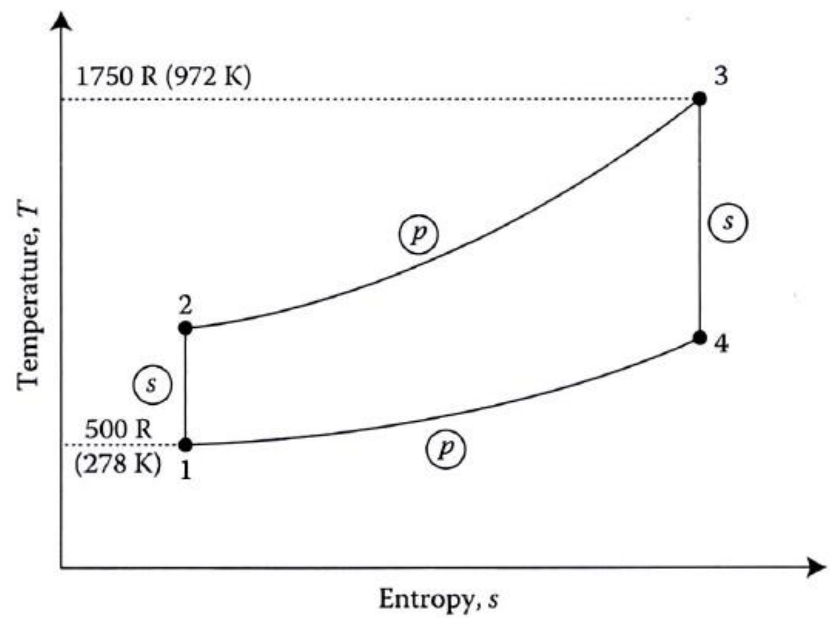
Contents

The engine cycle is named after George Brayton (1830–1892), the American engineer who developed it, although it was originally proposed and patented by Englishman John Barber in 1791. It is also sometimes known as the **Joule cycle**. The Ericsson cycle is similar to the Brayton cycle but uses external heat and incorporates the use of a regenerator. There are two types of Brayton cycles, open to the atmosphere and using internal combustion chamber or closed and using a heat exchanger.

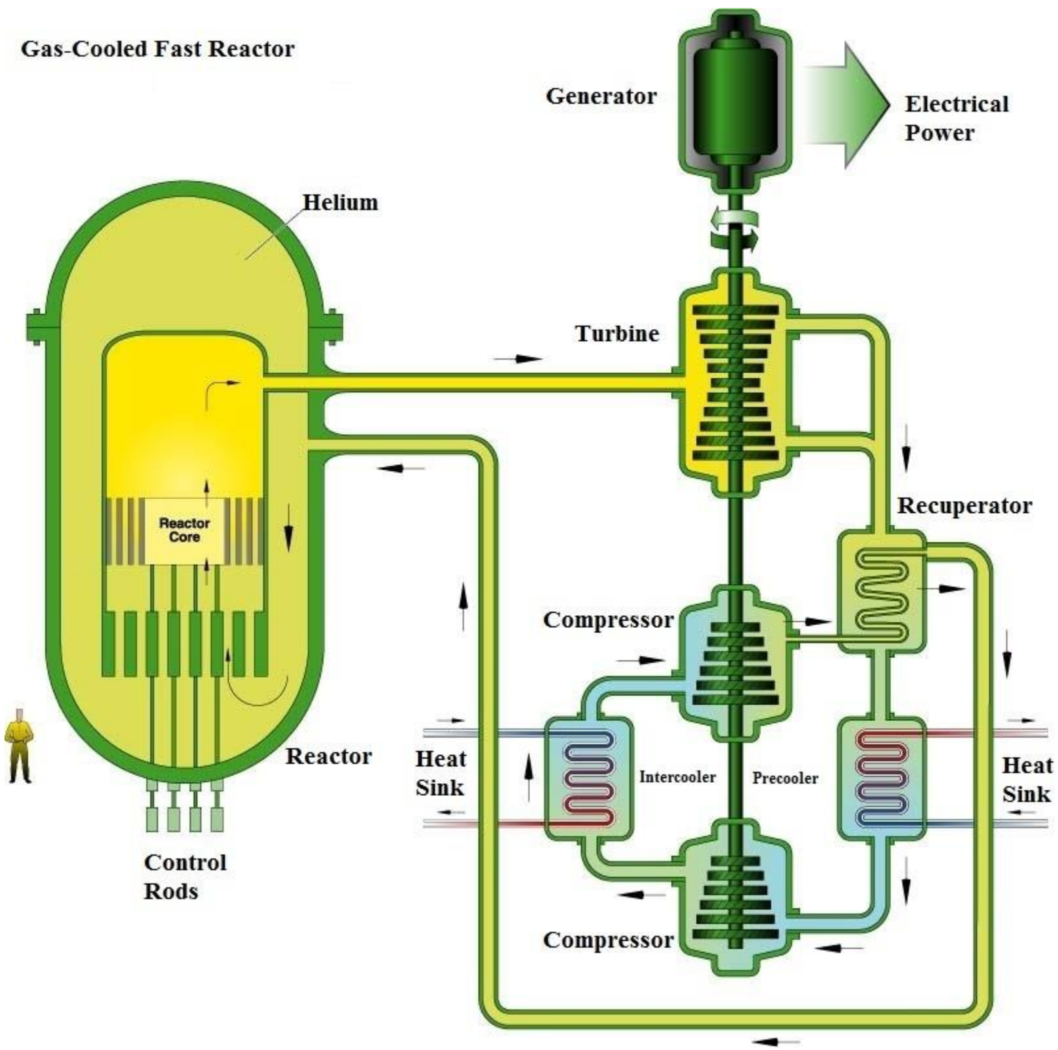
Simple Brayton Cycle

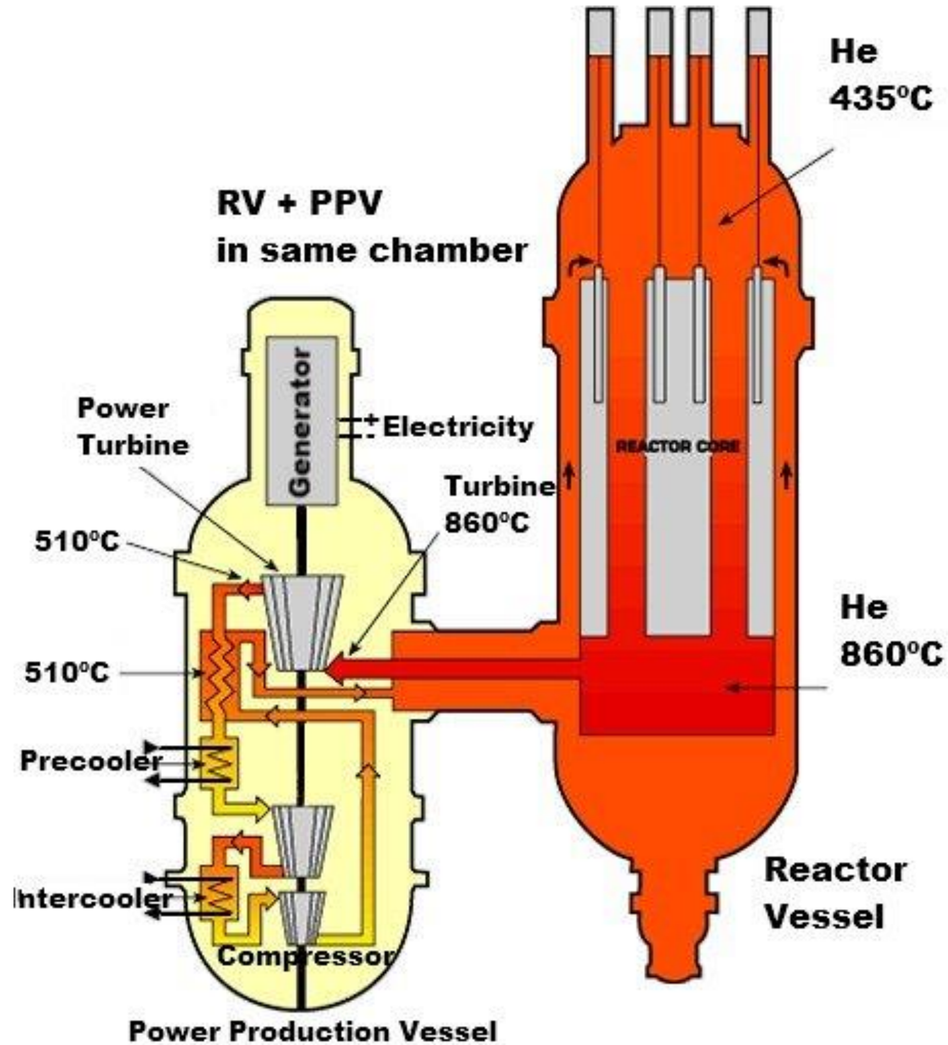
- Brayton Cycle
  - Reactor systems that employs gas coolants offer the potential for operating as direct Brayton cycle by passing the heated gas directly into a turbine.
  - Ideal for single-phase, steady-flow cycles with heat exchange and therefore is the basic cycle for modern gas turbine plants as well as proposed nuclear gas-cooled reactor plants.
  - The ideal cycle is composed of two reversible constant-pressure heat-exchange processes and two reversible, adiabatic work processes
  - The compressor work, or “backwork," is a larger fraction of the turbine work than is the pump work in a Rankine cycle.





- Brayton Cycle





Simple Brayton Cycle

EXAMPLE 7–12 Compressing a Substance in the Liquid versus Gas Phases

Determine the compressor work input required to compress steam isentropically from 100 kPa to 1 MPa, assuming that the steam exists as (*a*) saturated liquid and (*b*) saturated vapor at the inlet state.

(*a*)

$$v_1 = v_{f @ 100\,\text{kPa}} = 0.001043\ \text{m}^3/\text{kg} \qquad \text{(Table A–5)}$$

$$\begin{aligned} w_{\text{rev,in}} &= \int_1^2 v\,dP \cong v_1(P_2 - P_1) \\ &= (0.001043\ \text{m}^3/\text{kg})[(1000 - 100)\ \text{kPa}]\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) \\ &= \mathbf{0.94\ kJ/kg} \end{aligned}$$

(*b*)

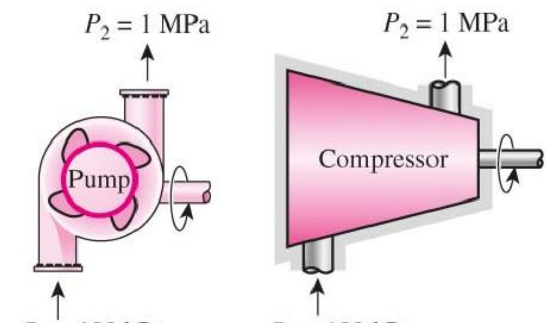
$$w_{\text{rev,in}} = \int_1^2 v\,dP = \int_1^2 dh = h_2 - h_1$$

$$\left.\begin{aligned} T\,ds &= dh - v\,dP \quad \text{(Eq. 7-24)} \\ ds &= 0 \quad \text{(isentropic process)} \end{aligned}\right\} \quad v\,dP = dh$$

$$\textit{State 1:} \quad \left.\begin{aligned} P_1 &= 100\ \text{kPa} \\ &\text{(sat. vapor)} \end{aligned}\right\} \quad \begin{aligned} h_1 &= 2675.0\ \text{kJ/kg} \\ s_1 &= 7.3589\ \text{kJ/kg}\cdot\text{K} \end{aligned} \quad \text{(Table A–5)}$$

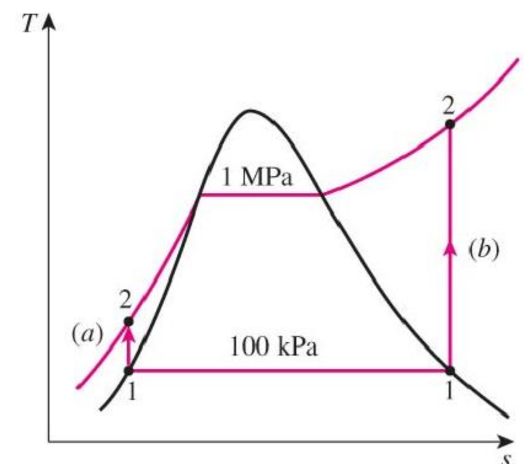
$$\textit{State 2:} \quad \left.\begin{aligned} P_2 &= 1\ \text{MPa} \\ s_2 &= s_1 \end{aligned}\right\} \quad h_2 = 3194.5\ \text{kJ/kg} \quad \text{(Table A–6)}$$

$$w_{\text{rev,in}} = (3194.5 - 2675.0)\ \text{kJ/kg} = \mathbf{519.5\ kJ/kg}$$



(*a*) Compressing a liquid

(*b*) Compressing a vapor



- Brayton Cycle Analysis
  - Pressure or compression ratio of the cycle

$$r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

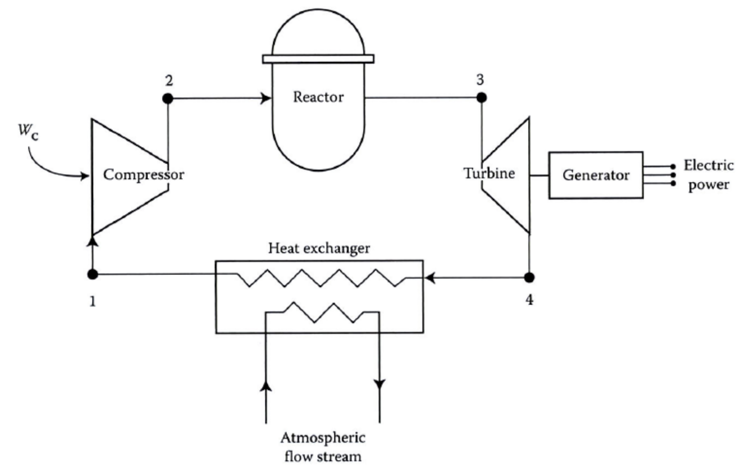
  - For isentropic processes with a perfect gas, constant c_p

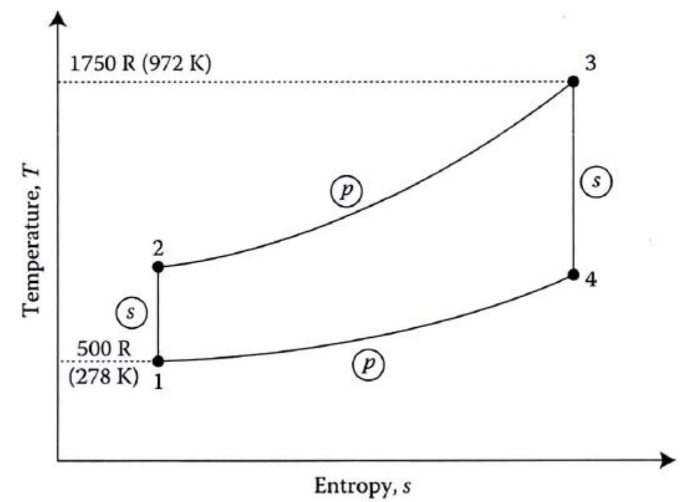
$$\left(\frac{T_2}{T_1}\right)_s = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = \left(\frac{v_1}{v_2}\right)^{(\gamma-1)} \quad ,\gamma \equiv c_p / c_v$$

$$Tv^{\gamma-1} = c \qquad Tp^{\frac{\gamma-1}{\gamma}} = c$$

  - For a perfect gas, because enthalpy is a function of temperature only and the specific heats are constant.

$$\Delta h = c_p \Delta T$$





Simple Brayton Cycle

- Brayton Cycle Analysis
  - Entropy change of ideal gas

From the first *T ds* relation

$$ds = \frac{du}{T} + \frac{P\,dv}{T} \qquad du = c_v\,dT \qquad P = RT/v$$

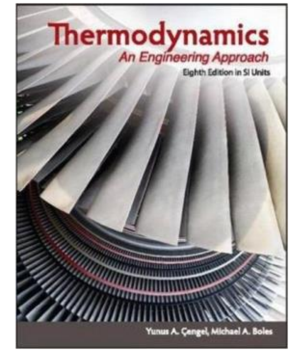
$$ds = c_v \frac{dT}{T} + R\frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{v,\mathrm{avg}} \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

From the second T ds relation

$$ds = \frac{dh}{T} - \frac{v\,dP}{T}$$



Y. A. Cengel

$$dh = c_p\,dT \qquad v = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

$$s_2 - s_1 = c_{p,\mathrm{avg}} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$(\mathrm{kJ/kg \cdot K})$$

- Brayton Cycle Analysis
  - Isentropic Processes of Ideal Gases

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \qquad R = c_p - c_v \;,\; \gamma = c_p / c_v ,\; R/c_v = \gamma - 1$$

$$0 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \Rightarrow \quad \ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1} = \ln \left(\frac{v_1}{v_2} \right)^{R/c_v}$$

$$\left(\frac{T_2}{T_1} \right) = \left(\frac{v_1}{v_2} \right)^{\gamma - 1}$$

For isentropic process, ideal gas

$$s_2 - s_1 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \qquad R = c_p - c_v \;,\; \gamma = c_p / c_v ,\; R/c_p = 1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma}$$

$$0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \Rightarrow \quad \ln \frac{T_2}{T_1} = \frac{R}{c_p} \ln \frac{P_2}{P_1} = \ln \left(\frac{P_2}{P_1} \right)^{\frac{R}{c_p}} = \ln \left(\frac{P_2}{P_1} \right)^{(\gamma - 1)/\gamma}$$

$$\left(\frac{T_2}{T_1} \right) = \left(\frac{P_2}{P_1} \right)^{(\gamma - 1)/\gamma}$$

For isentropic process, ideal gas

- Brayton Cycle Analysis
  - Isentropic Processes of Ideal Gases

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \qquad \left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \qquad \Rightarrow \qquad \left(\frac{P_2}{P_1}\right) = \left(\frac{v_1}{v_2}\right)^{\gamma}$$

$$Tv^{\gamma-1} = \text{constant}$$

$$TP^{(1-\gamma)/\gamma} = \text{constant}$$

$$Pv^{\gamma} = \text{constant}$$

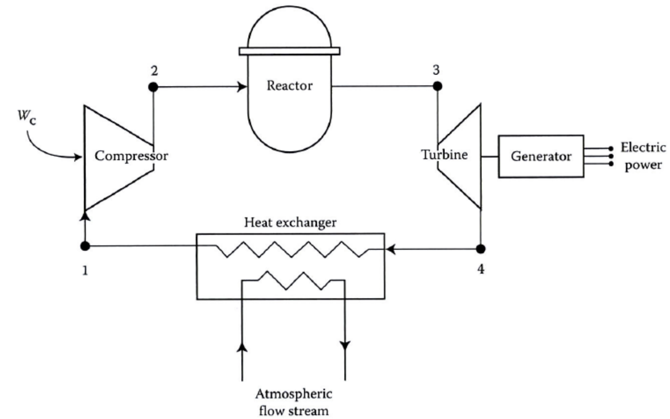
Valid for
- Ideal gas
- Isentropic process
- Constant specific heats

Simple Brayton Cycle

- Brayton Cycle Analysis
  - Turbine work

$$\dot{W}_T = \dot{m}c_p(T_3 - T_4) = \dot{m}c_pT_3\left(1 - \frac{T_4}{T_3}\right) = \dot{m}c_pT_3\left[1 - \frac{1}{(r_p)^{\gamma-1/\gamma}}\right]$$

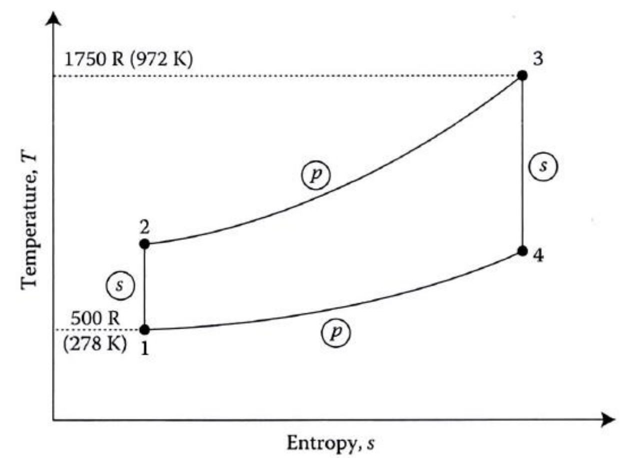
$$\left(\frac{T_4}{T_3}\right) = \left(\frac{P_4}{P_3}\right)^{(\gamma-1)/\gamma} \qquad r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4}$$



  - Compressor work

$$\dot{W}_{CP} = \dot{m}c_p(T_2 - T_1) = \dot{m}c_pT_1\left[\frac{T_2}{T_1} - 1\right] = \dot{m}c_pT_1[(r_p)^{\gamma-1/\gamma} - 1]$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \qquad r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4}$$



Simple Brayton Cycle

- Brayton Cycle Analysis
  - The heat input from the reactor

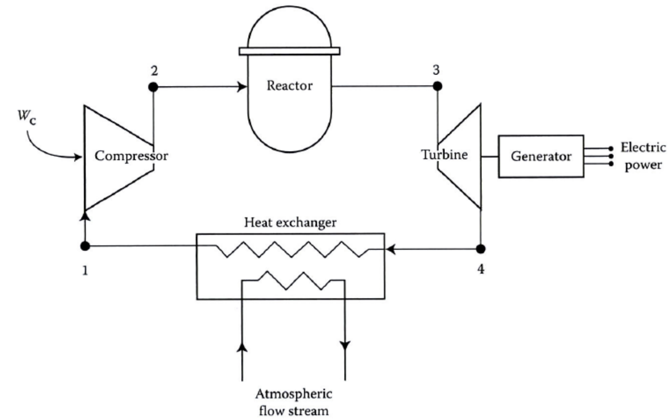
$$\dot{Q}_R = \dot{m}c_p(T_3 - T_2) = \dot{m}c_pT_1\left[\frac{T_3}{T_1} - (r_p)^{\gamma-1/\gamma}\right]$$

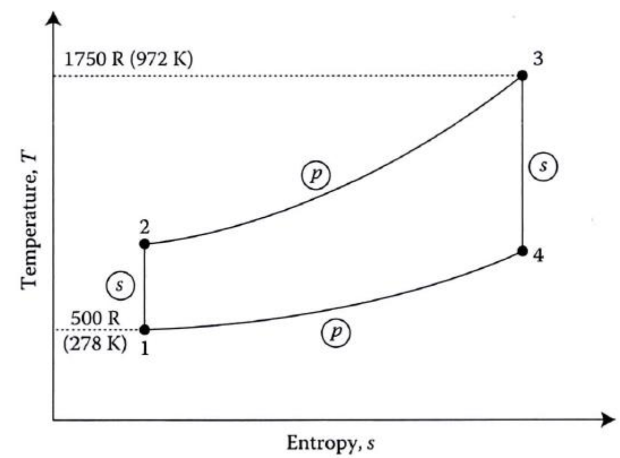
$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \qquad r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

  - The heat rejected by the heat exchanger

$$\dot{Q}_{HX} = \dot{m}c_p(T_4 - T_1) = \dot{m}c_pT_3\left[\frac{1}{(r_p)^{\gamma-1/\gamma}} - \frac{T_1}{T_3}\right]$$

$$\left(\frac{T_4}{T_3}\right) = \left(\frac{P_4}{P_3}\right)^{(\gamma-1)/\gamma} \qquad r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

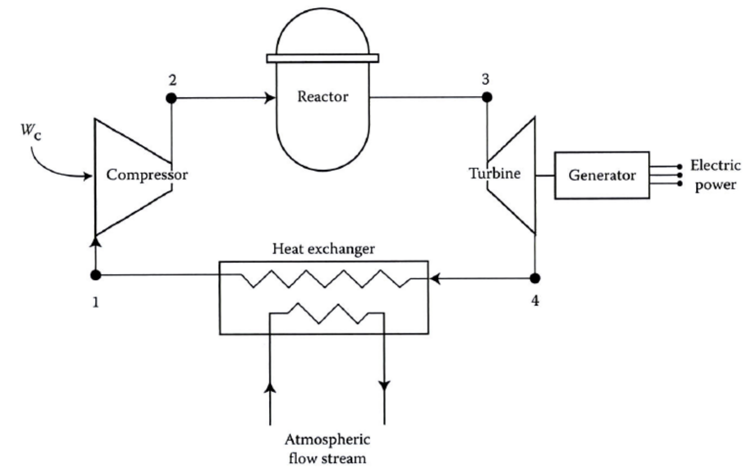




Simple Brayton Cycle

- Brayton Cycle Analysis
  - Maximum useful work

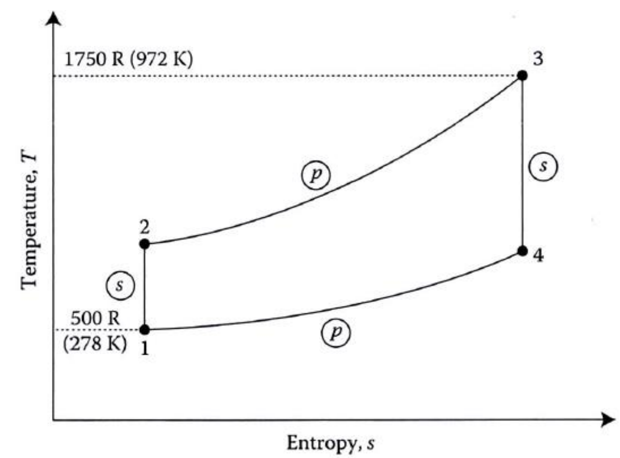
$$\dot{W}_{u,\,max} \equiv \dot{Q}_R = \dot{m}c_p T_1\left[\frac{T_3}{T_1} - (r_p)^{\gamma-1/\gamma}\right]$$



  - Thermodynamic efficiency

$$\zeta = \frac{\dot{W}_T - \dot{W}_{CP}}{\dot{W}_{u,max}} = \frac{T_3\left[1-(1/(r_p)^{\gamma-1/\gamma})\right] - T_1(r_p)^{\gamma-1/\gamma}\left[1-(1/(r_p)^{\gamma-1/\gamma})\right]}{T_1\left[(T_3/T_1)-(r_p)^{\gamma-1/\gamma}\right]}$$

$$= 1 - \frac{1}{(r_p)^{\gamma-1/\gamma}}$$



  - Optimum pressure ratio for maximum net work

$$(r_p)_{optimum} = \left(\frac{T_3}{T_1}\right)^{\gamma/2(\gamma-1)}$$

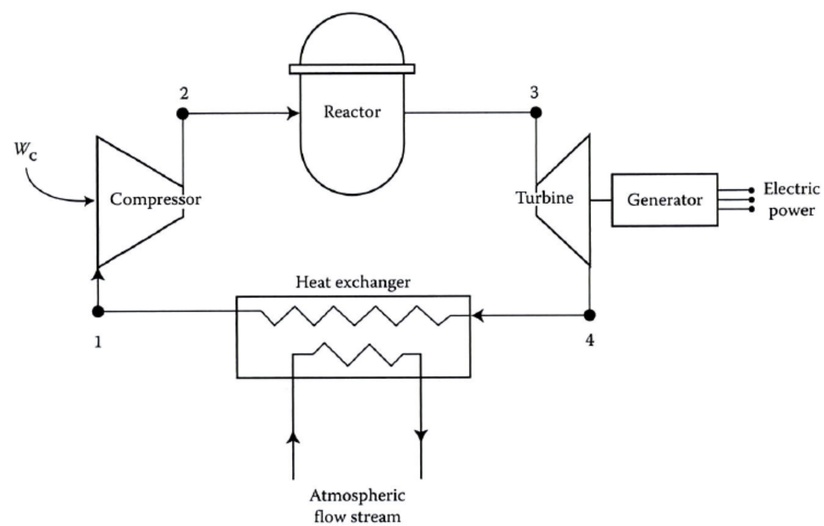
$$(W_T - W_{CP})' = 0$$

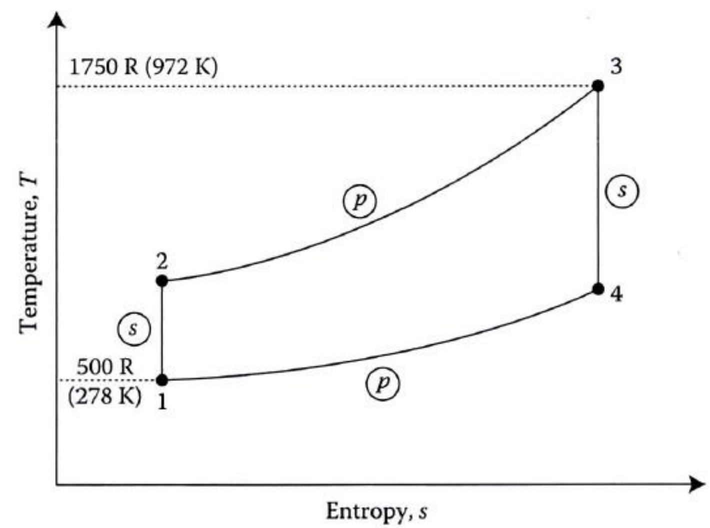
Example 6.7: First Law Thermodynamic Analysis of a Simple Brayton Cycle

PROBLEM

Compute the cycle efficiency for the simple Brayton cycle of Figures 6.8 and 6.24 for the following conditions:

1. Helium as the working fluid taken as a perfect gas with
   $c_p = 1.25$ Btu/lb °R (5230 J/kg K)
   $\gamma = 1.658$
   $\dot{m}$ in lb/s (English units) or kg/s (SI units)
2. Pressure ratio of 4.0
3. Maximum and minimum temperatures of 1750°R (972 K) and 500°R (278 K), respectively





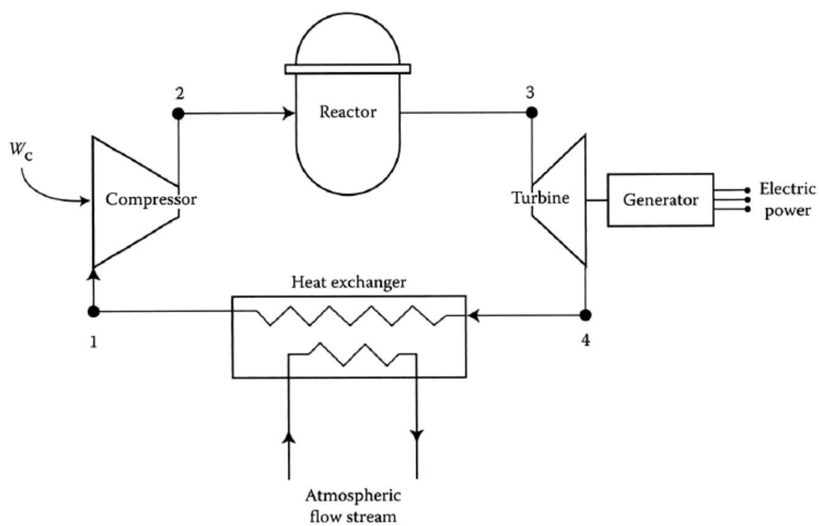
Example 6.7: First Law Thermodynamic Analysis of a Simple Brayton Cycle

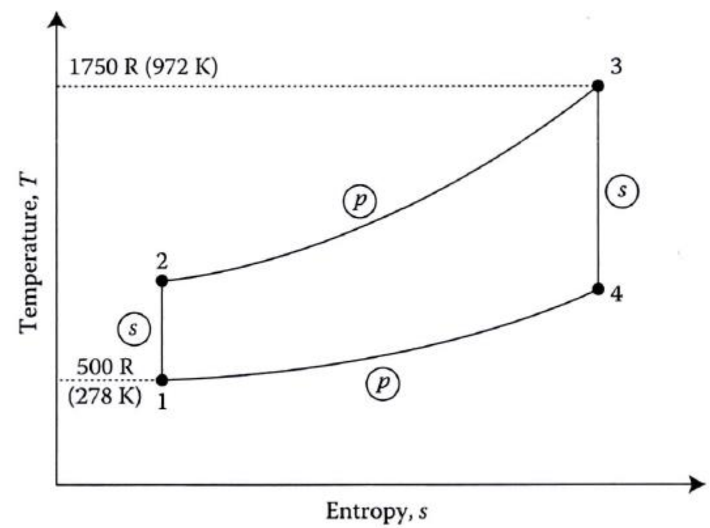
SOLUTION

$$\zeta = \frac{\dot{W}_T - \dot{W}_{CP}}{\dot{W}_{u,max}} = \frac{T_3\left[1-(1/(r_p)^{\gamma-1/\gamma})\right] - T_1(r_p)^{\gamma-1/\gamma}\left[1-(1/(r_p)^{\gamma-1/\gamma})\right]}{T_1\left[(T_3/T_1)-(r_p)^{\gamma-1/\gamma}\right]}$$

$$= \frac{(\dot{W}_T - \dot{W}_{CP})/\dot{m}}{\dot{W}_{u,max}/\dot{m}}$$

$r_p = 4 \quad \gamma = 1.658 \quad T_1 = 278\ K \quad T_3 = 972\ K$

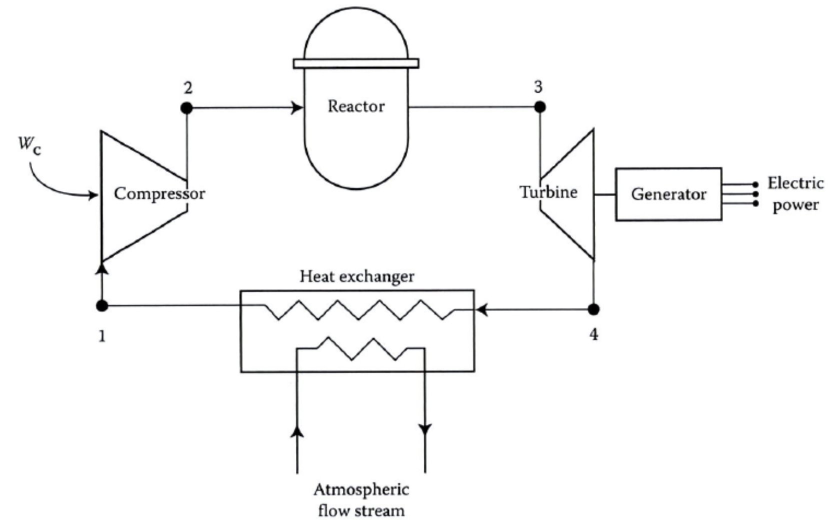




Example 6.7: First Law Thermodynamic Analysis of a Simple Brayton Cycle

$$\frac{\dot{W}_T}{\dot{m}} = c_p T_3\left[1 - \frac{1}{(r_p)^{\gamma-1/\gamma}}\right] = 1.25\ (1750)\left[1 - \frac{1}{(4.0)^{0.397}}\right]$$

$$= 925.9\,\text{Btu/lb}\ (2.150\,\text{MJ/kg})$$



$$\frac{\dot{W}_{CP}}{\dot{m}} = c_p T_1[(r_p)^{\gamma-1/\gamma} - 1] = 1.25\ (500)\ [(4.0)^{0.397} - 1]$$

$$= 458.67\,\text{Btu/lb}\ (1.066\,\text{MJ/kg})$$

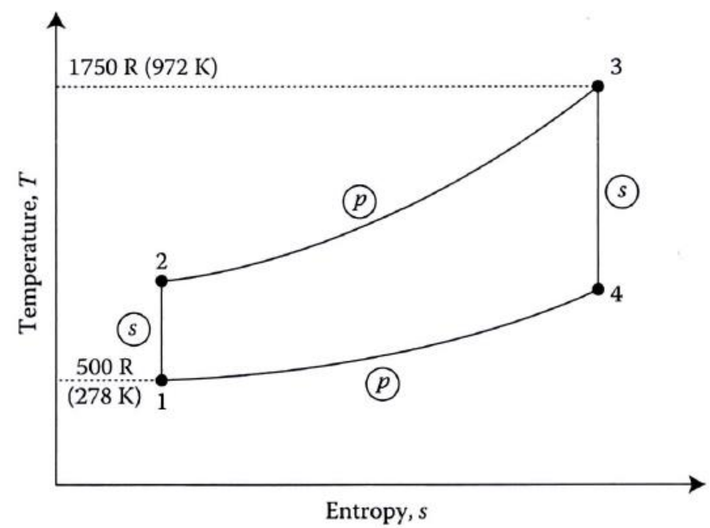
$$\frac{\dot{W}_{u,\max}}{\dot{m}} = c_p T_1\left[\frac{T_3}{T_1} - (r_p)^{\gamma-1/\gamma}\right] = 1.25\ (500)\left[\frac{1750}{500} - (4.0)^{0.397}\right]$$

$$= 1103.8\,\text{Btu/lb}\ (2.560\,\text{MJ/kg})$$

$$\zeta = \frac{(\dot{W}_T - \dot{W}_{CP})/\dot{m}}{\dot{W}_{u,\max}/\dot{m}} = \left(\frac{925.9 - 458.7}{1103.8}\right) 100\ \text{(English units)}$$

$$= \left(\frac{2.15 - 1.066}{2.56}\right) 100\ \text{(SI units)}$$

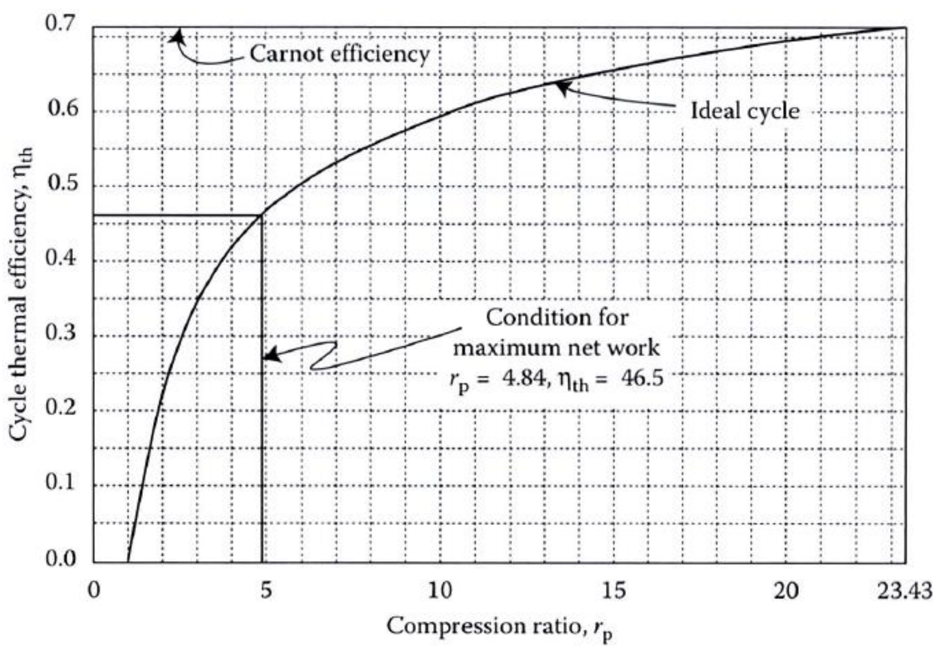
$$= 42.3\%$$



Example 6.7: First Law Thermodynamic Analysis of a Simple Brayton Cycle

$$\eta_{Carnot} = \frac{T_{max} - T_{min}}{T_{max}} = \frac{972K - 278K}{972K} = 0.714$$

$$(r_p)_{optimum} = \left(\frac{T_3}{T_1}\right)^{\gamma/2(\gamma-1)} = \left(\frac{972}{278}\right)^{1.658/2(0.658)} = 4.84$$



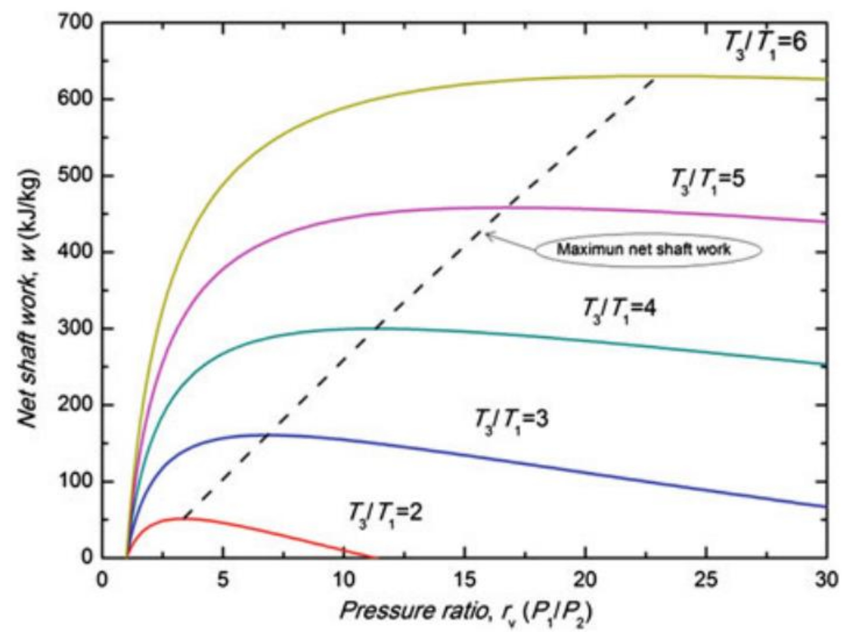
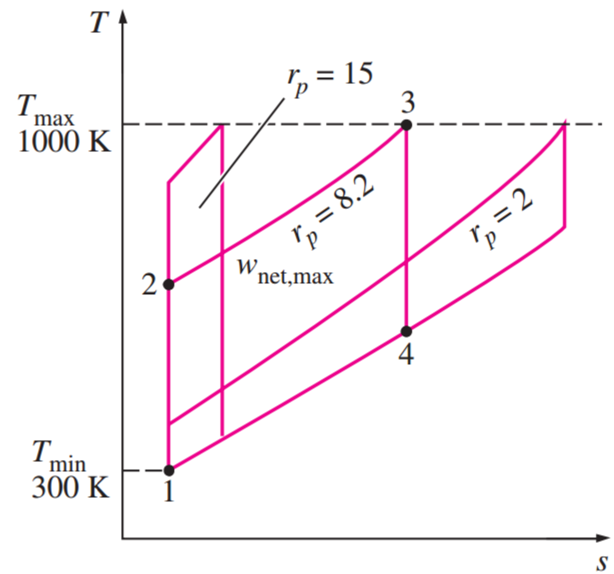


FIGURE 6.25 Thermal efficiency of an ideal Brayton cycle as a function of the compression ratio. $\gamma = 1.658$.

Example 6.7: First Law Thermodynamic Analysis of a Simple Brayton Cycle

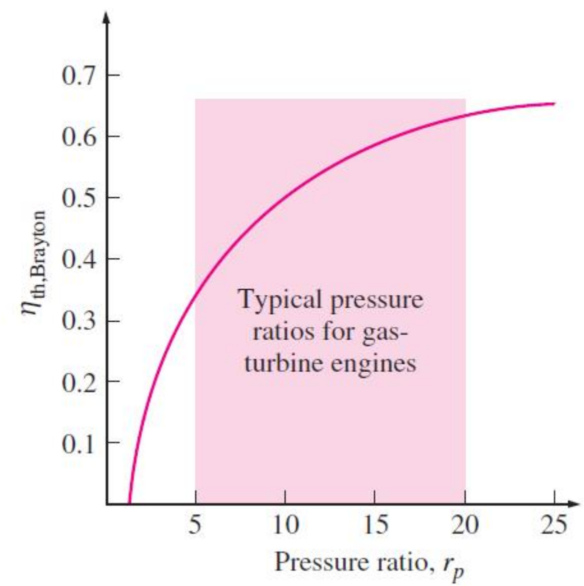


The highest temperature in the cycle occurs at the end of the combustion process (state 3), and it is limited by the maximum temperature that the turbine blades can withstand. This also limits the pressure ratios that can be used in the cycle.

For a fixed turbine inlet temperature *T*3, the net work output per cycle increases with the pressure ratio, reaches a maximum, and then starts to decrease. Therefore, there should be a compromise between the pressure ratio (thus the thermal efficiency) and the net work output.

With less work output per cycle, a larger mass flow rate (thus a larger system) is needed to maintain the same power output, which may not be economical.

In most common designs, the pressure ratio of gas turbines ranges from about 11 to 16.



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Example 6.8: Brayton Cycle with Real Components

Compute the thermal efficiency for the cycle depicted in Figure 6.26 if the isentropic efficiencies of the compressor and the turbine are each 90%. All other conditions of Example 6.7 apply.

SOLUTION

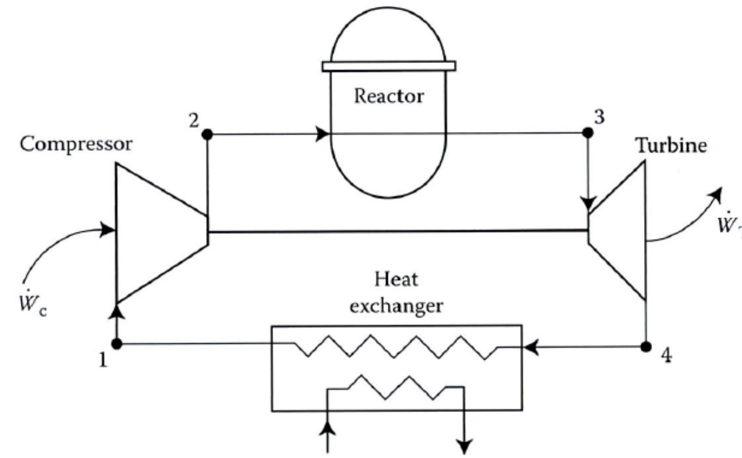
For $\dot{W}_T$:

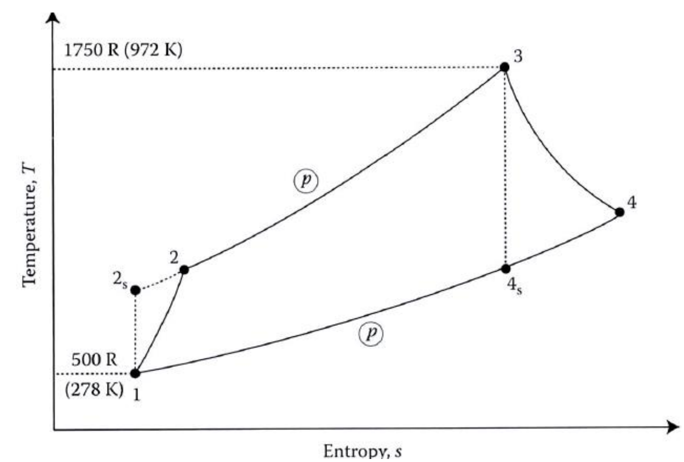
$$\eta_T = \frac{\text{Actual work out of turbine}}{\text{Ideal turbine work}} = \frac{W_T}{\dot{W}_{Ti}} = \frac{\dot{m}c_p(T_3 - T_4)}{\dot{m}c_p(T_3 - T_{4s})}$$

$$\therefore \dot{W}_T = \eta_T\dot{W}_{Ti} = \eta_T\dot{m}c_p(T_3 - T_{4s}) = \eta_T\dot{m}c_pT_3\left(1 - \frac{T_{4s}}{T_3}\right)$$

$$= \eta_T\dot{m}c_pT_3\left[1 - \frac{1}{(r_p)^{\gamma-1/\gamma}}\right] = \eta_T\dot{m}\ 925.9 = (0.9)(925.9)\ \dot{m}$$

$$= 833.3\ \dot{m}\ \text{Btu/s}\ (1.935\ \dot{m}\ \text{MJ/s or MW})$$





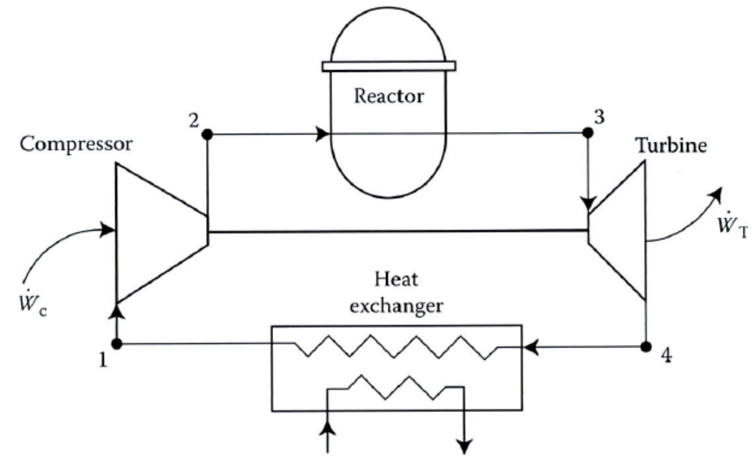
$$\left(\frac{T_4}{T_3}\right) = \left(\frac{P_4}{P_3}\right)^{(\gamma-1)/\gamma} \qquad r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

More Complex Brayton Cycles

Example 6.8: Brayton Cycle with Real Components

Compute the thermal efficiency for the cycle depicted in Figure 6.26 if the isentropic efficiencies of the compressor and the turbine are each 90%. All other conditions of Example 6.7 apply.

SOLUTION

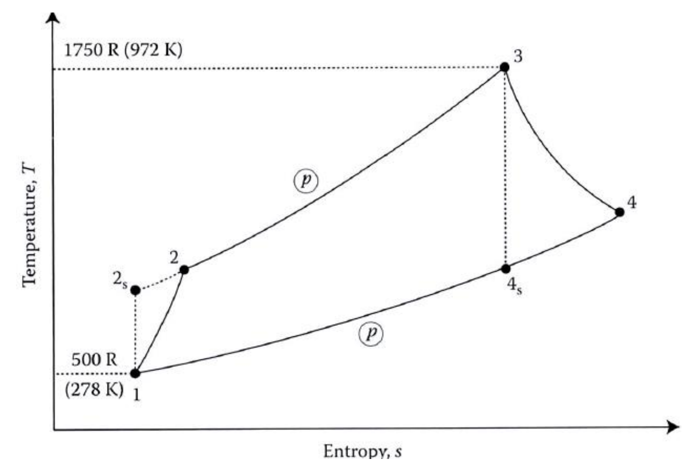


For $\dot{W}_{CP}$:

$$\eta_{CP} = \frac{\text{Ideal compressor work}}{\text{Actual compressor work}} = \frac{\dot{W}_{CPi}}{\dot{W}_{CP}} = \frac{\dot{m}c_p(T_{2s} - T_1)}{\dot{m}c_p(T_2 - T_1)}$$

$$\dot{W}_{CP} = \frac{\dot{m}}{\eta_{CP}} c_p(T_{2s} - T_1) = \frac{\dot{m}}{\eta_{CP}} c_p T_1\left(\frac{T_{2s}}{T_1} - 1\right) = \frac{\dot{m}\,458.7}{0.9}$$

$$= 509.7\ \dot{m}\text{ Btu/s } (1.184\ \dot{m}\text{ MW})$$



$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}$$

$$\dot{W}_{NET} = \dot{W}_T - \dot{W}_{CP} = \dot{m}\ (833.3 - 509.7)$$

$$= 323.6\ \dot{m}\text{ Btu/s } (0.752\ \dot{m}\text{ MW})$$

Example 6.8: Brayton Cycle with Real Components

Compute the thermal efficiency for the cycle depicted in Figure 6.26 if the isentropic efficiencies of the compressor and the turbine are each 90%. All other conditions of Example 6.7 apply.

SOLUTION

$$\dot{Q}_R = \dot{m}c_p(T_3 - T_2)$$

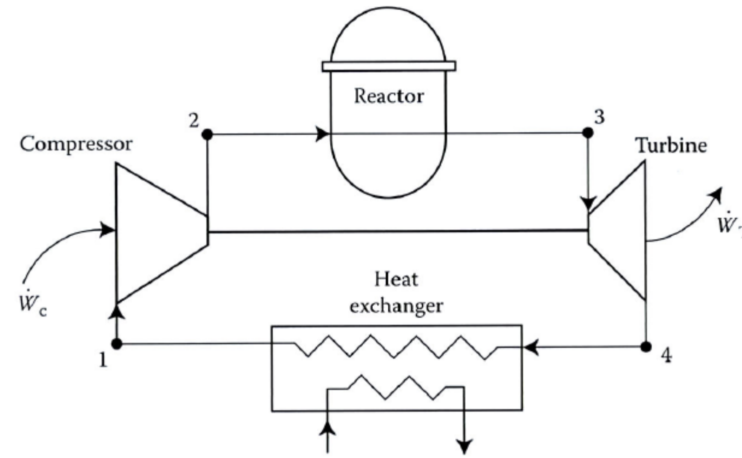
$$\dot{W}_{CPi} = \dot{m}c_p(T_{2s} - T_1) = 458.7\ \dot{m}\ \text{Btu/s} = 1.066\ \dot{m}\ \text{MW},$$

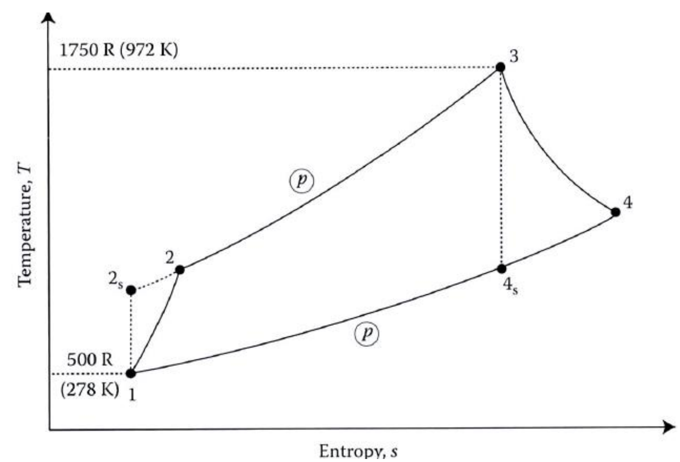
$$T_2 - T_1 = \frac{\dot{W}_{CP}}{\dot{m}c_p} = \frac{\dot{W}_{CPi}}{\dot{m}c_p\eta_{CP}} = \frac{458.7}{1.25(0.9)} = 407.7°\text{R}\ (226.5\text{K})$$

$$T_2 = 407.7 + T_1 = 407.7 + 500 = 907.7°\text{R}\ (504.3\ \text{K})$$

$$\dot{Q}_R = (1.25)(T_3 - T_2)\ \dot{m} = 1.25(1750 - 907.7)\dot{m} = 1052.9\ \dot{m}\ \text{Btu/s}\ (2.45\ \dot{m}\ \text{MW})$$

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} = \left(\frac{0.752}{2.45}\right) 100\ (\text{SI units}) = 30.7\%$$





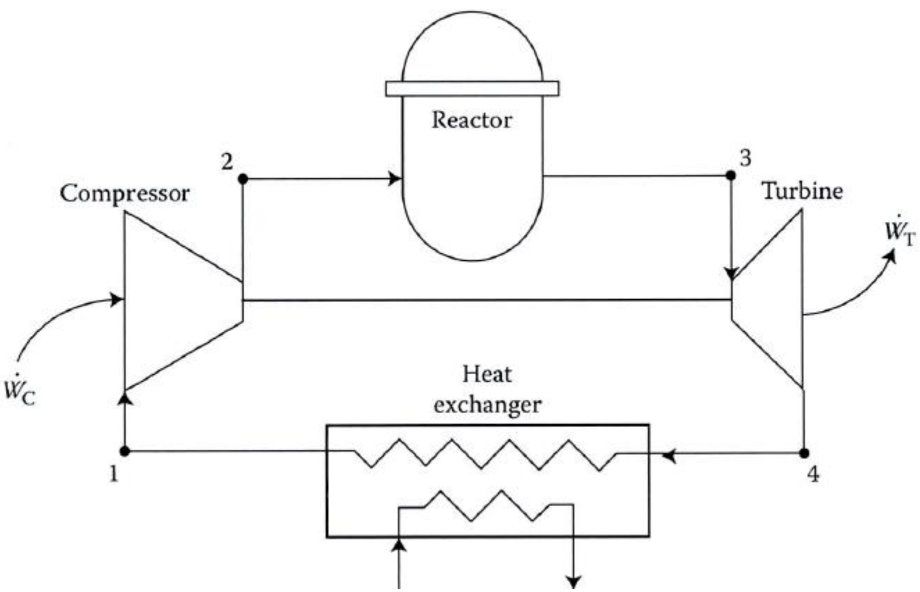
More Complex Brayton Cycles

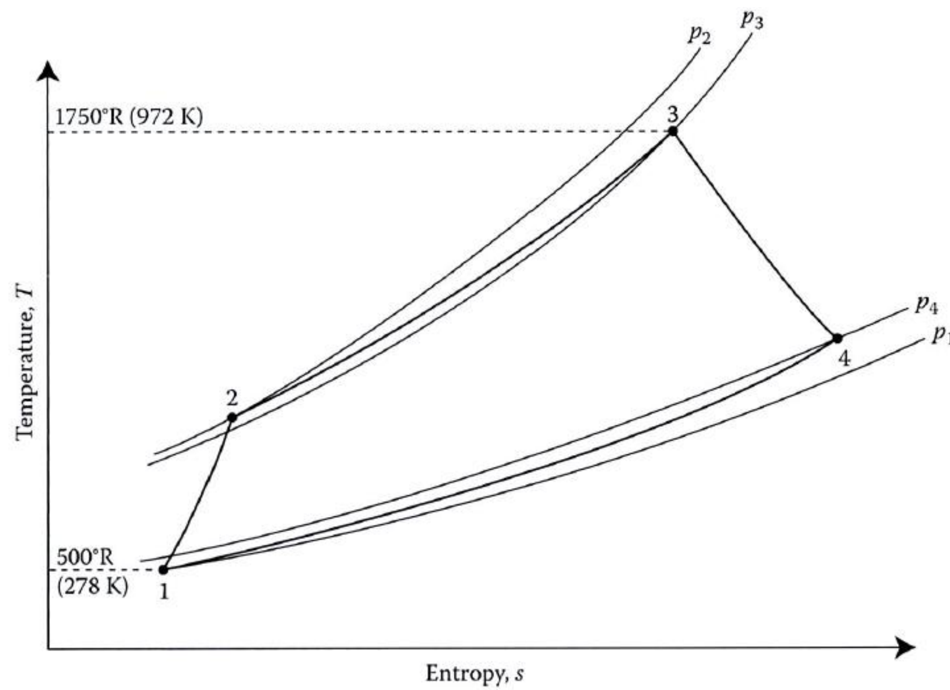
Example 6.9: Brayton Cycle Considering Duct Pressure Losses

PROBLEM Compute the cycle thermal efficiency considering pressure losses in the reactor and heat exchanger processes as well as 90% isentropic turbine and compressor efficiencies. The cycle is illustrated in Figure 6.27. The pressure losses are characterized by the parameter β where

$$\beta \equiv \qquad =$$

All other conditions of Example 6.7 apply.





Example 6.9: Brayton Cycle Considering Duct Pressure Losses

SOLUTION

$$\dot{W}_T = \eta_T \dot{m} c_p T_3 \left(1 - \frac{T_{4s}}{T_3}\right) = \eta_T \dot{m} c_p T_3 \left[1 - \frac{1}{(p_3/p_4)^{\gamma-1/\gamma}}\right]$$

$$\left(\frac{T_4}{T_3}\right) = \left(\frac{P_4}{P_3}\right)^{(\gamma-1)/\gamma}$$

$$r_p \equiv \frac{p_2}{p_1} \neq \frac{p_3}{p_4}$$

Because β is defined as $\left(\frac{p_4}{p_1} \cdot \frac{p_2}{p_3}\right)^{\gamma-1/\gamma}$

$$\beta \equiv \left(\frac{p_4}{p_1}\frac{p_2}{p_3}\right)^{\gamma-1/\gamma} = 1.05$$

$$\therefore \left(\frac{p_4}{p_3}\right)^{\gamma-1/\gamma} = \frac{(p_4/p_1 \cdot p_2/p_3)^{\gamma-1/\gamma}}{(p_2/p_1)^{\gamma-1/\gamma}} = \frac{\beta}{(r_p)^{\gamma-1/\gamma}}$$

$$\therefore \dot{W}_T = \eta_T \dot{m} c_p T_3 \left[1 - \frac{\beta}{(r_p)^{\gamma-1/\gamma}}\right] = 0.9\,\dot{m}(1.25)(1750)\left[1 - \frac{1.05}{(4)^{0.397}}\right] = 776.5\,\dot{m}\ \text{Btu/s}\ (1.803\,\dot{m}\ \text{MW})$$

Example 6.9: Brayton Cycle Considering Duct Pressure Losses

SOLUTION

$$\dot{W}_{CP} = \frac{\dot{m}c_p}{\eta_{CP}}(T_{2s} - T_1) = \dot{m}\frac{c_pT_1}{\eta_{CP}}\left(\frac{T_{2s}}{T_1} - 1\right) = \frac{\dot{m}c_pT_1}{\eta_{CP}}\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma} - 1\right] = \frac{\dot{m}(1.25)(500)}{0.9}(1.7338 - 1.0)$$

$$= 509.7\dot{m} \text{ Btu/s } (1.184\dot{m} \text{ MW})$$

$$r_p \equiv \frac{p_2}{p_1} \neq \frac{p_3}{p_4}$$

$$\dot{Q}_R = \dot{m}c_p(T_3 - T_2)$$

$$\eta_{CP} = \frac{\dot{m}c_p(T_{2i} - T_1)}{\dot{m}c_p(T_2 - T_1)} = \frac{\dot{W}_{CPi}}{\dot{W}_{CP}}$$ where $\dot{W}_{CPi}$ was calculated in Example 6.7.

$$\therefore T_2 - T_1 = \frac{\dot{W}_{CPi}}{c_p\eta_{CP}} = \frac{458.7}{(1.25)(0.9)} = 407.7°\text{R } (226.5\text{K})$$

$$\therefore T2 = 500 + 407.7 = 907.7°\text{R } (504.3 \text{ K})$$

$$\therefore \dot{Q}_R = \dot{m}c_p(1750 - 907.7) = 1052.9\dot{m} \text{ Btu/s } (2.45\dot{m} \text{ MW})$$

Example 6.9: Brayton Cycle Considering Duct Pressure Losses

SOLUTION

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} = \left(\frac{266.9}{1052.9}\right) 100 \text{ (English units)}$$

$$= \left(\frac{0.620}{2.45}\right) 100 \text{ (SI units)} \quad = 25.3\%$$

$$\eta_{Carnot} = \frac{T_{max} - T_{min}}{T_{max}} = \frac{972\text{K} - 278\text{K}}{972\text{K}} = 0.714$$

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} \quad 42.3\%$$

$$= 30.7\%$$

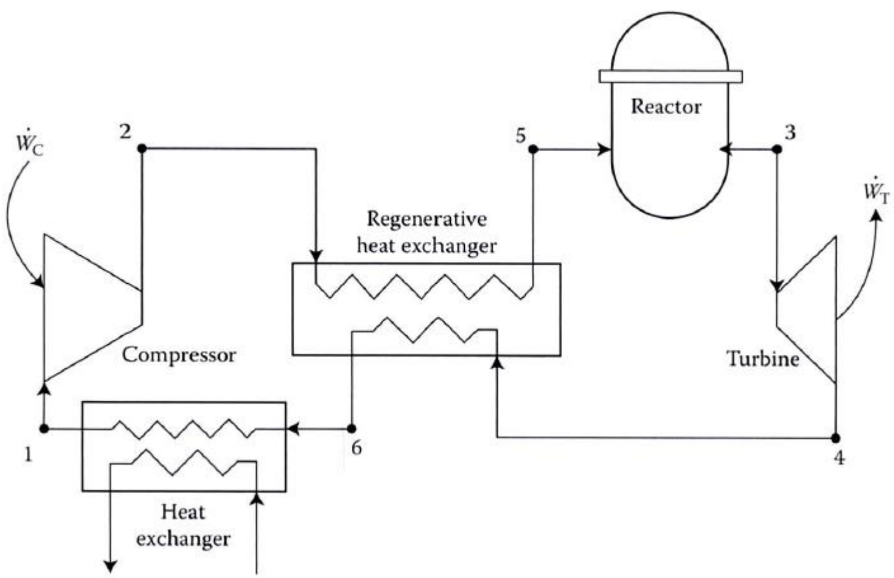
$$= 25.3\%$$

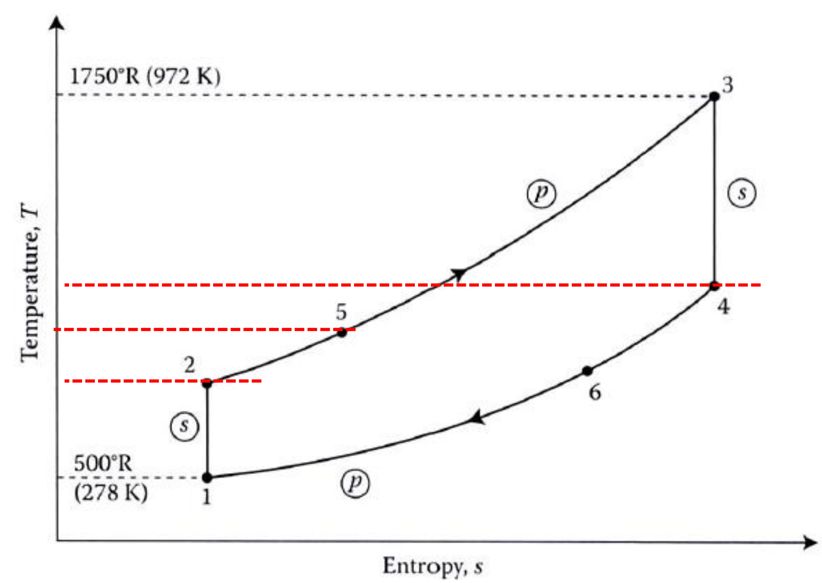
Example 6.10A: Brayton Cycle with Regeneration for Ideal Turbines and Compressors

PROBLEM Compute the cycle thermal efficiency first for ideal turbines and compressors but with the addition of a regenerator of effectiveness 0.95. The cycle is illustrated in Figure 6.28. Regenerator effectiveness is defined as the actual preheat temperature change over the maximum possible temperature change, that is,

$$\xi = \frac{T_5 - T_2}{T_4 - T_2}$$

All other conditions of Example 6.7 apply.





Example 6.10A: Brayton Cycle with Regeneration for Ideal Turbines and Compressors

Solution

$$\dot{W}_{C_p} = \dot{m}c_p(T_2 - T_1) = \dot{m}c_pT_1\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma} - 1\right] = 458.6\,\dot{m}\ \text{Btu/s}\ (1.066\,\dot{m}\ \text{MW})$$

(as in Example 6.7).

$$\dot{W}_T = \dot{m}c_p(T_3 - T_4) = \dot{m}c_pT_3\left[1 - \frac{1}{(r_p)^{\gamma-1/\gamma}}\right] = 925.9\,\dot{m}\ \text{Btu/s}\ (2.150\,\dot{m}\ \text{MW})$$

$$\dot{Q}_R = \dot{m}c_p(T_3 - T_5)$$

$$\xi\ \text{(effectiveness of regenerator)} = \frac{T_5 - T_2}{T_4 - T_2} = 0.95 \longrightarrow T_5 = (T_4 - T_2)(0.95) + T_2 = 0.95T_4 + 0.05T_2$$

$$\left(\frac{T_4}{T_3}\right) = \left(\frac{P_4}{P_3}\right)^{(\gamma-1)/\gamma} \qquad \left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \qquad r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

$$T_5 = (0.95)\left[\frac{T_3}{(r_p)^{\gamma-1/\gamma}}\right] + 0.05T_1(r_p)^{\gamma-1/\gamma} = (0.95)(0.5767)(1750) + (0.05)(500)(1.7338) = 1002.1\,°\text{R}(556.7\,\text{K})$$

$$Q_R = \dot{m}c_p(1750 - 1002.1) = 934.9\,\dot{m}\ \text{Btu/s}\ (2.172\,\dot{m}\ \text{MW})$$

Example 6.10A: Brayton Cycle with Regeneration for Ideal Turbines and Compressors

Solution

$$\dot{W}_{C_p} = \dot{m}c_p(T_2 - T_1) = \dot{m}c_pT_1\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma} - 1\right] = 458.6\,\dot{m}\text{ Btu/ s }(1.066\,\dot{m}\text{ MW})$$

(as in Example 6.7).

$$\dot{W}_T = \dot{m}c_p(T_3 - T_4) = \dot{m}c_pT_3\left[1 - \frac{1}{(r_p)^{\gamma-1/\gamma}}\right] = 925.9\,\dot{m}\text{ Btu/}s\text{ }(2.150\,\dot{m}\text{ MW})$$

$$\dot{Q}_R = \dot{m}c_p(T_3 - T_5)$$

$$\dot{W}_{NET} = \dot{W}_T - \dot{W}_{CP} = \dot{m}(925.9 - 458.7) = 467.2\,\dot{m}\text{ Btu/s }(1.084\,\dot{m}\text{ MW})$$

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} = \left(\frac{1.084}{2.172}\right)100\,(\text{SI units}) = 50.0\%$$

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} \quad 42.3\%$$

With regeneration

Without regeneration

More Complex Brayton Cycles

Example 6.10B: Brayton Cycle with Regeneration for Real Turbines and Compressors

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} = 38.3\ \%$$

With regeneration

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} = 30.7\ \%$$

Without regeneration

Example 6.11: Brayton Cycle with Regeneration for Ideal Turbines and Compressors at Elevated Pressure Ratio

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} = \left(\frac{0.993}{2.800}\right)100(\text{SI units}) =$$

With regeneration, $r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4} = 8$

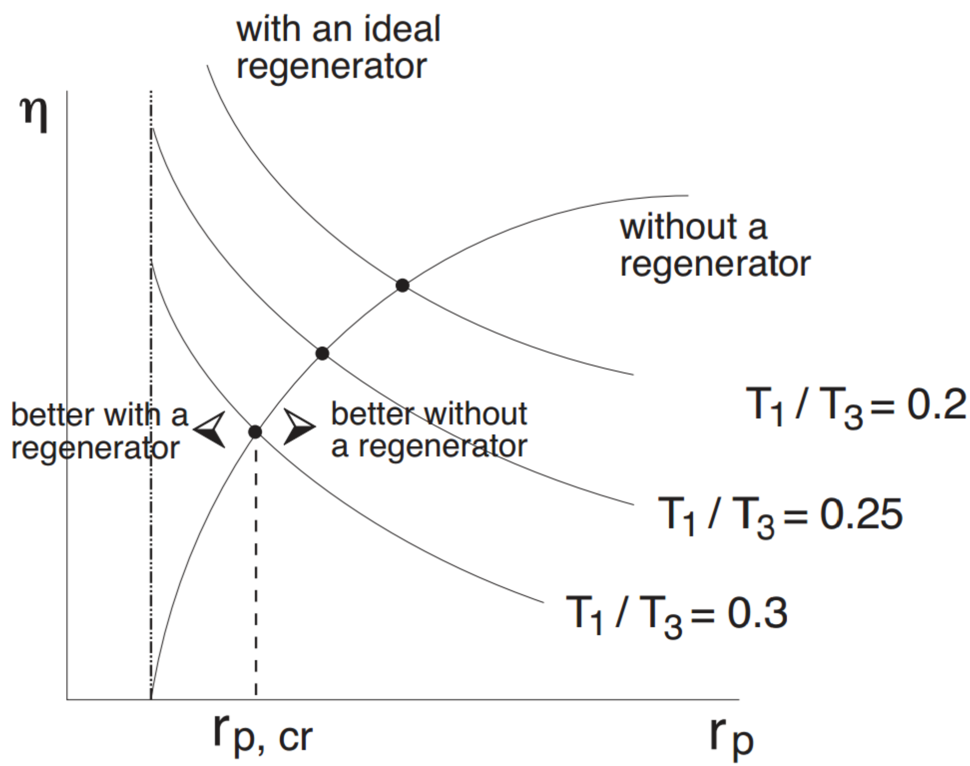
$T_2 > T_4$ Not desired!

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} = \left(\frac{1.084}{2.172}\right)100(\text{SI units}) = 50.0\%$$

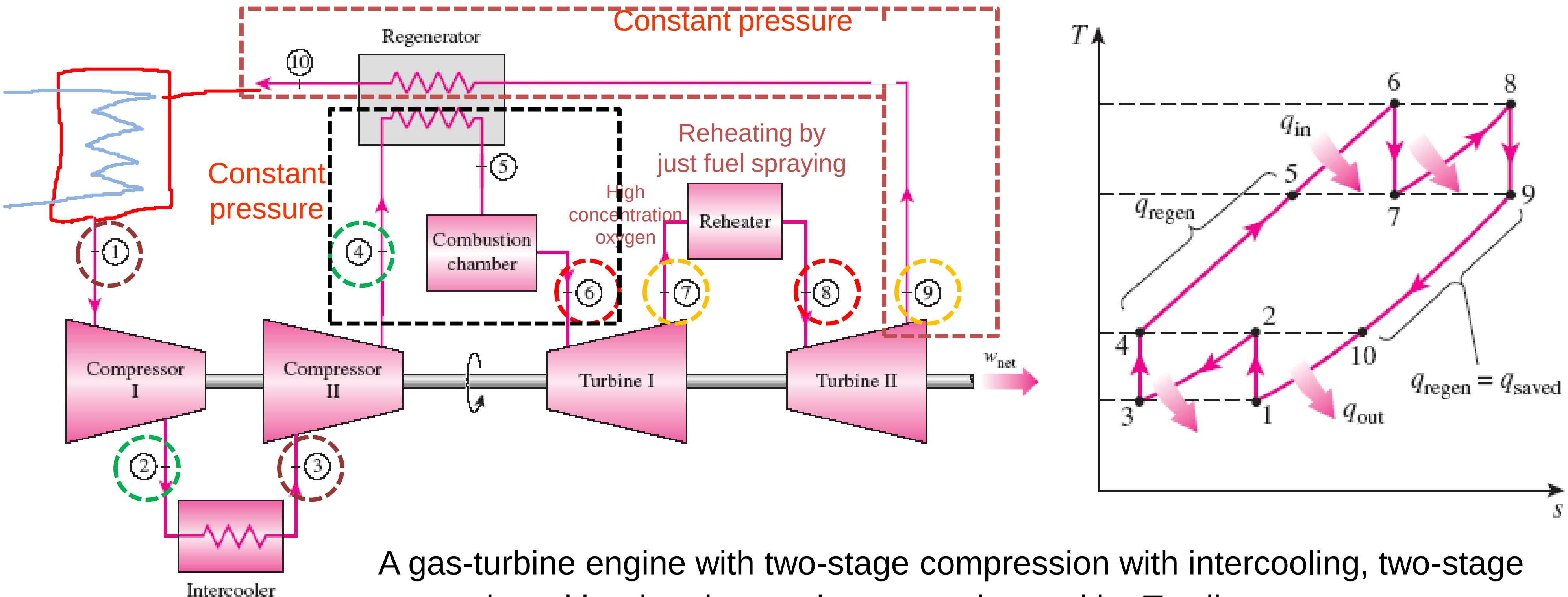
With regeneration, $r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4} = 4$

Without regeneration $r_p \equiv \frac{p_2}{p_1} = \frac{p_3}{p_4} = 8$

Example 6.11: Brayton Cycle with Regeneration for Ideal Turbines and Compressors at Elevated Pressure Ratio



More Complex Brayton Cycles

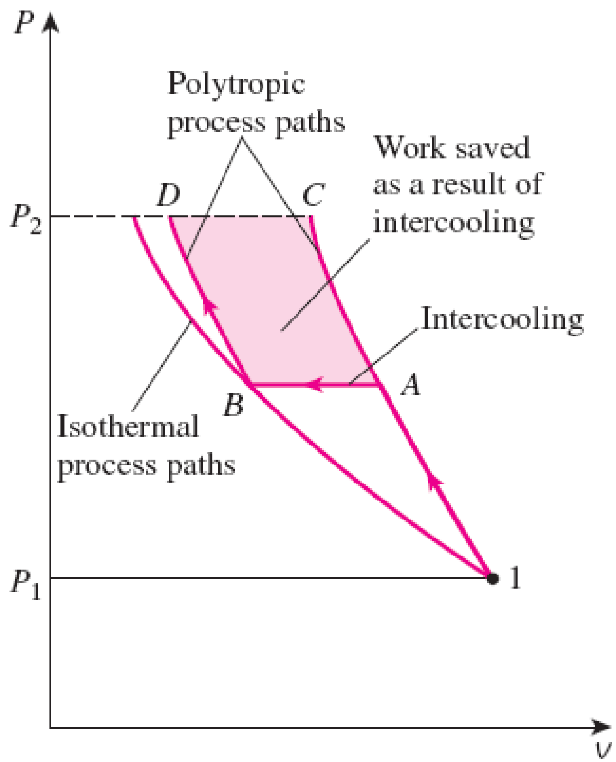


A gas-turbine engine with two-stage compression with intercooling, two-stage expansion with reheating, and regeneration and its *T-s* diagram.

More Complex Brayton Cycles

- **Net work of gas turbine = (turbine work output)**
  – (compressor work input)

- **Efficiency enhancement by**
  -> *Decreasing the compressor work input*
  -> *Increasing the turbine work output*



$$w_{\text{rev,in}} = \int_1^2 v\, dP$$

Steady flow compression or expansion work is proportional to the specific volume of fluid.

1. As the number of stages increases, the compression becomes nearly isothermal at the inlet temperature.
-> compression work decrease.

Intercooling

2. Similarly, turbine work between the two pressure levels can be increases by expanding the gas in stages and reheating it -> multistage expansion with reheating.

Reheating

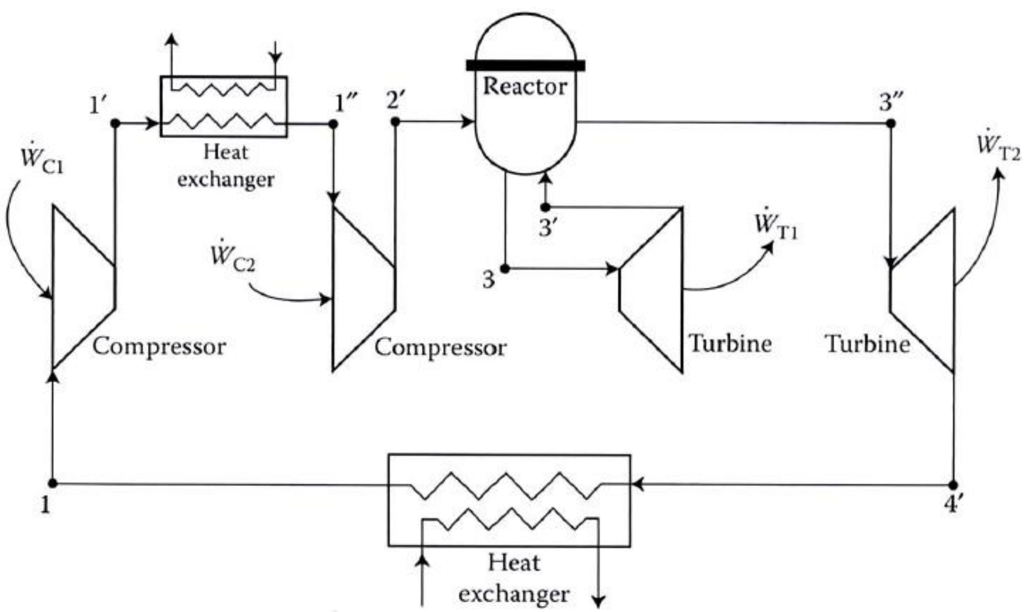
Example 6.14: Brayton Cycle with Reheat and Intercooling

PROBLEM Calculate the thermal efficiency for the cycle employing both intercooling and reheat as characterized below. The cycle is illustrated in Figure 6.29. All other conditions of Example 6.7 apply.

Intercooling: $\frac{p_1'}{p_1} = \frac{p_2}{p_1'} = r_p' \quad T_1'' = T_1$

$$r_p = 4 = \frac{p_2}{p_1} = \frac{p_1'}{p_1}\frac{p_2}{p_1'} = r_p'^2$$

Reheat: $\frac{p_3'}{p_4} = \frac{p_3}{p_3'} = r_p' \quad T_3'' = T_3$



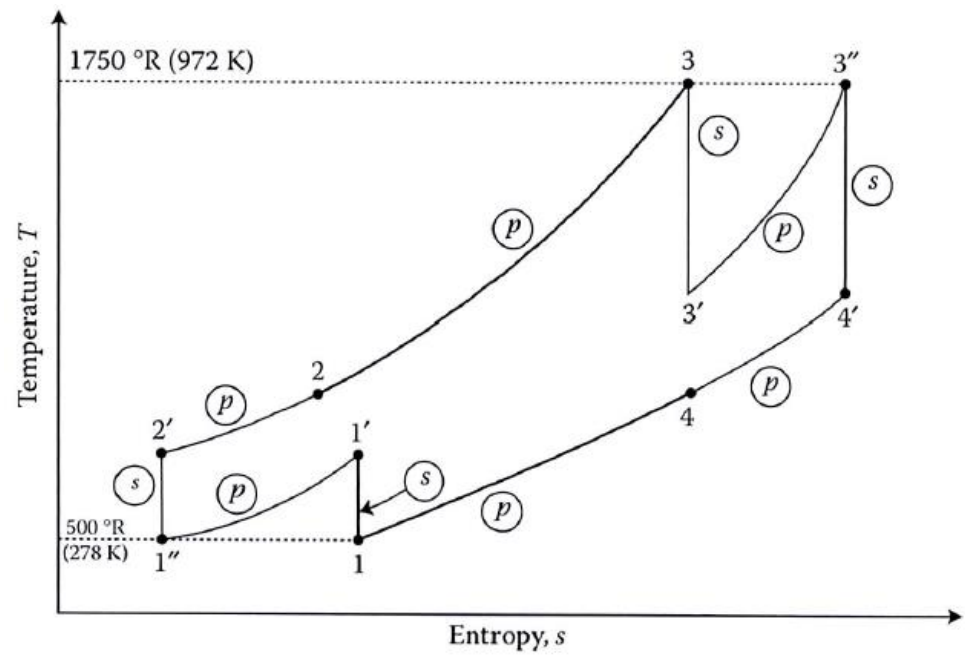


TABLE 6.9
Results of Brayton Cycle Cases of Examples 6.7 through 6.14

Parameter	Ex. 6.14
$\beta = \left(\frac{p_2 p_4}{p_3 p_1}\right)^{\gamma-1/\gamma}$	1.0
Component isentropic efficiency (η_s)	1.0
Regenerator effectiveness (ξ)	—
Pressure ratio (r_p)	4
Intercooling	$\frac{p_1'}{p_1} = \frac{1}{2}\frac{p_2}{p_1}$ $T_1'' = T$
Reheat	$\frac{p_3'}{p_4} = \frac{1}{2}\frac{p_3}{p_4}$ $T_3'' = T_3$
Turbine work ($\dot{W}_T/\dot{m}$) Btu/lb MJ/kg	1052.5 2.444
Compressor work ($\dot{W}_c/\dot{m}$) Btu/lb MJ/kg	395.96 0.920
Net work ($\dot{W}_{NET}/\dot{m}$ Btu/lb MJ/kg	656.5 1.524
Heat in ($\dot{Q}_R/\dot{m}$) Btu/lb MJ/kg	1890.8 4.391
Cycle thermal efficiency (η_{th})(%)	34.7

SOLUTION

$$\dot{W}_{CP} = \dot{m}c_p(T_1' - T_1) + \dot{m}c_p(T_2' - T_1'')$$

$$\dot{W}_{CP} = \dot{m}c_pT_1\left(\frac{T_1'}{T_1} - 1\right) + \dot{m}c_pT_1''\left(\frac{T_2'}{T_1''} - 1\right)$$

$$= \dot{m}c_pT_1[(r_p')^{\gamma-1/\gamma} - 1] + \dot{m}c_pT_1''\,[(r_p')^{\gamma-1/\gamma} - 1]$$

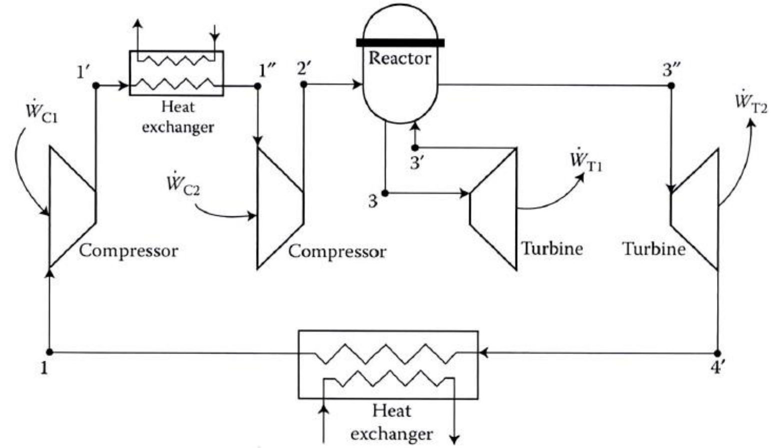
$$= 2\dot{m}c_pT_1[(r_p')^{\gamma-1/\gamma} - 1] = 2\dot{m}c_pT_1[(2)^{0.397} - 1]$$

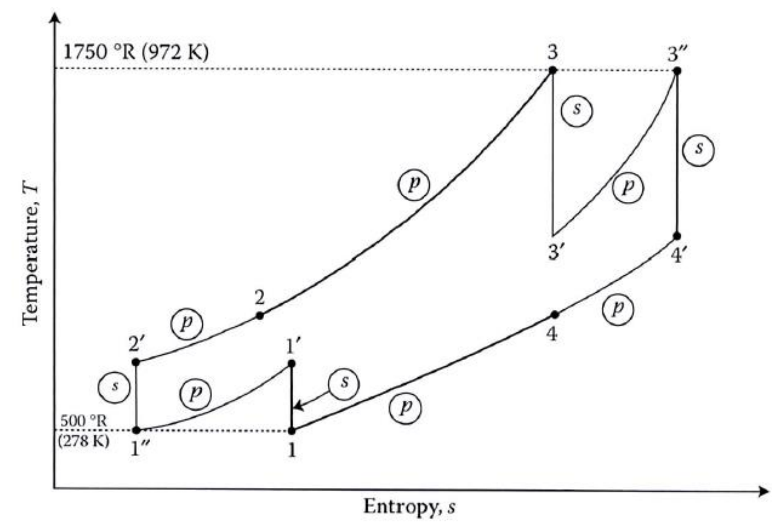
$$= 395.96\,\dot{m}\ \text{Btu/s}\ (0.920\,\dot{m}\ \text{MW})$$

Intercooling: $\frac{p_1'}{p_1} = \frac{p_2}{p_1'} = r_p' \quad T_1'' = T_1$

$$\frac{p_1'}{p_1} = \frac{1}{2}\frac{p_2}{p_1}$$
$$T_1'' = T$$

$$r_p = 4$$





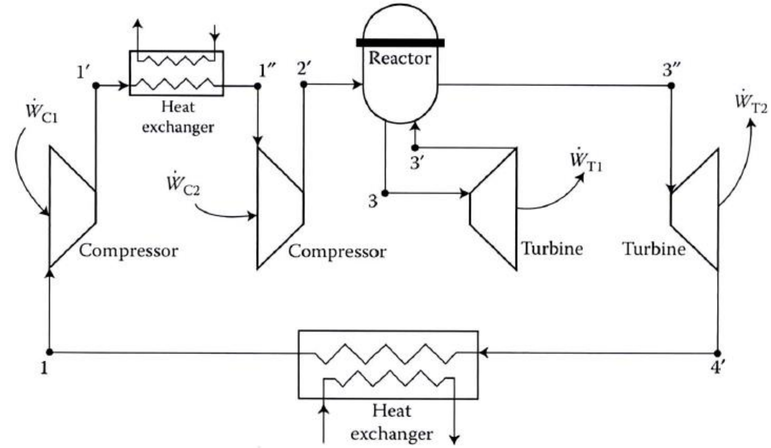
SOLUTION

$$\dot{W}_T = \dot{m}c_p(T_3 - T_3') + \dot{m}c_p(T_3'' - T_4') = \dot{m}c_pT_3\left(1 - \frac{T_3'}{T_3}\right) + \dot{m}c_pT_3''\left(1 - \frac{T_4'}{T_3''}\right)$$

Again for the isentropic case:

$$\dot{W}_T = 2\dot{m}c_pT_3\left[1 - \frac{1}{(r_p')^{\gamma-1/\gamma}}\right] = 2\dot{m}(1.25)1750\left[1 - \frac{1}{(2)^{0.397}}\right]$$

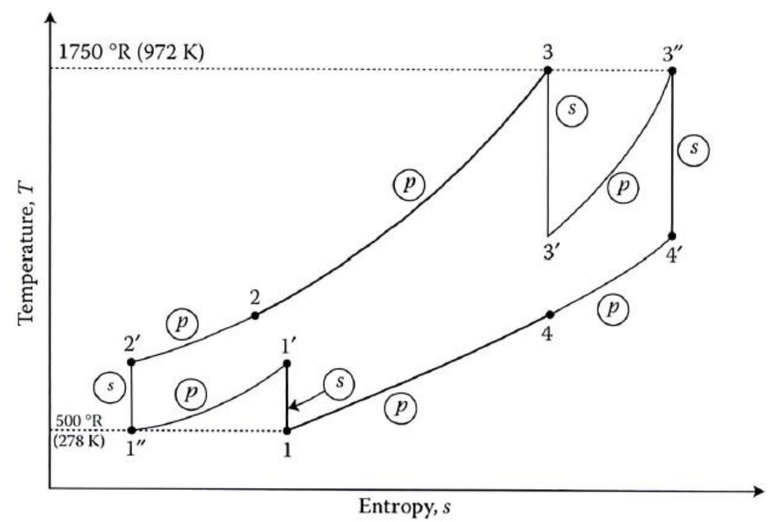
$$= 1052.5\,\dot{m}\text{ Btu/s } (2.444\,\dot{m}\text{ MW})$$



Reheat: $\dfrac{p_3'}{p_4} = \dfrac{p_3}{p_3'} = r_p' \quad T_3'' = T_3$

$$\frac{p_3'}{p_4} = \frac{1}{2}\frac{p_3}{p_4}$$
$$T_3'' = T_3$$

$r_p = 4$



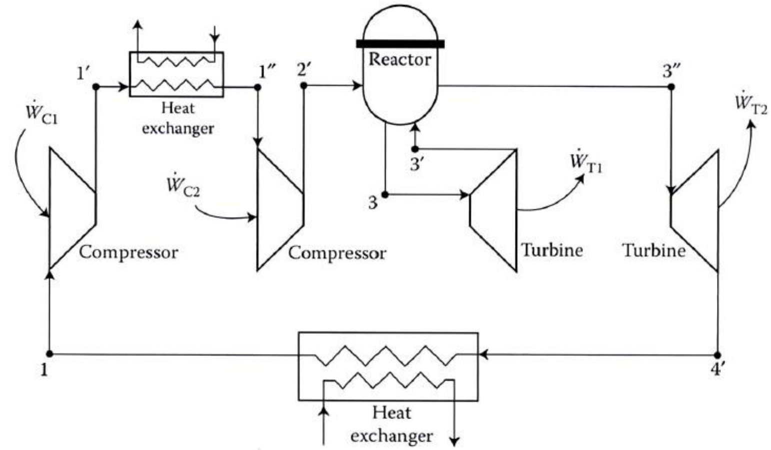
$$\frac{T_3'}{T_3} = \frac{1}{(r_p')^{\gamma-1/\gamma}} \qquad \therefore T_3' = \frac{T_3}{(r_p')^{\gamma-1/\gamma}} = \frac{1750}{(2)^{0.397}} = \frac{1750}{1.317}$$

$$= 1329°\text{R}(738.3\text{K})$$

More Complex Brayton Cycles

SOLUTION

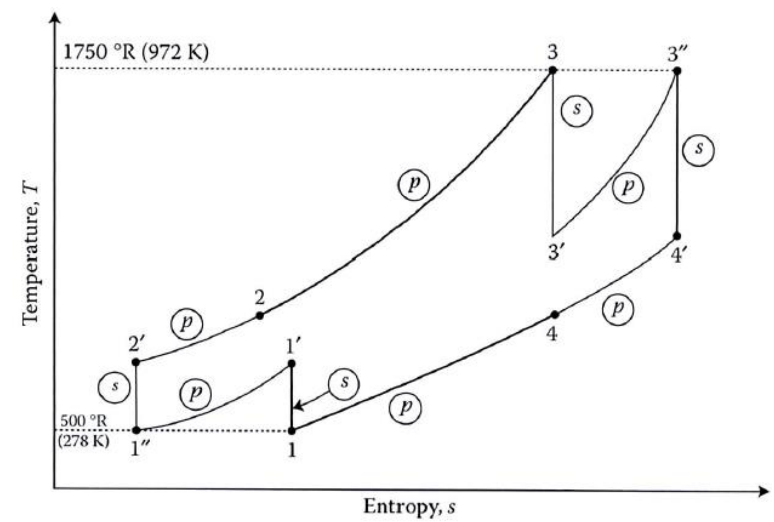
$$\dot{Q}_R = \dot{m}c_p(T_3 - T_2) + \dot{m}c_p(T_3'' - T_3')$$
$$= \dot{m}c_p[(1750 - 658.4) + (1750 - 1329.0)]$$
$$= 1890.8\,\dot{m}\ \text{Btu/s}\ (4.391\dot{m}\ \text{MW})$$



where $T_2' = T_1''(r_p')^{\gamma-1/\gamma}$ and $T_1'' = T_1 = 500°\text{R}\ (278\text{K})$

$$\therefore T_2' = (500°\text{R})2^{0.397} = 658.4°\text{R}(365.8\text{K})$$

$$\dot{W}_{NET} = \dot{W}_T - \dot{W}_{CP} = \dot{m}\,(1052.5 - 395.96)$$
$$= 656.5\,\dot{m}\ \text{Btu/s}\ (1.524\,\dot{m}\ \text{MW})$$



$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R} = \left(\frac{1.524}{4.391}\right)100\ (\text{SI units})$$
$$= 34.7\%$$