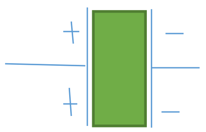
Lecture 5-Capacitors

:Conductor plates with insulator (vacuum, dielectric material such as ceramic, plastic, metal oxide) to give greater capacitance (or storage capacity) per unit area. (Mechanical analogy =Tank, Accumulator)



Capacitance $C=\varepsilon A/d$ [Farad] or [F]

d,A where ε=permittivity [F/m]

Application: Filtering or Conditioning, Smoothing, DC blocking, etc

<u>Laws for Capacitor</u>

1. Static description

Charge $Q=CV$ [Coloumb], or $V=Q/C=\int i\,dt/C$ and i=current

V C

Serial connection:

V_T V_1 V_2

$V_T=V_1+V_2=Q_1/C_1+Q_2/C_2=Q/C_T$ ($Q_1=Q_2=Q$ why?)

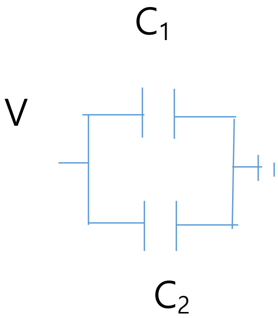
C_1 C_2

Thus $1/C_T=1/C_1+1/C_2$, and $C_T=C_1C_2/(C_1+C_2)=C_2/(1+C_2/C_1)$

$=C/2$ (if $C_1=C_2=C$)

$\fallingdotseq C_2$ (if $C_1 \gg C_2$) $\therefore$ <u>Smaller capacitor dominates</u>

Parallel connection



$Q_T=Q_1+Q_2=C_1V+C_2V=(C_1+C_2)V=C_TV$

Thus $C_T=C_1+C_2=C_1(1+C_2/C_1)=2C$ (if $C_1=C_2=C$)

$\fallingdotseq C_1$ (if $C_1 \gg C_2$)

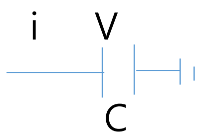
∴<u>Larger capacitor dominates</u>

2. Dynamic description

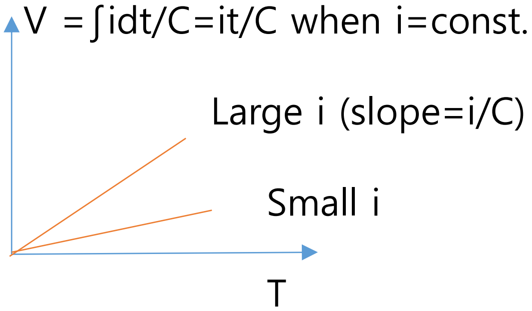
:Differentiation of static description w.r.t. time. That is,

$dQ/dt=i=CdV/dt$

∴ Current $\propto$ dV/dt where V= voltage across capacitor



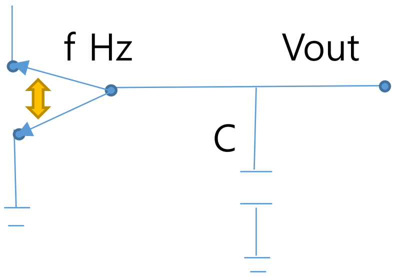
$V=\int idt/C=it/C$ when i=const. => linearity vs time



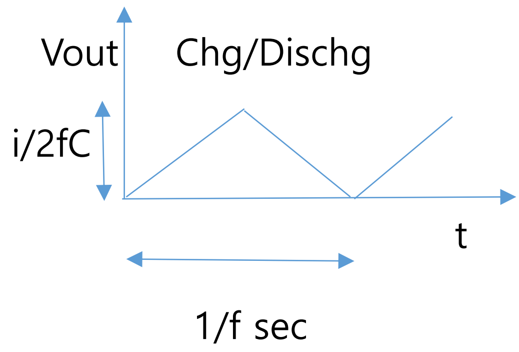
Thus bigger current gives faster voltage change.

Application 1 (Constant current input): Ramp generator

i=const.

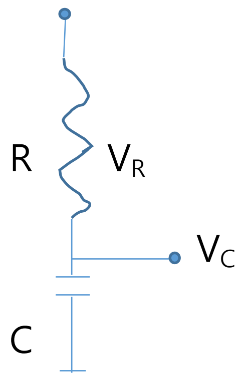


∴ $V_{out}=Q/C=it/C$



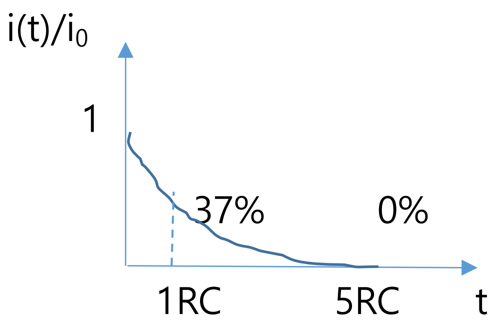
Application2 (Constant voltage input)

V_{in}=const.



$V_{in}=V_R+V_C=iR+\int idt/C$, assuming very little current flows into Vc

$RCdi/dt+i=0$ ∴ $i(t)=i_0 exp(-t/RC)$ and i_0= Vin/R

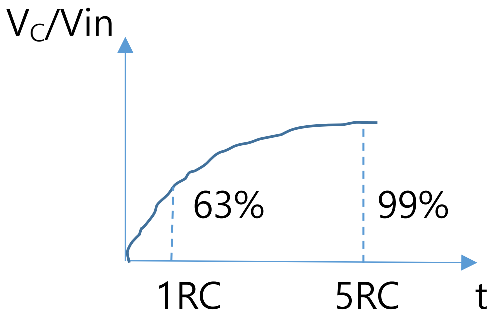


R	C	time
MΩ	μF	sec
KΩ	μF	ms

∴ After 5RC time, no current flows ->Z_C≡Impedance of capacitor=∞

$V_C = \int i dt/C = V_{in}(1-\exp(-t/RC)) \fallingdotseq V_{in}$ (after 5RC), or

$Vc = V_{in} Zc/(R+Zc) = V_{in}(1+R/Zc) \fallingdotseq V_{in}$ ($\because$ $Zc=\infty$)



R	C	time
MΩ	μF	sec
KΩ	μF	ms

Application3 (Varying voltage input or AC input)

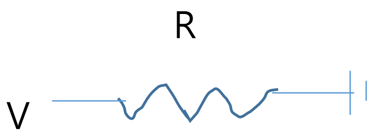
When V=AC (Alternating Current) voltage, it can be expressed as

$V = \Sigma V_i \exp(j\omega_i t) = \Sigma V_i(\cos\omega_i t + j\sin\omega_i t)$, and i=1 to ∞

For a specific ith component,

$V = V\exp(j\omega t) = V(\cos\omega t + j\sin\omega t)$ is applied to the following elements

(1) Resistor



$i = V/R = V\exp(j\omega t)/R$

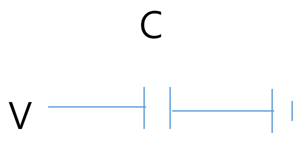
∴ Z≡Impedance or Generalized resistance

=Resistance (by resistor)+Reactance (by capacitor or inductor)

$\equiv \Delta V/\Delta i = V/i = R$ (ω independent)

∴ Resistor gives constant impedance for AC or DC

(2) Capacitor



$i=CdV/dt=Cj\omega V\exp(j\omega t)=j\omega CV$ ∴ $Z=\text{Impedance}=V/i=1/j\omega C=f(\omega)$

If $\omega \to 0$ then $Z \fallingdotseq \infty$ (Disconnect or DC blocking)

if $\omega \to \infty$ then $Z \fallingdotseq 0$ (Connect or Current free-flowing)

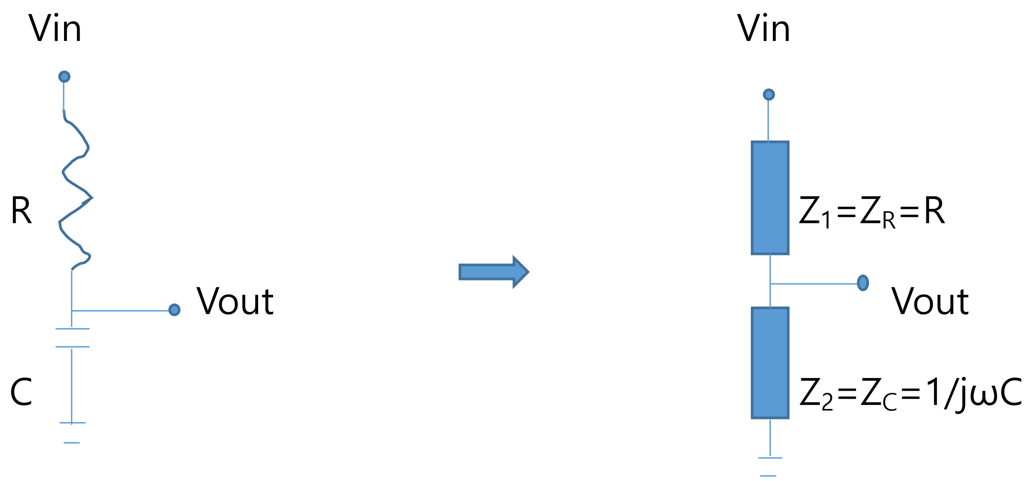
∴ Capacitor can give DC blocking or Filtering (ω dependant)

Application 4 (RC filters or RC voltage dividers)

<u>LPF (Low Pass Filter)</u>

:To pass low frequency component, or

To filter out the high frequency noise from the signal



The left RC circuit can be changed as the right circuit of voltage divider, considering that $Z_R=R$ and $Z_C=1/j\omega C$

Transfer function $=V_{out}/V_{in}=Z_2/(Z_1+Z_2)=(1/j\omega C)/(R+1/j\omega C)$

$=1/(1+j\omega RC)$, assuming very little current flows into Vout.

This is the complex number, thus magnitude, phase are of interest.

Magnitude $=|H|=|V_{out}/V_{in}|=1/\sqrt{1+(\omega RC)^2}$

Phase $=\angle H=\angle V_{out}/V_{in}=0-\angle(1+j\omega RC)=-\tan^{-1}(\omega RC)$

where ω = angular velocity of signal [rad/sec] $=2\pi f$

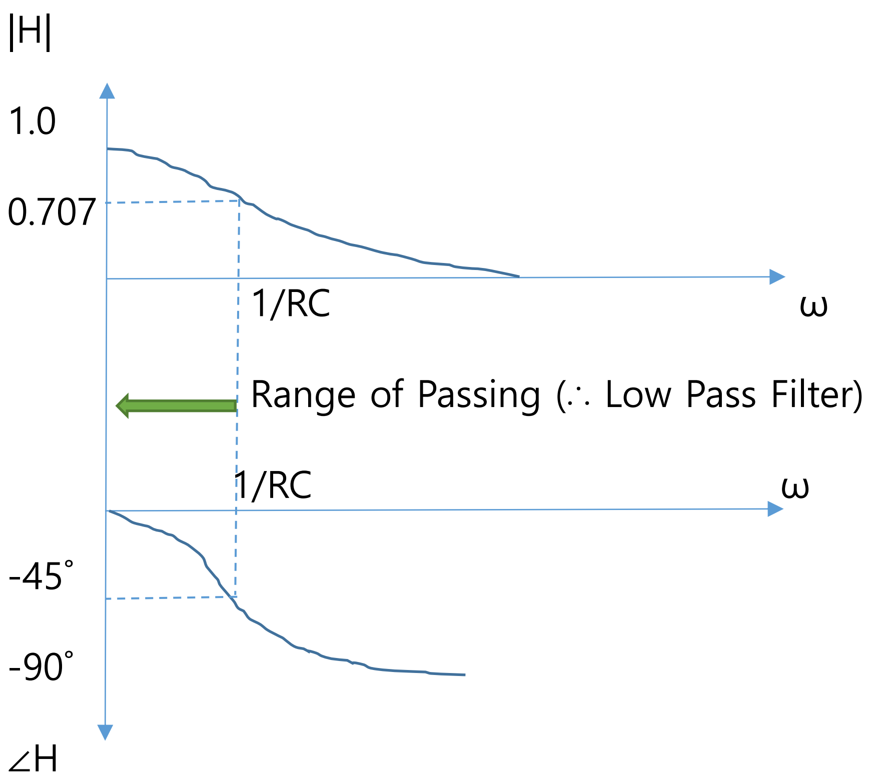
and f = frequency of signal [Hz]

Let's plot the magnitude and phase in the ω domain;

If $\omega<1/RC$; then $|H|\fallingdotseq 1$ and $\angle H\fallingdotseq 0^\circ$ (or $[-45^\circ, 0^\circ]$)

If $\omega=1/RC$ then $|H|=1/\sqrt{2}=0.707$ and $\angle H=-45^\circ$

If $\omega>1/RC$ then $|H|\fallingdotseq 0$ and $\angle H\fallingdotseq -90^\circ$



If $\omega \leq 1/RC$ then $|H| \geq 0.707$ and $|\angle H| \leq 45^{\circ}$, and they are acceptable for many engineering application. This circuit effectively can pass the signal of frequency range under 1/RC. Thus it is LPF, Low Pass Filter.

At $\omega = 1/RC$, $dB \equiv -20Log|H| = -20Log(1/\sqrt{2}) \fallingdotseq 3$, thus 1/RC is called as ω_{3dB} which is very important for application. At $\omega = \omega_{3dB}$, 70% amplitude of signal is delivered, while 30% of signal is attenuated, which is quite acceptable for many engineering application. Ex) ω_{3dB} for sensor?

This LPF is very useful tool for signal conditioning, or removing the high frequency noise from the signal.

<u>5 Design Procedures or 5 Steps for LPF</u>

1) Frequency identification of signal

   Measured signal can be analyzed by oscilloscope or frequency spectrum analysis. Then identify f_{signal} and f_{noise}

2) Choose f_{3dB}

   You can assign f_{3dB} at your design target, and f_{signal} can be conveniently chosen as the f_{3dB}

3) Choose R, C such that $1/RC = \omega_{3dB} = 2\pi f_{3dB}$ among commercially available components

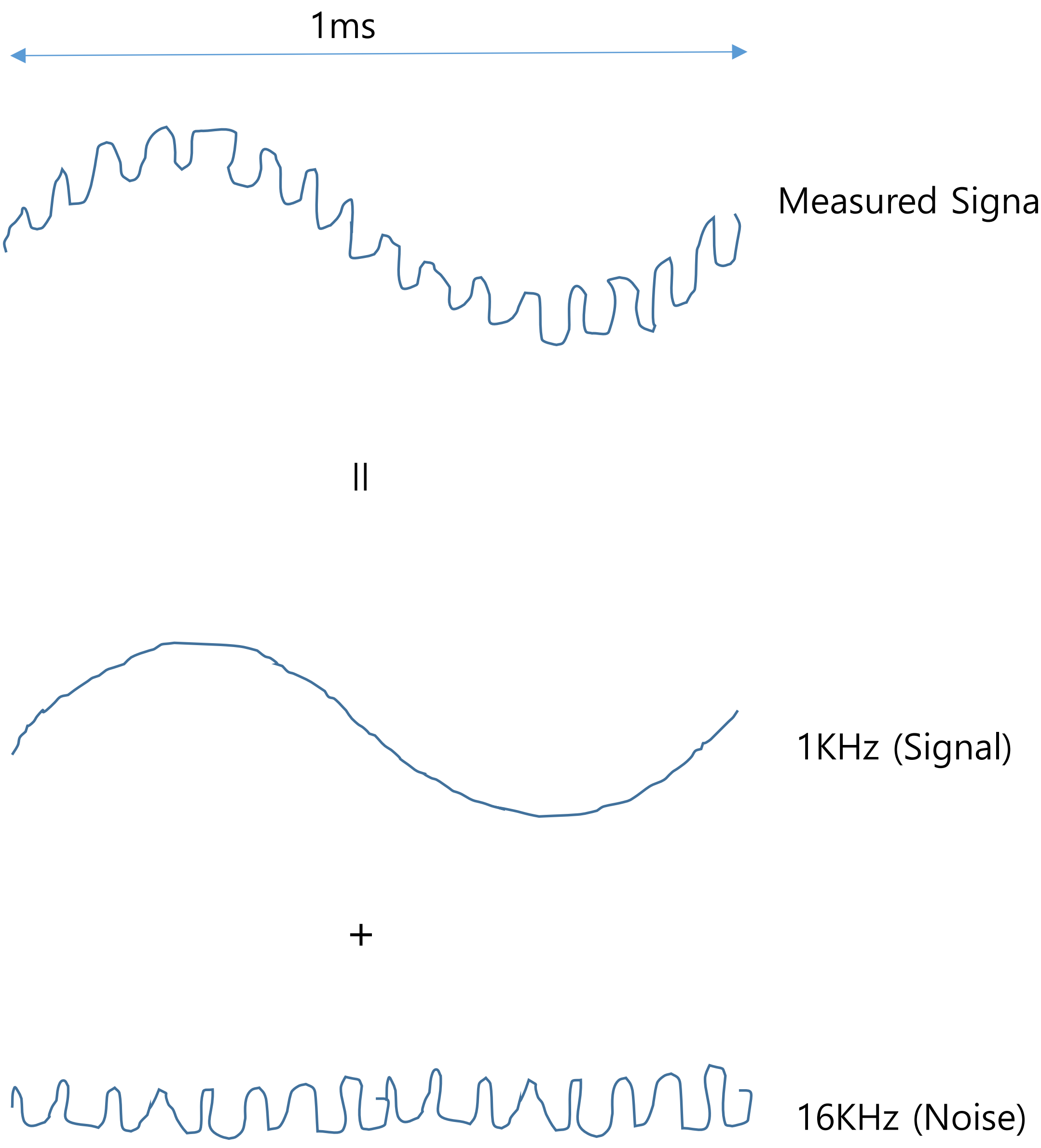
4) Use 10X rule for components choosing if necessary

5) Performance Verification by using Signal to Noise ratio, or S/N ratio, etc

Ex) LPF design

Design a LPF to remove the noise from the signal. This LPF will be connected to A/D converter of 100KΩ input impedance.

Oscilloscope shows the measured signal as follows;



1)Frequency identification

f_{signal}=1KHz, f_{noise}=16KHz by oscilloscope or spectrum analysis

2)Choose f_{3dB}

f_{3dB}=f=1KHz

(Q: What happen if we chose f_{3dB} as 16KHz?)

3)Choose R,C from $\omega_{3dB} = 1/RC$

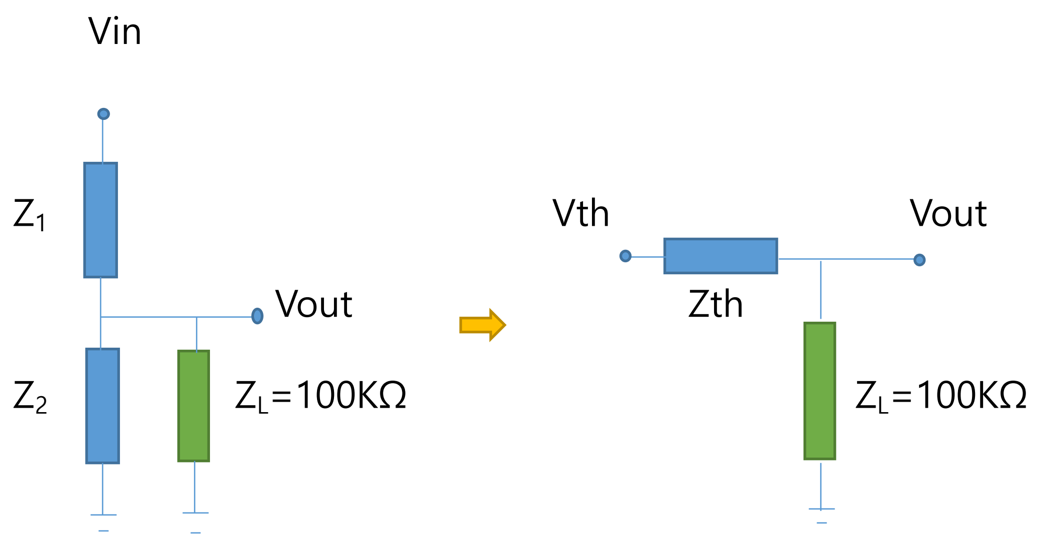
Thus $1/RC = \omega_{3dB} = 2\pi f_{3dB} = 6280$[rad/sec]

There are so many combinations to satisfy, thus we need more information from the 10X rule

4) 10X rule

This LPF is to drive the ADC of 100KΩ input impedance as follows;

And it can be transformed to Thevenin's equivalent circuit



$Zth = Z_1 \parallel Z_2 = Z_1 Z_2/(Z_1+Z_2) = (R/j\omega C)/(R+1/j\omega C) = R/(1+j\omega RC)$

Magnitude of Zth, $|Zth|=R/\sqrt{1+(\omega RC)^2} \leq R$

Thus maximum of Zth is R, and Zth is to drive the Z_L ; therefore it is quite reasonable to choose R as one tenth of Z_L(=100KΩ) by the 10X rule, satisfying $Z_{out} \leq Z_{in}/10$ in general application of voltage circuit.

Thus R =10KΩ, and C=1.59E-8F ≒ 0.02μF from commercial availability.

5) Verification

For 1KHz signal, or ω=2π(1000) rad/sec

$|H|=|V_{out}/V_{in}|=1/\sqrt{1+(\omega RC)^2}$ ≒ 0.623 (*why not 0.707 ?)*

$\angle H=-\angle(1+j\omega RC)=-\tan^{-1}(\omega RC)$ ≒ -51.5˚ (*why not -45˚ ?)*

For 16KHz noise, or ω=2π(16000) rad/sec

$|H|=|V_{out}/V_{in}|=1/\sqrt{1+(\omega RC)^2}$ ≒ 0.06 (6%)

$\angle H=-\angle(1+j\omega RC)=-\tan^{-1}(\omega RC)$ ≒ -87.2˚

Before LPF application, Given S=1, N=1, then S/N=1

After LPF application, S = 0.623, N=0.06, then S/N=0.623/0.06≒10.4

Thus S/N ratio changes from 1 to 10.4, which is more than 10 times improvement!

It is wonderful result for S/N ratio improvement by the LPF application.

Q: What happen if only Z_L is changed to 10KΩ, while others are unchanged?

➔ Hint : $H=V_{out}/V_{in}=1/\{1+R/R_L+j\omega RC\}$