

NUCLEAR SYSTEMS ENGINEERING

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❖ Contents of lecture

- Chapter 1 Principal Characteristics of Power Reactors

- Will be replaced by the lecture note
- Introduction to Nuclear Systems

Nuclear system

- Chapter 4 Transport Equations for Single-Phase Flow (up to energy equation)

- Chapter 6 Thermodynamics of Nuclear Energy Conversion Systems:
Nonflow and Steady Flow : First- and Second-Law Applications

- Chapter 7 Thermodynamics of Nuclear Energy Conversion Systems :
Nonsteady Flow First Law Analysis

Thermodynamics

- Chapter 3 Reactor Energy Distribution

- Chapter 8 Thermal Analysis of Fuel Elements

Heat transport
Conduction
heat transfer

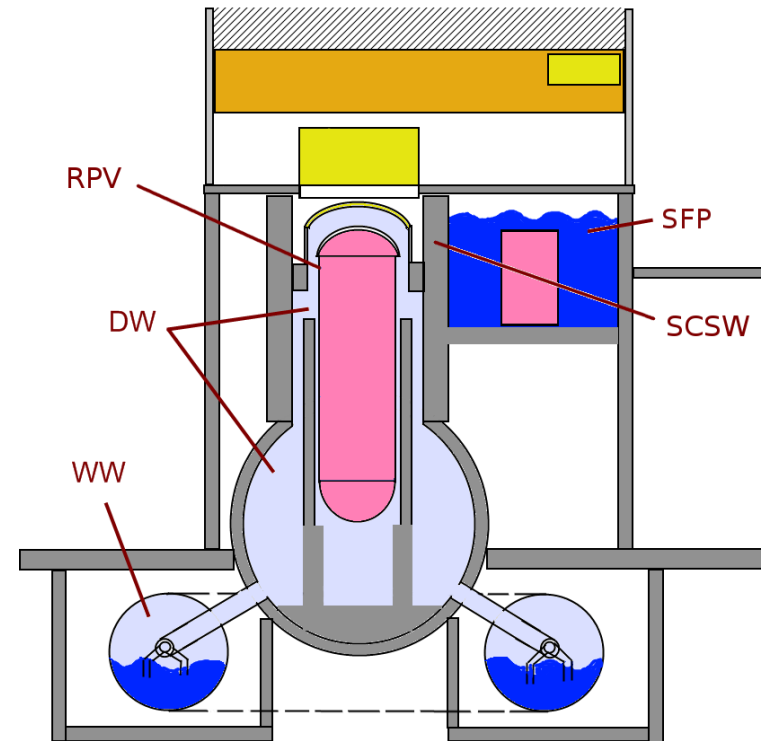
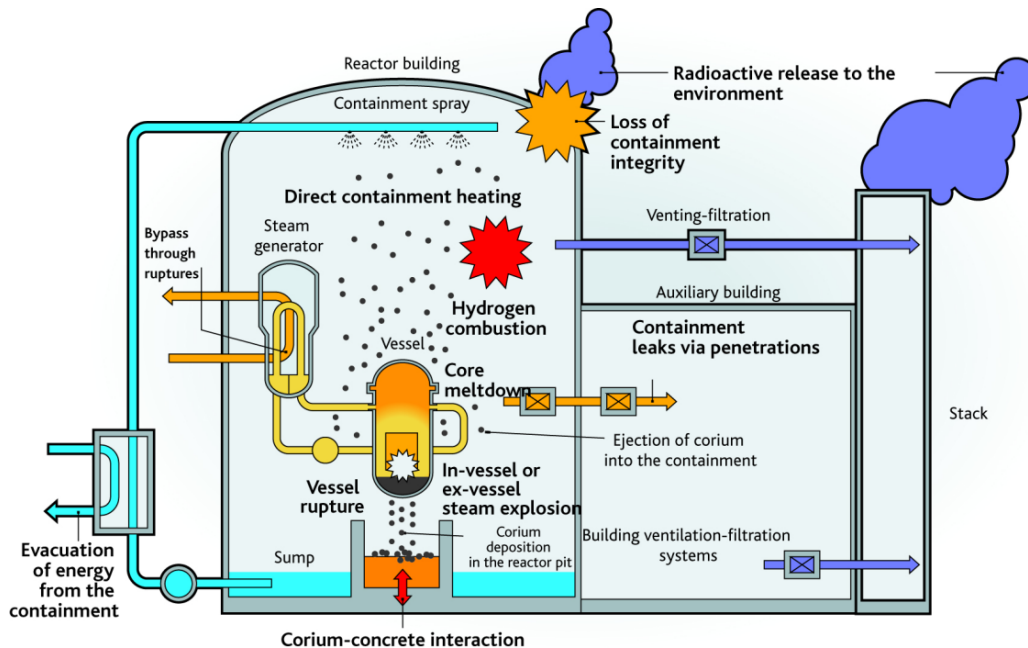
7. THERMODYNAMICS OF NUCLEAR ENERGY CONVERSION SYSTEMS: NON-STEADY FLOW FIRST LAW ANALYSIS

- ❖ Introduction
- ❖ Containment Pressurization Process
- ❖ Response of a PWR Pressurizer to Load Changes

Introduction

❖ Examples of time-varying flow processes

- Pressurization of the containment due to postulated rupture of the primary or secondary coolant systems
- Response of a PWR pressurizer to turbine load changes
- BWR suppression pool heatup by addition of primary coolant



Containment Pressurization Process

❖ Examples of time-varying flow processes

- The analysis of rapid mixing of a noncondensable gas and a flashing liquid has application in reactor safety;
 - for example, for the light-water reactor, one postulated accident is the release of primary or secondary coolant within the containment.
 - The magnitude of the peak pressure and the time to peak pressure are of interest for structural considerations of the containment.


❖ Fluid release in the containment

- Due to the rupture of either the primary or secondary coolant loops
- Final state of the water-air mixture depends on
 - (1) The initial thermodynamic state and mass of water in the reactor and the air in the containment;
 - (2) The rate of release of fluid into the containment and the possible heat sources or sinks involved;
 - (3) The likelihood of exothermic chemical reactions; and
 - (4) the core decay heat.

Containment Pressurization Process

TABLE 7.1

Factors to Consider during Analysis of Coolant System Ruptures

Possible Heat Sinks	Possible Heat Sources	Possible Fluid Added from External Sources
Primary System Rupture		
Containment walls and other cool surfaces	Stored heat	Emergency core cooling water
Active containment heat removal systems—air coolers, sprays, and heat exchangers	Decay heat	Feedwater (BWR) 
Steam generator secondary side	Other energy sources in core (e.g., Zr–H ₂ O reactions, H ₂ explosion)	
	Steam generator secondary side	
Secondary System Rupture		
Containment walls and other cool surfaces	Primary coolant through steam generator	Condensate makeup (PWR)
Active containment heat removal systems—air coolers, sprays, heat exchangers		

Containment Pressurization Process

❖ Fluid release in the containment

● In the following discussion

- Heat loss to structure
- Heat gain from the core
- Potential and kinetic energy effects are neglected.

● Control mass approach vs. control volume approach

❖ Control mass approach

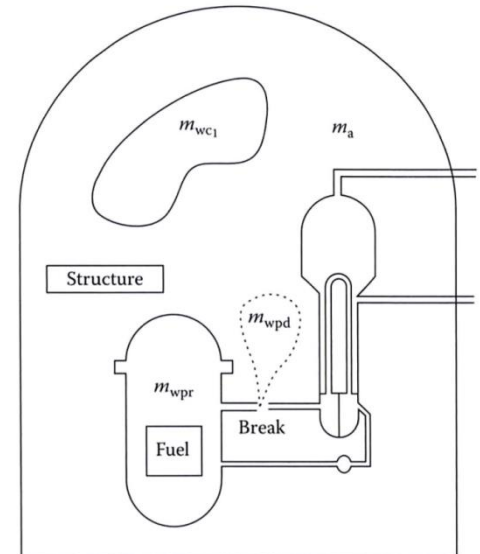
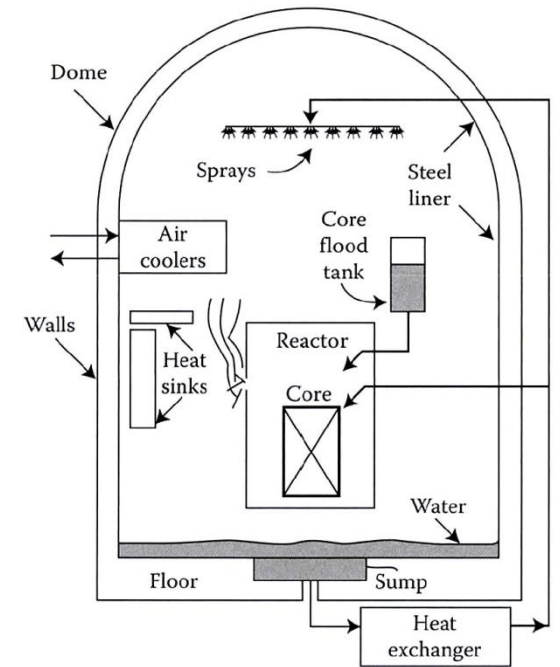
● Three subsystems

- Mass of air in containment, m_a ()
- Water vapor initially in the air of the containment, m_{wc1} ()
- Water initially in the primary system, m_{wp} ()

● At any given time,

$$m_{wp} = m_{wpd} + m_{wpr}$$

- d: discharged
- r: remained



Analysis of Final Equilibrium Pressure Conditions

❖ Simplification of the problem

- Consider only final conditions upon completion of the blowdown process.
- Establishment of pressure equilibrium
 - Between the containment vessel and the primary system
- Consider the total heat transfer rather than the rate of heat transfer.

❖ Control mass approach

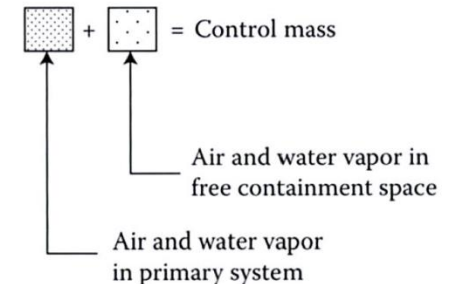
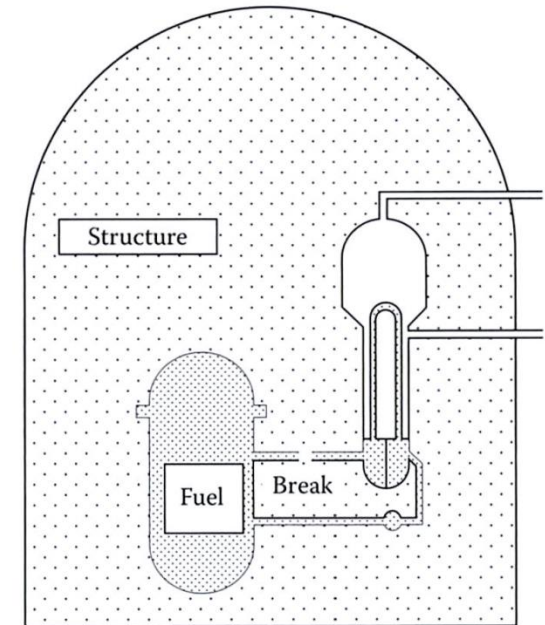
$$U_2 - U_1 = Q_{n-wpr} - \sum_i Q_{i-st}$$

$$U_2 = m_a u_{a_2} + (m_{wc_1} + m_{wpd_2}) u_{wc_2} + m_{wpr_2} u_{wpr_2}$$

$$U_1 = m_a u_{a_1} + m_{wc_1} u_{wc_1} + m_{wpr_1} u_{wpr_1}$$

- Simplified!

$$u_{wc_2} = u_{wp_2} \Rightarrow U_2 - U_1 = Q_{n-wpr} - \sum_i Q_{i-st}$$



where $U_2 = m_a u_{a_2} + (m_{wc_1} + m_{wp}) u_{wc_2}$

$$U_1 = m_a u_{a_1} + m_{wc_1} u_{wc_1} + m_{wp} u_{wp_1}$$

Analysis of Final Equilibrium Pressure Conditions

❖ Control volume approach

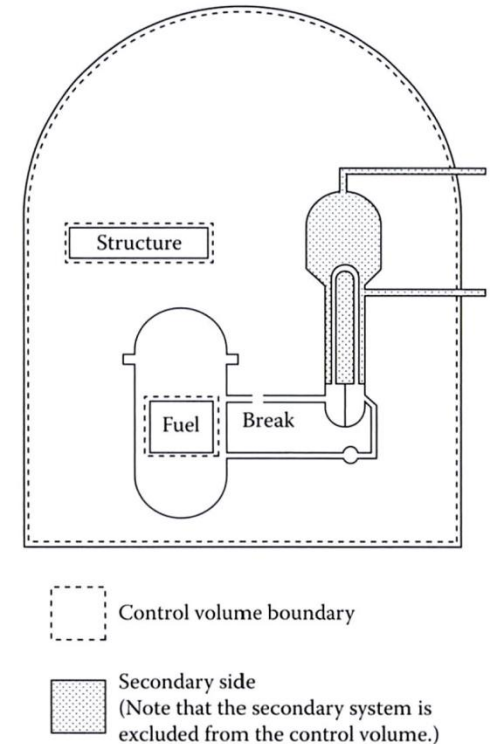
- At the final state, there are no entering or exiting flow streams, no shaft work, and no expansion work.

$$U_2 - U_1 = Q_{n-wpr} - Q_{c-st} \quad \text{where } U_2 = m_a u_{a_2} + (m_{wc_1} + m_{wp}) u_{wc_2}$$

$$U_1 = m_a u_{a_1} + m_{wc_1} u_{wc_1} + m_{wp} u_{wp_1}$$

❖ Governing equations for determination of final conditions

- We need subsidiary relations!
- The initial state (designated “1”)
 - For a PWR,
 - Pressure (p_{w1}), temperature (T_{w1}), mass (m_w)
 - Initial mass of air (m_a)
 - Initial mass of water vapor in the air (from relative humidity) (m_{wc1})
 - Negligible
- The final state
 - For a preexisting plant \Rightarrow containment volume is known \Rightarrow peak pressure is sought!
 - For a new plant \Rightarrow peak pressure is specified as a design limit \Rightarrow seek the containment volume!



Analysis of Final Equilibrium Pressure Conditions

❖ Analysis

- Redesignate the water initially in the containment air: m_{wa}
- Subscript (p) \Rightarrow sys

$$U_2 - U_1 = Q_{n-wpr} - Q_{c-st} \quad \text{where } U_2 = m_a u_{a2} + (m_{wc1} + m_{wp}) u_{wc2}$$

$$U_1 = m_a u_{a1} + m_{wc1} u_{wc1} + m_{wp} u_{wp1}$$

$$\left[(m_{wa} + m_{wsys}) u_{w2} - (m_{wa} u_{wa1} + m_{wsys} u_{wsys1}) \right] + \left[m_a (u_{a2} - u_{a1}) \right] = Q_{n-wsys} - Q_{c-st}$$

Final water energy
in the air and the RCS



Initial water energy
in the air and the RCS

$$\left[m_a (u_{a2} - u_{a1}) \right] = m_a c_{va} (T_2 - T_{a1})$$

$$m_w u_{w1} \equiv m_{wa} u_{wa1} + m_{wsys} u_{wsys1}$$

$$m_w u_{w2} \equiv (m_{wa} + m_{wsys}) u_{w2}$$

$$m_w (\boxed{u_{w2}} - u_{w1}) + m_a c_{va} (\boxed{T_2} - T_{a1}) = Q_{n-wsysr} - Q_{c-st}$$

Two unknowns in one equation!
 c_{va} : dependent variable of (T and P)

Analysis of Final Equilibrium Pressure Conditions

❖ Analysis

● Additional relations are needed!

■ Assumptions

- The volume of liquid water can be neglected.
- The air occupies the same total volume with liquid water plus vapor

$$V_T = V_c + V_p \quad \text{or} \quad V_c + V_s$$

- The water vapor and liquid exist at the partial pressure of the saturated water vapor.
 - Actually, the liquid is at a pressure equal to the total pressure.

■ Dalton's law of partial pressure

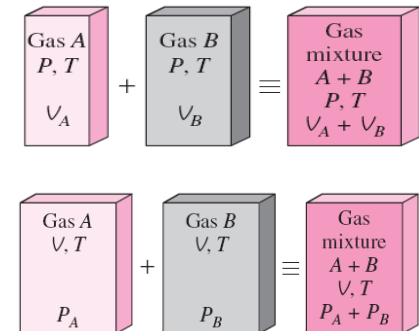
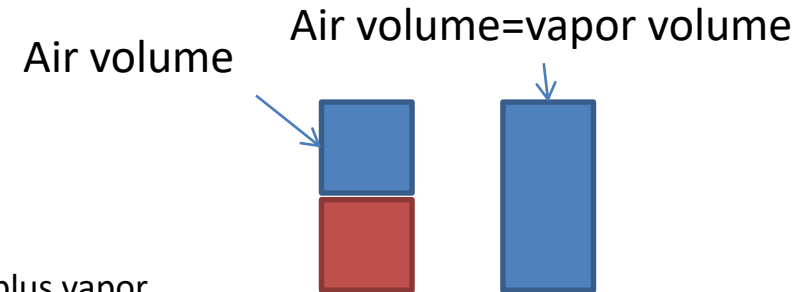
Dalton's law of additive pressures

$$p_2 = p_{w_2}(T_2) + p_{a_2}(T_2)$$

p_2 = final equilibrium pressure of the mixture

p_{w_2} = partial pressure of the saturated water vapor corresponding to T_2

p_{a_2} = partial pressure of air corresponding to T_2



Analysis of Final Equilibrium Pressure Conditions

❖ Analysis

● Additional relations are needed!

- Dalton's law of partial pressure

$$V_T = m_{w_2} \boxed{v_{w_2}(T_{2,\text{sat}})} \approx m_a v_a(T_2, p_{a_2}) \quad \leftarrow \text{Liquid volume is neglected!}$$

- Water in the final state: two-phase mixture

$$V_T = m_{w_2} [v_{f_2} + x_{\text{st}} v_{fg_2}(T_{2,\text{sat}})] \approx \frac{m_a R_a T_2}{p_{a_2}} \quad \leftarrow \text{Air is a perfect gas!}$$

- Initial air pressure: p_{a1}

- Total pressure: p_1
- Relative humidity: ϕ
- Dry bulb temperature: T_{a1}

$$p_{wa_1} = \phi p_{\text{sat}}(T_{a_1}) \quad p_{a_1} = p_1 - p_{wa_1}$$

- Air mass

- By the perfect gas law

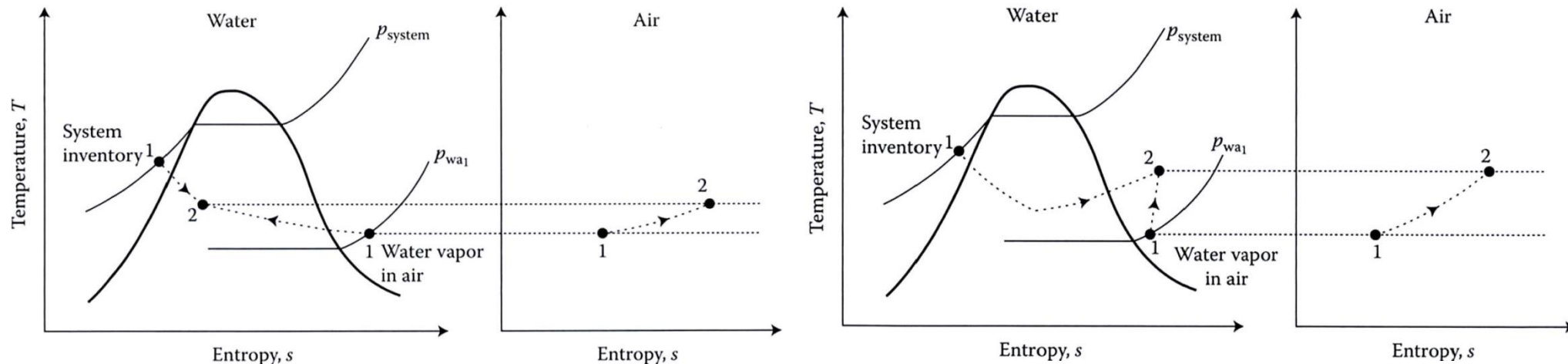
$$m_a = \frac{p_{a_1} V_c}{R_a T_{a_1}}$$

Analysis of Final Equilibrium Pressure Conditions

❖ Individual Cases

● Two general cases for the final condition

- Saturated water mixture in equilibrium with the air.
 - It is the expected result in the containment after a postulated large primary system pipe rupture.
 - Because the heat addition from the core is relatively small, we neglect it here for simplicity.
- Superheated steam in equilibrium with the air.
 - This case requires that heat be added to the thermodynamic system.
 - Such a situation could occur upon rupture of a PWR main steam line, as the intact primary system circulates through the steam generator and adds significant heat to the secondary coolant, which is blowing down into the containment.



Example 7.1 Containment Pressurization: Saturated Water Mixture in Equilibrium with Air Resulting from a PWR Primary System Rupture

PROBLEM A

Find the peak pressure, given the containment volume.

SOLUTION

Equations 7.15 and 7.18 are the governing equations, and numerical values for containment conditions are drawn from Table 7.2. There are several ways in which the final pressure can be determined. One method is a trial-and-error solution using the steam tables.

The approach is to assume a final temperature (T_2) and from Equation 7.18 calculate the static quality (x_{st}). This quality–temperature pair is checked in Equation 7.15, and the search is continued until these equations are simultaneously satisfied. A quality greater than 1 indicates that the equilibrium water condition is superheated, and this search technique fails. This result is unlikely for realistic reactor containment conditions.

Example 7.1

Fluid	Heat Addition during Blowdown (Joules)	Volume (m ³)	Pressure (MPa)	Temperature (K)	Quality (x_{st}) or Relative Humidity (Φ)
Example 7.1: Saturated Water Mixture in Equilibrium with Air as Final State					
Primary coolant water (initial)		$V_p = 354$	15.5	617.9	Assumed saturated liquid
Containment vessel air (initial)		$V_c = 50,970$	0.101	300.0	$\Phi = 80\%$
Mixture (final)	$Q = 0$	$V_T = 51,324$	0.523	415.6	$x_{st} = 50.5\%$

Example 7.1

❖ Recall the governing equations

$$m_w(u_{w_2} - u_{w_1}) + m_a c_{va} (T_2 - T_{a_1}) = \cancel{Q_{n-wsysr}} - \cancel{Q_{c-st}}$$

Energy conservation

$$V_T = m_{w_2} [v_{f_2} + x_{st} v_{fg_2} (T_{2,sat})] \approx \frac{m_a R_a T_2}{p_{a_2}}$$

Dalton's law

$u_{w_2} = f(x_{st}, P)$, $v_{f_2} = f(T_{2,sat})$, $v_{fg_2} = f(T_{2,sat})$ Three equations with three unknowns + steam table!

● Iterative solution method

- Assume a final temperature, $T_2 = 415 \text{ K}$!

$$m_w u_{w_1} \equiv m_{wa} u_{wa_1} + m_{wsys} u_{wsys_1}$$

$$m_w u_{w_2} \equiv (m_{wa} + m_{wsys}) u_{w_2}$$

● Parameters that we can evaluate from the given information

- $m_a = f(p_{a_1}, T_{a_1})$
 - $p_1 = p_{a_1} + p_{w_1} = 0.101 \text{ MPa}$
 - $p_{w_1} = \phi p_{sat}(T_1) \quad p_{w_1} = \Phi p_{sat}(T_1) = 0.8(3498 \text{ Pa}) = 2798 \text{ Pa}$

2829 Pa ?

Example 7.1

❖ Parameters that we can evaluate from the given information

- $m_a = f(p_{a1}, T_{a1})$

- $p_1 = p_{a1} + p_{w1} = 0.101 \text{ MPa}$

- $p_{w1} = \phi p_{\text{sat}}(T_1)$

$$p_{w1} = \Phi p_{\text{sat}}(T_1) = 0.8(3498 \text{ Pa}) = 2798 \text{ Pa}$$

$$p_{a1} = p_1 - p_{w1} = 101,378 - 2798 = 98,580 \text{ Pa}$$

$$m_a = \frac{p_{a1} V_c}{R_a T_{a1}} = \frac{(98,580) \text{ Pa} (50,970) \text{ m}^3}{(286) \text{ J/kg } ^\circ\text{K} (300) ^\circ\text{K}} = 5.9(10^4) \text{ kg}$$

- $m_{wa} = \frac{V_c}{v_{wa1}}$

- $v_{wa1} = f(p_{w1}, T_1) = 48.87 ?$

$$m_{wa} = \frac{V_c}{v_{wa1}} = \frac{50,970 \text{ m}^3}{50.02 \text{ m}^3/\text{kg}} = 1019 \text{ kg}$$

$$m_w u_{w1} \equiv m_{wa} u_{wa1} + m_{wsys} u_{wsys1}$$

$$m_w u_{w2} \equiv (m_{wa} + m_{wsys}) u_{w2}$$

Example 7.1

❖ Parameters that we can evaluate from the given information

$$m_w(u_{w_2} - u_{w_1}) + m_a c_{va}(T_2 - T_{a_1}) = Q_{n-wsysr} - Q_{c-st}$$

$$V_T = m_{w_2} [v_{f_2} + x_{st} v_{fg_2}(T_{2,sat})] \simeq \frac{m_a R_a T_2}{p_{a_2}}$$

$$u_{w_2} = f(x_{st}, P)$$

● x_{st} at state 2

$$x_{st} = \frac{(V_T/m_w) - v_{f_2}}{v_{fg_2}}$$

Assume a final temperature,
 $T_2 = 415 \text{ K!}$

- $m_w = m_{wp} + m_{wa} = 211372 \text{ [kg]}$
- $v_{f_2} = f(T_2) = 0.001082 \text{ [m}^3/\text{kg]}$
- $v_{fg_2} = v_{g_2} - v_{f_2} = f(T_2) = 0.483 \text{ [m}^3/\text{kg]}$

$$x_{st} = \frac{(V_T/m_w) - v_{f_2}}{v_{fg_2}} = \frac{(51,324)\text{m}^3/2.11(10^5)\text{kg} - 0.00108\text{m}^3/\text{kg}}{0.485\text{m}^3/\text{kg}} = 0.499$$

Example 7.1

❖ Parameters that we can evaluate from the given information

$$m_w(u_{w_2} - u_{w_1}) + m_a c_{va}(T_2 - T_{a_1}) = Q_{n-wsysr} - Q_{c-st}$$

$$m_w u_{w_1} \equiv m_{wa} u_{wa_1} + m_{wsys} u_{wsys_1}$$

$$m_w u_{w_2} \equiv (m_{wa} + m_{wsys}) u_{w_2}$$

● u_{w2} at state 2 (saturated state)

$$u_{w2} = u_{f2} + x_{st} u_{fg2}$$

$$u_{f2} = f(T_2) = 596,734 \text{ [J/kg]}$$

$$u_{fg2} = f(T_2) = 1,954,676 \text{ [J/kg]}$$

Initial vapor
partial pressure



● u_{w1}

$$u_{wp1} = f(P_1) = 1,603,772 \text{ [J/kg]}, u_{wa1} = f(p_{w1})$$

Assume a final temperature,
 $T_2 = 415 \text{ K!}$

● c_{va}

$$c_{va} = 718 \text{ [J/kgK]}$$

$$m_{wp}(u_{f2} + x_{st} u_{fg2} - u_{wp1}) + m_{wa}(u_{f2} + x_{st} u_{fg2} - u_{wa1}) + m_a c_{va}(T_2 - T_{a1}) = 0$$

$$2.1(10^5)[595,380 + x_{st} 1.95(10^6) - 1.6(10^6)] + 1.02(10^3)$$

$$\times [595,380 + x_{st} 1.95(10^6) - 2.41(10^6)] + 5.9(10^4)(719)[415 - 300] = 0$$

$$x_{st} = 0.505$$

Example 7.1

❖ Iterative solution method

$$x_{st} = 0.499$$

$$x_{st} = 0.505$$

Assume a final temperature,

$$T_2 = 415 \text{ K!}$$

- Change the temperature and repeat the procedure until the two qualities are equal!
- Final solution

$$T_2 = 415.6 \text{ K}, \quad p_{w2} = 0.386 \text{ MPa}$$

$$p_{a2} = \frac{m_a R_a T_a}{V_T} = \frac{5.9(10)^4 \text{ kg} (286 \text{ J/kg K}) (415 \text{ K})}{51,324 \text{ m}^3} = 1.37(10^5) \text{ Pa}$$

$$p_{a2} = p_{a1} \left(\frac{T_{a2}}{T_{a1}} \right) = 0.099 \frac{(415.6)}{300} = 0.137 \text{ MPa}$$

$$p_2 = p_{w2} + p_{a2} = 0.386 + 0.137 = 0.523 \text{ MPa}$$

For a preexisting plant

⇒ containment volume is known

⇒ peak pressure is sought!

❖ Homework

- Solve this problem by yourself! 반복계산 코딩으로!
- Solve the problem B by yourself!

참고

$\rho(P, T) - \rho_{sol} = 0$ 을 만족하는 T 찾기!

function example1


```
P=15.5e6;  
Tsol=250.0;  
Tinit=400.0;  
rho_sol=XSteam('rho_pT',P/1.0e5,Tsol);  
fun=@(Tinit)test(Tinit,P,rho_sol);  
T = fzero(fun,Tinit)
```

```
function result=test(T,P,rho_sol)
```

```
rho_spray=XSteam('rho_pT',P/1.0e5,T);  
result=rho_spray-rho_sol;
```

초기값 T 입력 시 아래 식을 만족시키는 T 찾기!

$$x_{st} = \frac{(V_T/m_w) - v_{f2}}{v_{fg2}}$$


$$m_w(u_{w_2} - u_{w_1}) + m_a c_{va}(T_2 - T_{a_1}) = 0$$

Example 7.1

PROBLEM B

Find the containment volume, given a design limit for the peak pressure (p_2).

5 bar, 5.5 bar

Example 7.2 Containment Pressurization: Superheated Steam in Equilibrium with Air Resulting from a PWR Secondary System Rupture

A rupture of a main steam line adds water to the containment while the intact primary system circulates water through the steam generator, transferring energy to the secondary water that is discharging into the containment. The amount of water added is primarily dependent on the size of the steam generator. For this example, typical PWR four-loop plant values have been used for the amount of secondary water added and energy transferred via the steam generator. These assumed values are listed in Table 7.2.

PROBLEM A

Find the peak pressure, given the containment volume.

Example 7.2

Conditions for Containment Examples

Fluid	Heat Addition during Blowdown (Joules)	Volume (m ³)	Pressure (MPa)	Temperature (K)	Quality (x_{st}) or Relative Humidity (Φ)
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Example 7.2: Superheated Steam in Equilibrium with Air as Final State

Secondary coolant water (initial)		$V_s = 89$	6.89	558	Assumed saturated liquid
Containment vessel air (initial)		$V_c = 50,970$	0.101	300	$\Phi = 80\%$
Mixture (final)	$Q = 10^{11}$	$V_T = 51,059$	0.446 (64.7 psia)	478	$\Phi = 17\%$

Example 7.2

❖ Superheated Steam in Equilibrium

Procedure in the textbook

- Treat the superheated steam as a perfect gas.



Do we really need this assumption?

$$p_2 = \frac{m_w R_w T_2}{V_T} + \frac{m_a R_a T_2}{V_T}$$

- $m_w = m_{wa} + m_{ws}$

$$m_w (u_{w2} - u_{w1}) + m_a c_{va} (T_2 - T_{a1}) = Q_{n-wsysr} - Q_{c-st}$$

$$u_{w2} = f(T_2, P_{2,steam})$$

- Unknowns

$$(T_2, p_2, u_{w2})$$

- Iterative solution method

- Assume T_2
- Calculate P_2
- Calculate u_{w2} and compare it with the steam table!

❖ Superheated Steam in Equilibrium

$$V_T = m_{w2} v_g(T_2, P_{2,steam}) \quad V_T = \frac{m_a R_a T_2}{P_{a2}} \quad u_{w2} = f(T_2, P_{2,steam})$$

$$m_w(u_{w2} - u_{w1}) + m_a c_{va}(T_2 - T_{a1}) = Q_{n-wsysr} - Q_{c-st}$$

● Unknowns

- $T_2, P_{2,steam}, P_{2,a}, u_{w2}$

● Iterative solution method

- Assume T_2
- Calculate P_{a2} and $P_{2,steam}$
- Calculate u_{w2} and compare it with the steam table!

❖ Homework #6

PROBLEM B

Find the containment volume, given a design limit for the peak pressure (p_2). 5 bar, 5.5 bar

- Select one problem in Example 7.1 (A or B)
- Select one problem in Example 7.2 (B if you selected 7.1 A or A otherwise)

❖ Similar primary system rupture analysis for NUSCALE

- Check the pressure increase in the containment vessel 3 problems

❖ Due date: 6/25