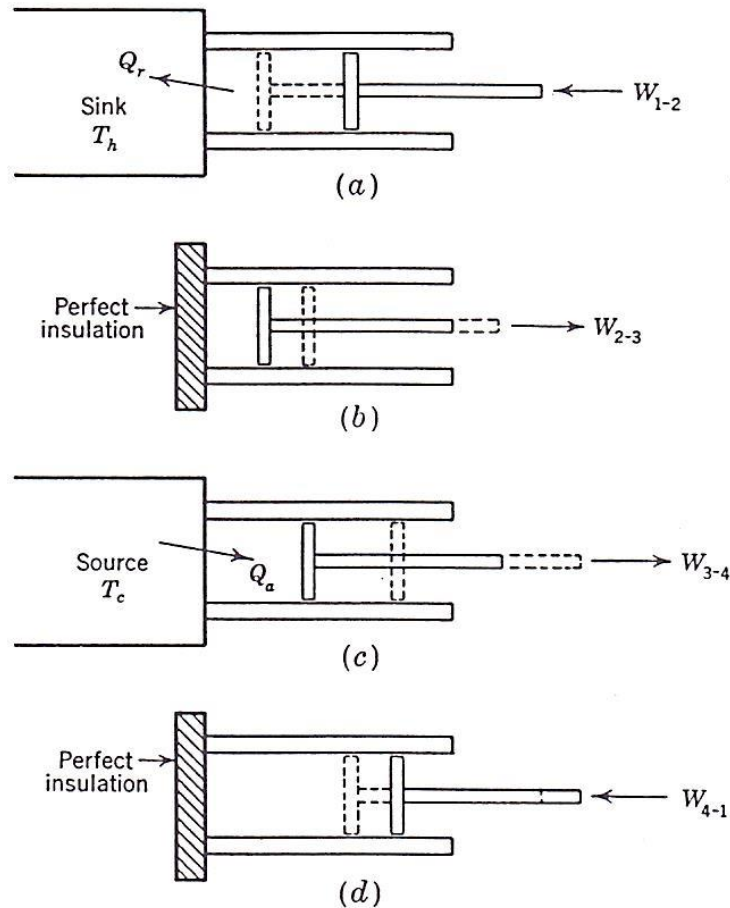# Cryogenic Engineering

Chapter 5.

Cryogenic Refrigeration Systems
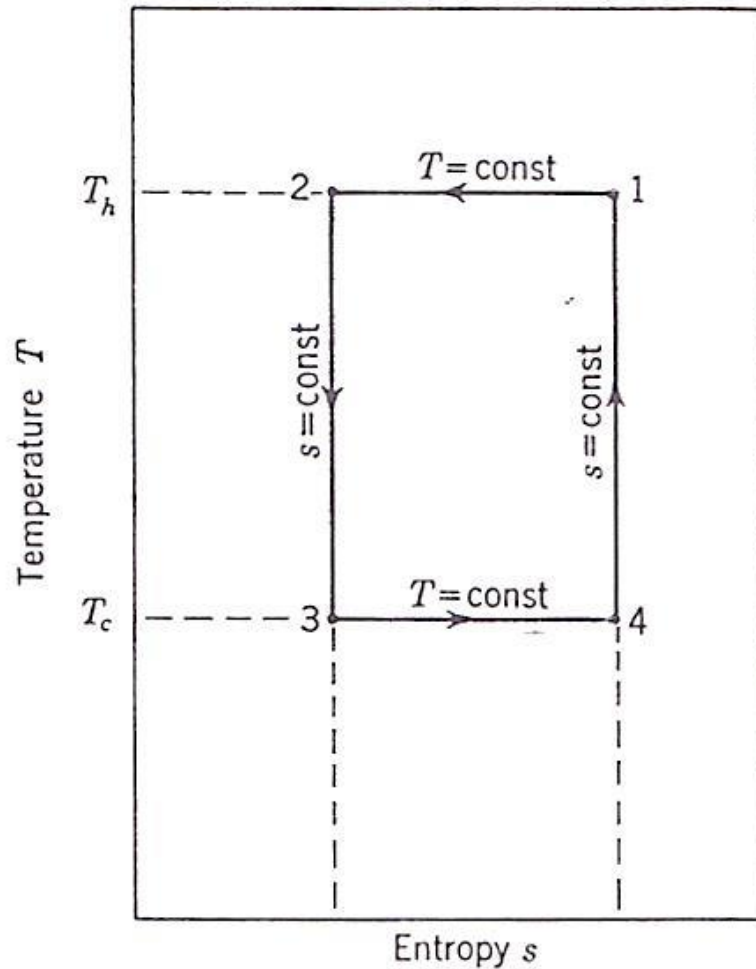
KIM, Min Soo

5.1 Thermodynamically Ideal Isothermal-Source

<Carnot Refrigeration>

(a) Reversible isothermal compression

(b) Reversible adiabatic expansion

(c) Reversible isothermal expansion with heat adsorption from the low temperature source

(d) Reversible adiabatic compression

5.1 Thermodynamically Ideal Isothermal-Source

<Carnot Refrigeration>

1-2 : Reversible isothermal compression with heat remover

2-3 : Reversible adiabatic expansion (isentropic)

3-4 : Reversible isothermal expansion with heat addition

4-1 : Reversible adiabatic compression (isentropic)

$$ds \equiv \left. \frac{\delta q}{T} \right)_{ideal}$$

5.1 Thermodynamically Ideal Isothermal-Source

- **Figure of Merit (FOM)**

  - $\text{FOM} = \text{COP}/\text{COP}_i$

  - COP is the coefficient of performance of the actual system

  - COP_i is the coefficient of performance of the thermodynamically ideal system

- **Coefficient of performance (COP)**

  - $\text{COP} = -\dfrac{Q_a}{W_{net}} = \dfrac{Q_a/m}{-W_{net}/m}$

  - $Q_{net} = W_{net}$
    $$= mT_h(s_2 - s_1) + mT_c(s_4 - s_3) = -(T_h - T_c)(s_1 - s_2)$$

$$\overset{0}{\oint \delta q = \oint du + \oint \delta\omega}$$

$$\boxed{\therefore \text{COP}_i = \dfrac{T_c}{T_h - T_c}}$$
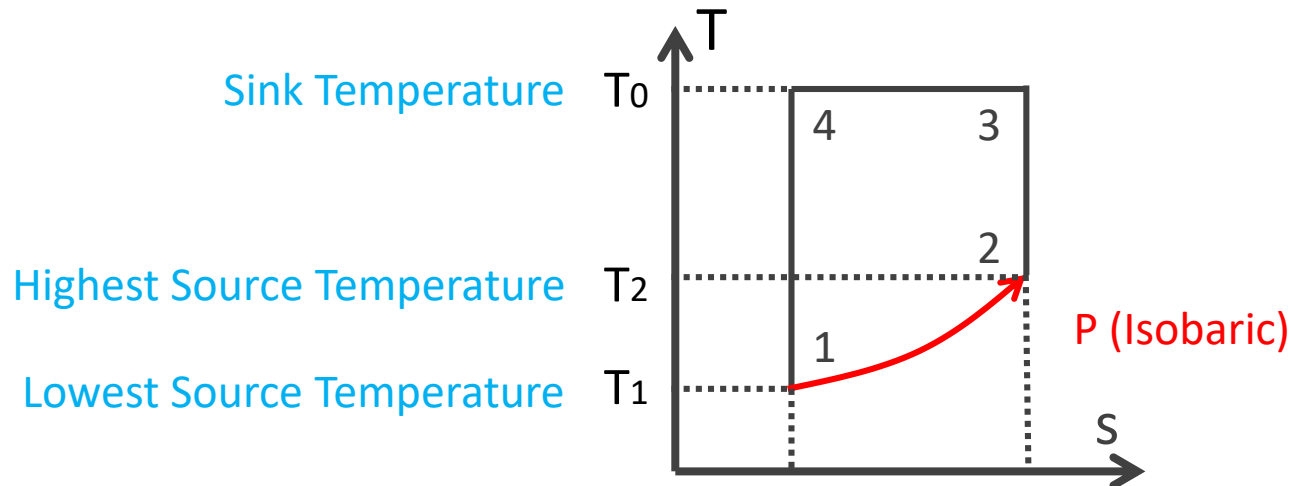
5.1 Thermodynamically Ideal Isothermal-Source

Source Temperature		
K	$COP_i = -Q_a/W_{net}$	$-W_{net}/Q_a$
111.7	0.5932	1.686
77.4	0.3477	2.876
20.3	0.07258	13.778
4.2	0.01420	70.43
1.0	0.003344	299.0
0.1	0.0003334	2999.0
0.01	0.0000333	29999.0

COP for a Carnot refrigerator operating between 300 K and a low temperature T_c

5.2 The thermodynamically Ideal Isobaric-Source System

- **Reversible isobaric-source refrigeration cycle**

  - It is for gas cooling. (not a liquefaction process)

  - There is no phase change, so energy is absorbed at a varying temperature.

The energy rejected from the system is given by

$$Q_{rejected} = mT_0(s_4 - s_3)$$

The energy absorbed from the source is given by

$$Q_{absorbed} = \int_1^2 mTds = m\int_1^2 (dh - vdp) = m(h_2 - h_1)$$

From the First Law of Thermodynamics and the definition of the COP,

$$COP = -\frac{Q_a}{W_{net}} = -\frac{Q_a}{(Q_r + Q_a)} = \frac{h_2 - h_1}{T_0(s_2 - s_1) - (h_2 - h_1)}$$

5.2 The thermodynamically Ideal Isobaric-Source System

For many cold-gas refrigerators, the pressures are sufficiently low that the working fluid may be assumed to behave as an ideal gas.
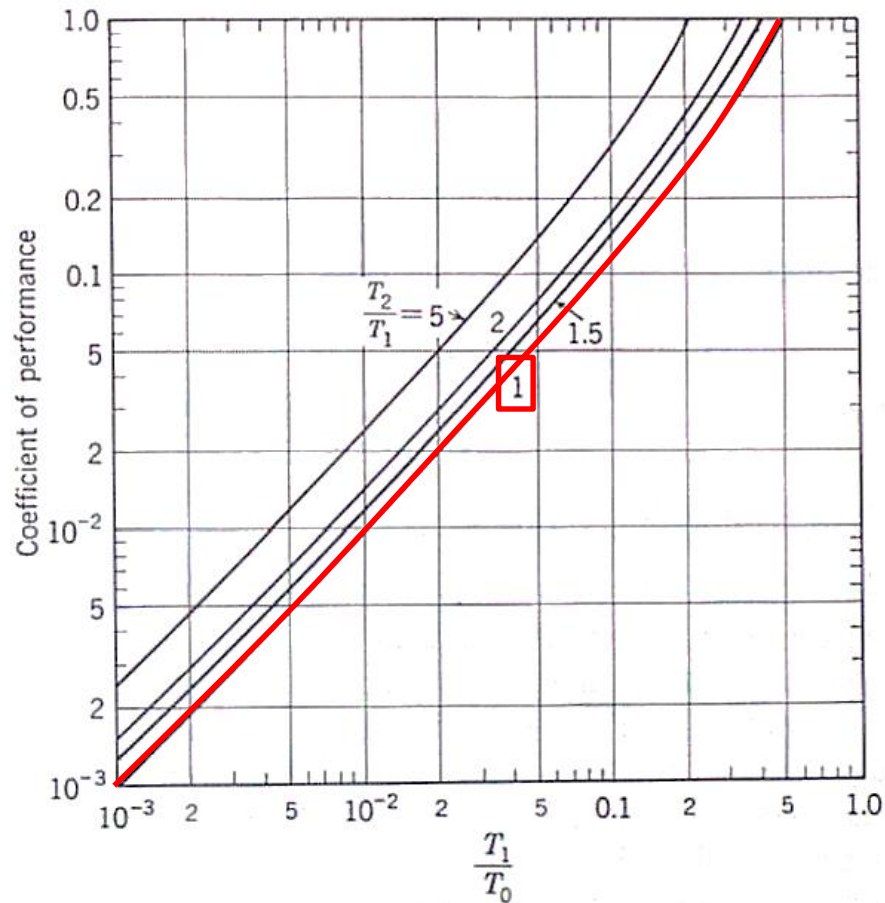
$$h_2 - h_1 = C_p(T_2 - T_1)$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$\therefore COP = \frac{\dfrac{T_2}{T_1} - 1}{\dfrac{T_0}{T_1} \ln \dfrac{T_2}{T_1} - \dfrac{T_2}{T_1} + 1}$$

(COP is independent of the refrigerant)

The equation of COP is plotted in below figure.

If $\dfrac{T_2}{T_1} \to 1$,

$$COP \to COP_{carnot}$$

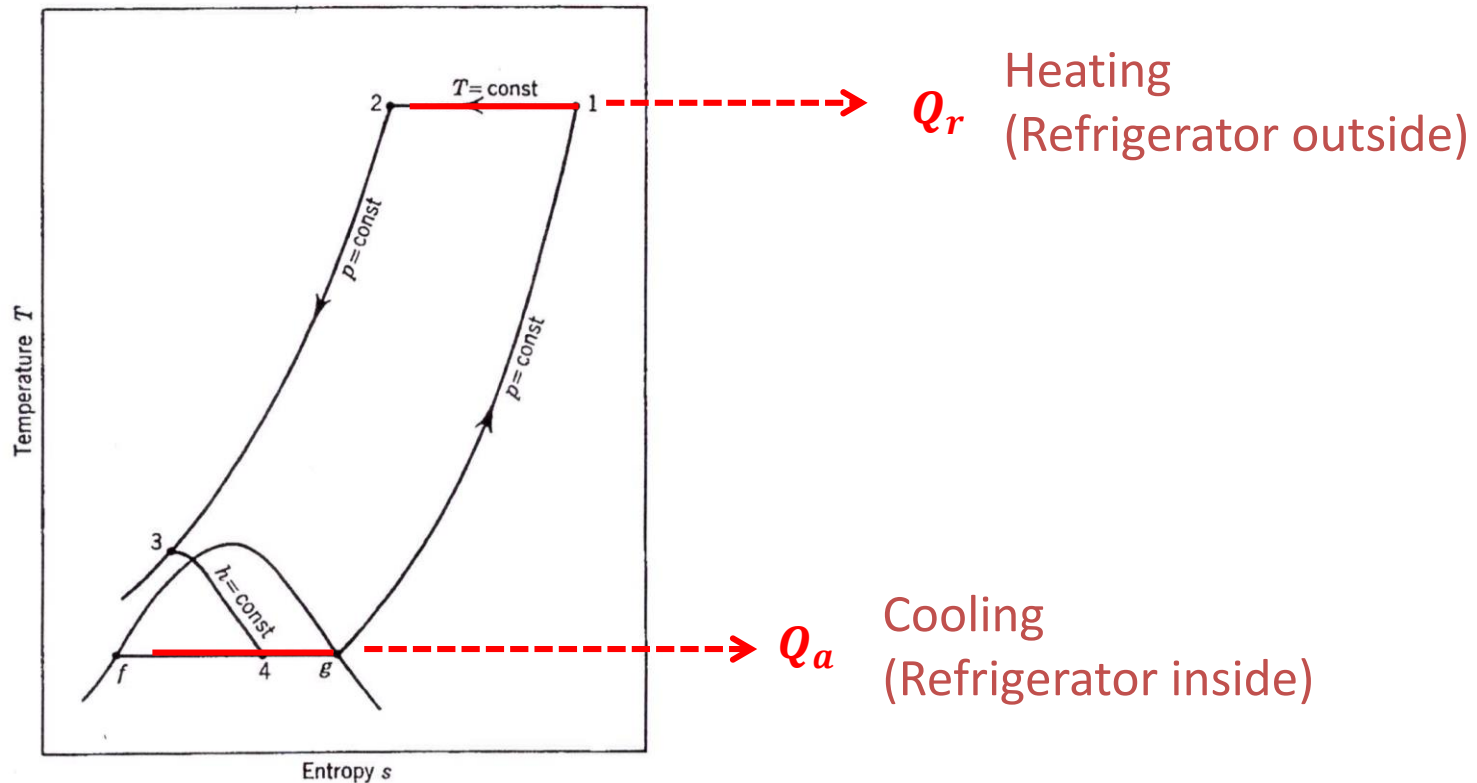(Referring chapter 3, Simple Linde-Hampson system)

Instead of withdrawing the liquid, **heat is absorbed to evaporate it**.
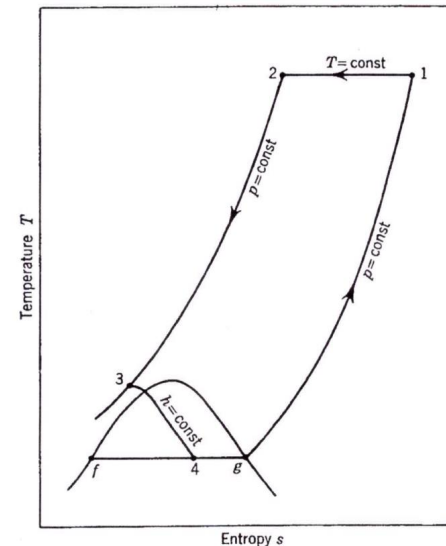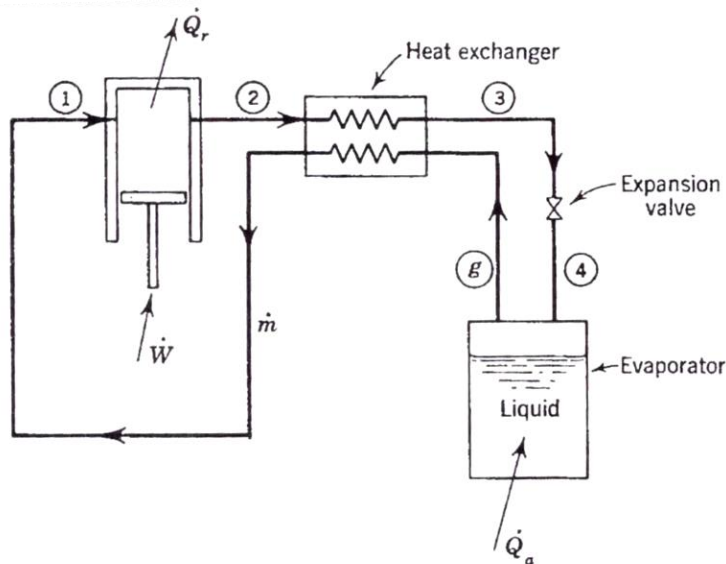
At control volume, the total energy absorbed from the source is given by

$$\dot{Q}_a = \dot{m}(h_1' - h_2)$$

5.3 Joule-Thomson Refrigeration System

Q_r Heating (Refrigerator outside)

Q_a Cooling (Refrigerator inside)

The heat exchanger effectiveness

$$\varepsilon = \frac{h_1' - h_g}{h_1 - h_g} = 1 - \frac{h_1 - h_1'}{h_1 - h_g}$$
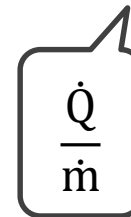
- The total energy absorbed

$$\frac{\dot{Q}}{\dot{m}} = (h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)$$

$\rightarrow$ $\quad \dfrac{\dot{Q}}{\dot{m}} > 0$ $\quad$ ($T_{warm.end} < T_{inversion}$ ... not for Ne, He, H2!)

$\rightarrow$ $\quad 1 - \dfrac{h_1 - h_2}{h_1 - h_g} < \varepsilon$

Below this value, the refrigerator will not work.
(This limit is determined by setting the effect equal to zero)

$$\frac{\dot{Q}}{\dot{m}}$$

5.3 Joule-Thomson Refrigeration System

- The work requirement

$$-\frac{\dot{W}}{\dot{m}} = \frac{T_2(s_1 - s_2) - (h_1 - h_2)}{\eta_{c.o}}$$

$\eta_{c.o}$ = overall efficiency of the compressor
(mechanical loss, friction, and etc.)

$$COP = \frac{-\dot{Q}_a}{\dot{W}} = \frac{\eta_{c.o}[(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)]}{T_2(s_1 - s_2) - (h_1 - h_2)}$$

$$COP_i = \frac{T_c}{T_h - T_c} \qquad \rightarrow \qquad FOM = \frac{COP}{COP_i}$$

5.3 Joule-Thomson Refrigeration System

▪ The temperature in the evaporator can be regulated by controlling the pressure by means of the expansion-valve setting.

Temperature	Pressure
65 K	17.4 kPa
115 K	1.939 MPa

Ex. When nitrogen is the working fluid

The temperature range is limited on the lower end by
1) the triple point
2) the difficulty in maintaining low vacuum pressure with large flow rates

If the pressure were lowered below the triple-point,
N_2 snow would form and clogging valve.

The temperature is limited on the high end by the critical point.

5.4 Refrigerator Optimization

- To design a refrigeration system
  - → Determine the effectiveness and cost of primary components.

① Compressor costs

$$C_c = C_1 \dot{W}_c = C_1 (\dot{W}_c / \dot{m}) \dot{m}$$

C_1 = Compressor cost per unit power requirement, including both operating and capital costs

② Heat exchanger costs

$$C_E = C_2 A$$

③ The sum of the compressor and heat exchanger costs

$$C_T = C_c + C_E$$

- The optimum condition → Derivative of the total cost equal to zero

$$\frac{dC_T}{di} = 0 = C_1 \left(\frac{\dot{W}_c}{\dot{m}} \right) \frac{d\dot{m}}{di} + C_2 \frac{dA}{di} \qquad ....(5.41)$$

$i = 1 - \varepsilon$ = heat exchanger ineffectiveness

5.4 Refrigerator Optimization

$$\dot{Q}_a = \dot{m} \left(h_1' - h_2 \right) \qquad \text{and} \qquad \varepsilon = \frac{h_1' - h_g}{h_1 - h_g}$$

$$\frac{\dot{Q}_a}{\dot{m}} = (h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)$$

$$\dot{m} = \frac{\dot{Q}_a}{(h_1 - h_2) - i(h_1 - h_g)} = \frac{\dot{Q}_a}{(h_1 - h_2)(H - i)}, \qquad \text{where } H = \frac{(h_1 - h_2)}{(h_1 - h_g)} = i_{max}$$

And, we can determine $\dfrac{d\dot{m}}{di}$, $\dfrac{dA}{di}$, $\emptyset$

$$\frac{d\dot{m}}{di} = \frac{\dot{Q}_a}{(h_1 - h_g)(H - i)^2} \qquad \qquad \text{....(5.42)}$$

5.4 Refrigerator Optimization

▪ Because of most of H/E used in cryogenic systems are counter flow, **surface area may be found from the effectiveness-NTU equation below**.

$$\epsilon = \frac{1 - \exp[-NTU(1 - C_R)]}{1 - C_R \exp[-NTU(1 - C_R)]} \quad \text{and} \quad NTU = \frac{UA}{C_{min}}$$

$$A = \frac{(\dot{m}C_C/U)}{(1 - C_R)} \ln\left(\frac{(1 - C_R + C_R i)}{i}\right)$$

$$\frac{dA}{di} = -\frac{(\dot{m}c_c/U)}{i(1 - C_R + C_R i)} \qquad \qquad \dots(5.43)$$

$$\emptyset = \frac{C_2(h_1 - h_g)(\dot{m}c_c/U)}{C_1(\dot{W}_c/\dot{m})\dot{Q}_a} \qquad \qquad \dots(5.44)$$

Making the substitutions from equations, 5.42~5.44, above into equation 5.41

$$(\emptyset - C_R)i^2 - (2\emptyset H + 1 - C_R)i + \emptyset H^2 = 0$$

5.4 Refrigerator Optimization

$$B_1 = \frac{2\emptyset H + 1 - C_R}{2(\emptyset - C_R)}$$

$$B_2 = \frac{\emptyset H^2}{\emptyset - C_R}$$

By using these equations, we can obtain the optimum ineffectiveness
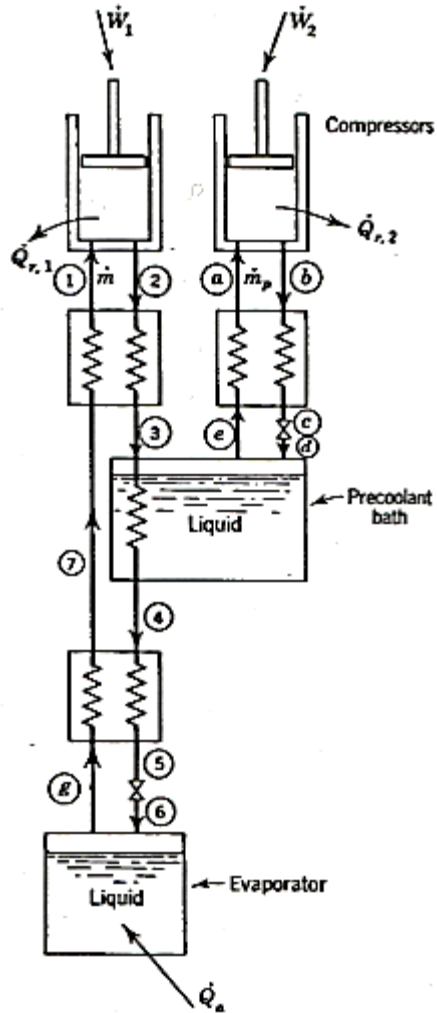
$$i_{opt} = B_1 - (B_1{}^2 - B_2)^{\frac{1}{2}}, \quad \text{where } i^2 - 2B_1 i + B_2 = 0$$

$$i_{opt} = \frac{C_R H^2}{2 C_R H + 1 - C_R}$$

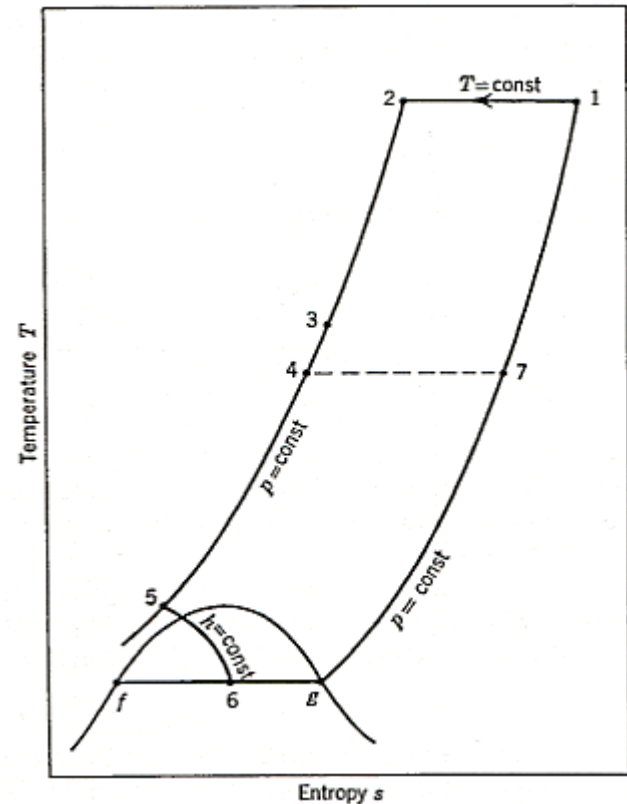If, we are given the heat exchanger "free", that is, if $C_2 = 0$, we find that $\emptyset = 0$ and $B_2 = 0$.

$$\text{So, } i_{opt} = 0$$

<Precooled Linde-Hampson refrigerator>

<Thermodynamic cycle for the precooled Linde-Hampson refrigerator>

5.5 Cascade or Precooled Joule-Thomson Refrigerators

For temperatures lower than those obtainable with liquid nitrogen, the only available working fluids are neon, hydrogen, helium. For these fluids, precooled system is useful.

$$\dot{Q}_a = \dot{m}(h'_1 - h_2) + \dot{m}_p(h'_a - h_b)$$

$$\dot{Q}_a/\dot{m} = (h'_1 - h_2) + z(h'_a - h_b)$$

$$z = \dot{m}_p/\dot{m}$$

$$\varepsilon = \frac{h'_1 - h_g}{h_1 - h_g}$$

$$\varepsilon_p = \frac{h'_a - h_e}{h_a - h_e}$$

$$\dot{Q}_a/\dot{m} = (h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g) + z[(h_a - h_b) - (1 - \varepsilon_p)(h_a - h_e)]$$

Applying the First Law to the cold exchanger and the evaporator, we obtain

$$\dot{Q}_a/\dot{m} = h_7' - h_4$$

Introducing the effectiveness of the cold and hot exchanger,

$$\varepsilon_c = \frac{h_7' - h_g}{h_7 - h_g} \, , \quad \varepsilon_h = \frac{h_1' - h_7}{h_1 - h_7}$$
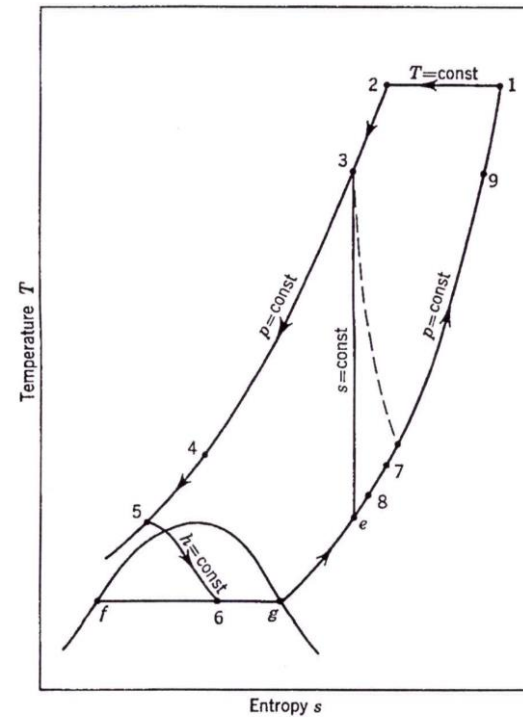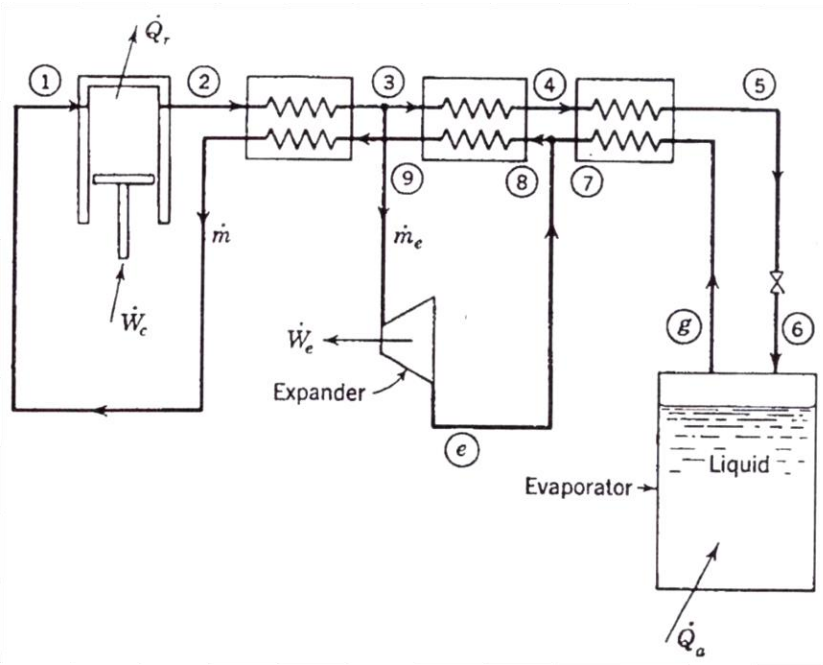
The refrigeration effect may be written as follows :

$$\dot{Q}_a/\dot{m} = (h_7 - h_4) - (1 - \varepsilon_c)(h_7 - h_g)$$

(Referring chapter 3, Claude system(Collins system))

COP is very low, but it can make very low temperature

At control volume, the total energy absorbed from the source is given by

$$\frac{\dot{Q}_a}{\dot{m}} = (h_1' - h_2) + x(h_3 - h_e')$$

$$= \frac{\dot{m}_e}{\dot{m}} \qquad = \eta_{a.d}(h_3 - h_e) \quad \text{※ Adiabatic efficiency}$$

$$-\frac{\dot{W}_{net}}{\dot{m}} = -\frac{\dot{W}_c}{\dot{m}} - x\frac{\dot{W}_e}{\dot{m}_e}$$
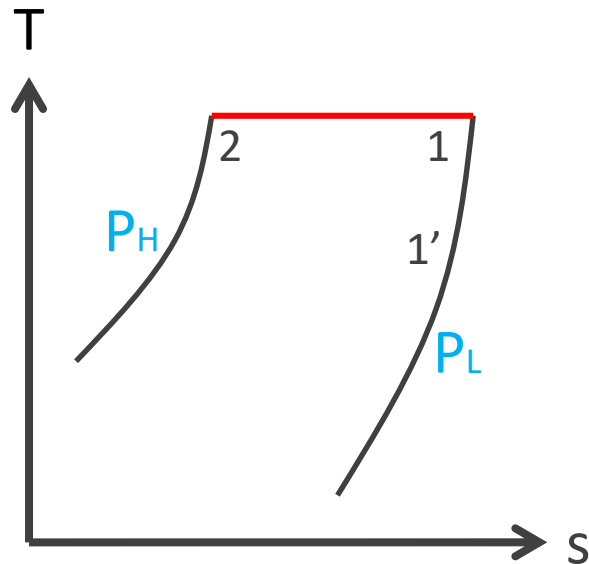
$$\text{※ } \dot{m}h_1 + \dot{Q}_R = \dot{m}h_2 + \dot{W}_c$$

$$= \frac{[T_1(s_1 - s_2) - (h_1 - h_2)]}{\eta_{co}} - x\eta_{em}\eta_{ad}(h_3 - h_e)$$

※ overall efficiency of the compressor
※ mechanical efficiency of the expander

▪ **The heat exchanger effectiveness problem**

Case 1→2,

$$Q_r = T_1(s_2 - s_1) < 0$$

$$h_1 + Q_r = h_2 + W_c,$$

$$W_c = T_1(s_2 - s_1) - (h_2 - h_1) < 0$$

Case 1'→2,

$$Q_r = \int_{1'}^{2} T ds \quad \dashrightarrow \text{ It can't be calculated!}$$

$$W_c = \int_{1'}^{2} T ds - (h_2 - h_1')$$

5.6 Expansion-Engine Refrigeration System

- **History of Claude refrigerator (1)**

    - 1966, Whitter
      Described the design utilizing two expansion engines similar to the Collins liquefier
    - 1968, Meier and Currie
   Described the performance used to maintain 4K while providing 1 watt of refrigeration

- **History of Claude refrigerator (2)**
 - Two significant modification

    - 1971, Johnson et al.
      using a "wet" expander (operating in the two-phase region to replace the expansion valve)
    - 1982, Minta and Smith
      using a low temperature compressor → Figure at next page

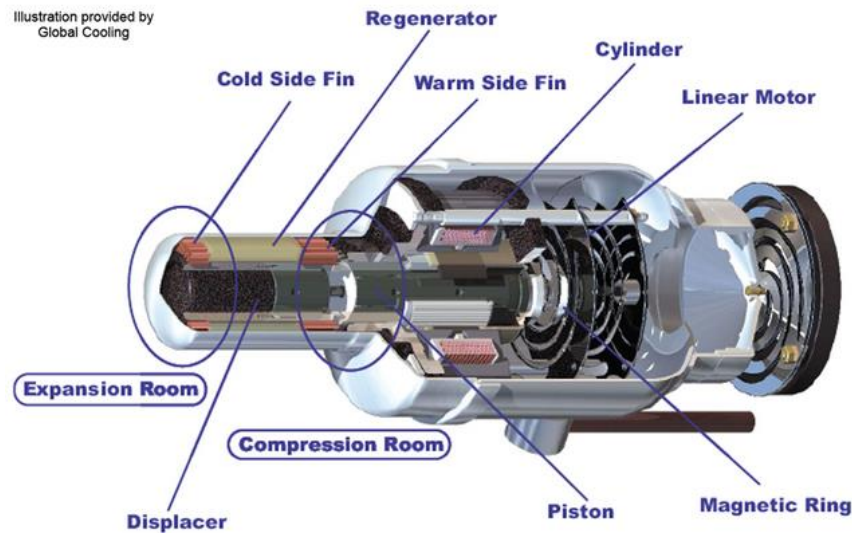- **Claude refrigerator with a wet expander and a saturated-vapor compressor**
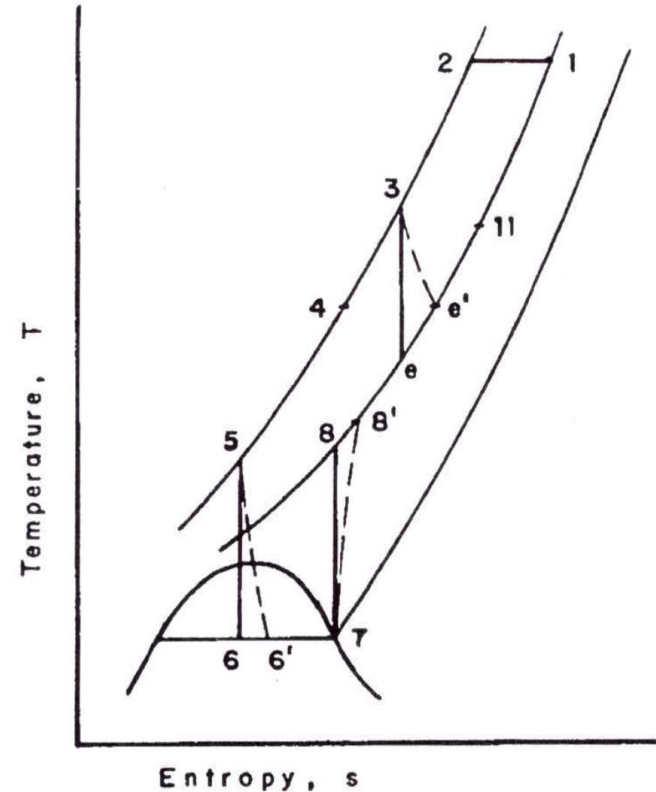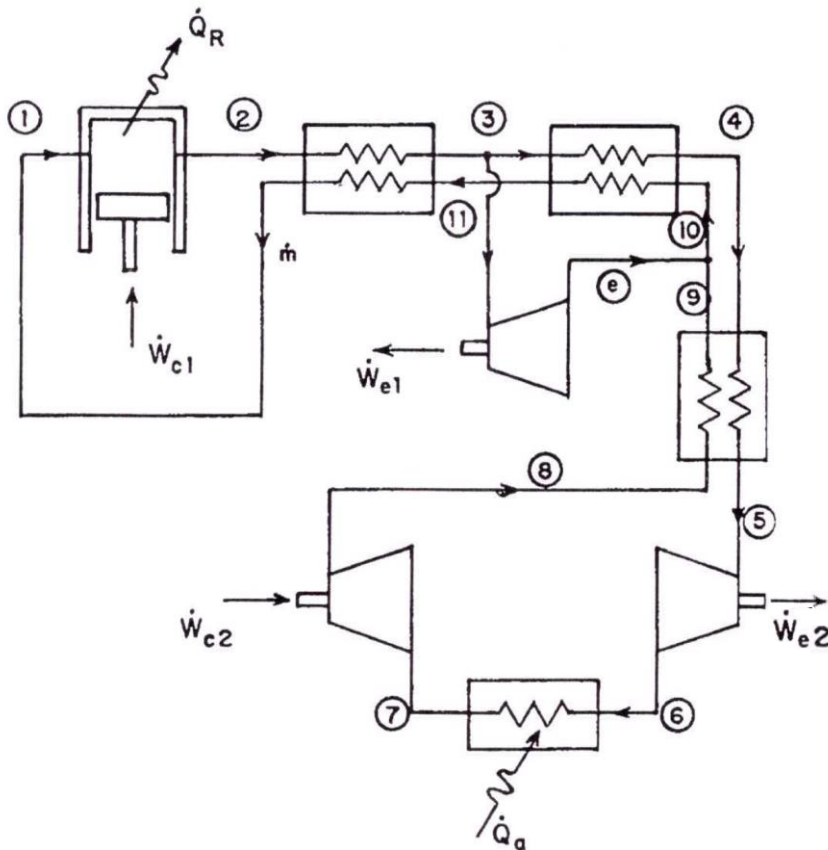
5.6 Expansion-Engine Refrigeration System

- **Characteristics of Claude refrigerator with a wet expander and a saturated-vapor compressor**

    - The wet(two-phase) expander is used primarily for helium system because the thermal capacity of the compressed gas is generally larger than the latent heat of the liquid phase

    - Unlike an air (or nitrogen) expansion engine in which the engine efficiency is seriously affected by the presence of liquid in the engine, operation of the helium expander in the two-phase region does not result in a serious deterioration of the engine performance.

    - This system required less heat exchanger surface than the conventional Claude refrigerator because the heat-transfer coefficients are higher when the cold gas stream pressure is increased.

- **Philips refrigerator?**

Illustration provided by Global Cooling

Regenerator · Cylinder · Linear Motor · Cold Side Fin · Warm Side Fin · Expansion Room · Compression Room · Displacer · Piston · Magnetic Ring

- An refrigeration device that operates on the reverse Stirling cycle, which was originally invented for use in a hot-air engine by a Scottish minister, Robert Stirling

Philips refrigerator (www.apexinst.com/)

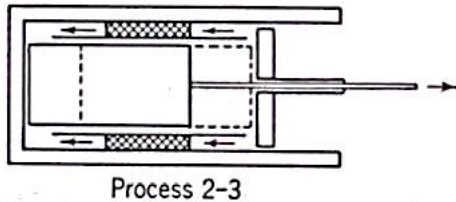# 5.7 Philips Refrigerator

▪ **Stirling engine?**

Stirling engine operates

- ▪ at high mean gas pressures (~200 bar)

- ▪ in an oil – free environment

- ▪ without valve gear

- ▪ Any source of thermal energy is possible

- ▪ Efficiency of Stirling engine is same as that of Carnot cycle in perfect regeneration

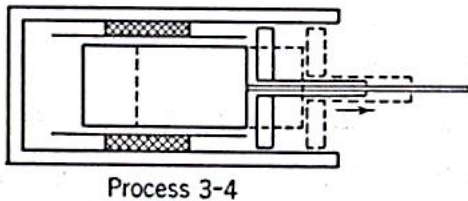- ▪ It requires two different temperature levels
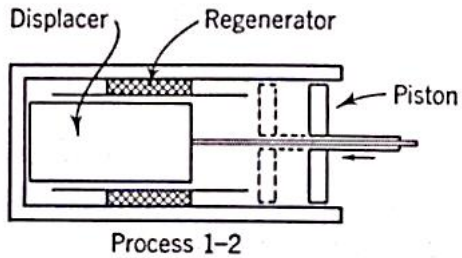
Process 1–2

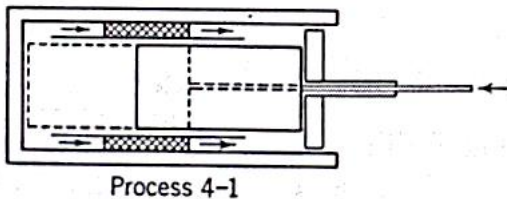1 → 2 : Isothermal compression

Process 2-3

2 → 3 : Const. volume cooling

Process 3-4
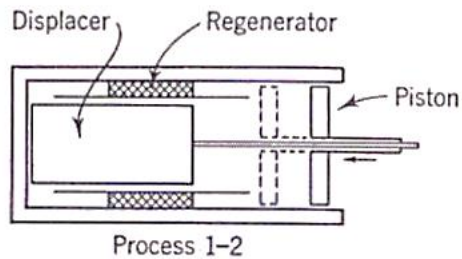
3 → 4 : Isothermal expansion

Process 4–1

4 → 1 : Const. volume heating
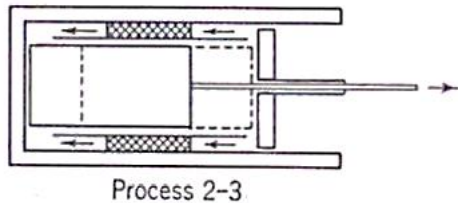
Process 1 → 2

The gas is compressed isothermally while rejecting heat to the high temperature sink Heat rejected :
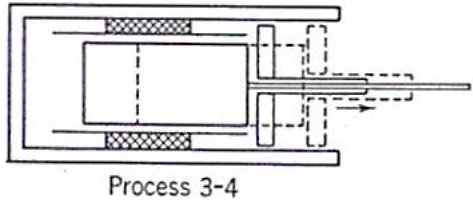
$$Q_r = mT_1(s_2 - s_1)$$

Process 2 → 3

The gas is forced through the regenerator by the displacer and cooled at constant volume during this process.
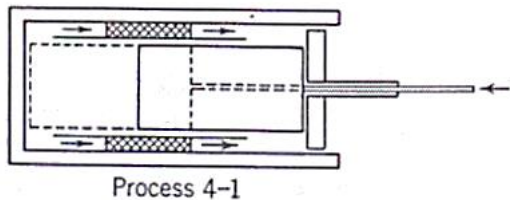
The removed energy is stored in the regenerator matrix

Process 3 → 4

The gas is expanded isothermally while absorbing heat from the low temperature source

Heat absorbed :

$$Q_a = mT_3(s_4 - s_3)$$
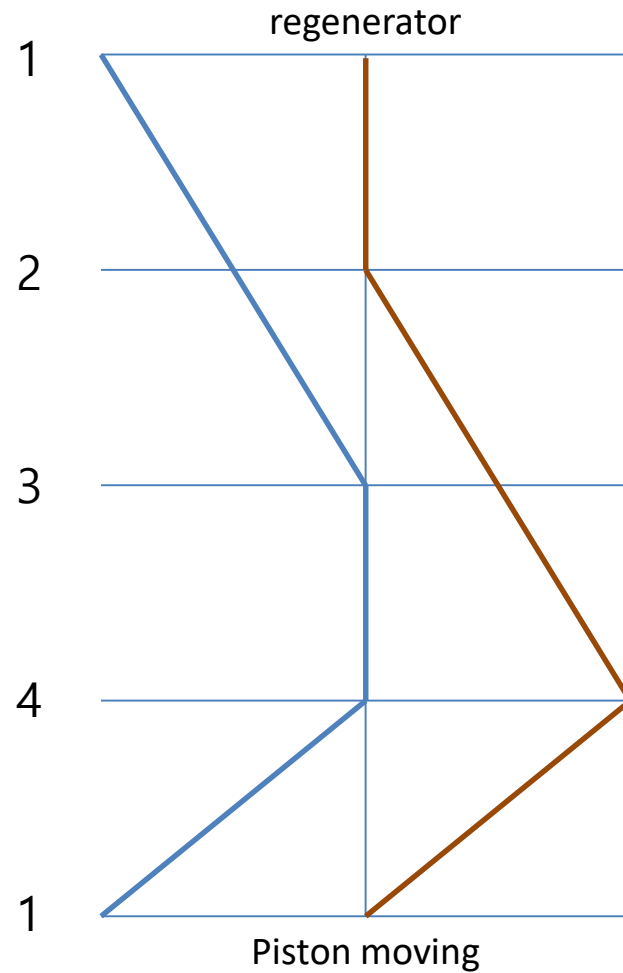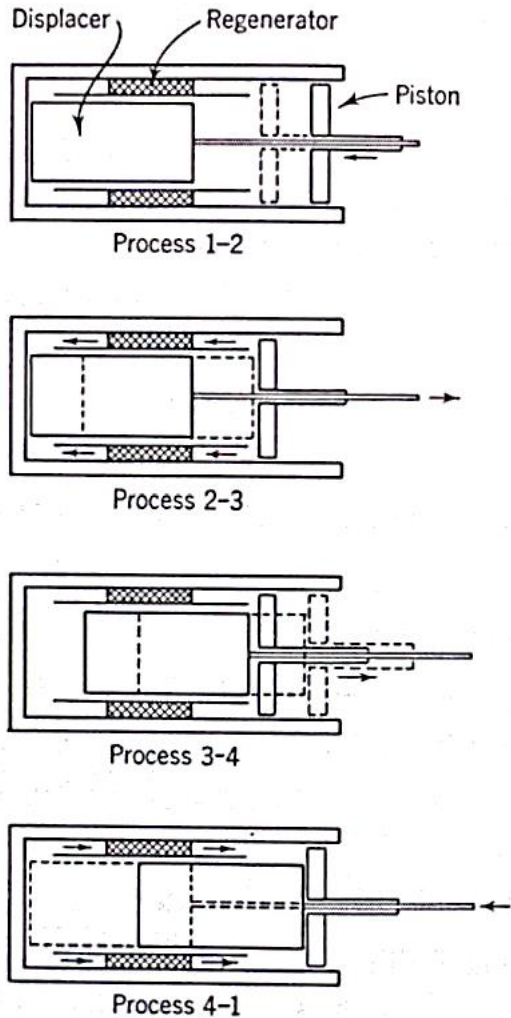

Process 4 → 1

The cold gas is forced through the regenerator by the displacer with heating as the energy stored during process 2-3 is transferred back to the gas.
Constant volume heating

In the ideal case (no heat in-leaks), heat is transferred to the refrigerator only during process 3-4, and heat is rejected from refrigerator only during process 1-2
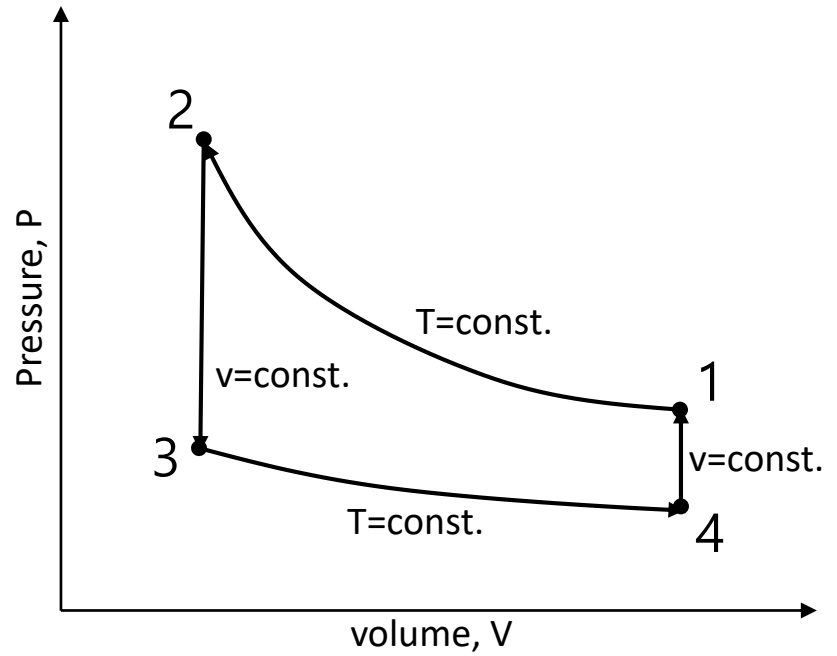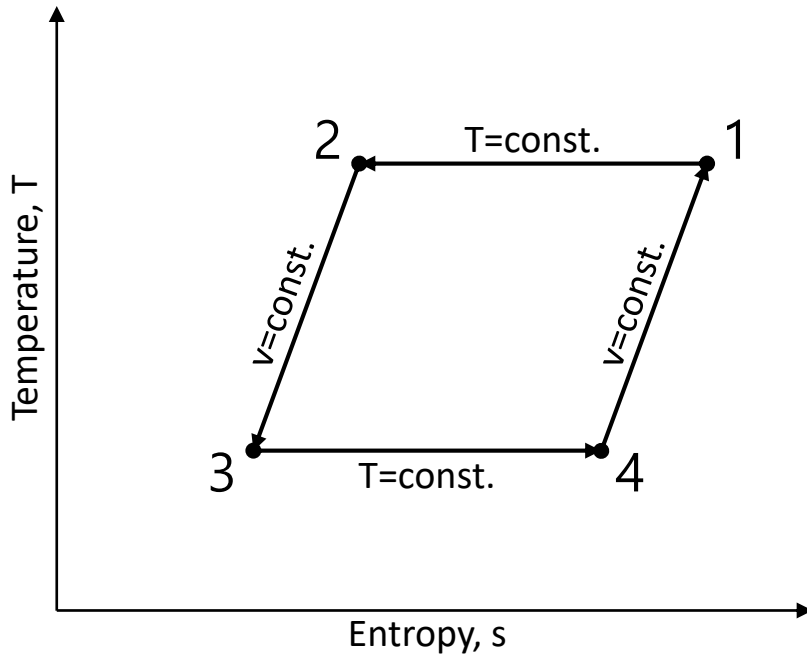
5.7 Philips Refrigerator

COP of the ideal Philips refrigerator

$$\text{COP} = \frac{-Q_a}{W_{net}} = \frac{-Q_a}{Q_a + Q_r} = \frac{1}{\dfrac{Q_r}{Q_a} - 1} = \frac{T_3}{T_1 \dfrac{(s_1 - s_2)}{(s_4 - s_3)} - T_3}$$

If working fluid behaves as an ideal gas,

$$s_1 - s_2 = c_v \ln\left(\frac{T_1}{T_2}\right) + R \ln\left(\frac{v_1}{v_2}\right) = R \ln\left(\frac{v_1}{v_2}\right) = R \ln\left(\frac{v_4}{v_3}\right)$$

$$= s_4 - s_3 \qquad (\because T_1 = T_2, \, T_3 = T_4, \, v_1 = v_4, \, v_3 = v_4)$$

Finally, COP of an ideal Philips refrigerator with an deal gas as the refrigerant becomes

$$\text{COP} = \frac{T_3}{T_1 - T_3}$$

This is the same expression as that for the COP of a Carnot refrigerator

5.8 Regenerator Effectiveness for the Philips **Refrigerator**

- The success of the Philips refrigerator
    → Large extent upon the effectiveness of the regenerator used in the system!

The actual energy absorbed from the low-temperature source becomes

$$Q_a = Q_{a,ideal} - \Delta Q$$

$$\varepsilon = \frac{Q_{actual}}{Q_{ideal}} = \frac{Q_{2-3,ideal} - \Delta Q}{Q_{2-3,ideal}}$$

$$\Delta Q = (1 - \varepsilon)Q_{2-3,ideal} = (1 - \varepsilon)mc_v(T_2 - T_3)$$

$$Q_{a,ideal} = mT_3(s_4 - s_3) = mRT_3\ln(v_4/v_3) = (\gamma - 1)mc_vT_3\ln(v_4/v_3)$$

$$\boxed{\frac{\Delta Q}{Q_{a,ideal}} = \frac{1 - \varepsilon}{\gamma - 1}\frac{T_2/T_3 - 1}{\ln(v_4/v_3)}}$$

$Q_{2-3,ideal} = ideal\ heat\ transferred\ from\ the\ gas\ to\ the\ regenerator\ duting\ process\ 2-3$
$m = mass\ of\ gas\ flowing\ through\ the\ regenerator$
$c_v = specific\ heat\ of\ the\ gas\ flowing\ through\ the\ regenerator$

5.9. Vuilleumier Refrigerator

- Vuilleumier Refrigerator (presented by Rudolph Vuilleumier in 1918)
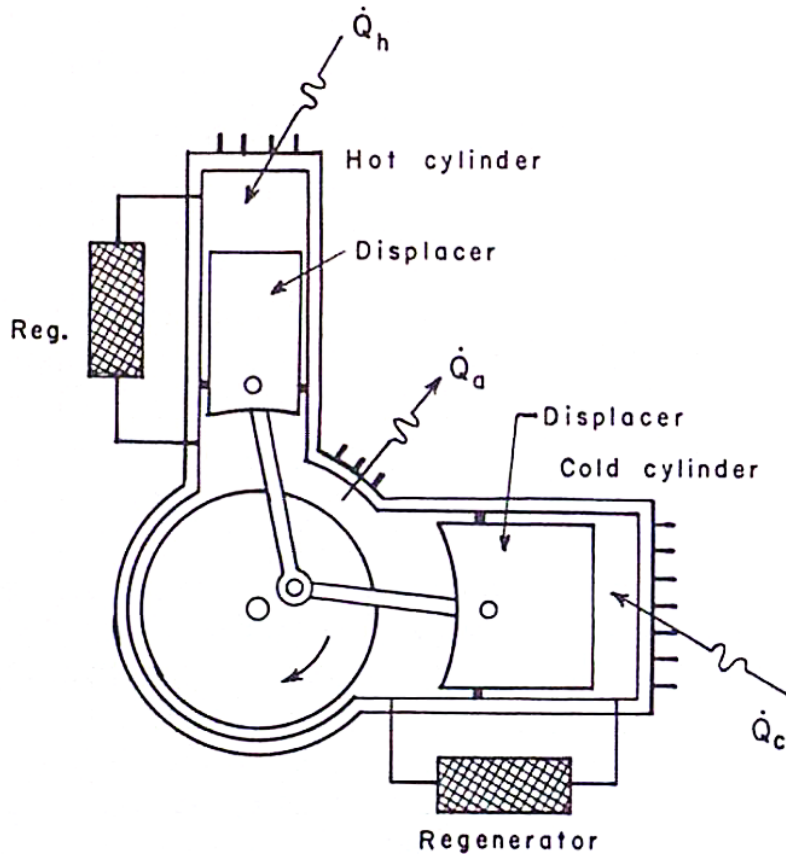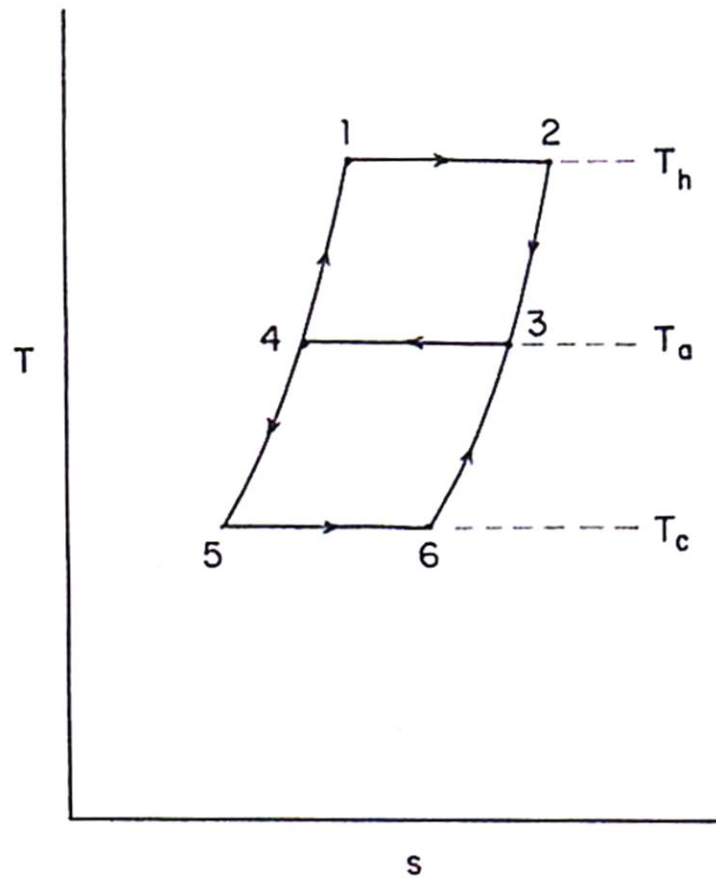

Fig. 5.16. Vuilleumier refrigerator schematic.

- Characteristics
  - Thermal Compression
  - Constant Volume
  - No External Work

- Advantage
  - heating only (solar, geothermal)
  - no driving engine

- Disadvantages
  - no harmonic motion
  - regenerator efficiency
  - other losses such as shuttle heat transfer

Fig. 5.17. Thermodynamic cycle for the ideal Vuilleumier refrigerator.

Process 1-2 : heat is added from a high-temperature source to the gas in the "hot" cylinder, displacer moves downward.

Process 4-1 : near-ambient temperature gas flows from the intermediate volume through the regenerator to the hot volume.

Process 2-3 : the displacer then moves upward and gas is displaced from the hot volume to the intermediate volume.

Process 3-4 : heat is rejected from the intermediate volume to maintain the temperature of the gas in the volume constant at T_a

5.9. Vuilleumier Refrigerator
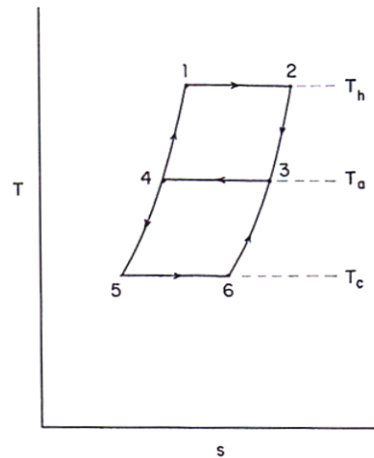
Process 5-6 : As the cold displacer is moved to the left, heat is absorbed by the gas in the cold volume from the low-temperature the same time, gas from the intermediate volume flows through the cold regenerator to the cold volume.

Process 4-5 : At the same time, gas from the intermediate volume flows through the cold regenerator to the cold volume.

Process 6-3 : the cold displacer then moves back to the right, and gas is displaced from the cold volume through the cold regenerator to the intermediate volume.

Fig. 5.17. Thermodynamic cycle for the ideal Vuilleumier refrigerator.
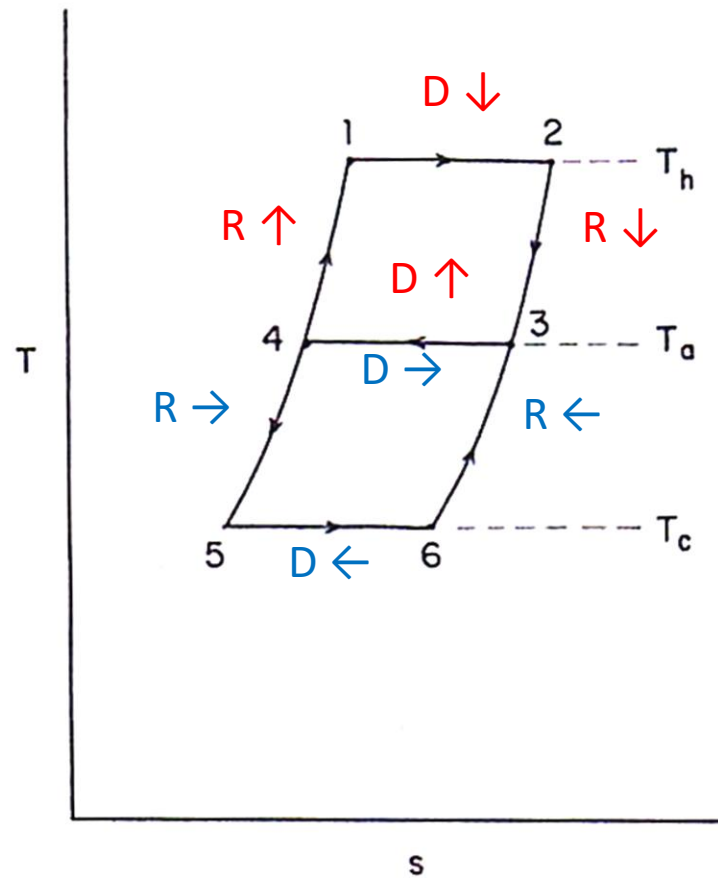
R : Regenerator
D : Displacer

- Numerical approach for Vuilleumier refrigerator

$$Q_h = m_h T_h (s_2 - s_1) = m_h R T_h \ln\left(\frac{\upsilon_2}{\upsilon_1}\right)$$

$$Q_c = m_c T_c (s_6 - s_5) = m_c R T_c \ln\left(\frac{\upsilon_6}{\upsilon_5}\right)$$

$$Q_a = (m_h + m_c) T_a (s_4 - s_3) = -(m_h + m_c) R T_a \ln\left(\frac{\upsilon_3}{\upsilon_4}\right)$$

$$\upsilon_2 = \upsilon_3 = \upsilon_6 \quad \text{and} \quad \upsilon_1 = \upsilon_4 = \upsilon_5$$

$$Q_h + Q_c + Q_a = 0$$

$$\rightarrow \quad m_h RT_h \ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + m_c RT_c \ln\left(\frac{\upsilon_2}{\upsilon_1}\right) - (m_h + m_c)RT_a \ln\left(\frac{\upsilon_2}{\upsilon_1}\right) = 0$$

$$\rightarrow \quad \frac{m_c}{m_h} = \frac{T_h - T_a}{T_a - T_c}$$

Definition of COP of this system

$$COP = \frac{Q_c}{Q_h} = \frac{m_c T_c}{m_h T_h}$$

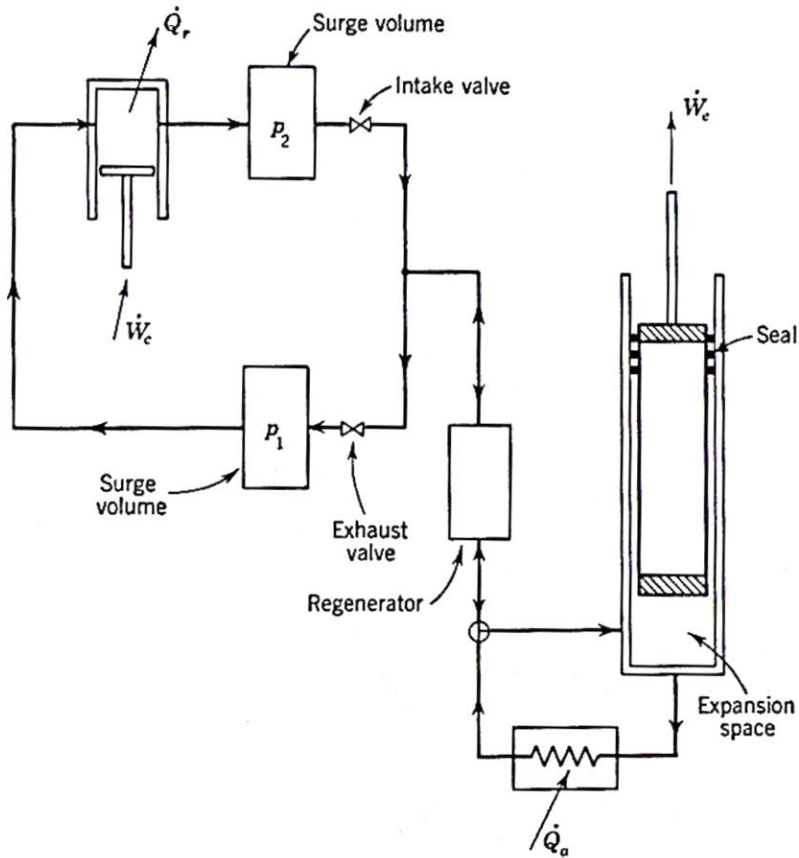$$\rightarrow \quad COP = \frac{T_c(T_h - T_a)}{T_h(T_a - T_c)}$$

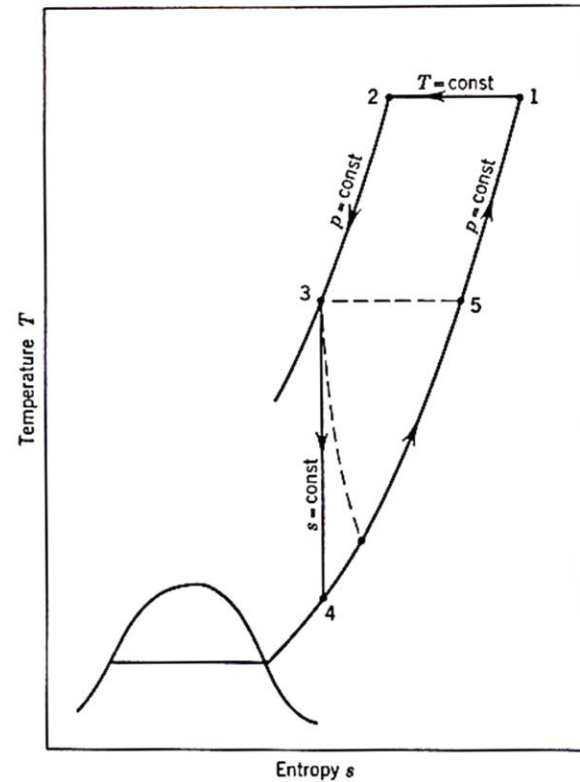Fig. 5.18. Solvay refrigerator schematic.

Fig. 5.19. Path traced out by a unit mass of gas on the T-s plane for the Solvay refrigerator.

5.10. Solvay Refrigerator

Process 1-2: With the piston at the bottom of its stroke, the inlet valve is opened. The high-pressure gas flows into the regenerator, in which the gas is cooled, and the system pressure is increased from a low pressure P_1 to a higher pressure P_2

Process 2-3: With the inlet valve still open, the piston is raised to draw a volume of gas into the cylinder. The gas has been cooled during its flow through the regenerator.

Process 3-4: The inlet valve is closed, and the gas within the cylinder is expanded (isentropic in the ideal case) to the initial pressure P_1. As the gas expands, it does work on the piston, and energy is removed from the gas as work. The temperature of the gas therefore decreases.

Process 4-5: The exhaust valve is opened, and the piston is lowered to force the cold gas out of the cylinder. During this process, the cold gas passes through a heat exchanger to remove heat from the region to be cooled.

Process 5-1: The gas finally passes out through the regenerator, in which the cold gas is warmed back to room temperature.

5.10. Solvay Refrigerator

	$\dot{W}_c/\dot{W}_e$	$\dot{Q}_r/\dot{Q}_a$	Intake valve	Exhaust valve	Regenerator	Expansion space
1-2	$\dot{W}_c$ (-) (consuming)	Release	Open	Close	No flow (cooling)	Bottom $P_1 \rightarrow P_2$
2-3			Open	Close	P_2 in (cooling)	Bottom
3-4	$\dot{W}_e$ (+)		Close	Close	No flow	Raise $P_2 \rightarrow P_1$
4-5		absorb	Close	Open		Down
5-1			Close	Open	P_1 in (heating)	

5.10. Solvay Refrigerator

- Net work requirement

$$-\frac{\dot{w}_{net}}{\dot{m}} = \frac{T_2(s_1 - s_2) - (h_1 - h_2)}{\eta_{c,o}} - \eta_{e,m}\eta_{ad}(h_3 - h_4)$$

- Energy removed from low T source is given by

$$\eta_{ad} = \frac{h_3 - h_4'}{h_3 - h_4} = 1 - \frac{h_4' - h_4}{h_3 - h_4}$$

$$\rightarrow \quad \frac{Q_a}{m} = h_5 - h_4' = h_5 - h_4 - (1 - \eta_{ad})(h_3 - h_4)$$

5.11 Gifford– McMahon Refrigerator (1960)

- Gifford-McMahon refrigerator

  - Gifford-McMahon (GM) coolers have found widespread application in many low-temperature systems; e.g., in MRI and cryo-pumps.

  - This system consists of a compressor, a cylinder closed at both ends, a displacer within the cylinder, and a regenerator.

Gifford- McMahon refrigerator schematic

McMahon refrigerator

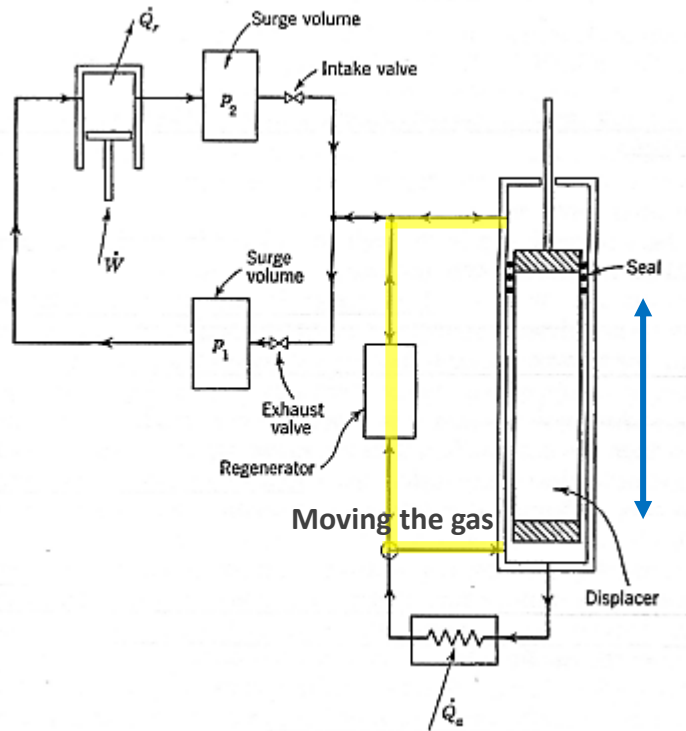*Gifford- McMahon refrigerator (www.cryomech.com)*

5.11 Gifford– McMahon Refrigerator (1960)

- What's different from Solvay refrigerator?

  - No work is transferred from the system during the expansion process.
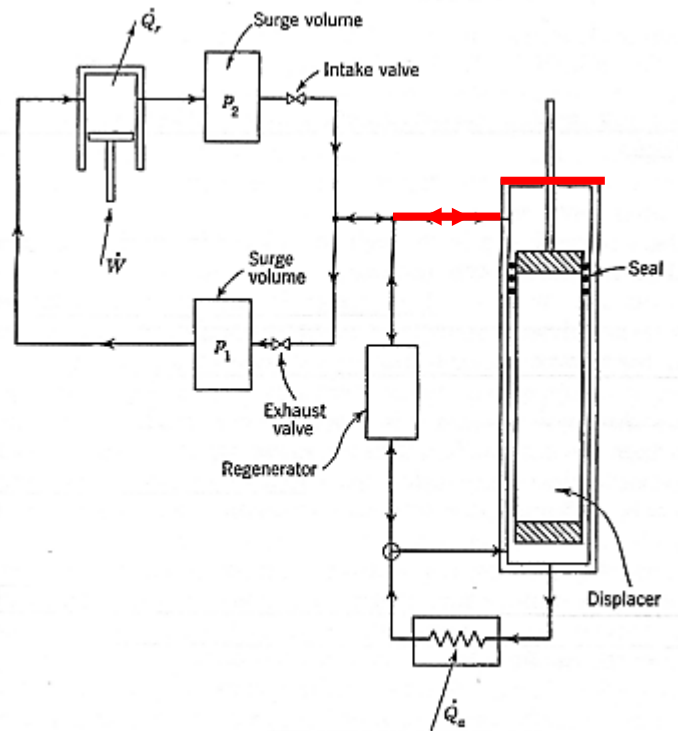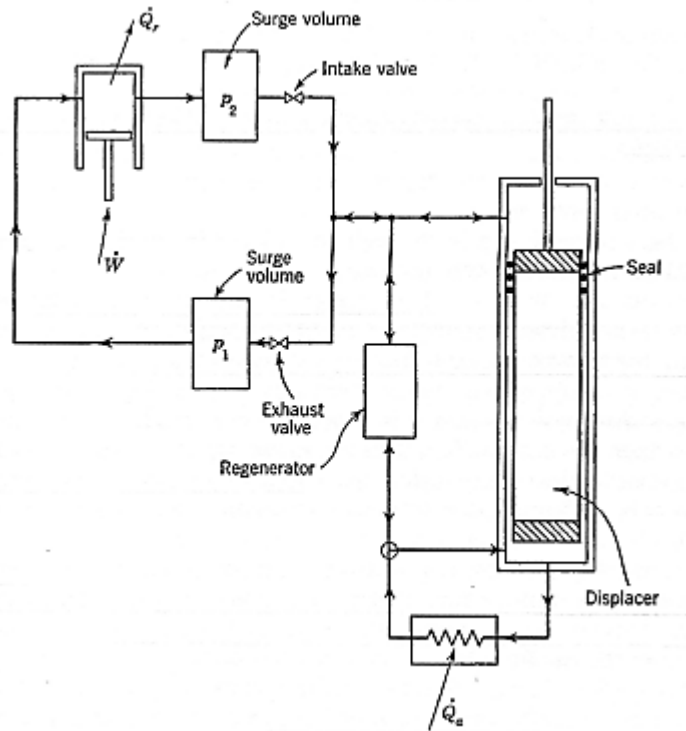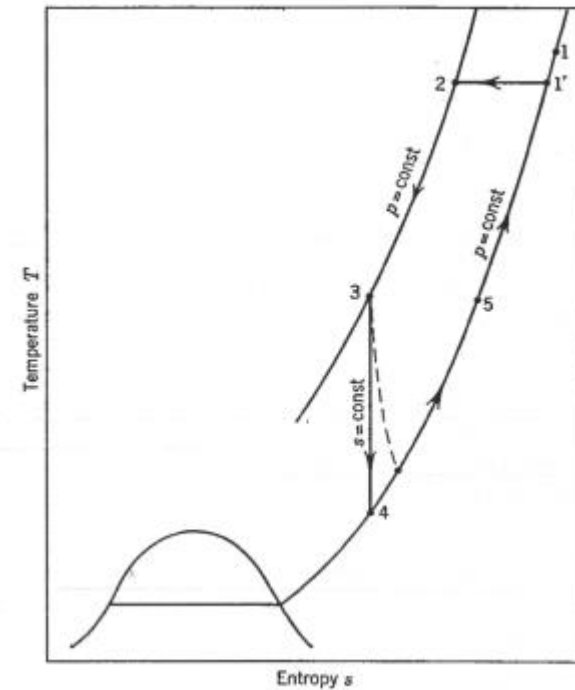

Gifford- McMahon refrigerator schematic

The displacer serves the purpose of moving the gas from one expansion space to another and would do zero net work in the ideal case of zero pressure drop in the regenerator.
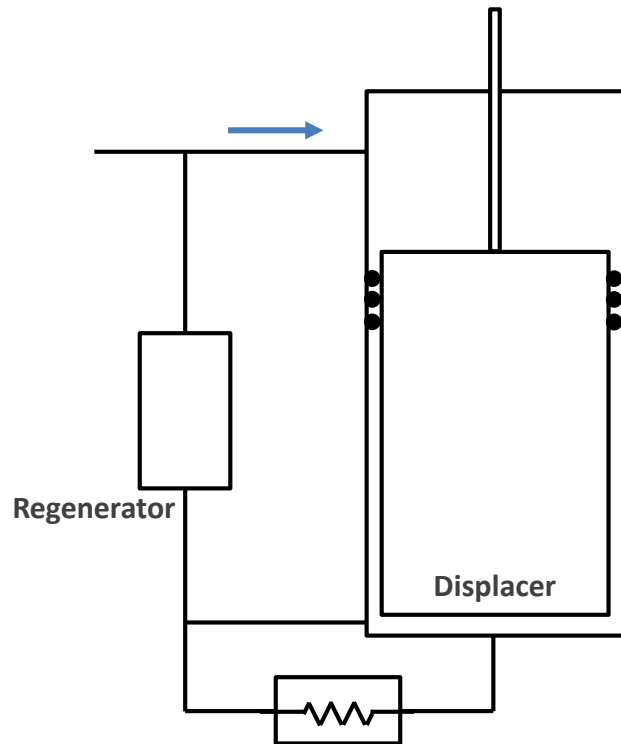
Gifford- McMahon refrigerator schematic

Path traced out by a unit mass of gas on the T-s plane for the Gifford- McMahon refrigerator

Process 1 → 2

Regenerator

Displacer

- The displacer at the bottom of the cylinder.

- The inlet valve is opened.

- The exhaust valve closed.

- The upper expansion space is increased from a low pressure P_1 to a higher pressure P_2.

Process 2 → 3

Cooled
down

Regenerator

Displacer

- The displacer is moved to the top of the cylinder.

- The inlet valve is still opened.

- The exhaust valve is closed.

- The gas that was in the upper expansion space down through the regenerator to the lower expansion space.

- The gas is cooled as it passes through the regenerator.

Process 3 → 4

Cooled down

Displacer

Regenerator

- The displacer at the top of the cylinder.

- The inlet valve is closed.

- The exhaust valve is opened.

- The gas within the lower expansion.

  space expands to the initial pressure P_1.

- The gas within the lower expansion.

  space drops to a low temperature.

Process 4 → 5

Regenerator

Displacer

Q_a

- The displacer is moved to the bottom of the cylinder.

- The inlet valve is closed.

- The exhaust valve is opened.

- The cold gas flows through a heat exchanger in which heat is transferred.

Process 4 → 5

Warmed back

Regenerator

Displacer

Q_a
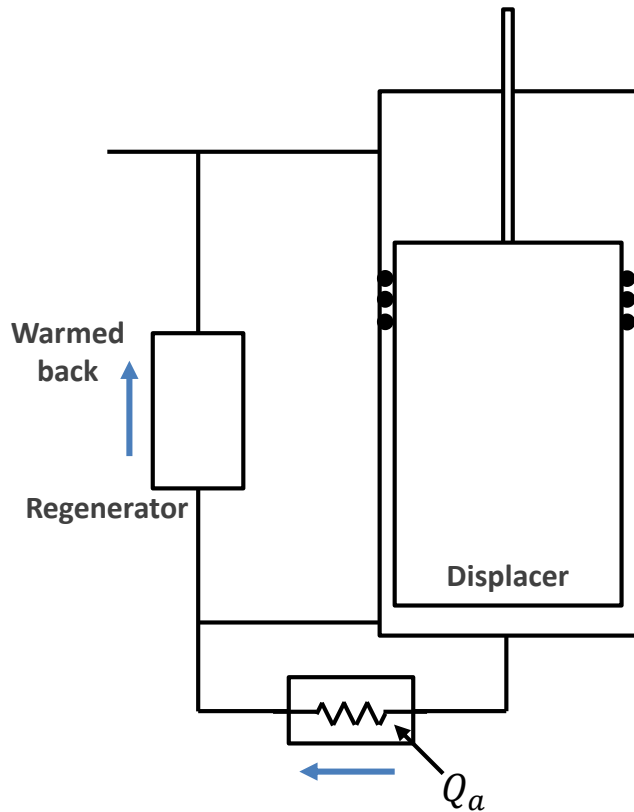
- The cold gas flows through the regenerator, in which the gas is warmed back.

5.11 Gifford– McMahon Refrigerator (1960)

Process	Comp-ressor	Intake	Exhaust	Displacer	Upper expansion space	Lower expansion space	Regenerator
1-2	↑	O	C	Bottom	P1 -> P2	P1 -> P2	
2-3		O	C	↑	→ (Gas flow direction)		Gas is cooled, V decreased -> Extra supply of hot gas
3-4		C	O	Top	P2 -> P1	P2 -> P1	
4-5		C	O	↓	←		Cold gas flows regenerator to heat up
5-1							Gas is warmed back to near ambient temperature

5.11 Gifford– McMahon Refrigerator (1960)

- The net work requirement for this system

$$-\frac{W}{m} = \frac{[T_1(s_1 - s_2) - (h_1 - h_2)]}{\eta_{c,o}}$$

- The energy removed from the low-temperature source

$$Q_a/m = \left(\frac{m_e}{m}\right)(h_5 - h'_4) = \eta_{ad}\left(\frac{m_e}{m}\right)(h_5 - h_4)$$

- Because the volume of the expansion space remains constant during the expansion process

$$\frac{m_e}{m} = \rho'_4/\rho_3$$

- m_e is the mass of gas within the lower expansion space at the end of the expansion precess 3−4
- m is the total mass of gas compressed

■ Factors that contribute to a loss in performance of the Gifford-McMahon refrigerator

- Regenerator efficiency should be higher than 98%.

- Thermal conduction down the displacer and housing.

- Shuttle heat transfer.

- Finite volume within the regenerator.
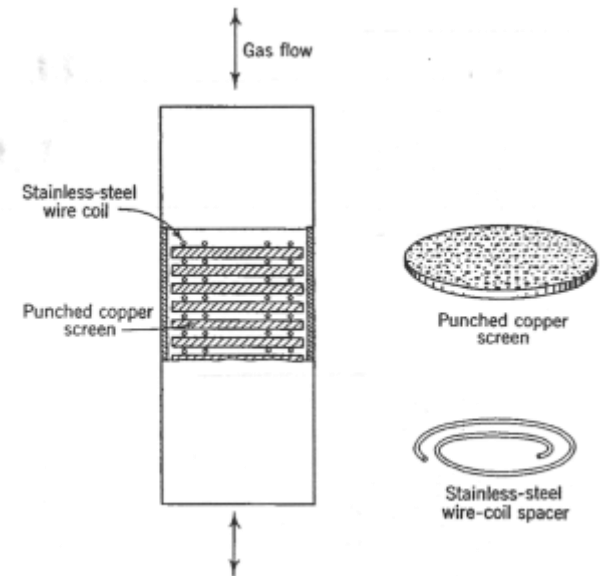
Regenerator schematic. The stainless-steel wire spacer is used to reduce the longitudinal conduction heat transfer within the matrix

5.11 Gifford– McMahon Refrigerator (1960)

- Advantage of Solvay and Gifford-McMahon refrigerator

    - The engine valves and displacer piston seals are at room temperature, so low-temperature sealing problems are eliminated.

- Solvay

    - COP is inherently higher than that of the Gifford-McMahon system because more energy is removed from the working fluid by the external-work-producing process.
    - Expanding gas moves the displacer.

- Gifford-McMahon

    - No leakage past the displacer because small pressure difference between across the displacer seals.
    - Motion transmission system of displacers is simple and little force.

▪ Multistage Gifford-McMahon refrigerator
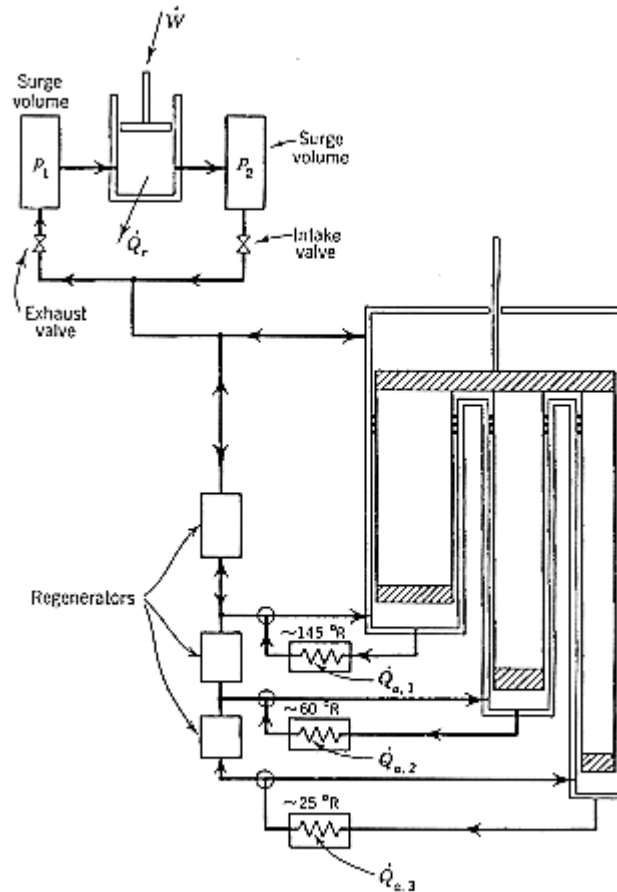
Three-stage Gifford-McMahon refrigerator

5.12 Regenerators

- The regenerator is a critical component in refrigeration systems.

  - If we apply the First law to a differential element of gas flowing through the regenerator

$$h(T_s - T)(A/L)dx - \dot{m}c_p \frac{\partial T}{\partial x}dx = \rho(V_g/L)c_p \frac{\partial T}{\partial \tau}dx$$

$h = $ heat $-$ transfer coefficient between the flowing gas and solid material within the regenerator

$T = $ gas temperature at location x and time τ within regenerator

$T_s = $ temperature of matrix at location x and time τ

$A = $ heat $-$ transfer surface area of matrix

$L = $ length of regenerator

$\dot{m} = $ mass flow rate of gas through the regenerator

$c_p = $ specific heat of the gas flowing through the regenerator

$\rho = $ density of the gas

$V_g = $ void volume or gas volume within the regenerator

Apply the dimensionless variables:

$$\xi = x/L$$
$$\eta = (\tau/P) - (\rho V_g / \dot{m}LP)x = [\tau - (x/L)\tau_d]/P$$
$$\tau = P\eta + \tau_d\xi$$

$$P = heating\ or\ cooling\ period\ for\ the\ regenerator$$
$$\tau_d = dwell\ time(fluid\ particle\ remain\ time\ in\ the\ regenerator$$

Use these relations and rearrange the equations,

$$h(T - T_s)\left(\frac{A}{L}\right)dx = (\frac{m_s}{L})c_s\frac{\partial T_s}{\partial \tau}dx$$

$$T - T_s = \frac{L}{hA}(-\dot{m}c_p\frac{\partial T}{\partial x} - \rho V_g c_p\frac{\partial T}{\partial \tau}\frac{1}{L})$$

$$= -\frac{\dot{m}c_p}{hA}(L\frac{\partial T}{\partial x} + \frac{\rho V_g}{\dot{m}}\frac{\partial T}{\partial \tau})$$

$$\tau_d$$

Dwell time: Time that a fluid particle
remains within the regenerator

5.12 Regenerators

Apply the First law to a differential element of solid material within the regenerator,

$$h(T - T_s)\left(\frac{A}{L}\right)dx = (\frac{m_s}{L})c_s\frac{\partial T_s}{\partial \tau}dx$$

It can be written in the form

$$\boxed{T - T_s = \frac{m_s c_s}{hAP}\frac{\partial T_s}{\partial \eta} = (\frac{\dot{m}c_p}{hA})(\frac{m_s c_s}{\dot{m}c_p P})\frac{\partial T_s}{\partial \eta}}$$

5.12 Regenerators

- One method of regenerator analysis was proposed by Hausen (1976).

Reduced length: $\Lambda_h = \dfrac{h_h A_h}{\dot{m}_h c_h} , \Lambda_c = \dfrac{h_c A_c}{\dot{m}_c c_c}$

Reduced period: $\Pi_h = \dfrac{h_h A_h P_h}{m_{sh} c_s} , \Pi_c = \dfrac{h_c A_c P_c}{m_{sc} c_c}$

- If $\Pi_h = \Pi_c$, regenerator is regarded as symmetry

- $\dfrac{A_h}{\Pi_h} = \dfrac{A_c}{\Pi_c}$ means the regenerator is balanced

5.12 Regenerators

- The effectiveness of a counter-flow regenerator by Coppage and London(1953)

  - From NTU method, $\frac{UA}{C_{min}}$ from ordinary of heat exchanger formula

$$N_{tu,0} = \frac{1}{C_{min}}[\frac{1}{h_h A_h} + \frac{1}{h_c A_c}]^{-1}$$

$$C_R = C_{min}/C_{max} = (\dot{m}c_p)_{min}/(\dot{m}c_p)_{max}$$

Matrix capacity can be defined as $C_m = m_s c_s / P_0 C_{min}$

Total period for the regenerator. $P_0 = P_h + P_c$

  - The regenerator effectiveness is defined by

$$\varepsilon = \dot{Q}/C_{min}(T_{h1} - T_{c1})$$

$\dot{Q} = $ average heat $-$ transfer rate
$T_{h1} = $ inlet temperature of the hot fluid steams
$T_{c1} = $ inlet temperature of the cold fluid streams

5.13 Magnetic Cooling

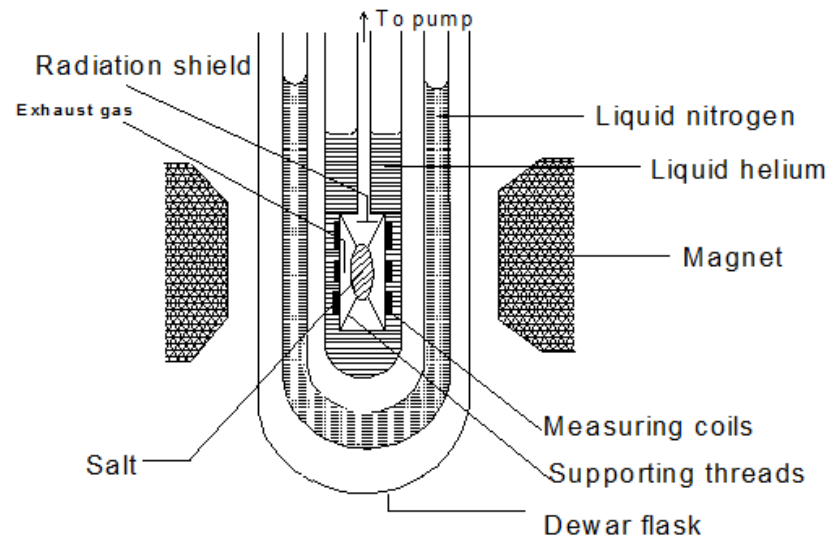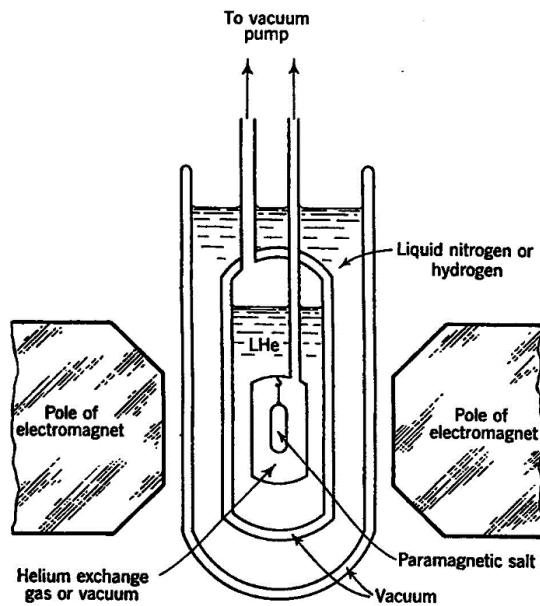

Fig. Apparatus for carrying out adiabatic demagnetization process

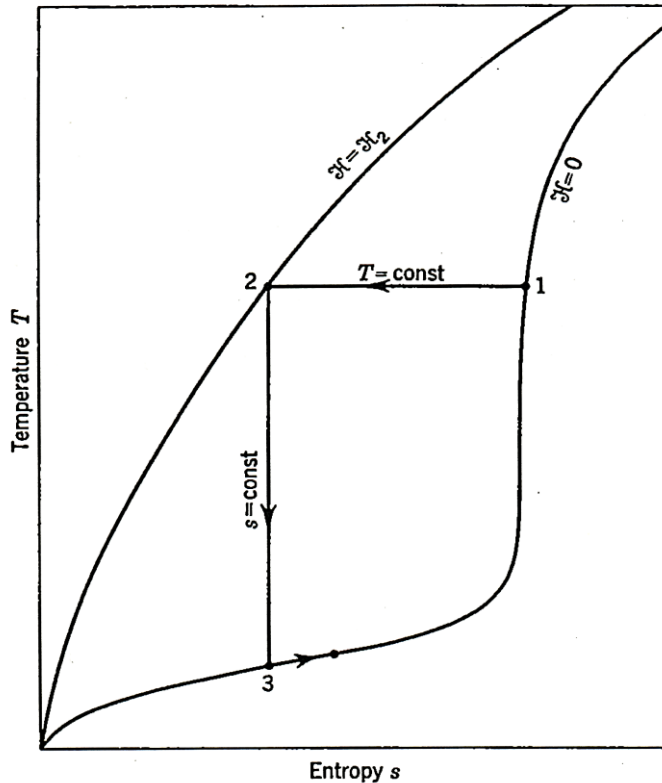Fig. T-s diagram of the magnetic cooling

- Vapor compression cycle (Refrigerant)

  1 – 2 : gas compression at constant T

  – decrease entropy (increase order)

  2 – 3 : reversible adiabatic expansion

  – same entropy (no change in order)

  – gas molecules farther apart

  (random molecular velocity ↓)

5.13 Magnetic Cooling

- Paramagnetic salt

  - No magnetic field dipole : random even at low T

  - With magnetic field at constant T

    - dipole align (ordered, s ↓)

    - heat of magnetization ↑

  - If magnetic field is removed

    - the same entropy

    - alignment should be disordered

    - T should be lowered

    - Adiabatic, demagnetization

5.14 Thermodynamics of Magnetic Cooling

- The magnetic process may be analyzed thermodynamically

$$Tds = du - \mu_0 \mathcal{H} \, d\zeta$$

$\mu_0 = 4\pi \times 10^{-7}$T-m/A=permeability of free space in SI units
$\mathcal{H}$ = the magnetic field intensity
ζ =the magnetic moment per unit mass

For a pure substance,

$$Tds = dh - vdp = c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp$$

For a paramagnetic substance,

$$Tds = c_{\mathcal{H}} dT + \mu_0 T\left(\frac{\partial \zeta}{\partial T}\right)_{\mathcal{H}}$$

$c_{\mathcal{H}}$ = the specific heat at constant magnetic field intensity
(analogous to c_p for a pure substance)

5.14 Thermodynamics of Magnetic Cooling

- Magnetocaloric coefficient
  (analogous to the isentropic expansion coefficient)

$$\mu_M = \left(\frac{\partial T}{\partial \mathcal{H}}\right)_{\mathcal{H}} = -\frac{\mu_0 T}{c_{\mathcal{H}}}\left(\frac{\partial \zeta}{\partial T}\right)_{\mathcal{H}}$$

- The Curie law
  : a sort of "ideal-gas law" for paramagnetic materials

$$\zeta = \frac{\mathcal{C}\mathcal{H}}{T}$$

$\mathcal{C}$: Curie constant

5.14 Thermodynamics of Magnetic Cooling

Table 5.4. Curie constants for paramagnetic salts

Paramagnetic salt	Ionic Weight, M g/mol or lb_m/mole	Density kg/m³	Density lb_m/ft³	Gas Constant, R J/kg-K	Gas Constant, R Btu/lb_m-°R	Curie Constant, $\mathscr{C}$ K-m³/kg
Cerium ethyl sulfate	678			12.263	2.93×10^{-3}	25.1×10^{-6}
Cerium magnesium nitrate	765			10.868	2.60×10^{-3}	5.22×10^{-6}
Chromium methylammonium alum	492	1645	103	16.899	4.04×10^{-3}	30.5×10^{-6}
Chromium potassium alum	499	1830	114	16.662	3.98×10^{-3}	45.8×10^{-6}
Copper potassium sulfate	442	2220	139	18.811	4.49×10^{-3}	12.21×10^{-6}
Gadolinium sulfate	373	3010	188	22.290	5.32×10^{-3}	263.3×10^{-6}
Iron ammonium alum	482	1710	107	17.250	4.12×10^{-3}	114.1×10^{-6}
Manganese ammonium sulfate	391	1830	114	21.264	5.08×10^{-3}	149.2×10^{-6}
Titanium cesium alum	589	2000	125	14.116	3.37×10^{-3}	4.21×10^{-6}

By permission from M. W. Zemansky, *Heat and Thermodynamics*, 4th ed. McGraw-Hill Book Co., New York, 1957.

- Magnetic moment of the paramagnetic material

  - When the magnetic field is increased to very high values or the temperature is lowered to very low values, the Curie law obviously does not apply because there must be limit to the magnetization that a substance can undergo.

$$\zeta = {}^1\!/_2 \; ng\mu_B B(\xi)$$

  - Brillouin function defines the expression for the magnetic moment of paramagnetic material.

$$B(\xi) = \text{Brillouin function}$$

$$B(\xi) = (2J + 1)\coth[(2J + 1)\xi] - \coth(\xi) \quad \text{where} \quad \xi = g\mu_B\mu_0\mathcal{H}/2kT$$

  - Entropy of the paramagnetic salt may be determined using Brillouin expression as,

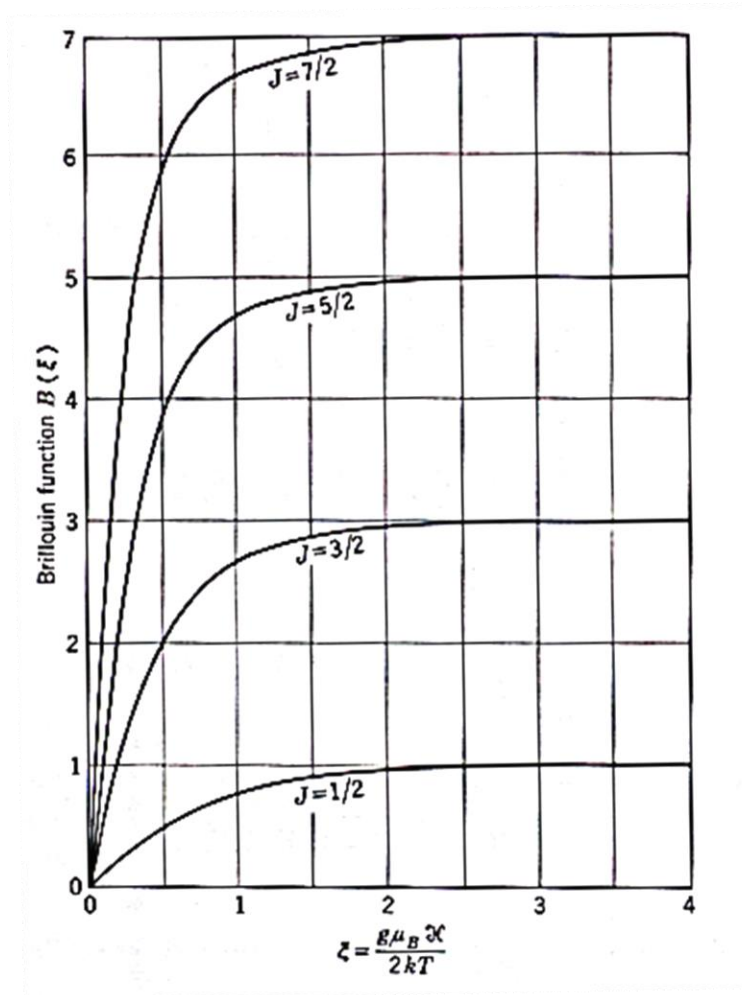$$s = R\ln\left\{\frac{\sinh[2J + 1)\xi]}{\sinh\xi}\right\} - R\xi B(\xi)$$

Fig. Brillouin function

5.16 Magnetic Refrigeration System

- Application of adiabatic demagnetization in maintaining T < 1.0 K
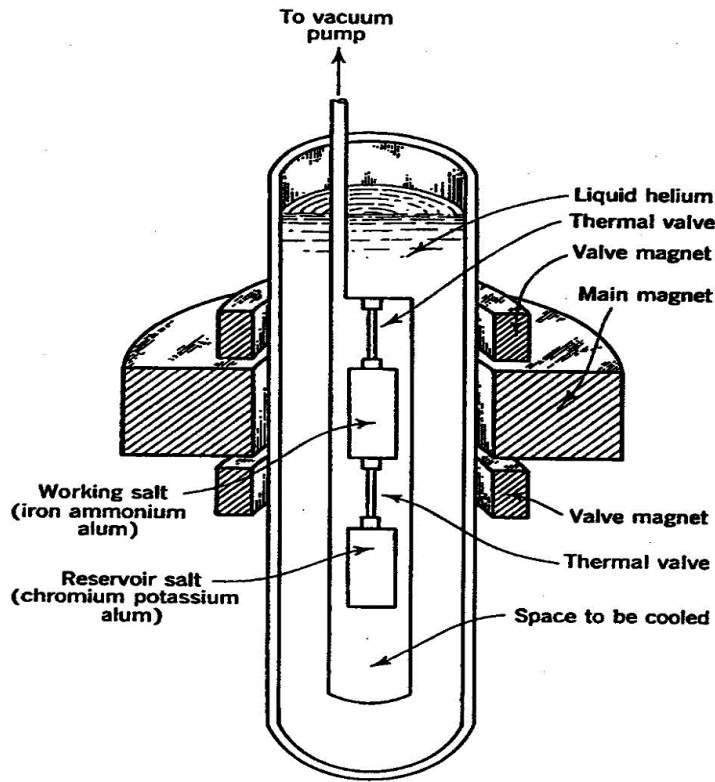
  Effective operation in zero gravity
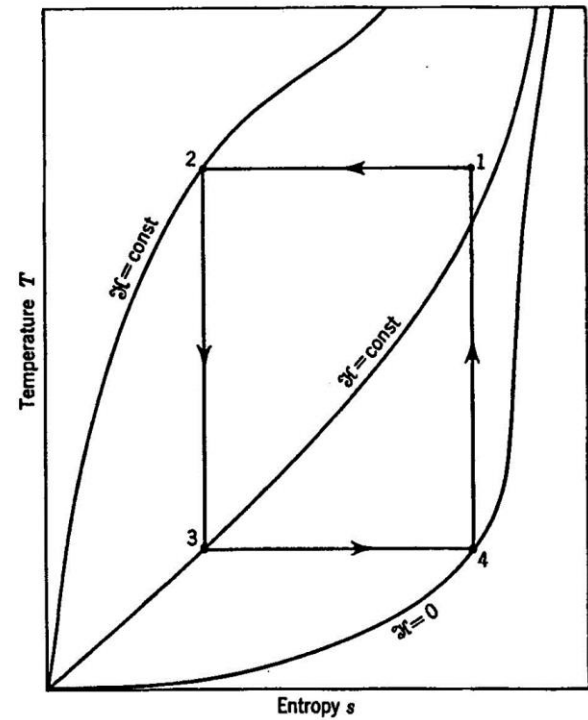

Fig. Magnetic refrigerator schematic

Fig. Thermodynamic cycle for the magnetic refrigerator

5.16 Magnetic Refrigeration System

▪ The sequence of operations for the magnetic refrigerator

Process 1-2: The magnetic field is applied to the working salt while the Upper thermal valve is open and the lower thermal valve is closed. When the upper thermal valve is open, heat may be transferred from the working salt to the liquid helium bath, thereby maintaining the salt temperature fairly constant. The thermal valve between the working salt and the reservoir salt is closed so that heat will not flow back into the low-temperature reservoir during this process.

Process 2-3: Both thermal valves are closed, and the magnetic field around the working salt is reduced adiabatically to some intermediate value. During this process, the temperature of the working salt decreases.

Process 3-4: the thermal valve between the working salt and the reservoir salt is opened, and the field around the working salt is reduced to zero while heat is absorbed isothermally by the working salt from the reservoir salt.

Process 4-1: Both thermal valves are closed, and the magnetic field around the working salt is adiabatically increased to its original value.

5.16 Magnetic Refrigeration System

	$\mathcal{H}$	Upper thermal valve	Lower thermal valve
$1-2$	applied to working salt $\uparrow$ (heat to the LH$_e$ bath)	open	closed
$2-3$	(T of the working salt decreases) $\downarrow$	closed	closed
$3-4$	$\downarrow$ (to zero) (heat is absorbed isothermally)	closed	open
$4-1$	$\uparrow$	closed	closed

- Irreversibility

    - heat transfer from the ambient

    - finite time rate of change of magnetic field : Entropy, Temperature

5.16 Magnetic Refrigeration System

- COP of the Magnetic refrigeration system

$$Q_a = mT_3(s_4 - s_3)$$

$$Q_r = mT_1(s_2 - s_1) = -mT_2(s_4 - s_3)$$

$$W_{net} = Q_a + Q_r = -m(T_1 - T_3)(s_4 - s_3)$$

$$COP = -\frac{Q_a}{W_{net}} = \frac{mT_3(s_4 - s_3)}{m(T_1 - T_3)(s_4 - s_3)} = \frac{T_3}{T_1 - T_3}$$

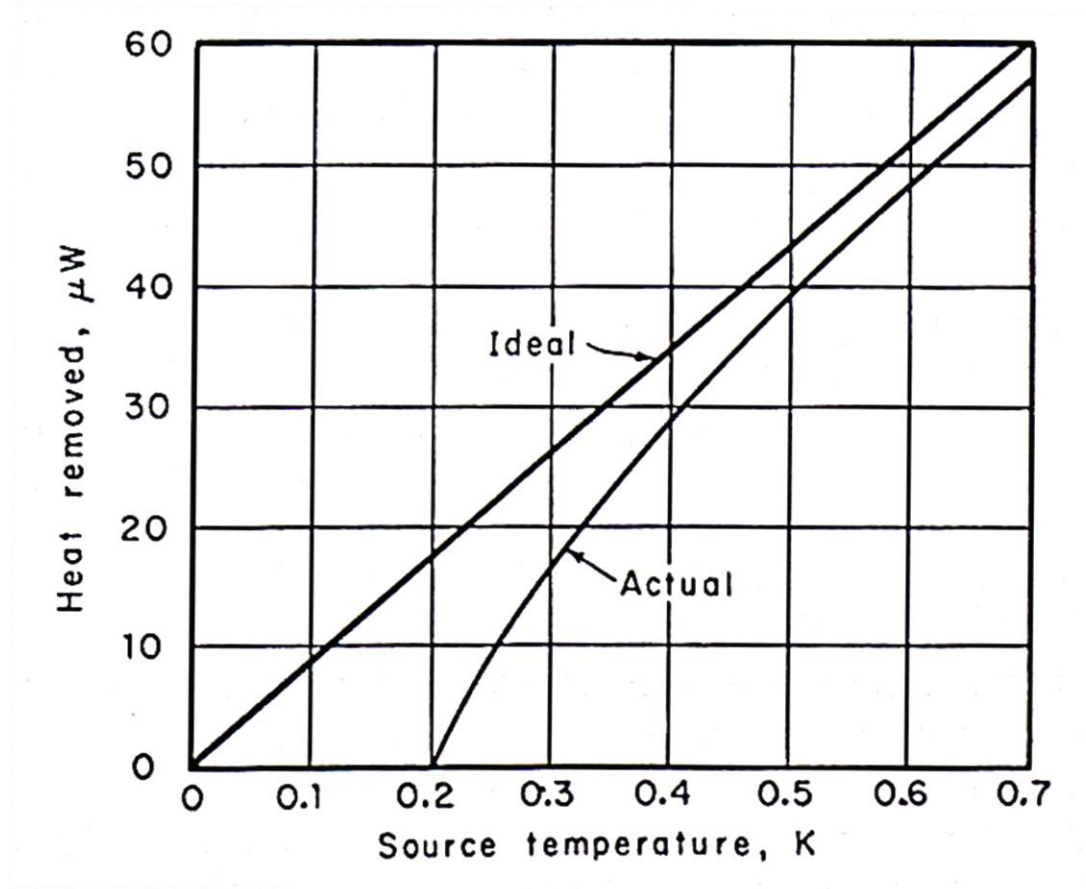COP for the ideal magnetic refrigerator is the same as that for a Carnot refrigerator

Fig. Actual and ideal performance of the magnetic refrigerator
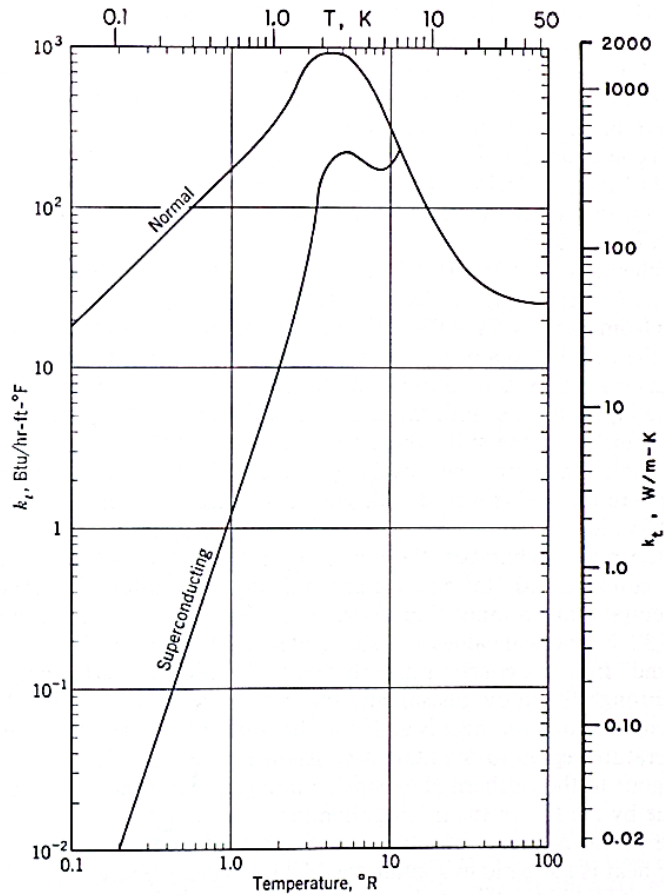
Fig. Thermal conductivity of lead in the normal and superconducting states.

- k in the superconducting state ~ 1 (closed) at 0.5 K – no magnetic field

- -k in the normal state   ~ 200 (open)   – with magnetic field

■ Cooling could be achieved by means of dilution of He^3 by superfluid He^4 was first suggested by H.London(1951)

The refrigeration effect of the dilution refrigerator may be determined by application of the First law to the mixing chamber:

$$\dot{Q}_a = \dot{n}_3(h_m - h_i)$$

$$h_m = C_1 T_m^2, \qquad \text{where } C_1 = 94 \ J/mol - K^2$$
$$h_i = C_2 T_i^2, \qquad \text{where } C_2 = 12 \ J/mol - K^2$$

Fig. $He^3 - He^4$ dilution refrigerator unit

Fig. $He^3 - He^4$ dilution refrigerator schematic
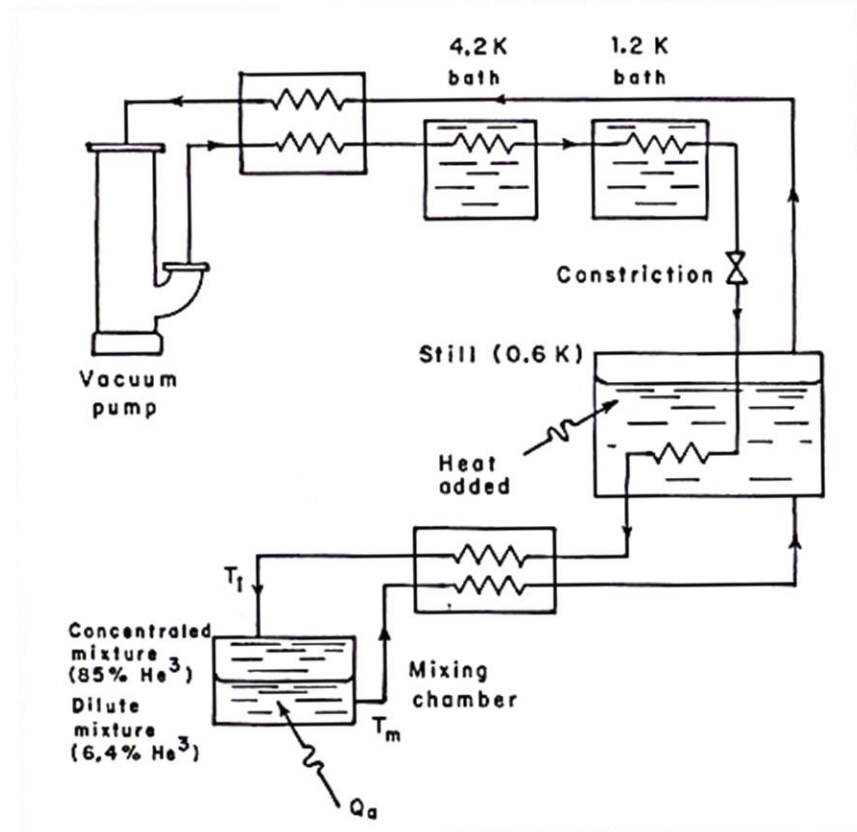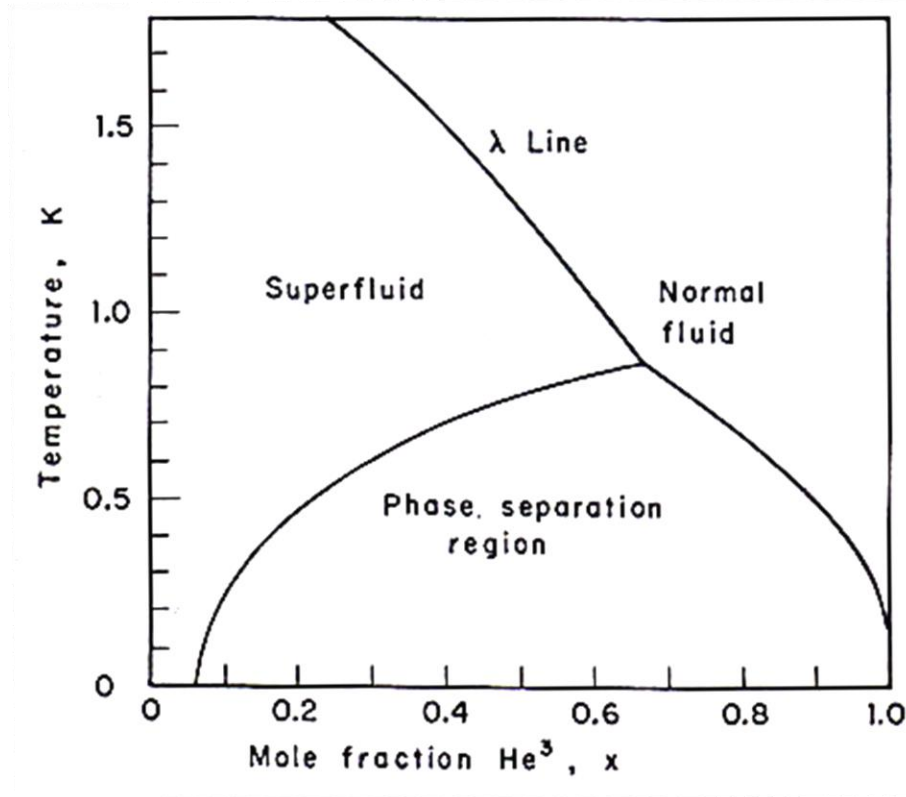
Phase diagram for $He^3 - He^4$ mixtures

5.19 Thermoacoustic Refrigeration

■ **Thermoacoustic Refrigeration (열음향냉동)**

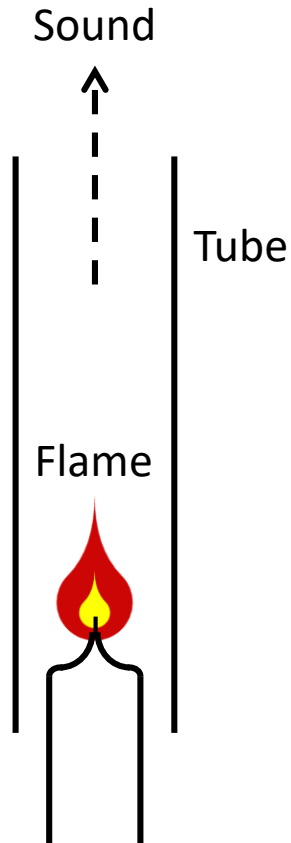Adiabatic compression and expansion of fluid(gas) by <u>standing wave</u>

- Higgins (1777)

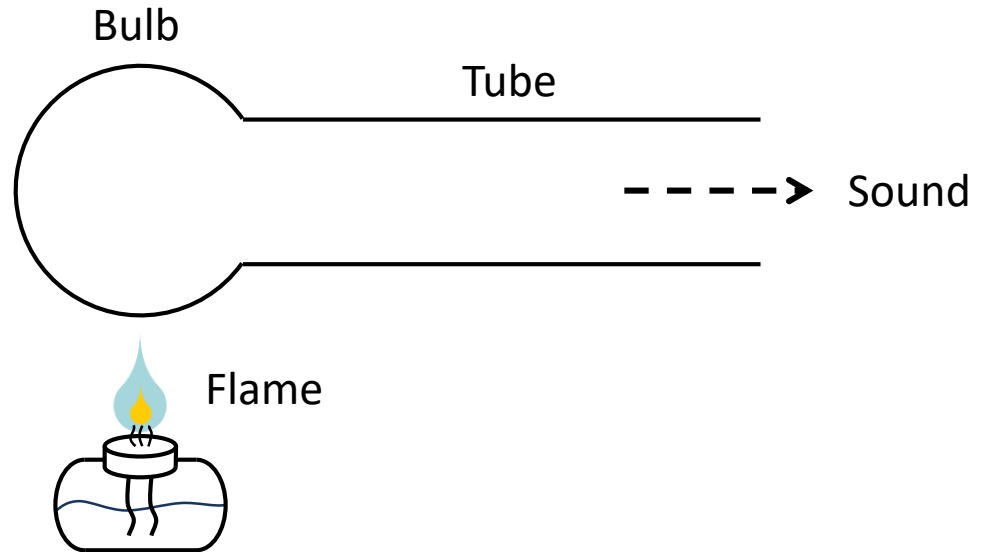 Bryon Higgins was able to excited pipe oscillations in a large tube with two open ends.

- Sondhauss (1850)

  Sondhauss experimented with a open-close tube.
  Heating the bulb at the close end produce sound.

5.19 Thermoacoustic Refrigeration

Sound

Tube

Flame

- Higgins (1777)

Bulb

Tube

Sound

Flame

- Sondhauss (1850)

▪ **Wheatly (1982) – Thermoacoustic refrigerator developed**

    <Theory> Adiabatic compression & expansion

$$T ds \overset{\nearrow\ 0}{=} C_v dT + P dv$$

$$0 = C_v dT + \frac{RT}{v} dv \quad \text{(for ideal gas)}$$

$$0 = \frac{dT}{T} + (k-1) \frac{dv}{v}$$

$$T v^{k-1} = c \quad \left(\text{where } k = \frac{C_p}{C_v}, \quad C_p - C_v = R \ \rightarrow \ k C_v - C_v = R \ \rightarrow \ C_v = \frac{R}{k-1} \right)$$
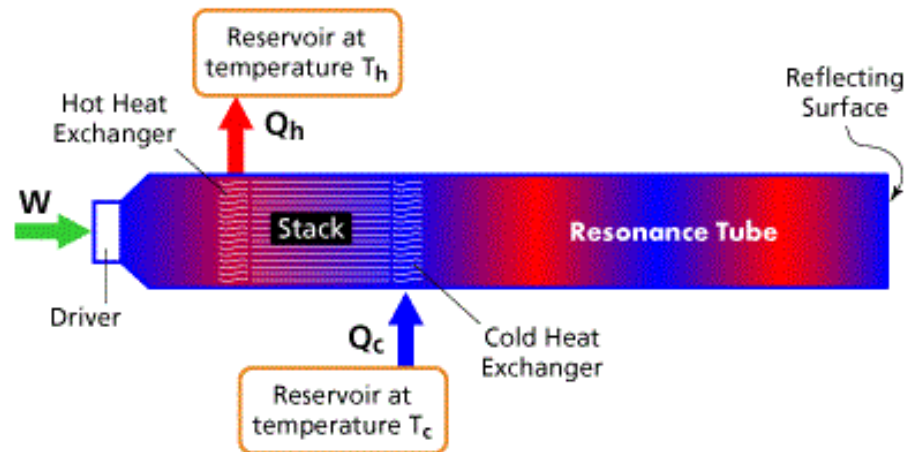
$$P v^k = c$$

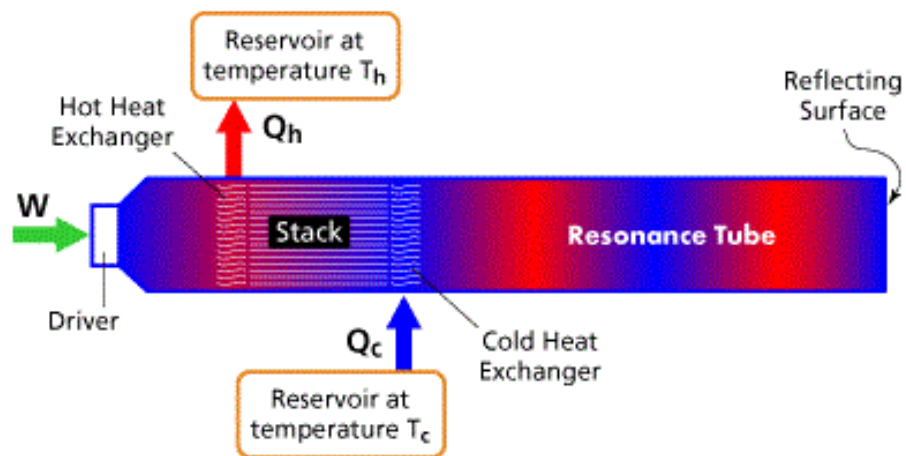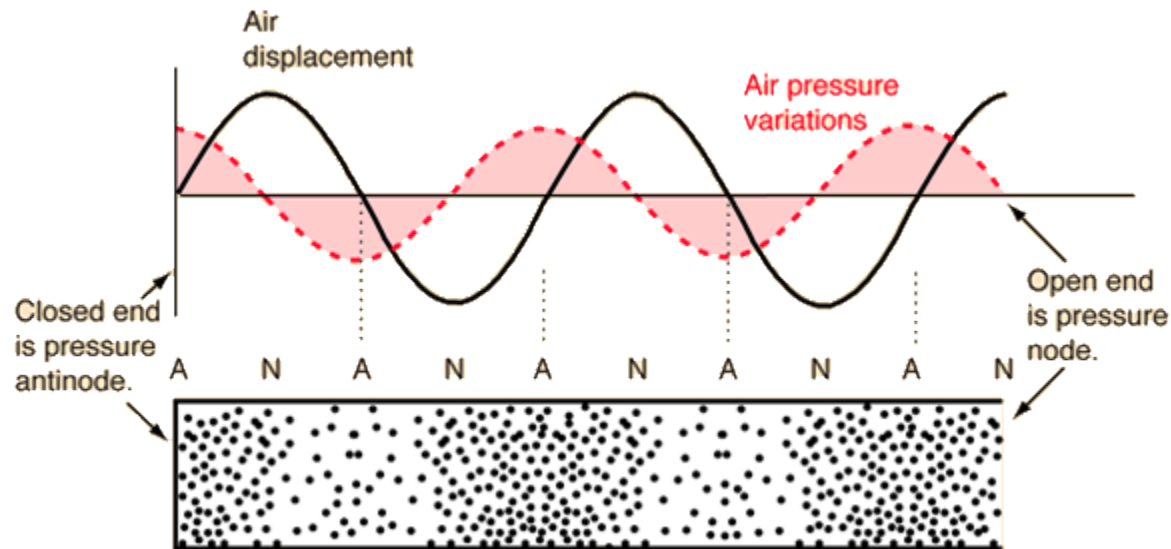$$T \left(\frac{T}{P} \right)^{k-1} = \frac{T^k}{P^{k-1}} = c$$

■ **Components**

- Acoustic driver (Speaker)

- Resonance tube (Resonator)

- Stack of plates

- Cold and hot heat exchanger

■ Shuttle heat transfer
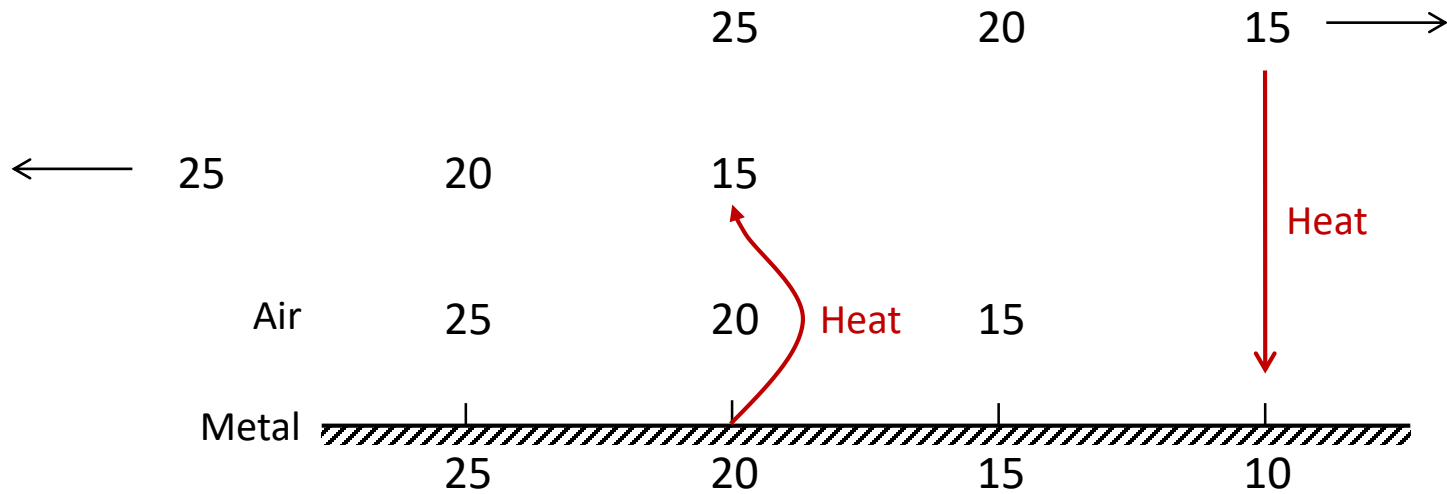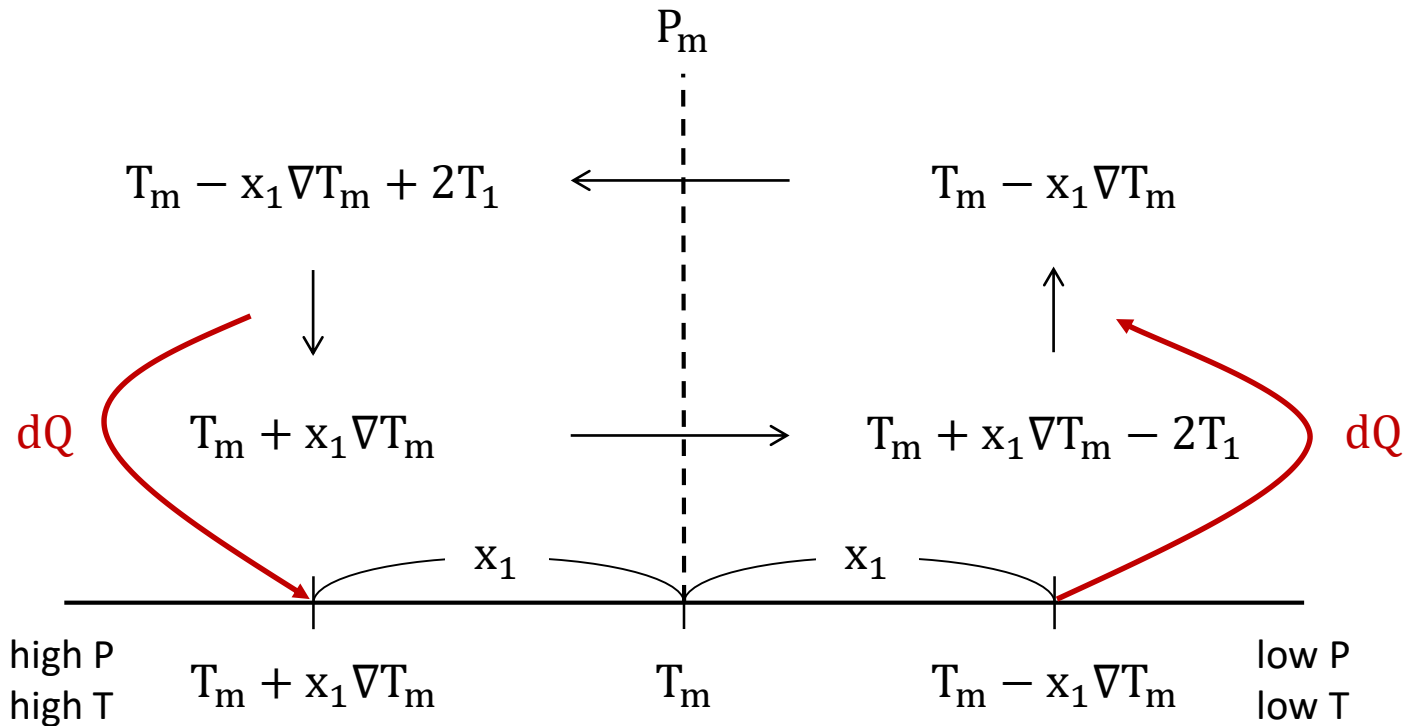
P_m

$T_m - x_1 \nabla T_m + 2T_1$ ← | → $T_m - x_1 \nabla T_m$

dQ $T_m + x_1 \nabla T_m$ → $T_m + x_1 \nabla T_m - 2T_1$ dQ

x_1 x_1

high P
high T $T_m + x_1 \nabla T_m$ T_m $T_m - x_1 \nabla T_m$ low P
low T

5.19 Thermoacoustic Refrigeration

- Advantages of thermoacoustic refrigeration

  - Simplicity

  - Reliability

  - Low cost

  - Minimal use of moving part

  - No harmful chemicals

- Disadvantages of thermoacoustic refrigeration

  - Capacity of thermoacoustic refrigerator very low ([W] scale)

  - COP of thermoacoustic refrigerator low (< 0.1)