CHBE 401 20 SEP 11.

- friction

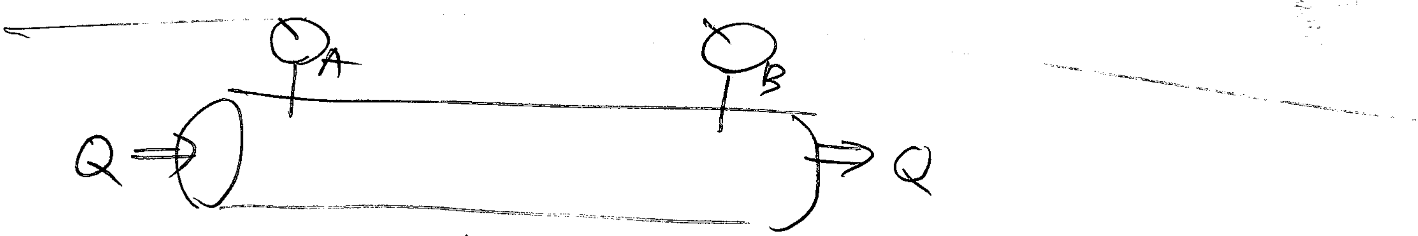
- viscosity

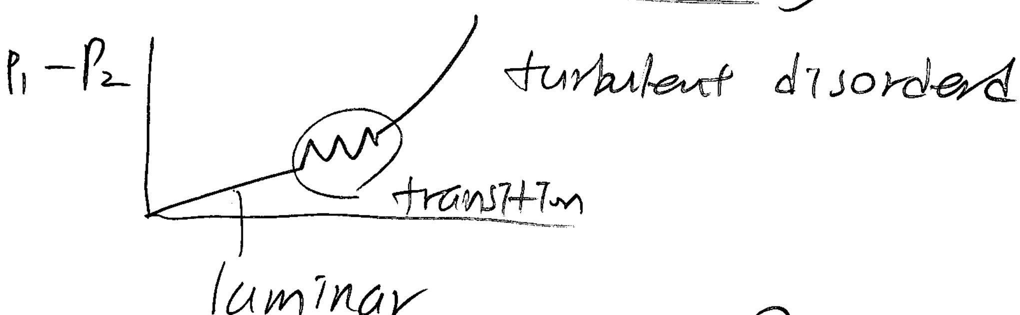
- def. friction factor

- laminar to turbulent

- Equivalent diameter

-

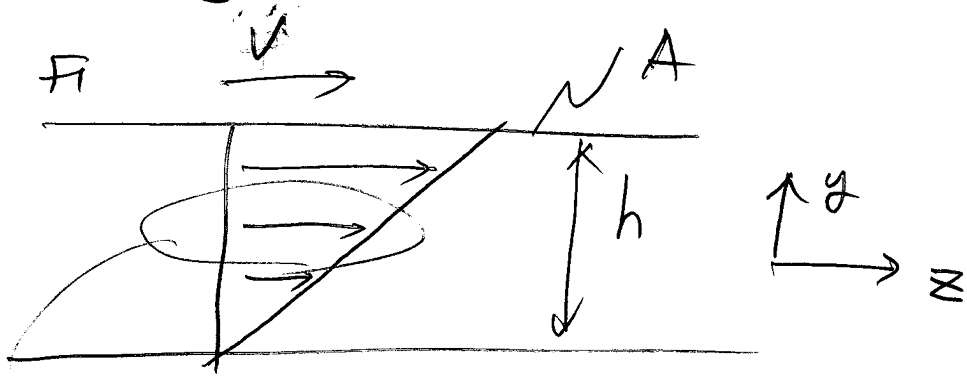




$$\frac{P_1 - P_2}{\rho} = \cancel{\frac{u_1^2 - u_2^2}{2}}^{\,0} - \cancel{\rho g (h_1 - h_2)}^{\,0} = f$$

friction losses

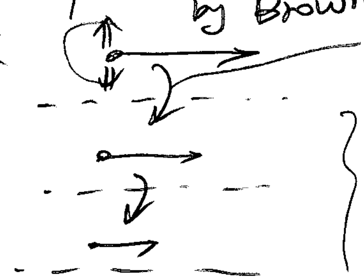
Velocity distribution



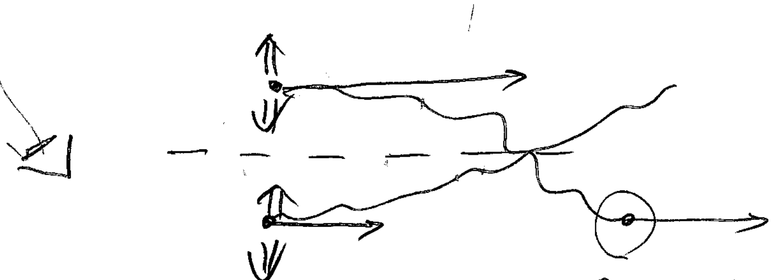
$$\frac{F_i}{A} = \tau = \mu \frac{V}{h} = \mu \frac{\partial v}{\partial y}$$

Random velocity by Brownian motion

transfer of momentum across streamline



"imaginary" layer



Carried momentum across streamline

Net momentum transfer through liquid.

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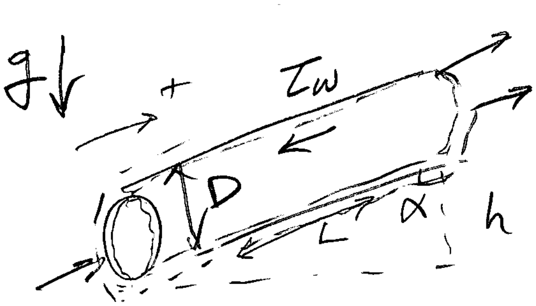


$$\tau_w = \frac{\text{frictional force}}{\text{area}} \text{ of pipe wall}$$

Bernoulli Eqn (E.B)

$$\frac{P_1 - P_2}{\rho} + \rho g (h_1 - h_2) = f \quad \cdots ①$$

$$\rightarrow P_1 - P_2 = \rho g h + \rho f$$

inlet surface, outlet surface, volume

$$0 = P_1 \frac{\pi}{4} D^2 - P_2 \frac{\pi}{4} D^2 - \rho g \frac{\pi D^2}{4} L \cos\alpha \quad \text{(M.B)}$$

$(u_1 = u_2$ this momentum flux disappear

$$- \tau_w (\pi D L)_{\text{surface}}$$

↳ acting against flow.

$g\cos\alpha$, α, g

$\rightarrow L\cos\alpha = h$

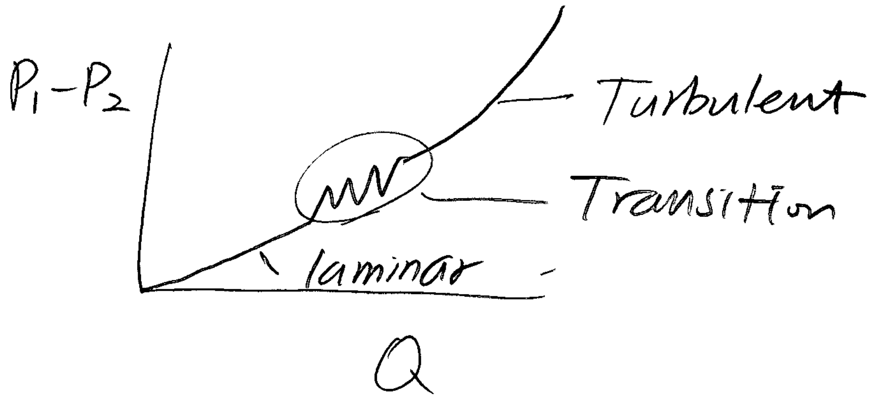
$$\rightarrow 0 = P_1 - P_2 - \rho g h - \tau_w 4 \frac{L}{D}$$

$$\rightarrow P_1 - P_2 = \rho g h + \tau_w 4 \frac{L}{D} \quad \cdots ②$$

τ_w: assume constant. reasonable for a cylinderical pipe.

Compare ① & ②

$$\rho f = \tau_w 4 \frac{L}{D} \longrightarrow \boxed{f = \frac{\tau_w}{\rho} 4 \frac{L}{D}}$$



Too complicate

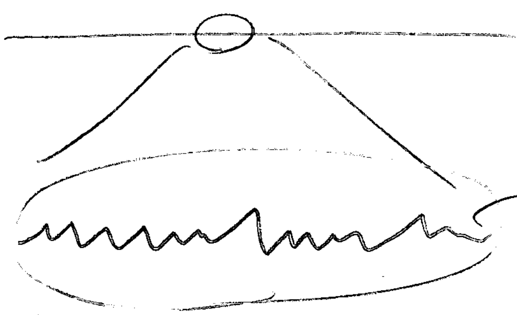
How can we handle τ_w ?

Dimensional analysis

velocity

$$\tau_w = \mathcal{F}(\rho, \mu, D, L, u, \varepsilon)$$

Roughness



surface roughness
can measure w/ profilameter

$$\frac{\tau_w}{\rho u^2}, \quad \frac{\rho u D}{\mu}, \quad \frac{L}{D}, \quad \frac{\varepsilon}{D}$$ ← Dimensionless quantity

$$\frac{\tau_w}{\rho u^2} = \mathcal{F}\left(\frac{\rho u D}{\mu}, \frac{\varepsilon}{D}, \cancel{\boxed{\frac{L}{D}}}\right)$$

↳ Not a factor
affect total loss but
not loss per unit length.

$$\frac{\tau_w}{\rho u^2} = \mathcal{G}\Big(\underbrace{\frac{\rho u D}{\mu}}_{=\ \text{Reynolds \#}},\ \underbrace{\frac{\epsilon}{D}}_{\text{Dimless Roughness}} \Big)$$

$$N_{Re} \text{ or } Re = \frac{\rho u D}{\mu} = \frac{\rho u^2}{\mu \frac{u}{D}} = \frac{\text{Inertial force (convective momentum flux)}}{\text{Viscous force}}$$

Historically,

chemical engineer uses

Fanning friction factor

$$\frac{\tau_w}{\frac{1}{2}\rho u^2} \equiv f_{Fi}\left(Re, \frac{\epsilon}{D}\right)$$

$$\boxed{\text{Losses/mass } f = \frac{\tau_w}{\rho} 4 \frac{L}{D} \quad \text{from "pipe"}}$$

$$\tau_w = \frac{1}{2}\rho u^2 f_{Fi}\left(Re, \frac{\epsilon}{D}\right)$$

$$f = \frac{1}{2}\rho u^2 f_{Fi}\left(Re, \frac{\epsilon}{D}\right) \frac{4}{\rho}\frac{L}{D}$$

$$= 2u^2 f_{Fi}\left(Re, \frac{\epsilon}{D}\right) \frac{L}{D}$$

Note that it appears in total loss

MOODY FRICTION FACTOR ~ Mechanical Engineer uses..

$$f_M = \frac{4\tau_w}{\frac{1}{2}\rho u^2} = 4 f_{FI}$$

→ This guys developed his own f.f.

$$f = f_M\left(Re, \frac{\epsilon}{D}\right) \frac{1}{2} u^2 \frac{L}{D}$$

Preserve $\frac{1}{2}$ Kinetic E

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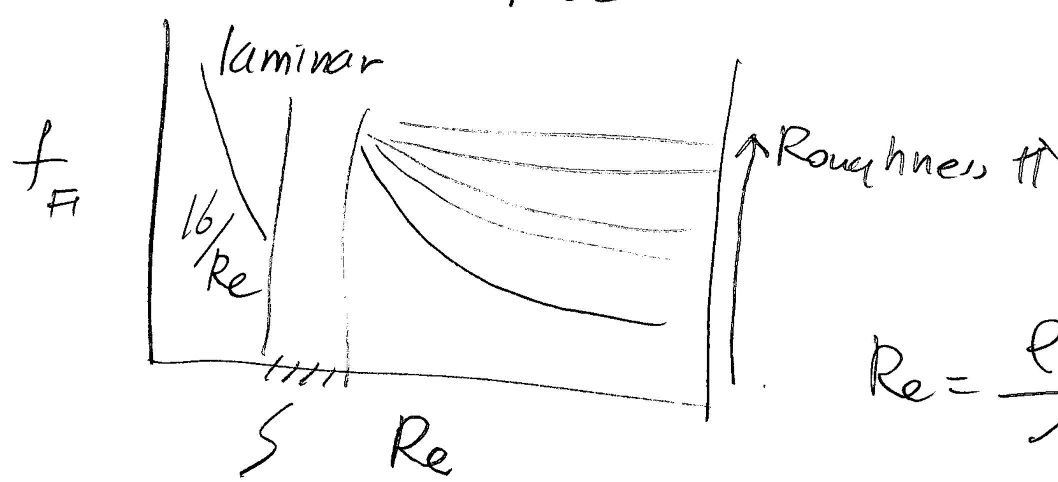
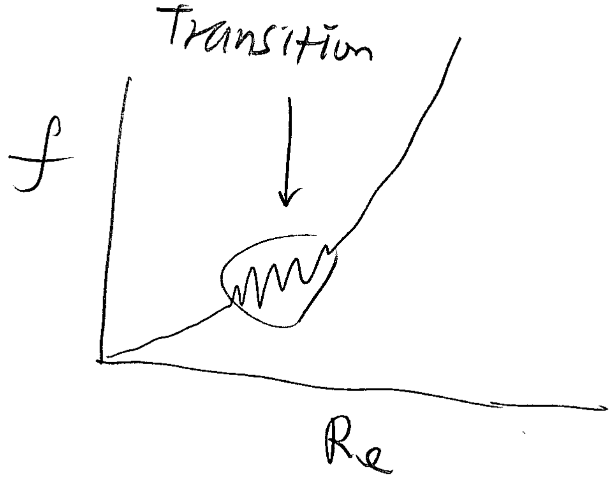
shows figures MOODY's diagram
Wilkes p135



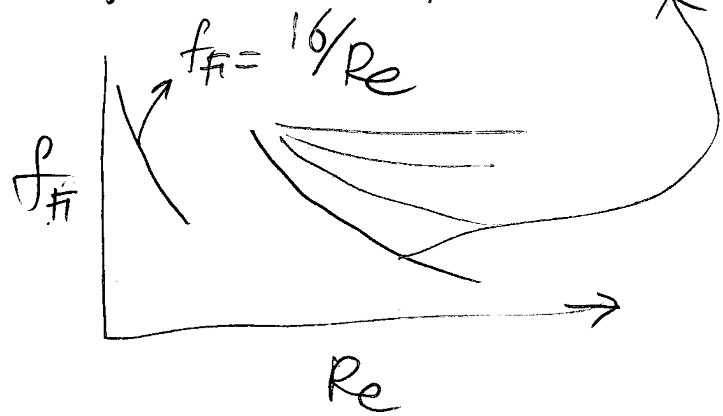
$$Re = \frac{\rho u D}{\mu}$$

Transition



Wilkes <u>Eqs 3.39–3.41</u> (Friction factor formulas)

$$f_{Fi} = 0.079\, Re^{-1/4}$$ for smooth pipe



$$f_{Fi} = G\left(Re, \frac{\epsilon}{D}\right)$$ table

<u>Nominal size</u> $\neq$ Outside diameter $\neq$ Inside diameter

dep on <u>schedule #</u>

thickness

P138 Table 3.3

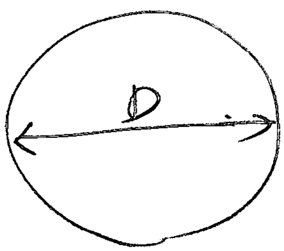
→ check <u>chemical Eng. handbooks</u>

Non circular duct

⇒ Need Concept Equivalent diameter

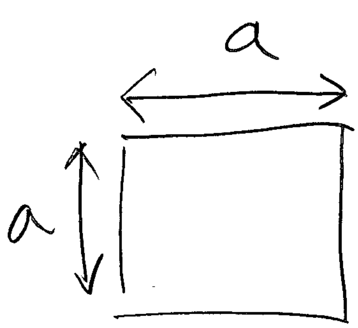
$$D_e = 4 \times \frac{\text{Cross sectional area}}{\text{wetted perimeter}}$$

↑ Perimeter where fluid is in Contact w/ solid walls



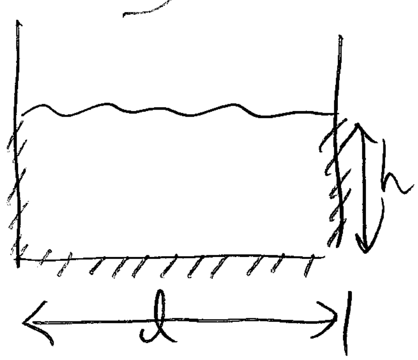
$$D_e = \frac{4 \frac{\pi D^2}{4}}{\pi D} = D$$

↑ Equivalent diameter



$$D_e = \frac{4a^2}{4a} = a$$

open channel



$$D_e = 4 \frac{hL}{(2h+L)}$$

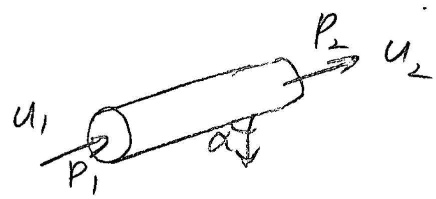
$$Re = \frac{\rho u D_e}{\mu}$$

Equivalent diameter

$$\text{Roughness ratio} = \frac{\epsilon}{D_e}$$

Lecture 10

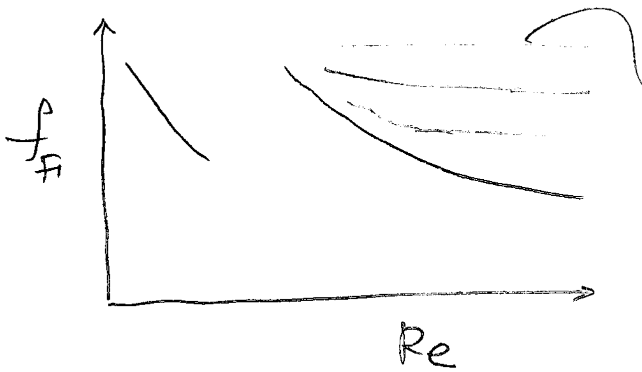
Summary



- f (friction loss per unit mass) $= 2u^2 f_{FI}(Re, \frac{\epsilon}{D}) \frac{L}{D}$

  Assume $u_1 = u_2$ & $\rho_1 = \rho_2$

- f_{FI} (fanning friction factor) $= \begin{cases} 16/Re & \text{(laminar)} \\ 0.079\, Re^{-1/4} & \text{(turbulent) w/ smooth pipe} \end{cases}$



If ϵ/D is high f_{FI} is virtually constant.

- Effective diameter

$$D_e = 4 \frac{S_c}{P}$$

S_c: cross sectional area

P: wetted perimeter.

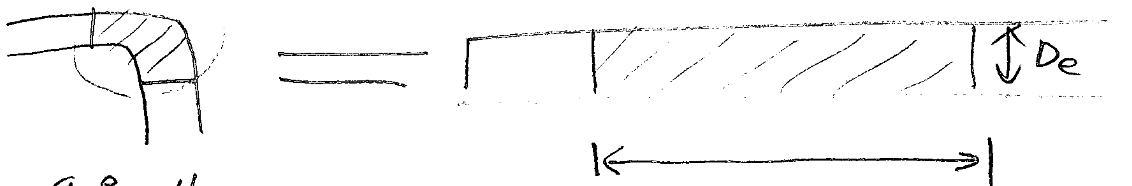
- $Re = \dfrac{\rho u D_e}{\mu}$

Roughness ratio $= \dfrac{\epsilon}{D_e}$

(see slide)

Pressure drop across pipe fittings (see slide)

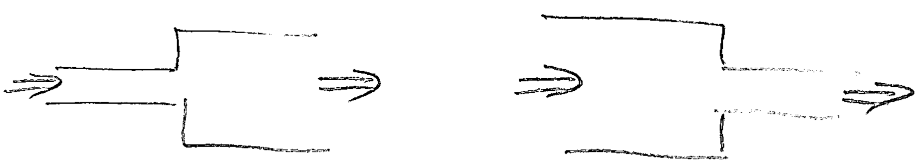
⇒ These fittings causes the flow to deviate from its normal straight course and induce additional turbulence and frictional dissipation.



90° elbow fitting

$L_e = 30 D_e$

↑ Equivalent Length



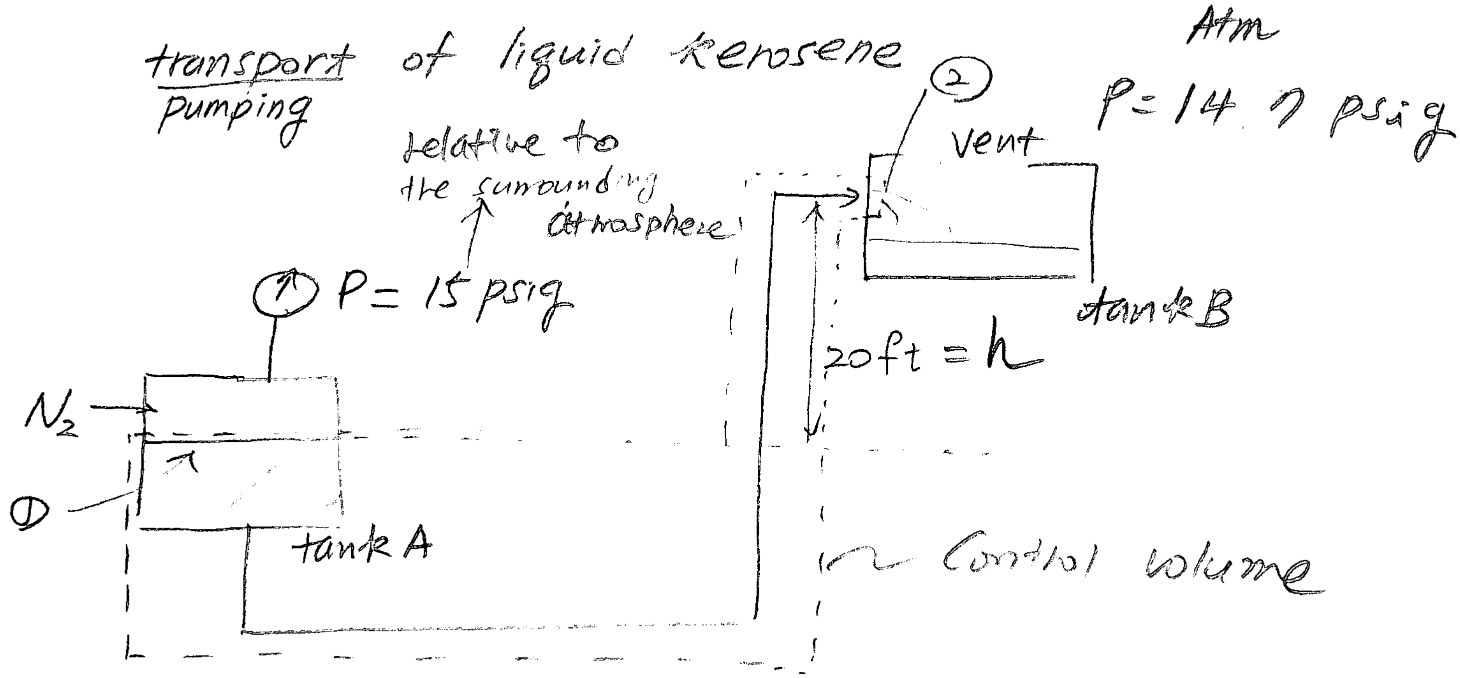
Sudden expansion   Sudden Contraction

L_e is estimated based on $(D_e)_{min}$.

Today, we will go over some examples

Wilkes problem 3.9

transport of liquid kerosene
pumping



* Data

① Liquid kerosene at 75°F

- $\rho = 51.0$ lbm/ft^3, • $\mu = 4.35$ lbm/~~ft~~ hr

② Pipe geometry

- Le (effective length) = 150 ft
  include fitting

- 2 in Schedule 40 pipe, commercial steel

  $D = 2.067$ in $\qquad h_t = 0.154$ in

  $\epsilon = 0.00015$ ft $\qquad$ (1 ft = 12 inch)

  $\frac{\epsilon}{D} = \frac{0.00015 \text{ ft}}{(2.067/12) \text{ ft}} = 8.7 \times 10^{-4}$

* Question

what is the flow rate at ② unit gallon per min (gpm) ?

Procedure

- Energy balance over C.V.

$$\frac{P_1 - P_2}{\rho} + \frac{u_1^2 - u_2^2}{2} + g(z_1 - z_2) = -\cancel{W_s} + f$$

0 no shaft work

Wilkes says neglect kinetic energy $u^2/2$

Let's try & verify later (Consistency check)

w/o kinetic E

$$\frac{P_1 - P_2}{\rho} - gh = f = f_{Ft}\, 2u^2 \frac{L}{D} \quad \cdots (1)$$

$$\Rightarrow \quad u = \sqrt{\left[\frac{P_1 - P_2}{\rho} - gh\right] \frac{D}{2 f_{Ft} L_e}}$$

Note) $1\ lb_f = 1\ lb_m \cdot 32.2 \frac{ft}{s^2}$

Careful about units!

- $$\frac{P_1 - P_2}{\rho} = \frac{15 \frac{lb_f}{in^2} \times 144 \frac{in^2}{ft^2}}{51 \frac{lb_m}{ft^3} \times \left(\frac{1}{32.2} \frac{lb_f}{lb_m \cdot ft/s^2}\right)}$$

$$= 1364 \frac{ft^2}{s^2}$$

- $$gh = 32.2 \frac{ft}{s^2} \cdot 20\ ft = 644\ ft^2/s^2$$

- $$\frac{D}{2L_e} = \frac{1}{2} \cdot \frac{2.067\ in \cdot \frac{1}{12} \frac{ft}{in}}{150\ ft} = 5.74 \times 10^{-4}$$

Then $u = \sqrt{(1364 - 644) \cdot 5.74\times10^{-4} \cdot \frac{1}{f_{FI}}}$

$= \frac{0.643}{\sqrt{f_{FI}}} \ \frac{ft}{s} \quad \ldots (2)$

While $Re = \frac{\rho u D}{\mu} = \frac{(51.0 \frac{lbm}{ft^3})(2.067/12\ ft)}{(4.38/3600 \frac{lbm}{ft \cdot s})} u$

$= 7220\ \underbrace{u}_{ft/s} \quad \ldots (3)$

---

check problem

3 unks u, Re & f_{FI}

Available eqn (2) & (3)

We need one more!

either [Colebrook eq & $f_F = 16/Re$
or
Moody's diagram

See keynote.

From (2) & (3) & $\frac{\epsilon}{D}$

$Re = 7220\, u$, $u = \dfrac{0.643}{\sqrt{f_{F1}}}$, $\frac{\epsilon}{D} = 8.71 \times 10^{-4}$

$\sim 1 \times 10^{-3}$

Use keynote to explain (Assume turbulent)

I.G $f_{F1} = 0.005$

$$u = \frac{0.643}{\sqrt{0.005}} = 9.09$$

$$Re = (7200)(9.09) = 65,654$$

f_{F1} (from diagram) $= 0.0058$

⋮

$$u = 8.44 \text{ ft/s}$$

Then flow rate becomes

$$Q = \frac{u \pi D^2}{4} = \dots = 88.3 \text{ gal/min}$$

$(7.48 \text{ gal} = 1 \text{ ft}^3)$

Lecture 10 Prob Cont.

problem is not done!

Was it ok to neglect K.E?

$$\begin{cases} \frac{u^2}{2} = 35.6 \ \frac{ft^2}{s^2} \\ gh = 644 \ \frac{ft^2}{s^2} \\ \frac{\Delta p}{\rho} = 1364 \ \frac{ft^2}{s^2} \end{cases}$$

$$\Rightarrow \quad \frac{u^2}{2} \ll \left(\frac{\Delta p}{\rho} - gh\right)$$

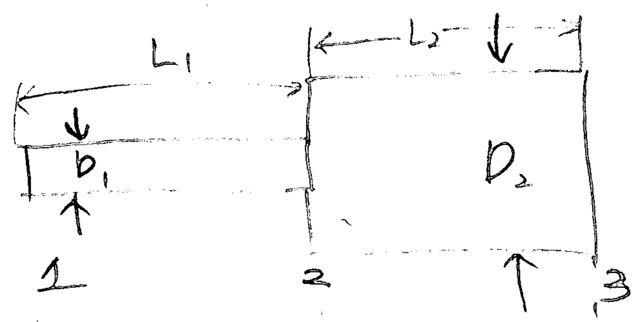
So the assumption was OK.

Lecture 10

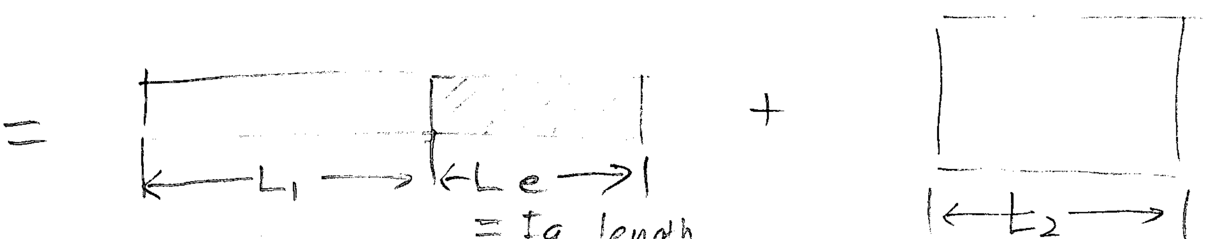
More example >

What about if the pipe expands?

How do we compute the friction coeff & loss?

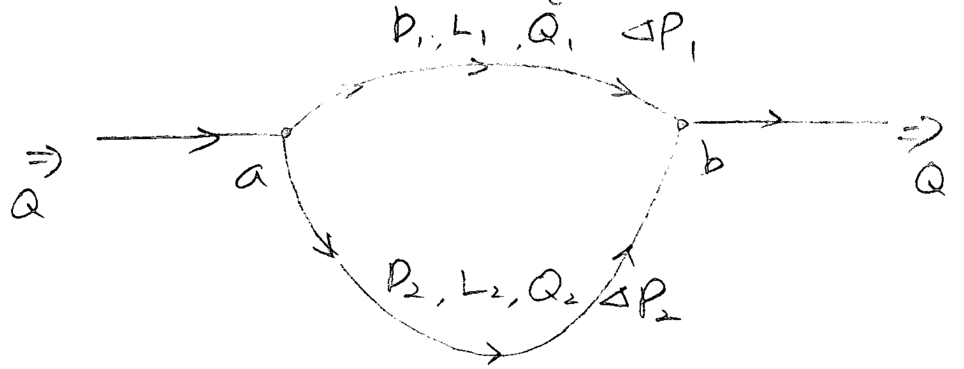


split into two problem $1 \rightarrow 2$ & $2 \rightarrow 3$



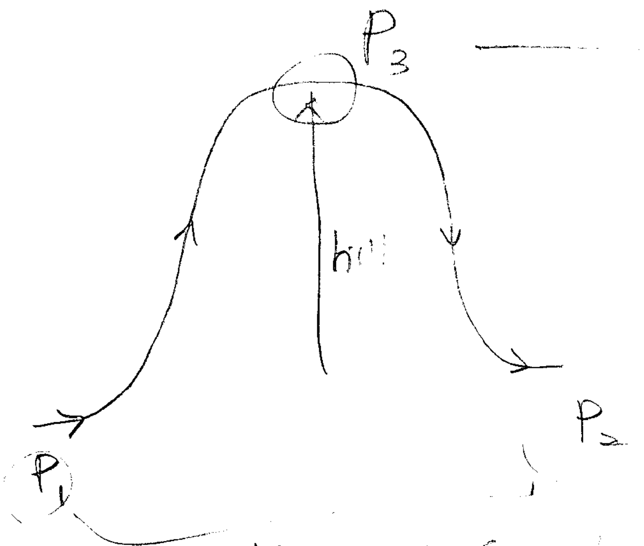
= Eq. length for sudden expansion based on D_1

what about splitting pipes?



$$\begin{cases} \Delta P_1 = \Delta P_1 \\ Q = Q_1 + Q_2 \end{cases}$$

How about this ?



→ P_3 needs to be above P_{vap} otherwide fluid would vaporize.

→ cavitation can become dangerous it can corrode pipeline.

We can't consider these two pressure.

Turbulent drag [illegible] in pipe

$$\mathcal{F}\text{ (loss per unit mass)} = \frac{P_1 - P_2}{\rho}$$

$$= \frac{32 f_{Fl} Q^2 L}{\pi^2 D^5}$$

$$\left(u = \frac{Q}{\pi D^2/4} = \frac{4Q}{\pi D^2} \right.$$
$$\mathcal{F} = 2u^2 f_{Fl} \frac{L}{D}$$

We are trying to plot $\mathcal{F}$ vs Q at fixed geometry & fluid properties
(L, D) (ρ, μ)

What is f_{Fl} (Fanning friction factor)

Recall $f_{Fl} = \begin{cases} \frac{16}{Re} & Re \lesssim 2000 \text{ laminar} \\ \text{Const} & Re \gtrsim Re_c \end{cases}$ ← depends on $\frac{\epsilon}{D}$ (roughness ratio)

Estimate Reynolds number

$$Re = \frac{\rho u D}{\mu} = \frac{4\rho Q}{\pi \mu D}$$

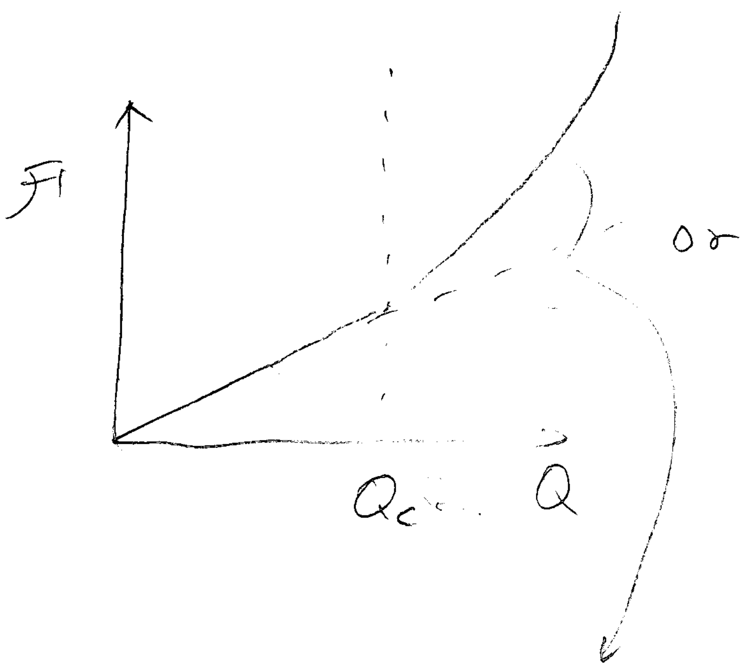
↑ In cylindrical pipe

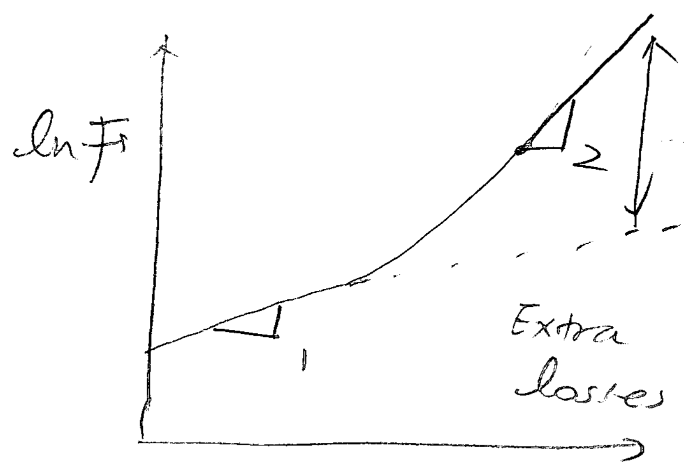
$$\left(u = \frac{4Q}{\pi D^2} \right)$$

Therefore

$$\frac{128\,\mu L}{\pi \rho D^4} Q$$

$$F_f = \begin{cases} \dfrac{32\left(\dfrac{16}{\frac{4\rho Q}{\pi \mu D}}\right) Q^2 L}{\pi^2 D^5} & \sim \ Q \text{ dependency} \\[2em] \dfrac{32 \ \text{Const} \ Q^2 L}{\pi^2 D^5} & \sim \ Q^2 \text{ dependency} \end{cases}$$



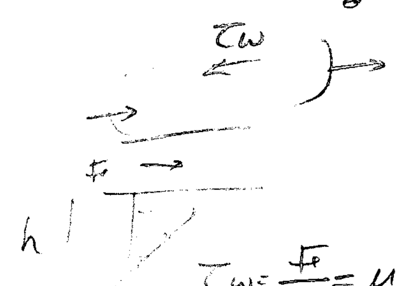


Apparently another mechanism
that leads to increase energy dissipation.

What happened to turbulent flow?

$$f = 2u^2 f_{Fl} \frac{L}{D} =$$

$$f_{Fl} = \frac{\tau_w}{\rho} 4 \frac{L}{D}$$



$$\tau_w = \frac{F}{A} = \mu \frac{V}{h}$$