Cryogenic Engineering

Chapter 6.

Measurement Systems for Low Temperatures

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6.1. Temperature Measurement Methods

- **Temperature measurement from temperature property**

  ① Mercury in a capillary tube

  ② Pt (platinum) wire resistance

  ③ Ideal gas pressure

  ④ Thermoelectric EMF (voltage)

  ⑤ Equilibrium pressure of gas

  ⑥ Difference in thermal expansion

  ⑦ Speed of sound

  ⑧ Magnetic susceptibility

▪ Temperature measurement from temperature property

① Mercury in a capillary tube

< Mercury in a capillary tube >

Mercury in glass	-39 ^{o}C to +357 ^{o}C
Pressurized mercury in glass	-39 ^{o}C to +500 ^{o}C
Pressurized mercury in quartz	-39 ^{o}C to +800 ^{o}C
Alcohol in glass	-120 ^{o}C to +60 ^{o}C
Pentane in glass	-200 ^{o}C to +30 ^{o}C

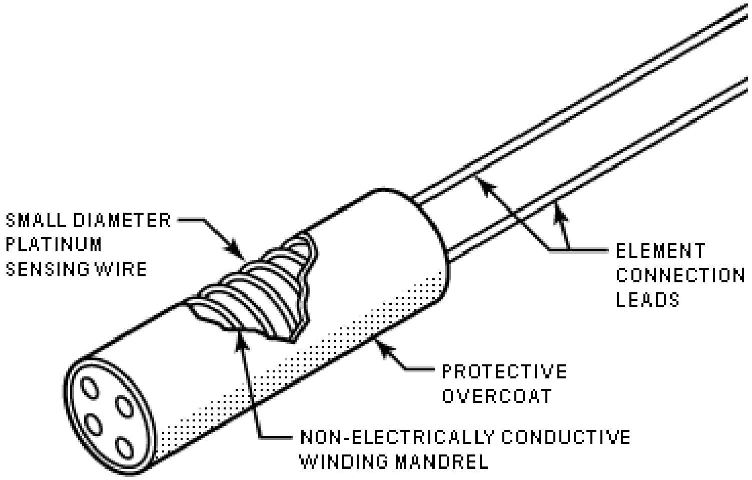
< The ranges of the most common liquid-in-glass thermometers >

6.1. Temperature Measurement Methods

- **Temperature measurement from temperature property**

② Pt (Platinum) wire resistance

< Platinum >



< wire-wound sensing element >

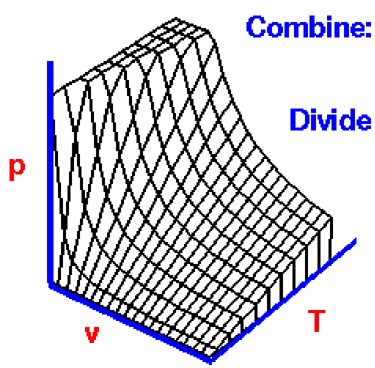
- **Temperature measurement from temperature property**

③ Ideal gas pressure

Pressure = ρ, Pressure = P, Temperature = T, Volume = V, Mass = M

Boyle: For a given mass, at constant temperature, the pressure times the volume is a constant (PV =C)

Charles and Gay-Lussac: For a given mass, at constant pressure, the volume is directly proportional to the temperature.



Combine: $pV/T = n\bar{R}$ $\bar{R}$ = 8.31 J / mole / K (Universal)

$pV = n\bar{R}T$ n = number of moles

Divide by mass: $pv = \frac{n\bar{R}T}{M}$ Specific Volume = v

$v = \frac{\text{volume}}{\text{mass}} = \frac{1}{\rho}$

$pv = RT$ or $p = R\rho T$

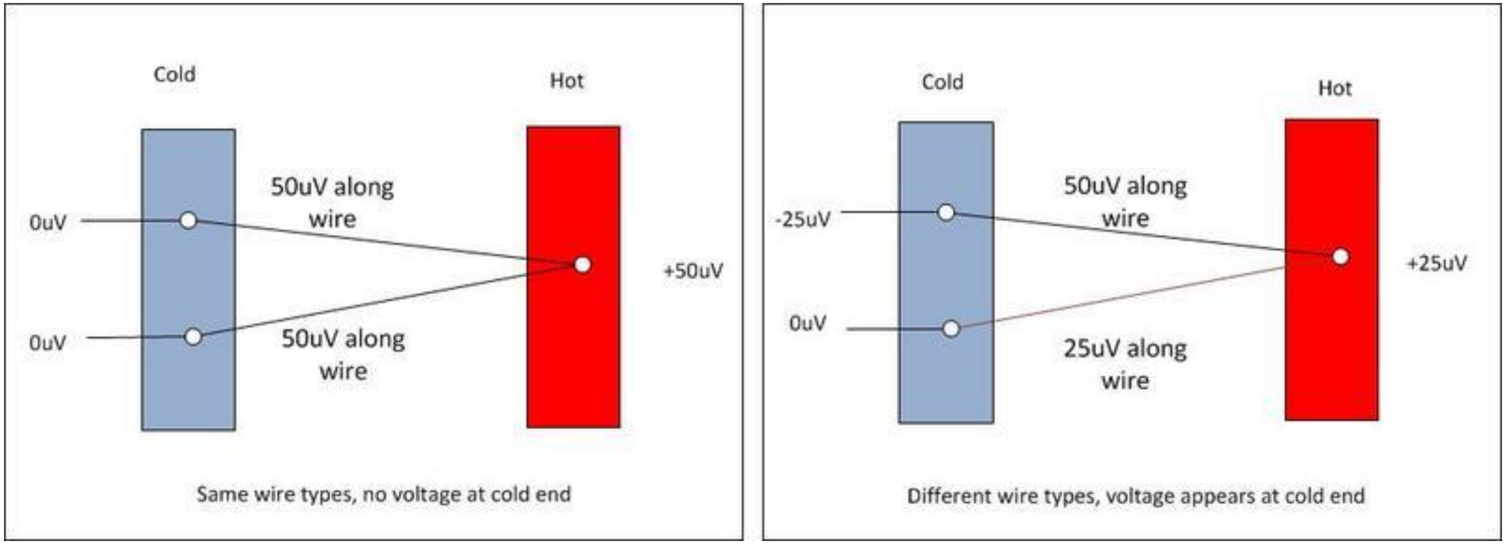
R = Constant value for each gas

= .286 kJ/ kg / K (for air)

6.1. Temperature Measurement Methods

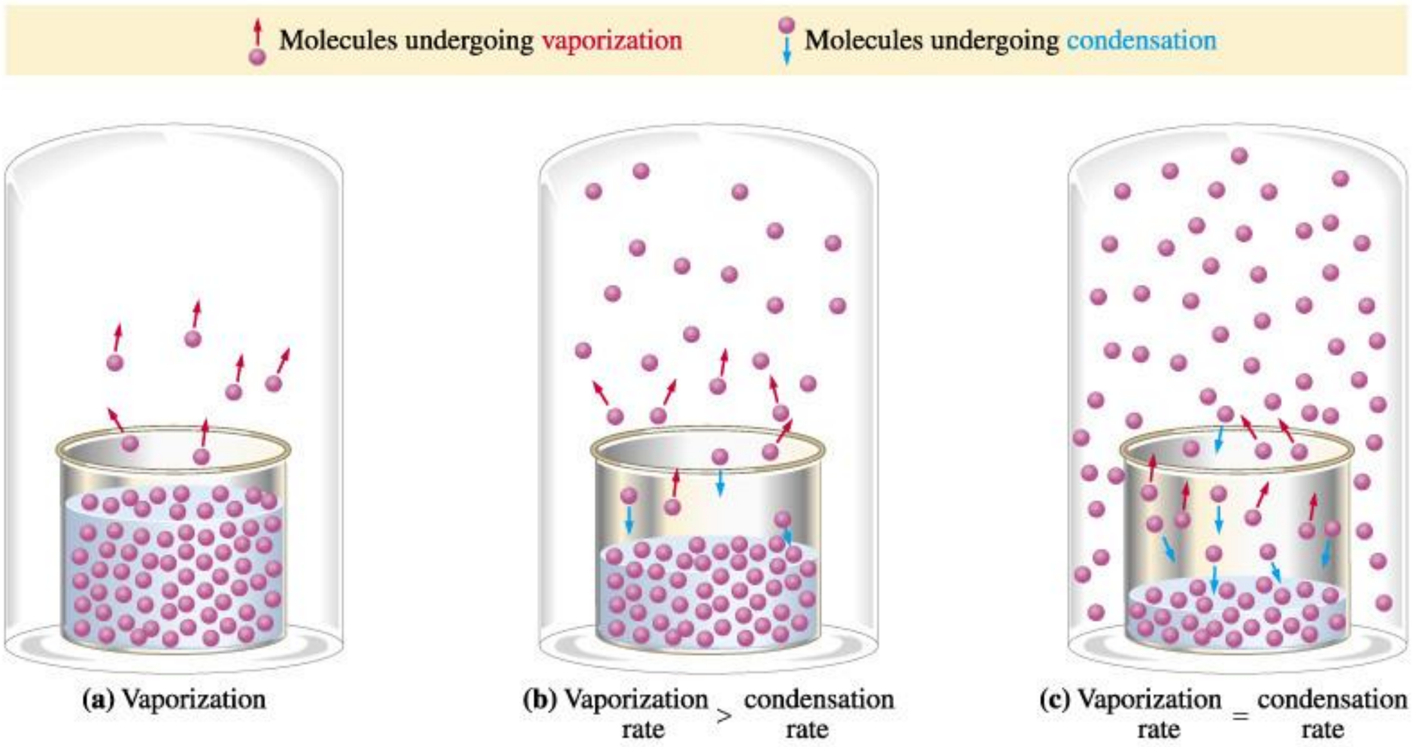
- **Temperature measurement from temperature property**

④ Thermoelectric EMF(Electro Motive Force)



- **Temperature measurement from temperature property**

⑤ Equilibrium pressure of gas

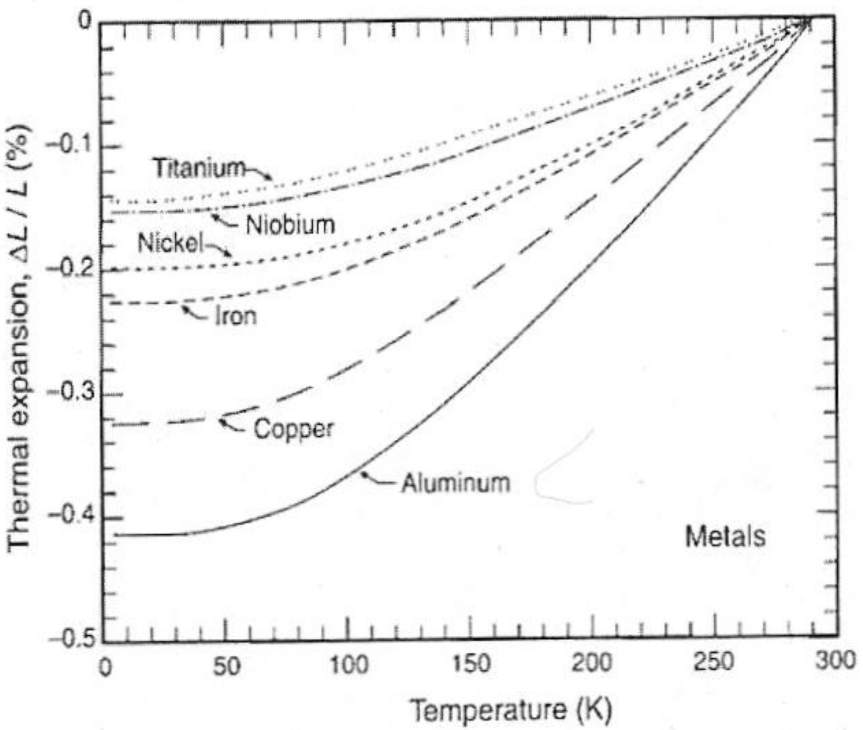


- **Temperature measurement from temperature property**

⑥ Difference in thermal expansion

Material	Fractional expansion per degree C x10^-6	Fractional expansion per degree F x10^-6
Glass, ordinary	9	5
Glass, pyrex	4	2.2
Quartz, fused	0.59	0.33
Aluminum	24	13
Brass	19	11
Copper	17	9.4
Iron	12	6.7
Steel	13	7.2
Platinum	9	5
Tungsten	4.3	2.4
Gold	14	7.8
Silver	18	10

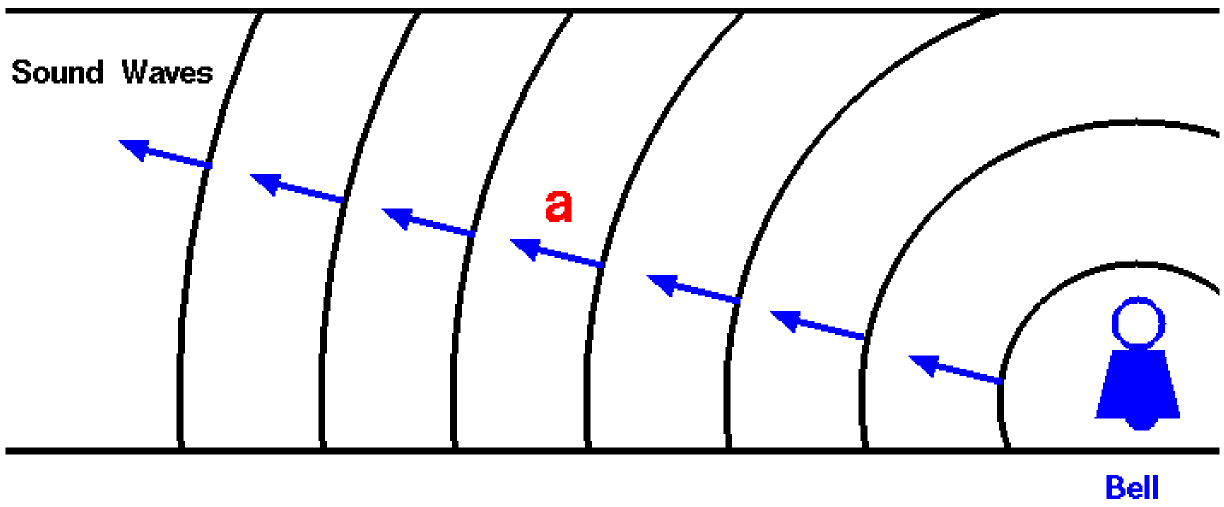
< Thermal Expansion Coefficients at 20 ℃ >



< Thermal linear expansion of common metals>

- **Temperature measurement from temperature property**

⑦ Speed of sound



Speed of sound (a) depends on the type of medium and the temperature of the medium.

$$a = \sqrt{\gamma R T}$$

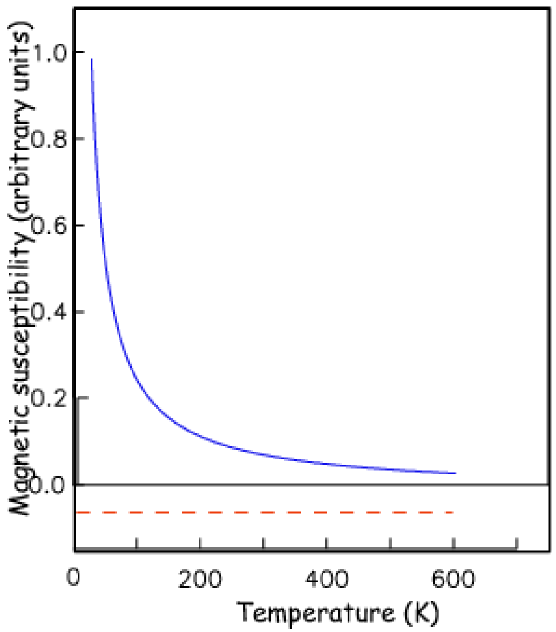
γ = ratio of specific heat (1.4 for air at STP)

R = gas constant (286 m^2/s^2/K for air)

T = absolute temperature (273.15 + °C)

▪ Temperature measurement from temperature property

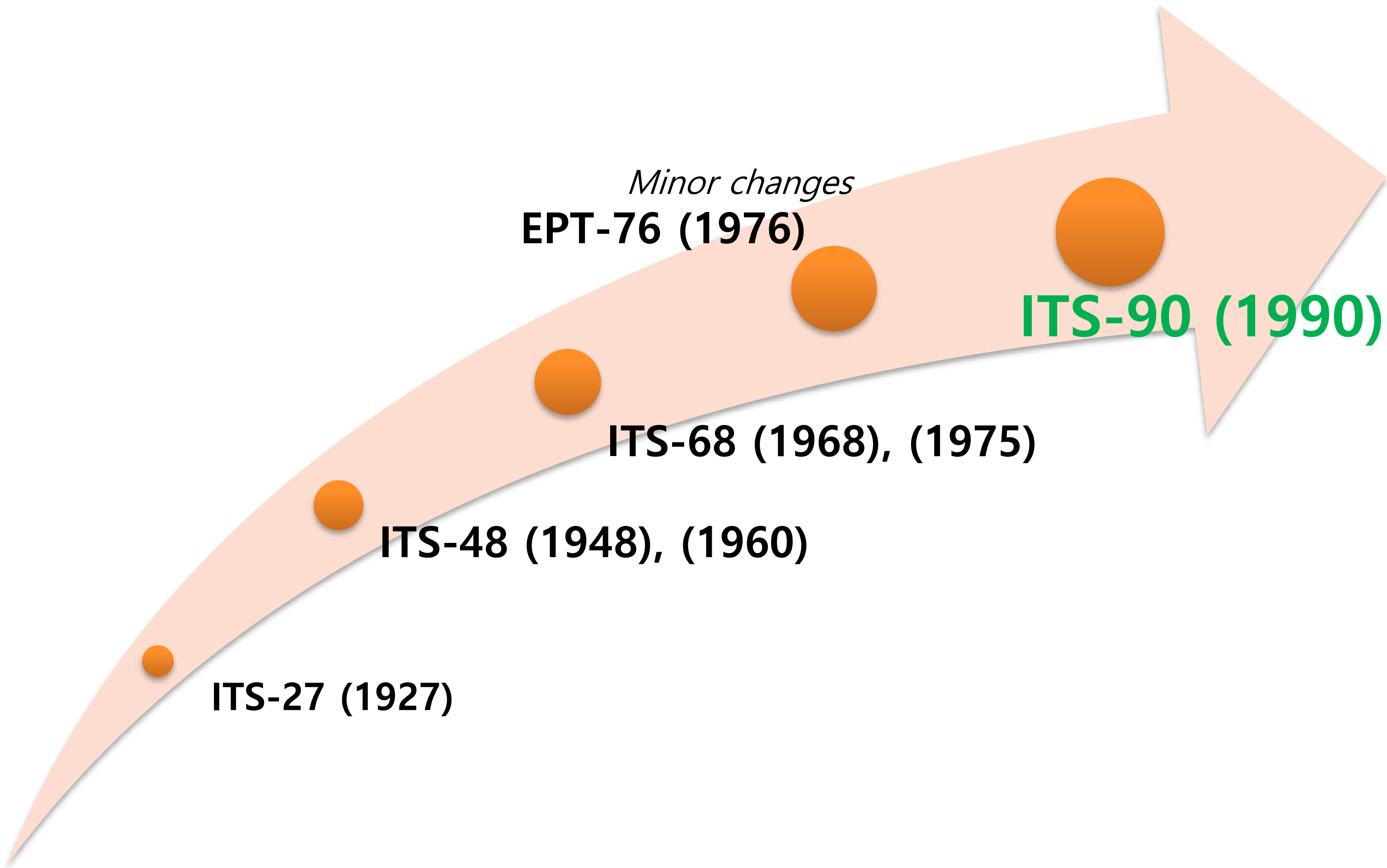
⑧ Magnetic susceptibility



< Paramagnetic (solid line) and diamagnetic (dashed line) magnetic susceptibility as a function of temperature >

- Magnetic susceptibility, quantitative measure of the extent to which a material may be magnetized in relation to a given applied magnetic field.
- magnetization essentially involves a certain measure of magnetism (dipole moment) per unit volume.

6.2. Temperature Scales and Fixed Points



6.2. Temperature Scales and Fixed Points

- **ITS-27 (1927)**

  - The ITS-27 was based on six fixed points.

  - It is an international temperature scale adopted to overcome the difficulties of directly measuring thermodynamic temperature by gas thermometry and the problems of using different temperature scales among countries.

  - The temperature scale was standardized by defining various measurement points between the boiling temperature of oxygen and the solidification temperature of gold according to the measurement reproducibility of the temperature scale.

6.2. Temperature Scales and Fixed Points

- **ITS-48 (1948) or IPTS-48 (1960)**

  - ITS-48 changed the lower limit of the platinum resistance thermometer from -190 degrees to -182.97 degrees, the boiling temperature of oxygen.

  - The constants used in the interpolation formula were also changed for platinum resistance thermometers and thermocouples.

  - The 1960 meeting revised the terminology of the temperature scale from the International Temperature Scale (ITS) to the International Practical Temperature Scale (IPTS).

  - When it was discovered that the freezing temperature of water, 0 degrees, was not constant, it was changed to the triple point of water.

6.2. Temperature Scales and Fixed Points

- **IPTS-68 (1968), IPTS-69:75 (1975)**

  - IPTS-48 has undergone many revisions.
  - The lower peak of the temperature definition was lowered to 13.81 K, the temperature of the triple point of equilibrium hydrogen.
  - The solidification temperature of equilibrium hydrogen, the boiling temperature, and the triple point of oxygen were newly added as points.

- **IPTS-90 (1990)**

  - The International Temperature Scale of 1990 (ITS-90) is an equipment calibration standard specified by the International Committee of Weights and Measures (CIPM) for making measurements on the Kelvin and Celsius temperature scales.
  - It is an approximation of thermodynamic temperature that facilitates the comparability and compatibility of temperature measurements internationally.
  - ITS-90 is the most recent of a series of International Temperature Scales adopted by the CIPM since 1927.

6.2. Temperature Scales and Fixed Points

- **ITS-68**

Fixed Point	Temperature (K)
NMP of gold	1337.58
NMP of silver	1235.08
NMP of zinc	692.73
NMP of water	373.15
Standard-triple point of water	273.16
NMP of oxygen	90.188
TP of oxygen	54.361
NBP of neon	27.102
NBP of hydrogen	20.28
B.P. of hydrogen at 25 torr	17.042
TP of hydrogen	13.81

NMP: Normal Melting Point or freezing point
NBP: Normal Boiling Point
TP: Triple Point

6.2. Temperature Scales and Fixed Points

- **ITS-90**

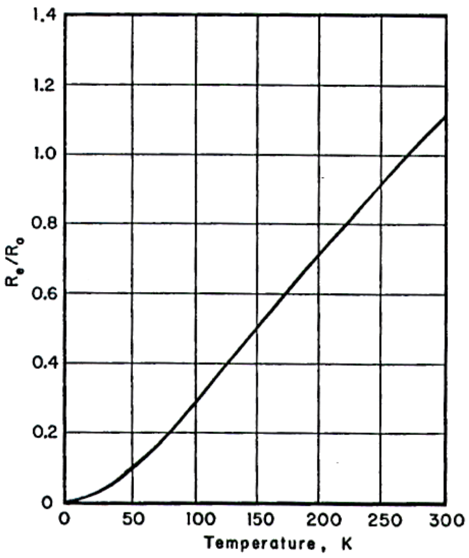
Fixed Point	T_{90} (K)	t_{90} (°C)
Triple point of H_2 at equilibrium	13.8033	-259.3467
Triple point of Ne	24.5561	-248.5939
Triple point of O_2	54.3584	-218.7916
Triple point of Ar	83.8058	-189.3442
Triple point of Hg	234.3156	-38.8344
Triple point of water	**273.16**	**0.01**
Equilibrium M of Ga	302.9146	29.7646
Equilibrium F of In	429.7485	156.5985
Equilibrium F of Zn	692.677	419.527
Equilibrium F of Al	933.473	660.323
Equilibrium F of Cu	1357.77	1084.62

- **Callendar Dusen equation**

$$R_e / R_0 = 1 + At + Bt^2 + Ct^3 (t - 100)$$

Typical values for platinum thermometers are

R_0=25 Ω, A=3.946*10^{-3} ℃$^{-1}$, B= -1.108*10^{-6} ℃$^{-2}$, C= 3.33*10^{-12} ℃$^{-4}$



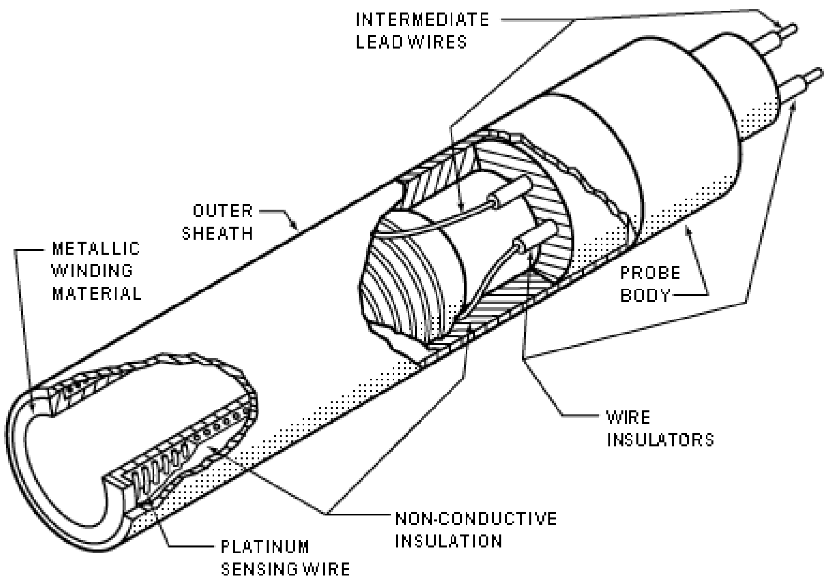
Choose materials for temperature range

- Platinum (Pt): below 630.74 ℃

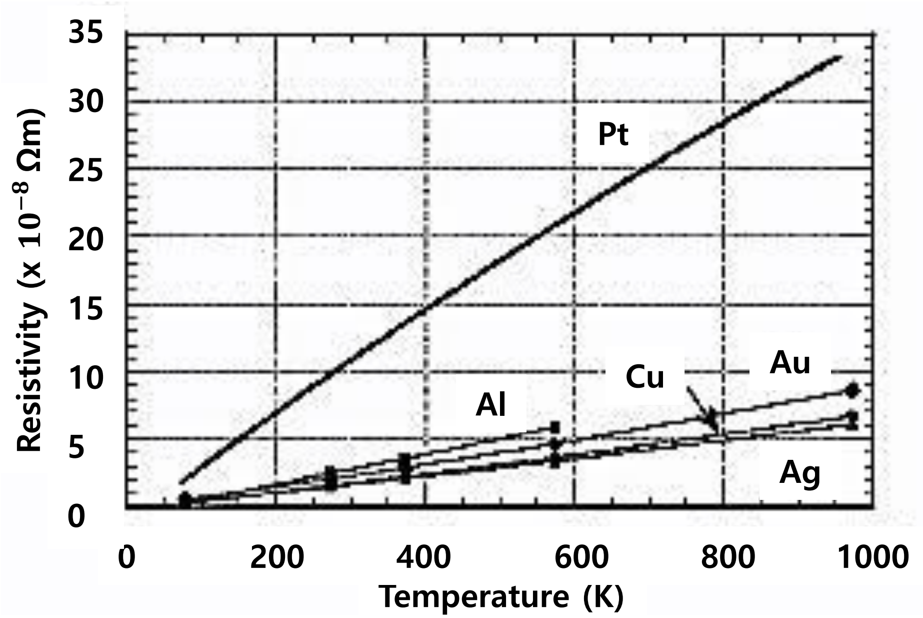
- Indium (In): low temperature range

< Reduced electric resistance ratio for platinum. R_0 is the electric at 0 ℃ >

- **Platinum resistance thermometer**



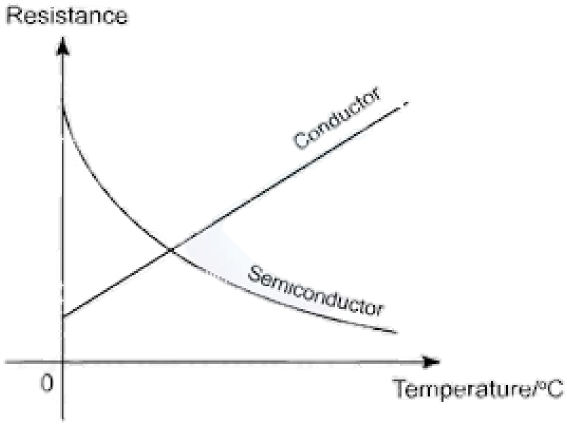
< The wire-wound sensing element >



< The resistivity of five metallic elements >

6.4. Semiconductor Resistance Thermometers

- **Electrical conductivity of semiconductors is temperature dependent**
  - High temperature : exponential proportion of the absolute temperature
  - Low temperature : due to the presence of impurities
- **Material of semiconductor thermometer**
  - Germanium : widely used
  - Carbon : for low temperature thermometer



< The resistivity change of a conductor and semiconductor against the temperature >

6.5. Thermocouples

One junction of thermocouple pair is placed at the point to be measured, the other junction is placed in a reference temperature region.

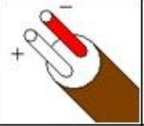
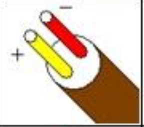
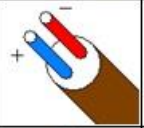
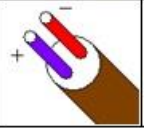
$$e = a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$$

$$t = b_1 e + b_2 e^2 + b_3 e^3 + b_4 e^4$$

t : the difference in temperature

e : the thermocouple output E.M.F value

* Disadvantage : Output E.M.F is quite small

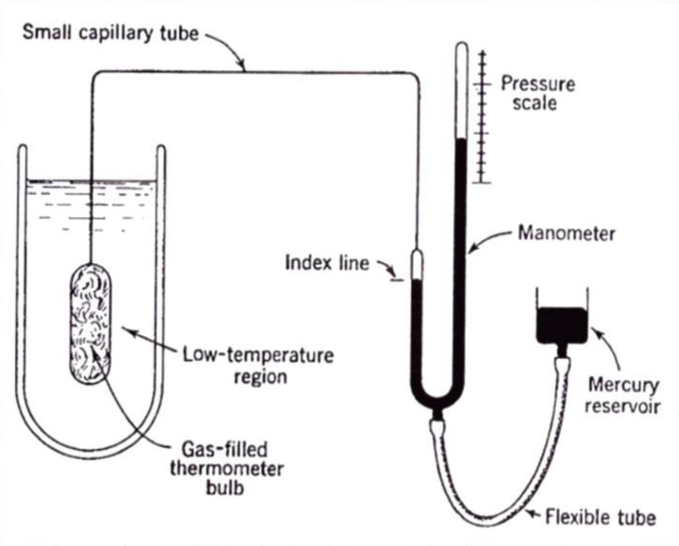
Type	Material		Color Code	Range (°C)	
Thermocouple Grade	Positive Wire	Negative Wire		Minimum	Maximum
J	Iron	Constantan		0	750
K	Chromel	Alumel		-200	1250
T	Copper	Constantan		-200	350
E	Chromel	Constantan		-200	900

6.6. Constant-Volume Gas Thermometer

The constant-volume thermometer may be calibrated by measuring the gas pressure T_s at a standard temperature p_s.

From ideal gas law : $\mathbf{T = p(\frac{T_s}{p_s})}$

For accurate work : $$\mathbf{T = \frac{p(\frac{T_s}{p_s})}{1 + (1 - \frac{p}{p_s})(\frac{V_0}{V})(\frac{T_s}{T_0})} = K_1 p(\frac{T_s}{p_s})}$$



< Constant-volume gas thermometer >

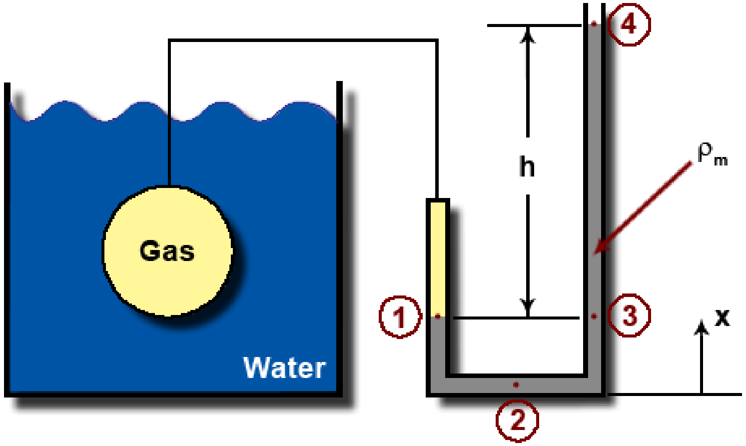
6.7. Vapor-Pressure Thermometer

The vapor pressure(saturation pressure) is a definite function of the temperature of the liquid, so this property can be used in determining the temperature.

$$\ln\left(\frac{p}{p_0}\right) = C_1 - \frac{C_2}{T} - C_3 \ln\left(\frac{T}{T_0}\right) - C_4 T + C_5 T^2$$

- Advantage : Great sensitivity in the applicable temperature range
- Disadvantage :Limited range

T_0 is normal boiling point, C values are constants in the vapor-pressure relationship



< Vapor-pressure thermometer >

6.8. Magnetic Thermometer

For temperature measurement below 1.0 K.

From Curie law, define a magnetic temperature T^*

$$T^* = \frac{C}{\chi}\ ; C = \text{The Curie constant}, \chi = \text{the magnetic susceptibility}$$

Following are some correlations of magnetic temperature with the absolute temperature.

- Cerium magnesium nitrate, for $\mathbf{T \geq 0.004\ K}$

$$\mathbf{T^* = T + (0.236 + \frac{0.004137}{T})(10^{-3})}$$

- Chromic methylammonium alum, for $T \geq 0.070$ K

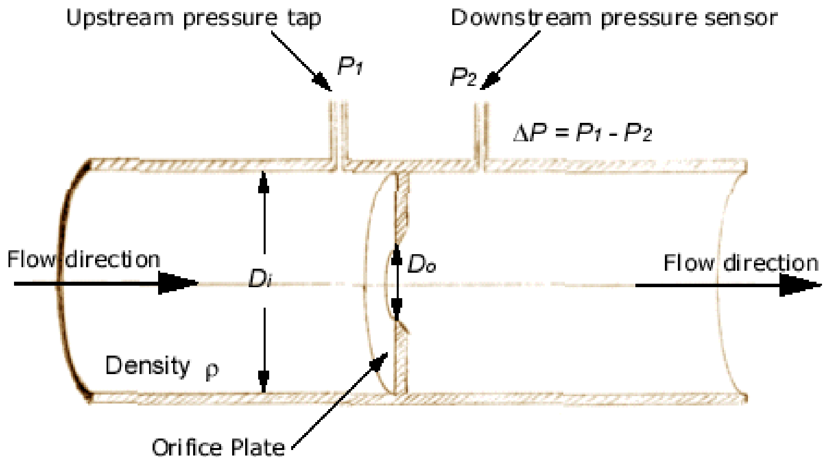
$$\mathbf{T^* = T + 0.00250 + \frac{0.002422}{T}}$$

- Chromium potassium alum, for $\boldsymbol{T \geq 0.10\ K}$

$$\mathbf{T^* = T + 0.000862 + \frac{0.002057}{T}}$$

6.9. Orifice Meters

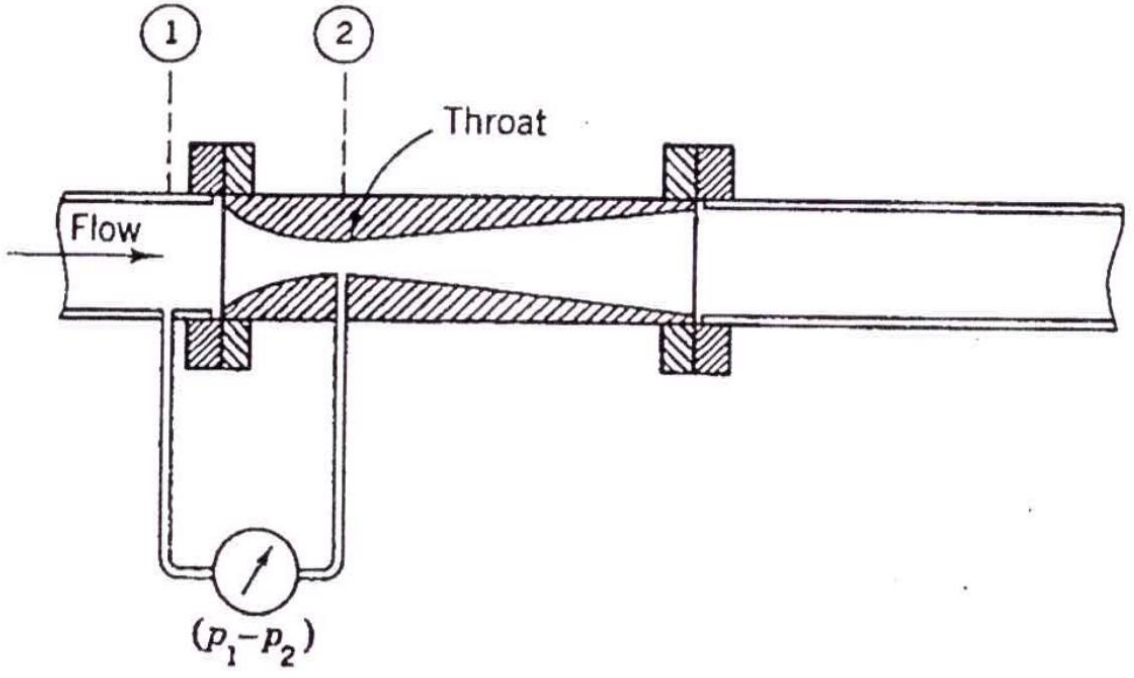
A fluid passing though an orifice constriction will experience a drop in pressure across the orifice. This change can be used to measure the flowrate of the fluid. To calculate the flowrate of a fluid passing through an orifice plate, enter the parameters below.



$$\Delta p = \frac{1}{2}\,\rho\,Q^2\,\frac{1}{{A_0}^2}\left[1-\left(\frac{A_o}{A_i}\right)^2\right]$$

< Orifice meter >

6.10. Venturi Meter



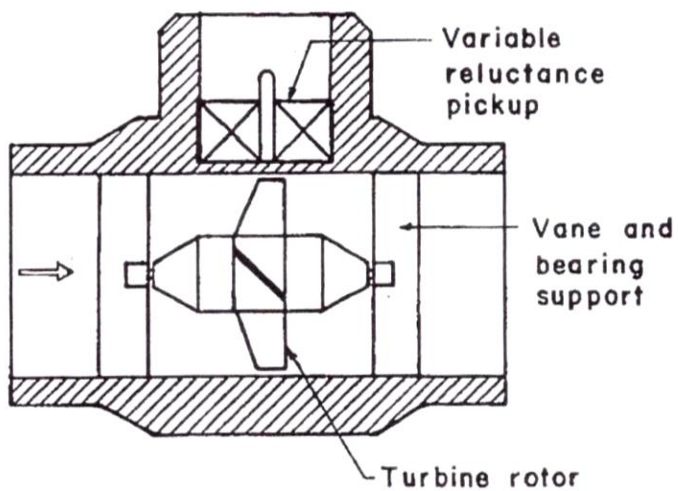
< Venturi meter >

$$P_1 - P_2 = \frac{\rho}{2}\left({v_2}^2 - {v_1}^2\right)$$

6.11. Turbine Flowmeters

- **Advantages of turbine flow meter**

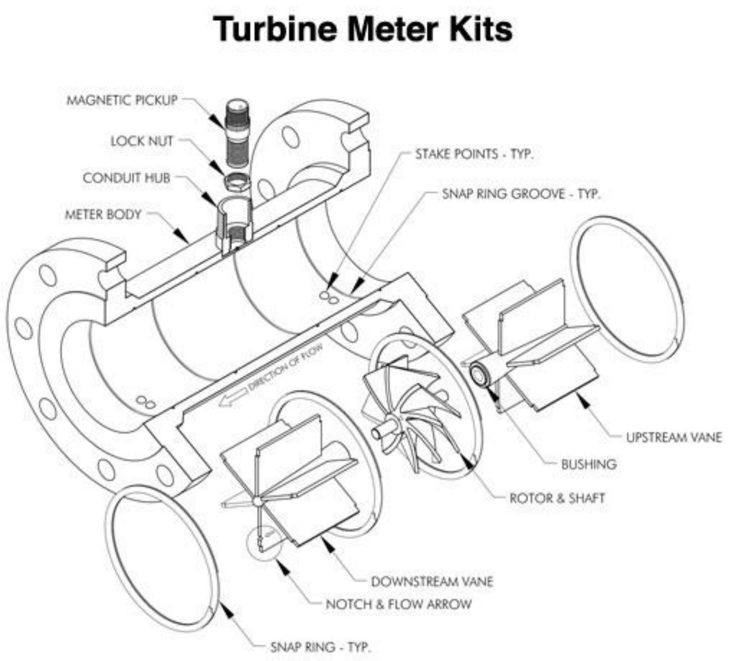
  - Simple, durable structure
  - Easy to install and maintain
  - Turbine meters are able to operate under a wide range of temperatures and pressures
  - Low pressure drop across the flow meter
  - Most effective in applications with steady, high-speed flows

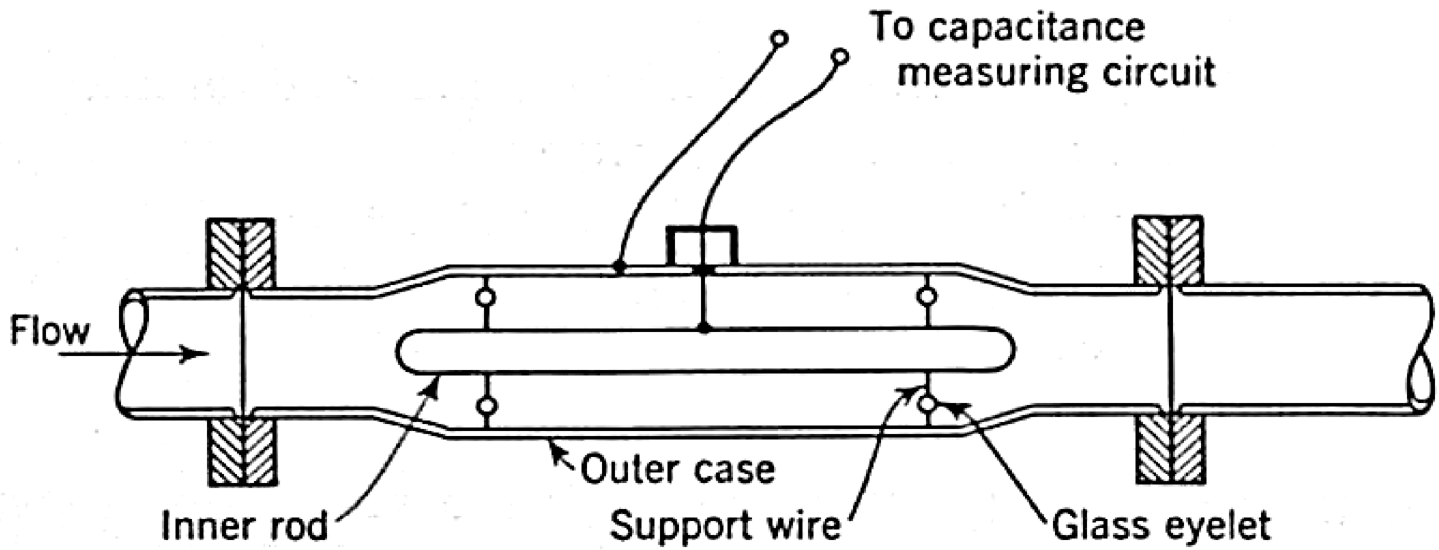


< Turbine Flowmeter >

- **Disadvantages of turbine flow meter**

  - Require constant backpressure in order to avoid cavitation
  - Accuracy adversely affected by bubbles in liquids
  - Sensitive to changes in fluid viscosity





< Fluid-Quality Measurement >

6.13. Hydrostatic Gauges

- **Hydrostatic pressure is related to the liquid level L_f by**

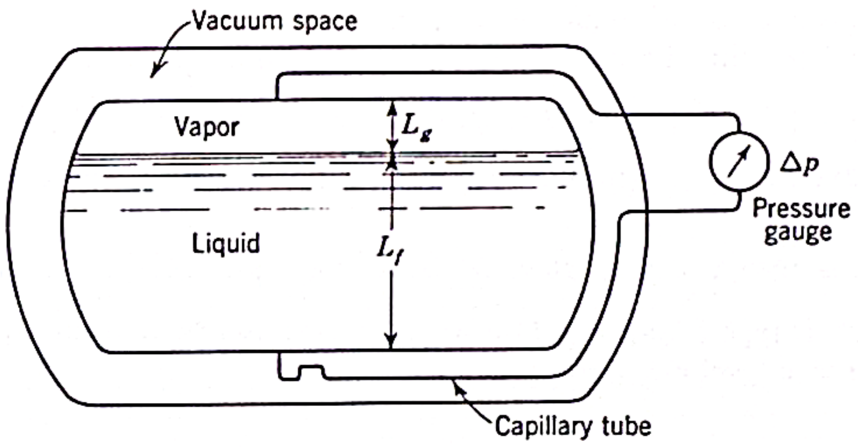


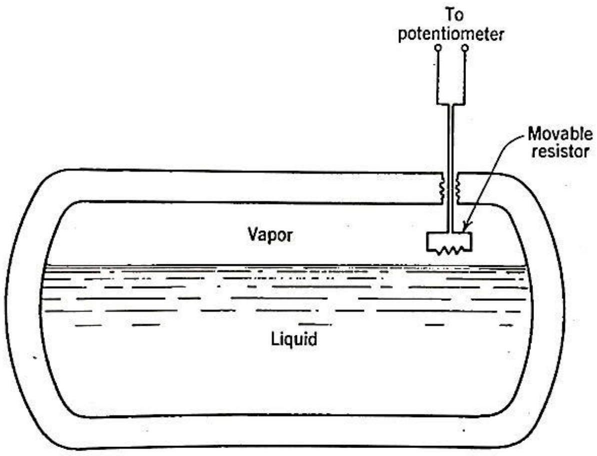
Fig. 6.13. Hydrostatic liquid-level gauge.

$$\Delta P = \frac{\rho_f L_f g}{g_c} + \frac{\rho_g L_g g}{g_c}$$

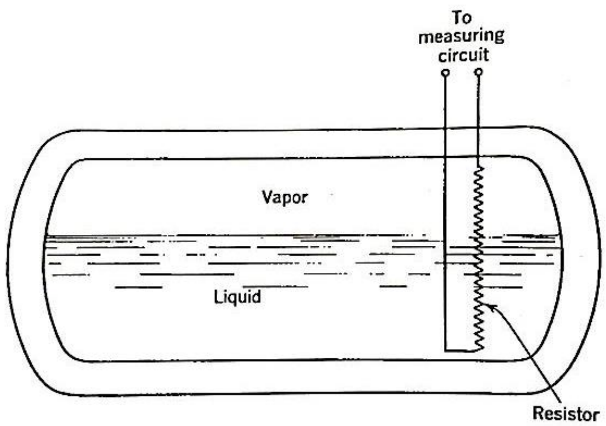
L_f = height of the liquid column
L_g= L-Lf = height of the vapor column
L = inside diameter of the vessel
ρ_f = liquid density
ρ_g = liquid density(saturated)
g = local acceleration due to gravity
g_c = conversion factor in Newton's Second Law of Motion

6.14. Electric-Resistance Gauges

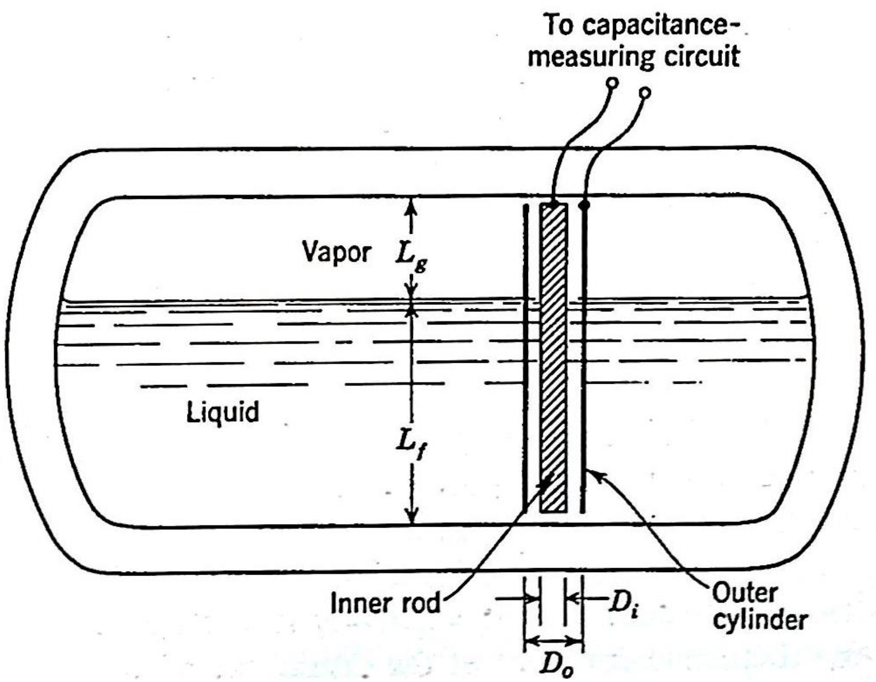
- **Movable electric resistance liquid-level gauge**



- **Fixed electric resistance liquid-level gauge**



6.15. Capacitance Liquid-Level Probes



< Capacitance liquid-level gauge >

$$L_f = \frac{C \ln\left(\frac{D_0}{D_i}\right)}{2\pi(\varepsilon_f - \varepsilon_g)\varepsilon_0} - \frac{\varepsilon_g L}{\varepsilon_f - \varepsilon_g}$$

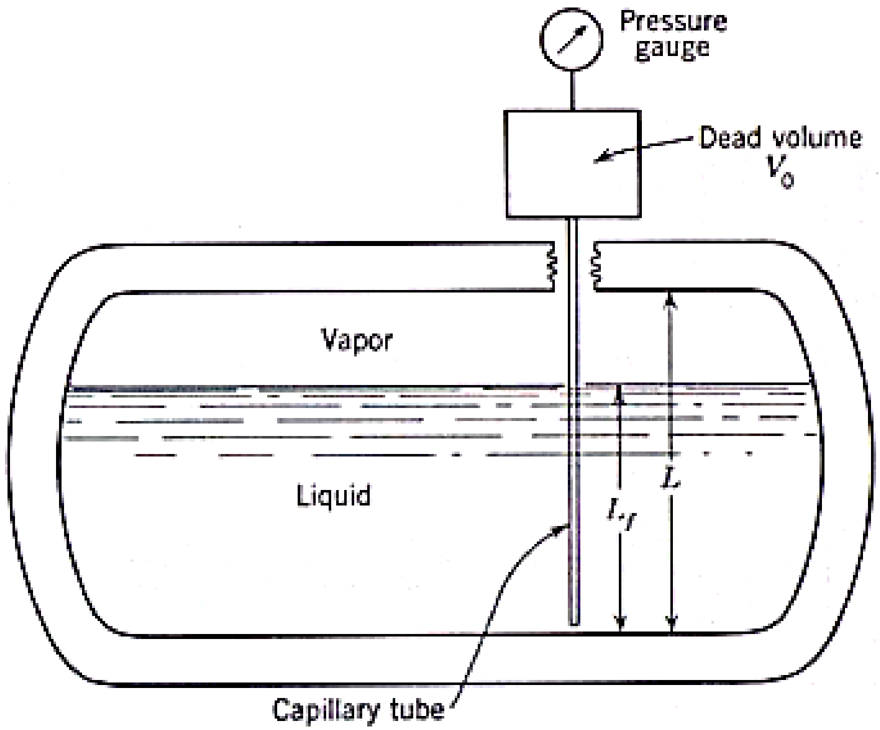
$\varepsilon_0 = 8.8542 * 10^{-12} \frac{F}{M}$ = permittibity of free space

ε_g = The dielectric constants for the vapor

ε_f = The dielectric constants for the fluid

C = The total capacitance for the gauge

6.16. Thermodynamic Liquid-Level Gauge



< Thermodynamic liquid-level gauge >

$$\frac{L_f}{L} = \frac{[(m - \rho_0 V_0)/A_c L] - \rho_g}{\rho_f - \rho_g}$$