Classical Second Law of Thermodynamics (2)

(Lecture 7)

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열역학 **(M2794.001100.002)**

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(*) Some texts and figures are borrowed from Sonntag & Borgnakke unless noted otherwise.

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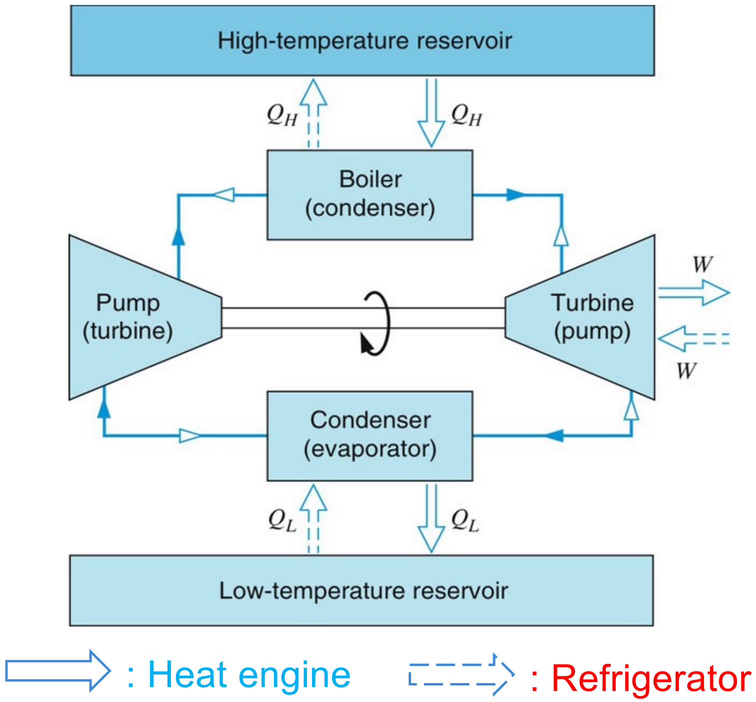
5.5 The Carnot Cycle

If the efficiency of all heat engines is less than 100%, what is the most efficient cycle we can have?

➔ Carnot cycle is the most efficient, reversible cycle that can operate between two constant-temperature reservoirs.



(Heat engine that operates on Carnot cycle)

1 : Boiler, Reversible isothermal process

(Q_H is transferred to the system)

2 : Turbine, Reversible adiabatic process

(W output, Working Fluid : $T_H \rightarrow T_L$)

3 : Condenser, Reversible isothermal process

(Q_L is rejected from the system)

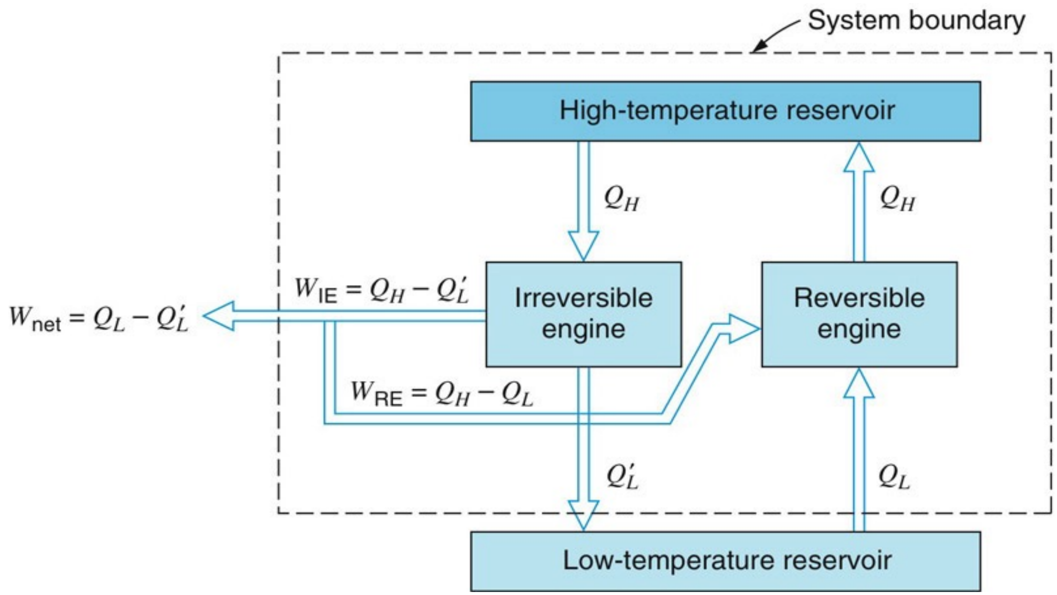
4 : Pump, Reversible adiabatic process

(W input, Working Fluid : $T_L \rightarrow T_H$)

5.6 Two Propositions Regarding the Efficiency of a Carnot Cycle

- First Proposition
  "It is impossible to construct an engine that operates between two given reservoirs and is more efficient than a reversible engine (or Carnot engine) operating between the same two reservoirs."

$$\eta_{irr} < \eta_{rev}$$



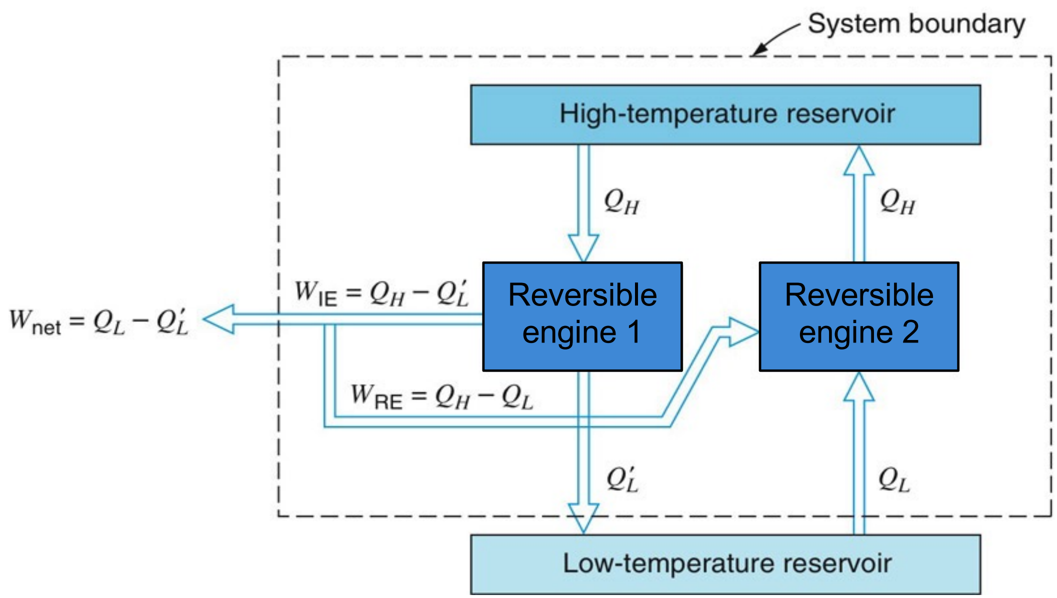
$$if \quad \eta_{irr} > \eta_{rev},$$

$$Q_L' < Q_L$$

Classical Second Law of Thermodynamics

- Second Proposition
  "All engines that operate on the Carnot cycle between two given reservoirs have the same efficiency, independent of working substance."

$$\eta_{rev} = f(T_H, T_L)$$



5.7 The Thermodynamic Temperature Scale

- From the equality of the efficiencies of Carnot cycles,

$$\eta_{rev} = f(T_H, T_L)$$

Here, the efficiency of the heat engine using a Carnot cycle is given as,

$$\eta_{\text{thermal}} = 1 - \frac{Q_L}{Q_H} = 1 - \psi(T_L, T_H)$$

- Define the thermodynamic scale of the absolute temperature as,

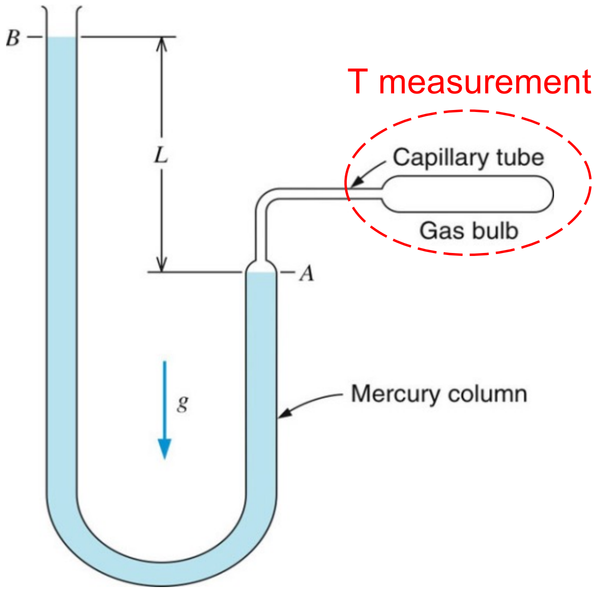
$$\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$$

Then, this leads to

$$\eta_{\text{thermal}} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H} \quad \text{(for Carnot cycle only)}$$

5.8 The Ideal-Gas Temperature Scale

- Use of constant-volume gas thermometer of an ideal gas:
  - Let the gas bulb be placed in the T measurement location.
  - Let the mercury column be adjusted so that the level of mercury stands at the reference mark A.
  - Read the height *L* to figure out pressure, thus leading to temperature.



Constant-volume gas thermometer

$$Pv = RT$$

when $v = \text{const}$,

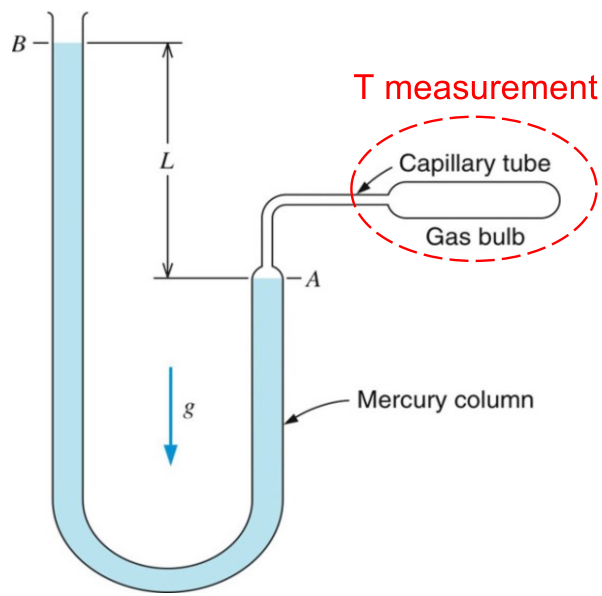
$$\frac{T}{T_{t,P}} = \frac{P}{P_{t,P}} \Rightarrow T = 273.16\left(\frac{P}{P_{t,P}}\right)$$

(where $T_{t,P}$: triple point temperature of water = 273.16 K

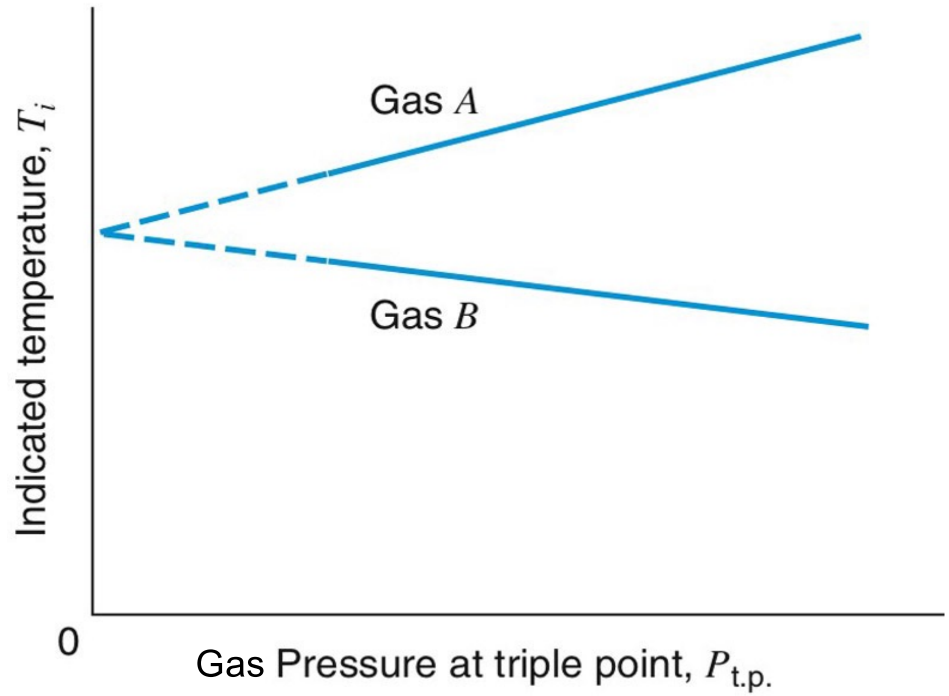
$P_{t,P}$: gas pressure at 273.16 K, NOT triple point pressure of water)

Used as the reference point of the ideal-gas temperature scale

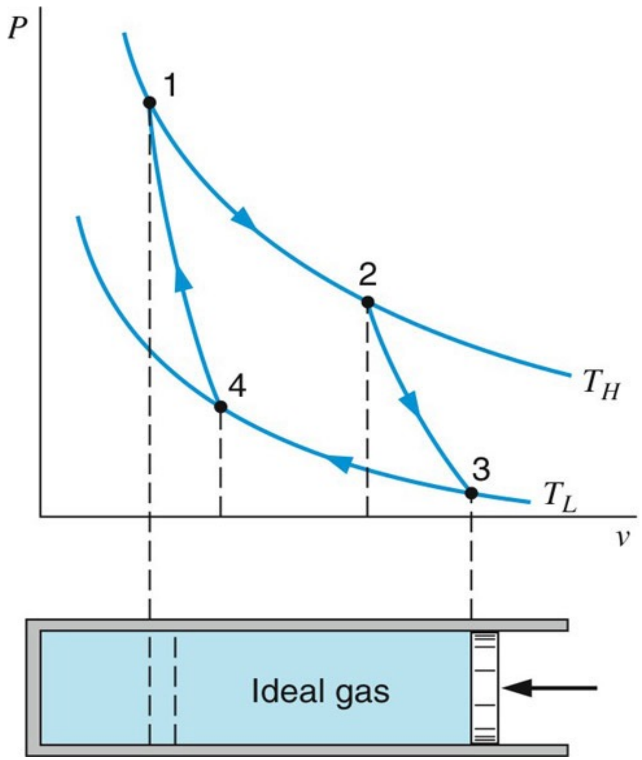
- To evaluate the ideal-gas temperature by using a real gas,
  - Conduct a series of the same T measurement at different pressures.
  - The ideal-gas temperature can be calculated by extrapolation of the curve to zero pressure.



Constant-volume gas thermometer



- Let's demonstrate that the ideal-gas temperature scale (from ideal gas EOS) is identical to the thermodynamic temperature scale (from Carnot cycle and the second law).



Carnot cycle in P-v diagram

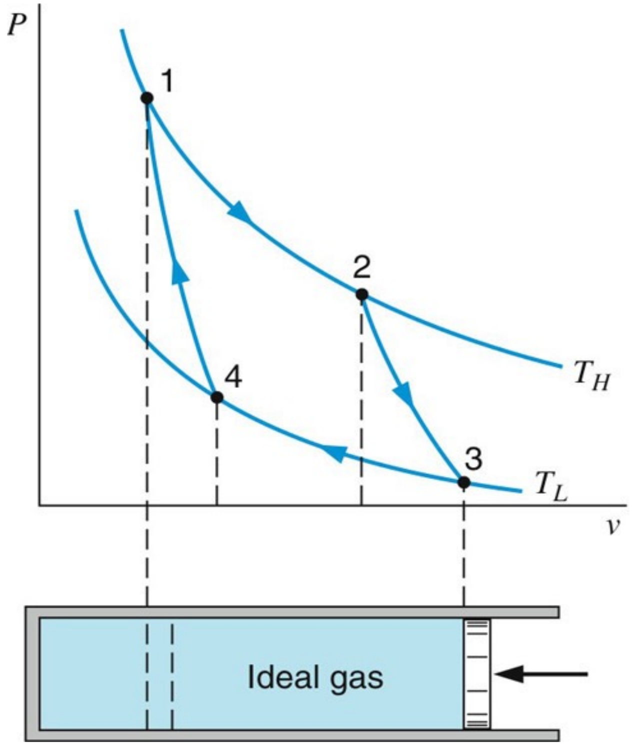
1st law for ideal gas :

Ideal gas T

$$\delta q = du + \delta w = C_{v0} dT + \left(\frac{RT}{v} \right) dv$$

$$\because \delta w = P dv = \left(\frac{RT}{v} \right) dv, \;\; du = C_{v0} dT$$

Classical Second Law of Thermodynamics



$$\delta q = C_{v0}dT + \left(\frac{RT}{v}\right)dv$$

①→②: Reversible isothermal heat addition

$$\int_1^2 \delta q = \int_1^2 C_{v0}dT + \int_1^2 \left(\frac{RT}{v}\right)dv \rightarrow q_H = q_{12} = RT_H \ln\frac{v_2}{v_1}$$

②→③: Reversible adiabatic expansion

$$\int_2^3 \frac{\delta q}{T} = \int_2^3 \frac{C_{v0}}{T}dT + \int_2^3 \left(\frac{R}{v}\right)dv \rightarrow 0 = \int_2^3 \frac{C_{v0}}{T}dT + R\ln\frac{v_3}{v_2}$$

③→④: Reversible isothermal heat rejection

$$\int_3^4 \delta q = \int_3^4 C_{v0}dT + \int_3^4 \left(\frac{RT}{v}\right)dv \rightarrow -q_L = q_{34} = RT_L \ln\frac{v_4}{v_3}$$

④→①: Reversible adiabatic compression

$$\int_4^1 \frac{\delta q}{T} = \int_4^1 \frac{C_{v0}}{T}dT + \int_4^1 \left(\frac{R}{v}\right)dv \rightarrow 0 = \int_4^1 \frac{C_{v0}}{T}dT + R\ln\frac{v_1}{v_4}$$

Classical Second Law of Thermodynamics

➔ Here,

$$0 = \int_2^3 \frac{C_{v0}}{T} dT + R\ln\frac{v_3}{v_2} \rightarrow \int_{T_H}^{T_L} \frac{C_{v0}}{T} dT = -R\ln\frac{v_3}{v_2} \rightarrow \int_{T_L}^{T_H} \frac{C_{v0}}{T} dT = R\ln\frac{v_3}{v_2}$$

$$0 = \int_4^1 \frac{C_{v0}}{T} dT + R\ln\frac{v_1}{v_4} \rightarrow \int_{T_L}^{T_H} \frac{C_{v0}}{T} dT = -R\ln\frac{v_1}{v_4} \rightarrow \int_{T_L}^{T_H} \frac{C_{v0}}{T} dT = R\ln\frac{v_4}{v_1}$$

➔ Thus, $\frac{v_3}{v_2} = \frac{v_4}{v_1}$ or $\frac{v_3}{v_4} = \frac{v_2}{v_1}$

➔ Finally,

$$\frac{q_H}{q_L} = \frac{RT_H \ln\frac{v_2}{v_1}}{RT_L \ln\frac{v_3}{v_4}} = \frac{T_H}{T_L}$$

Ideal gas T

Same definition as the thermodynamic temperature scale

5.9 Ideal versus Real Machines

- The thermal efficiency or coefficient of performance of a real device is always lower than that of a Carnot cycle system.

$$\eta_{\text{thermal}} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

$$\beta = \frac{Q_L}{Q_H - Q_L} \underset{\text{Carnot}}{=} \frac{T_L}{T_H - T_L}$$

(Carnot Cycle)

$$\beta' = \frac{Q_H}{Q_H - Q_L} \underset{\text{Carnot}}{=} \frac{T_H}{T_H - T_L}$$

$$\eta_{\text{real thermal}} = 1 - \frac{Q_L}{Q_H} \leq 1 - \frac{T_L}{T_H}$$

$$\beta_{\text{real}} = \frac{Q_L}{Q_H - Q_L} \leq \frac{T_L}{T_H - T_L}$$

(Real Device)

$$\beta'_{\text{real}} = \frac{Q_H}{Q_H - Q_L} \leq \frac{T_H}{T_H - T_L}$$