445.204

Introduction to Mechanics of Materials

(재료역학개론)

Chapter 8: Section forces in beam

(Ch. 10 in Shames)

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Contents

- Section forces, axial forces, bending moment
- Direct formulations
- Differential relationship among bending moment, shear force and load
- Shear force and bending moment diagrams

Shear force, axial force, and bending moment in beam

- Beam – Thin prismatic (slender) members loaded transversely (or perpendicular to the centerline)
- In the beam problems, the major concern is the components of the resultant force system from the applied loads acting on cross sections of the beam.

Shear force, axial force, and bending moment in beam

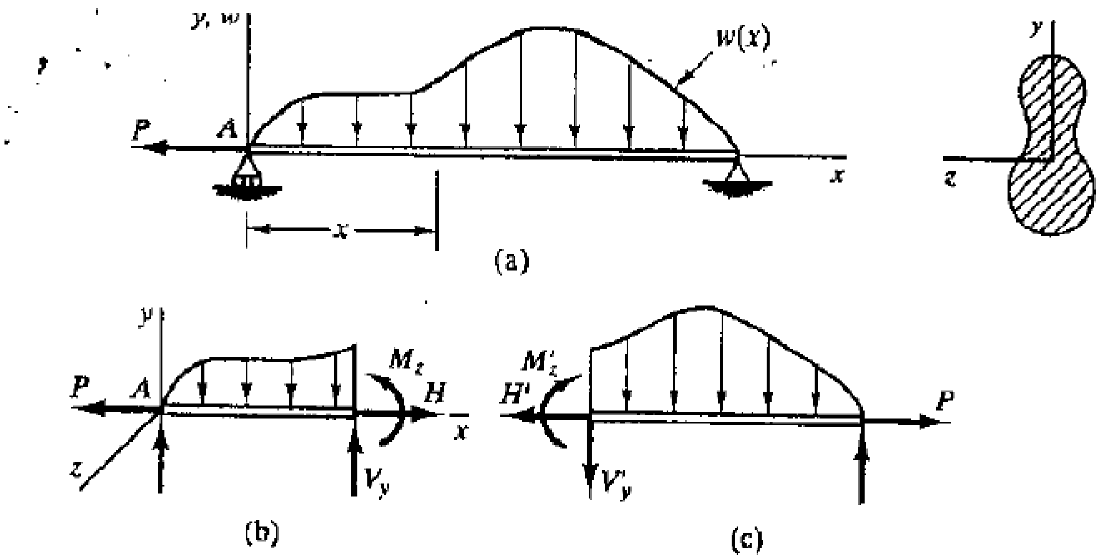


Figure 10.1. Left- and right-hand free-body diagrams exposing a section of the beam.

V_y	Shear force
M_z	Bending moment
H	Axial force

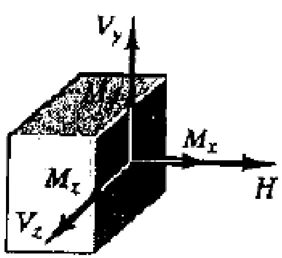
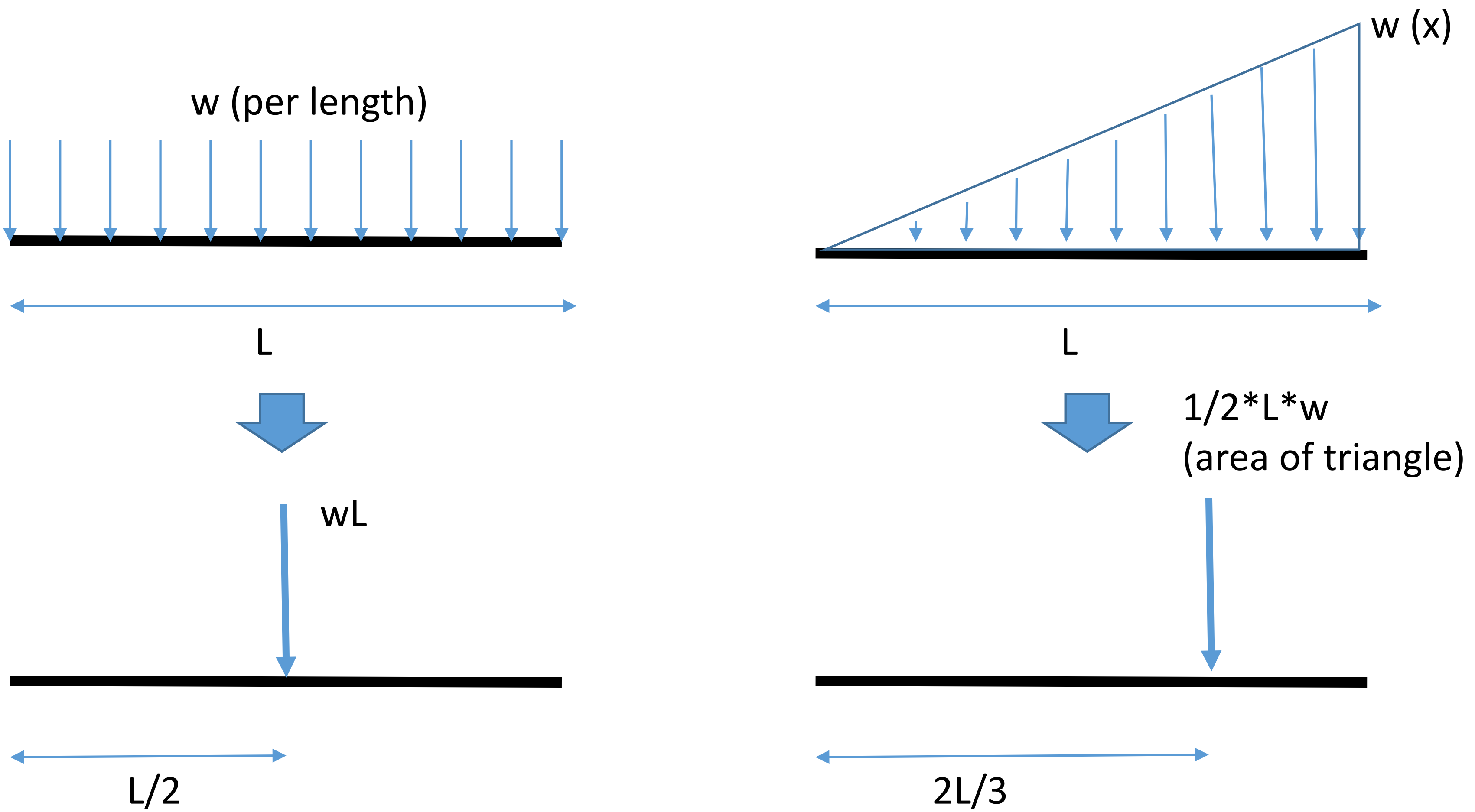


Figure 10.2. Section forces for three-dimensional loading.

TIP: Replacement of distributed load with single point load



Shear force, axial force, and bending moment in beam

Sign conventions

1) Stress sign convention

The axial force, shear force, or bending moment acting on a beam cross-section is positive if it acts on a positive face and is directed in a positive coordinate direction

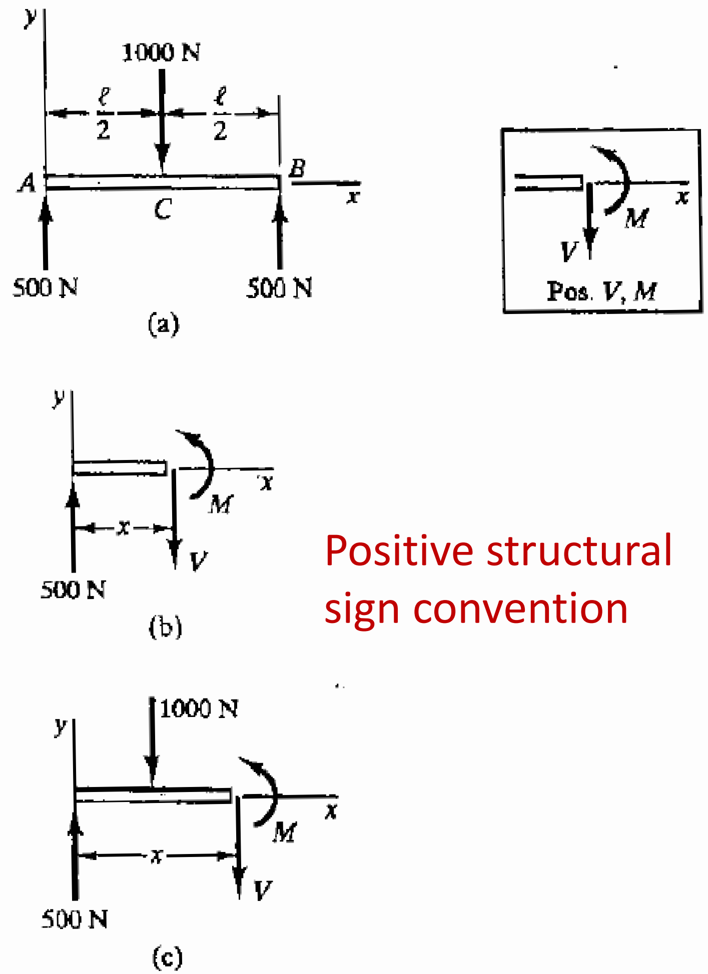
c.f. convention for stress tensor!

Our textbook!

2) Structural sign convention

An axial force or bending moment acting on a beam cross-section is positive if it acts on a positive face and is directed in a positive coordinate direction. The shear force is positive if it acts in the negative coordinate direction on a positive face.

Example 10.1



Positive structural sign convention

Figure 10.3. Simple beam problem requiring two domains.

$$0 < x < \frac{l}{2}$$

$$V = 500\ N$$

$$M = 500x\ Nm$$

$$\frac{l}{2} < x < l$$

$$V = -500\ N$$

$$M = 500(l - x)\ Nm$$

Note: specify limits of the domain to exclude discontinuity in either V or M.

Example 10.2

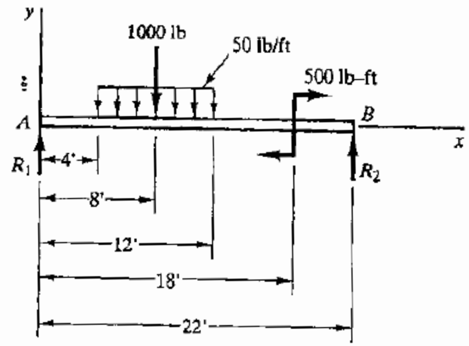


Figure 10.4. Simply supported beam.

By summing to zero moments about each end of the beam, we readily can find from statics

$$R_1 = 868 \text{ lb} \qquad R_2 = 532 \text{ lb}$$

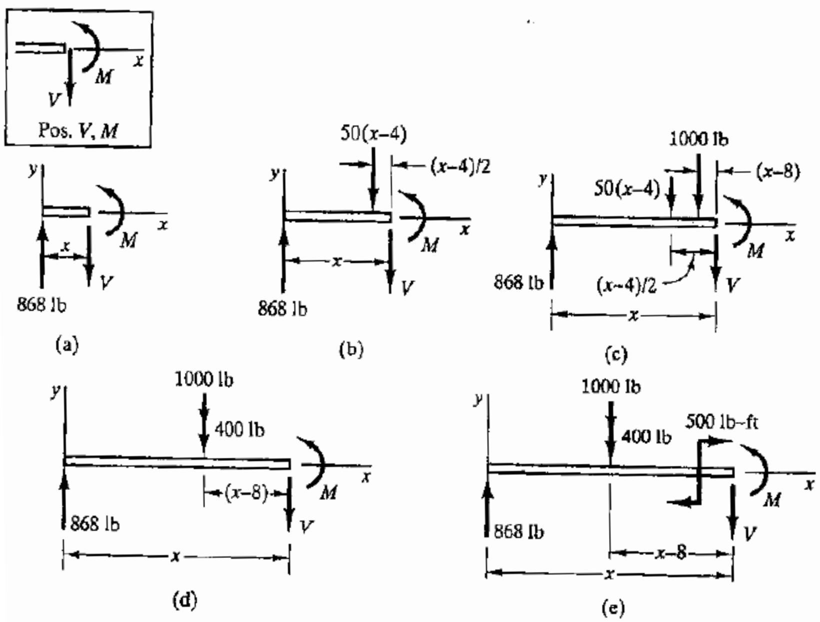


Figure 10.5. Free-body diagram for different domains.

$$0 < x \le 4$$

$$V = 868\, lb$$

$$M = 868x \; ft \cdot lb$$

$$4 \le x < 8$$

$$V = -50x + 1068\, lb$$

$$M = -25x^2 + 1068x - 400 \; ft \cdot lb$$

$$8 < x \le 12$$

$$V = -50x + 1068\, lb$$

$$M = -25x^2 + 68x + 7600 \; ft \cdot lb$$

$$12 \le x < 18$$

$$V = -532\, lb$$

$$M = -532x + 11200 \; ft \cdot lb$$

$$18 < x < 22$$

$$V = -532\, lb$$

$$M = -532x + 11700 \; ft \cdot lb$$

Direct Formulations of Shear-Force and Bending-Moment Equations

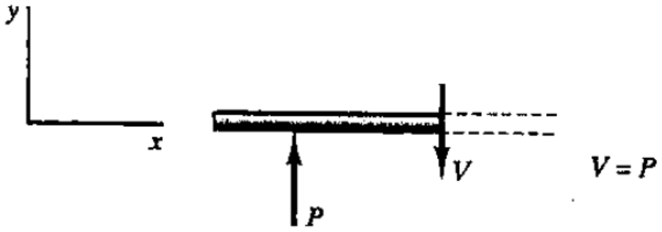
Objective

One can write the shear-force and bending moment equations in a more direct manner without drawing the simple but bothersome free-body diagrams

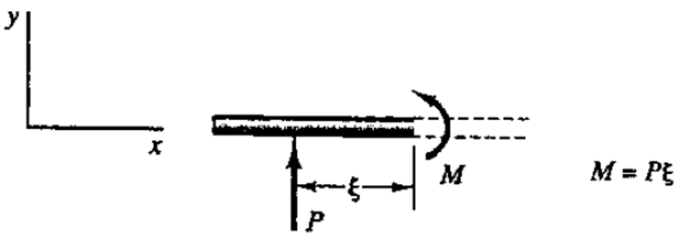
First, Do NOT adhering to the Sign Conventions!!

Only think about Equilibrium!!

Then, check the Sign!



(a)



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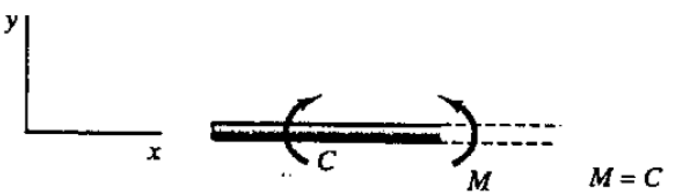

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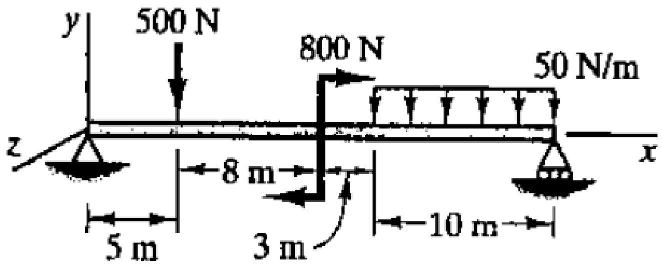
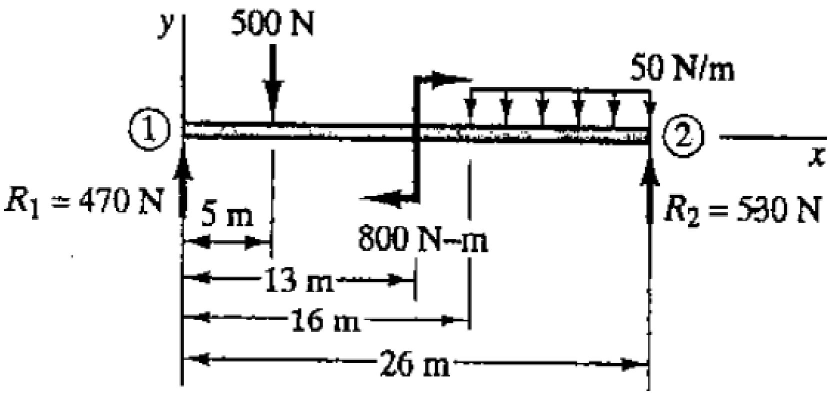
Direct Formulations of Shear-Force and Bending-Moment Equations – Example 10.4



(a)



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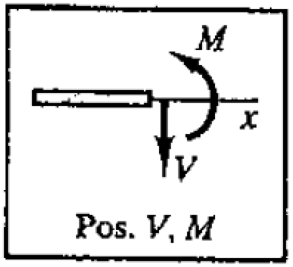

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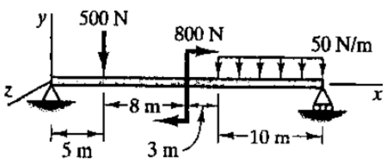
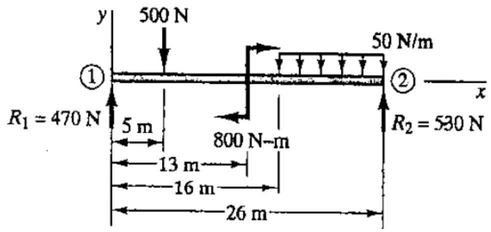
Direct Formulations of Shear-Force and Bending-Moment Equations – Example 10.4

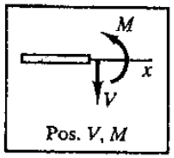


(a)



(b)

First step: derive supporting forces R1 and R2 first using the FBD



(c)

We will however use the free-body diagram of the entire beam [Fig. 10.10(b)] to evaluate the supporting forces R_1 and R_2. Equating moments to zero at the ends of the beam, we get, using the right-hand rule of statics,

$$R_1 = 470\text{ N} \qquad R_2 = 530\text{ N}$$

We may now directly give the shear-force V and bending-moment M while viewing Fig. 10.10(b). Thus:

$\underline{0 < x < 5:}$

$$V = R_1 = 470\text{ N}$$
$$M = 470x\text{ N-m}$$

$\underline{5 < x < 13:}$

$$V = 470 - 500 = -30\text{ N}$$
$$M = 470x - 500(x-5)\text{ N-m}$$

$\underline{13 < x \leq 16:}$

$$V = -30\text{ N (same as previous interval)}$$
$$M = 470x - 500(x-5) + 800\text{ N-m}$$

$\underline{16 \leq x < 26:}$

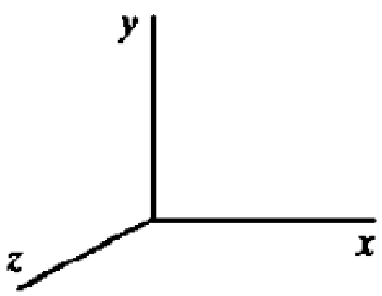
$$V = 470 - 500 - 50(x-16) = -30 - 50(x-16)\text{ N}$$
$$M = 470x - 500(x-5) + 800 - \frac{50(x-16)^2}{2}\text{ N-m}$$

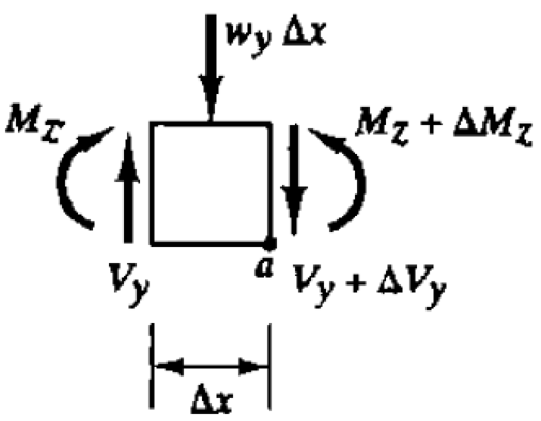
Second step: use direct formulations

Differential Relations for Bending Moment, Shear Force and Load

Objective

Understand differential relationships between the applied load, shear force, and bending moment and their integrals, which lead to the rapid sketch shear-force and bending-moment diagrams for simple but common loads, allowing for the quick assessment of critical values of moment and shear force along the beam.

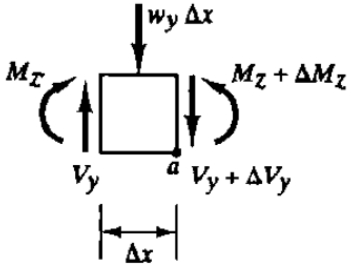




Structural convention

(c.f. other conventions can be also used)

Differential Relations for Bending Moment, Shear Force and Load



Force and moment equilibrium

$$\sum F_y = 0 \qquad V_y - \left(V_y + \Delta V_y\right) - w_y \Delta x = 0$$

$$\sum M_z = 0 \qquad -M_z - V_y \Delta x + \left(w_y \Delta x\right)\left(\beta \Delta x\right) + \left(M_z + \Delta M_z\right) = 0$$

$$\Delta x \to 0 \qquad \frac{dV_y}{dx} = -w_y \qquad \frac{dM_z}{dx} = V_y \qquad \Rightarrow \qquad \frac{d^2 M_z}{dx^2} = -w_y\left(x\right)$$

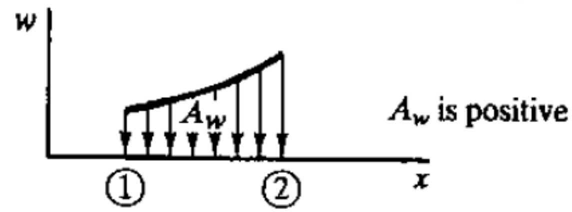
Differential Relations for Bending Moment, Shear Force and Load

Integral relationships

$$\left(V_y\right)_2-\left(V_y\right)_1=-\int_1^2 w_y\,dx \qquad \left(V_y\right)_2=\left(V_y\right)_1-\int_1^2 w_y\,dx$$

$$\left(M_z\right)_2-\left(M_z\right)_1=\int_1^2 V_y\,dx \qquad \left(M_z\right)_2=\left(M_z\right)_1+\int_1^2 V_y\,dx$$

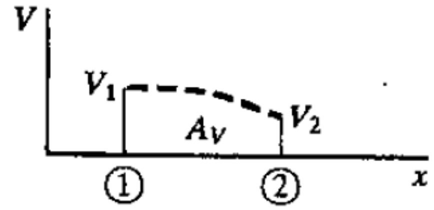
Sketching Shear-Force and Bending-Moment Diagrams



(a)

$$\frac{dV}{dx} = -w$$

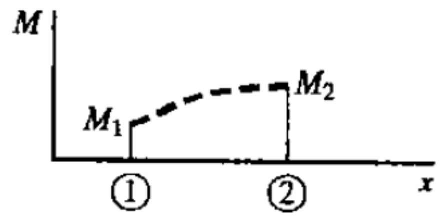
$$V_2 = V_1 - \int_1^2 w\,dx$$



(b)

$$\frac{dM}{dx} = +V$$

$$M_2 = M_1 + \int_1^2 V\,dx$$

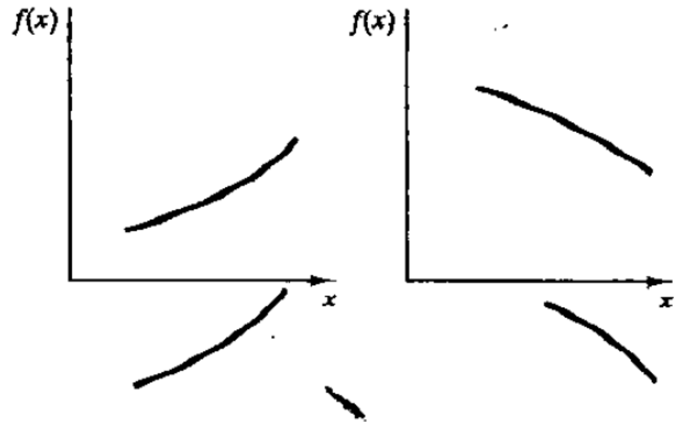


(c)

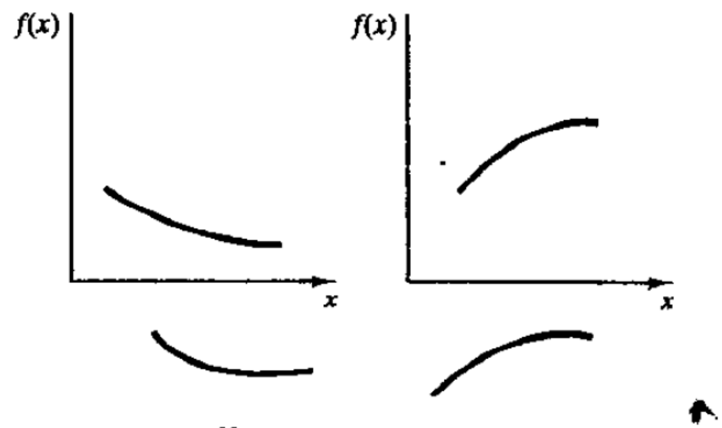
Sketching Shear-Force and Bending-Moment Diagrams

$$\left|\frac{dV_y}{dx}\right| = \left|-w_y\right|$$

- The magnitude of loading w_y is constant, then the magnitude of the slope of the shear force curve is constant; the curve is straight line.
- The magnitude of loading is increasing, then the magnitude of the slope of the shear force is increasing; the curve is steepening.
- The magnitude of the loading is decreasing, then the magnitude of the slope of the shear force curve is decreasing; the curve is flattening.



(a). $\left|\frac{df}{dx}\right|$ is *increasing*; slope is *steepening*

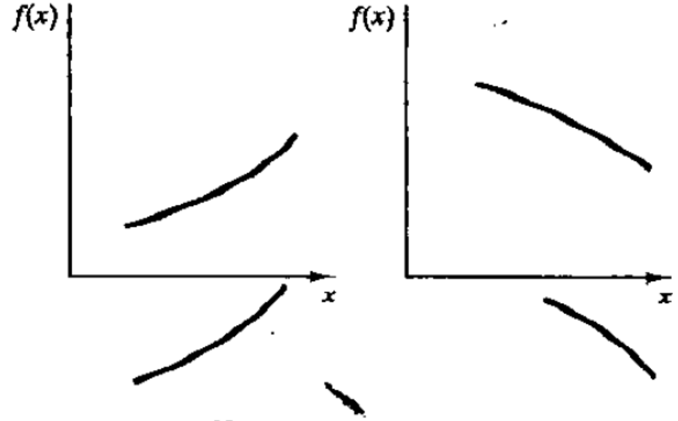


(b). $\left|\frac{df}{dx}\right|$ is *decreasing*; slope is *flattening*

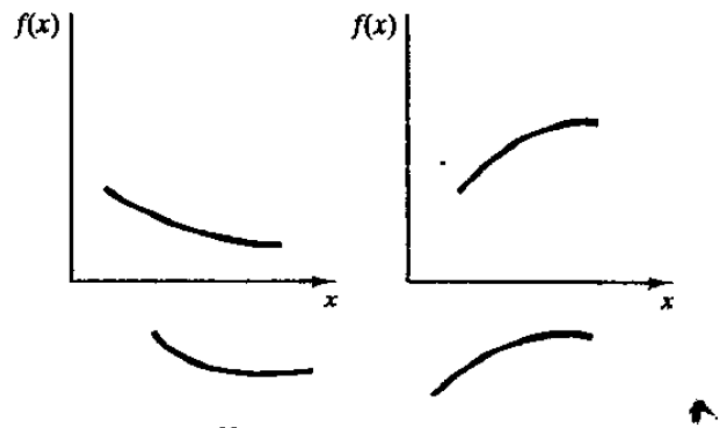
Sketching Shear-Force and Bending-Moment Diagrams

$$\left|\frac{dM_z}{dx}\right| = \left|V_y\right|$$

- The magnitude of shear force V_y is constant, then the magnitude of the slope of the moment force curve is constant; the curve is straight line.
- The magnitude of shear force is increasing, then the magnitude of the slope of the moment is increasing; the curve is steepening.
- The magnitude of the shear force is decreasing, then the magnitude of the slope of the shear moment is decreasing; the curve is flattening.



(a). $\left|\frac{df}{dx}\right|$ is *increasing*; slope is *steepening*



(b). $\left|\frac{df}{dx}\right|$ is *decreasing*; slope is *flattening*

Sketching Shear-Force and Bending-Moment Diagrams

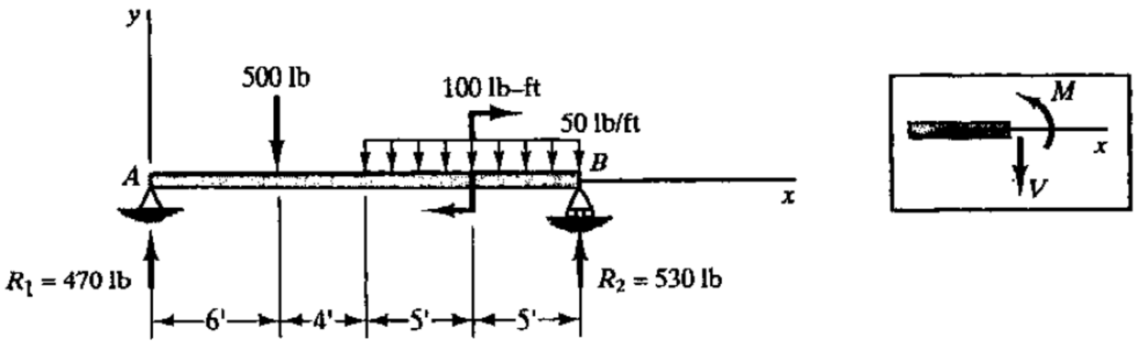


Figure 10.16. Simply supported beam.

$0 < x < 6$:

$$V = R_1 = 470 \text{ lb}$$
$$M = R_1 x = 470x \text{ ft-lb}$$

$6 < x \le 10$:

$$V = 470 - 500 = -30 \text{ lb}$$
$$M = 470x - 500(x - 6) \text{ ft-lb}$$

$10 \le x < 15$:

$$V = 470 - 500 - 50(x - 10) \text{ lb}$$
$$M = 470x - 500(x - 6) - 50(x - 10)^2 / 2 \text{ ft-lb}$$

$15 < x < 20$:

$$V = 470 - 500 - 50(x - 10) \text{ lb}$$
$$M = 470x - 500(x - 6) - 50(x - 10)^2 / 2 + 100 \text{ ft-lb}$$

Sketching Shear-Force and Bending-Moment Diagrams

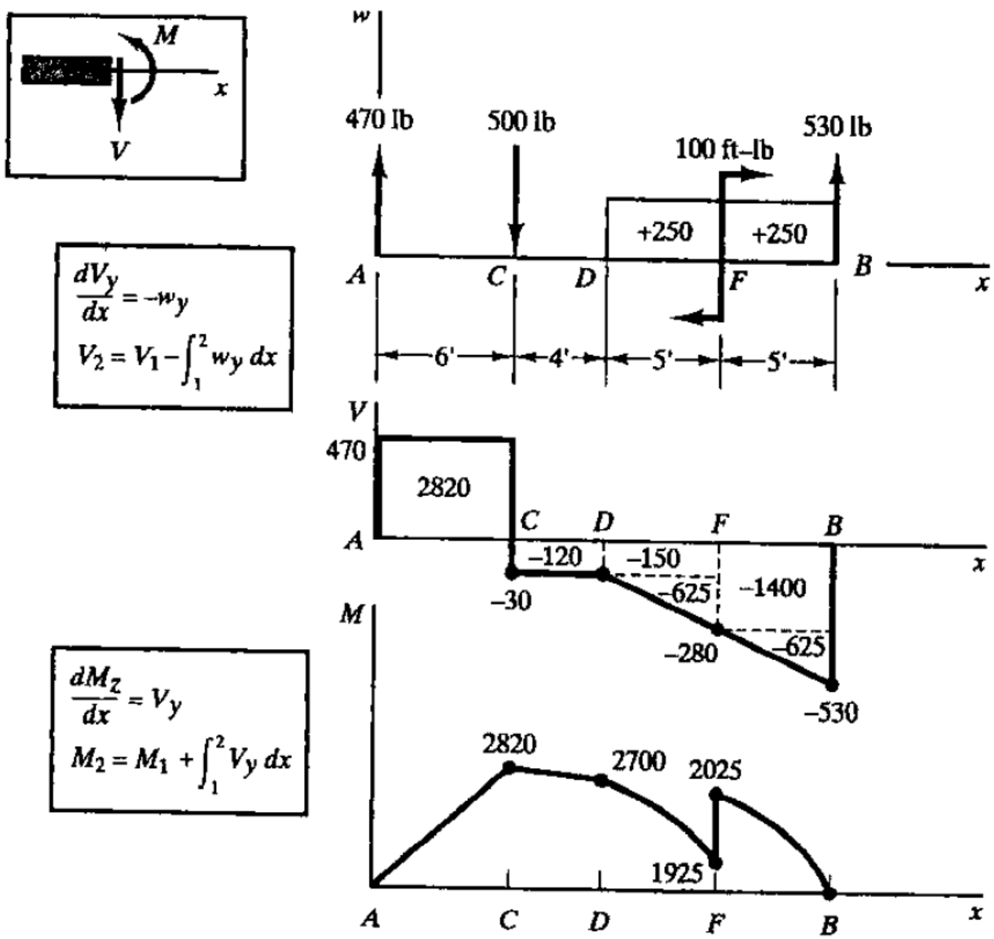
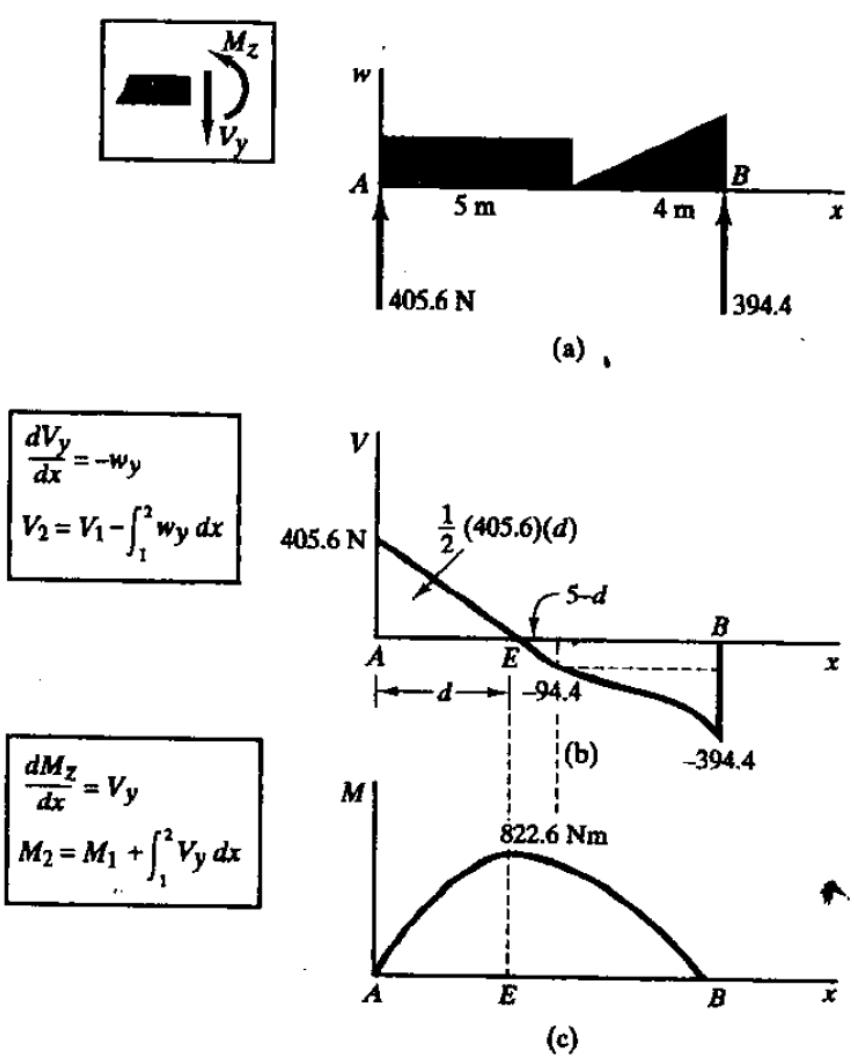


Figure 10.17. Shear and bending moment diagrams.

Find possible max. moment



Homework (Try it by yourself!)

Example 10.7

- Combine the solutions by equation and drawing !