Lecture Note 7: Band Pass Filters (BPF)

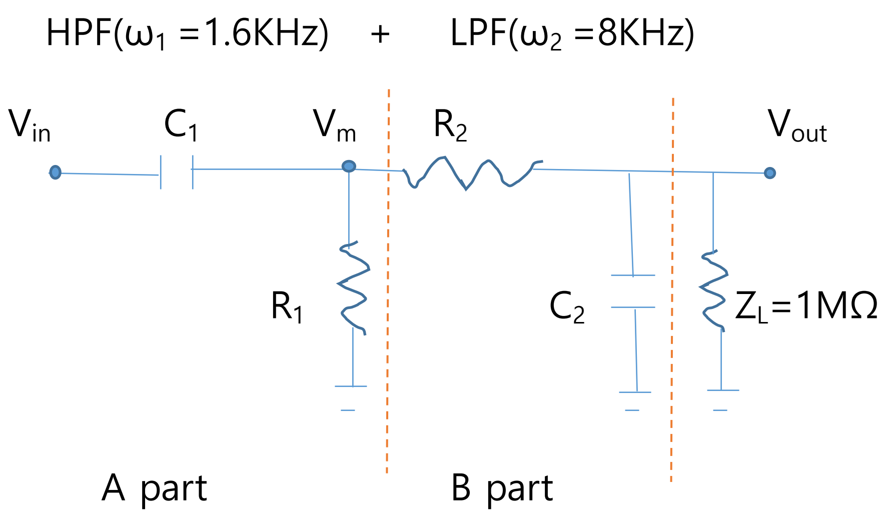
:To filter out unwanted noise from the signal in bandwidth, or

To pass the signal between the target bandwidths of frequency

The band pass filter can be implemented as the serial connection of the HPF and LPF, such as HPF+LPF (or LPF+HPF).

Ex) Design a band pass filter to pass signals between 1.6KHZ and 8KHz. This filter is to drive a device of 1MΩ input impedance.

Logically, a HPF passing signal over 1.6KHz($=\omega_{3dB}=\omega_1$) is serially connected to a LPF passing signal under 8KHz($=\omega_{3dB}=\omega_2$), then the band pass filtering can be implemented based on HPF+LPF as follows;



For HPF part, V_m/V_{in} is the transfer function, thus

$V_m/V_{in}=ZR_1/(ZC_1+ZR_1)=R_1/(1/j\omega C_1+R_1)=j\omega R_1C_1/(1+j\omega R_1C_1)$

For the LPF part, transfer function is V_{out}/V_m, thus

$V_{out}/V_m=ZC_2/(ZR_2+ZC_2)=(1/j\omega C_2)/(R_2+1/j\omega C_2)=1/(1+j\omega R_2C_2)$

Therefore the total transfer function, $H=V_{out}/V_{in}$ and

$H=V_{out}/V_{in}=(V_m/V_{in})\ (V_{out}/V_m)=j\omega R_1C_1/\{(1+j\omega R_1C_1)(1+j\omega R_2C_2)\}$

Magnitude$=|H|=\omega R_1C_1/\sqrt{[\{1+(\omega R_1C_1)^2\}\{1+(\omega R_2C_2)^2\}]}$

where $\omega_1=\omega_{3dBH}=1/R_1C_1=(1600)(2\pi)$, $\omega_2=\omega_{3dBL}=1/R_2C_2=(8000)(2\pi)$

$R_2C_2/R_1C_1=\omega_1/\omega_2=1/5$, $R_1C_1/R_2C_2=\omega_2/\omega_1=5$

Phase$=\angle H=\ \angle(j\omega R_1C_1)\ -\ \angle(1+j\omega R_1C_1)\ -\ \angle(1+j\omega R_2C_2)$

$=90^\circ-\tan^{-1}(\omega R_1C_1)-\tan^{-1}(\omega R_2C_2)$

Let's plot the magnitude and phase in the ω domain.

If $\omega<1/R_1C_1$ then $|H|\fallingdotseq 0$ and $\angle H\fallingdotseq 90^\circ$

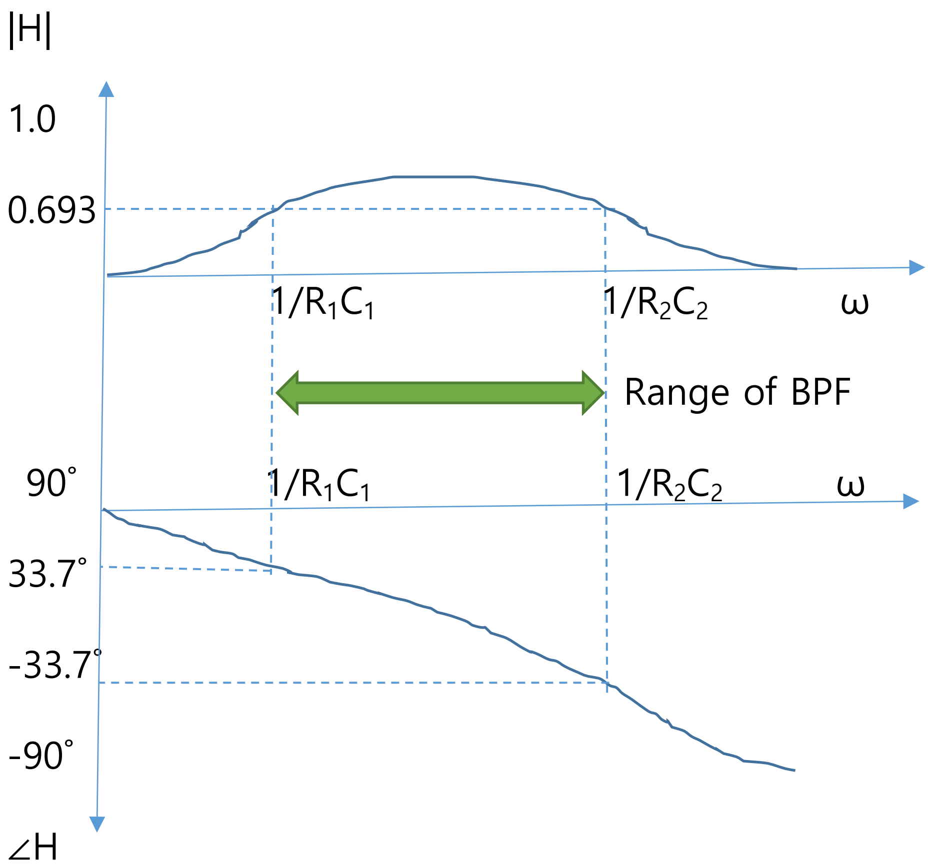
If $\omega=1/R_1C_1$ then $|H|=1/\sqrt{[2\{1+(R_2C_2/R_1C_1)^2\}]}=0.693$ and

$\angle H=90-45-\tan^{-1}(0.2)=33.7^\circ$

If $\omega=1/R_2C_2$; then $|H|=5/\sqrt{[1+(R_1C_1/R_2C_2)^2\}(2)]}=0.693$ and

$\angle H=90-45-\tan^{-1}(5)=-33.7^\circ$

If $\omega>1/R_2C_2$ then $|H|\fallingdotseq 0$ and $\angle H\fallingdotseq -90^\circ$



If $\omega < 1/R_1C_1$ then $|H| \fallingdotseq 0$ and $\angle H \fallingdotseq 90^\circ$

If $1/R_1C_1 < \omega < 1/R_2C_2$ then $0.693 < |H| < 1$ and $|\angle H| < 33.7^\circ$

If $\omega > 1/R_2C_2$ then $|H| \fallingdotseq 0$ and $\angle H \fallingdotseq -90^\circ$

Thus it is the Band Pass Filter (BPF) to pass signal between the band.

The design procedures can be similarly applied to the BPF

5 Design Procedures or 5 Steps for BPF

1) Frequency identification of signal

   Measured signal can be analysed by oscilloscope or frequency spectrum analysis. Then identify f_{signal} f_1 f_2

2) Choose $f_{3dBH,}$ f_{3dBL}

   You can assign f_{3dBH} as f_1 , f_{3dBL} as f_2

3) Choose R_1,C_1,R_2,C_2 such that $1/R_1C_1=\omega_1 =2\pi f_1$

   $1/R_2C_2=\omega_2=2\pi f_2$ among commercially available components

4) Use 10X rule for components choosing if necessary

5) Performance Verification by using Signal to Noise ratio, or S/N ratio, etc

Ex) Design the BPF as above.

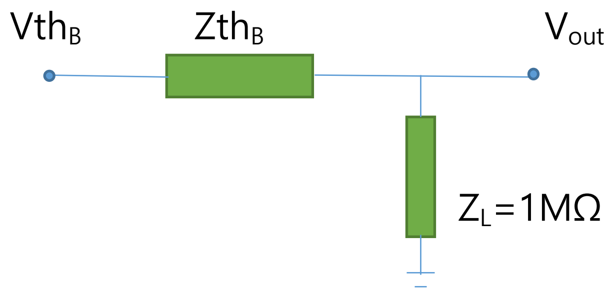
Four unknowns: R_1 C_1 R_2 C_2

$1/R_1C_1=\omega_1=2\pi f_1=2(3.14)(1600)=10{,}048(rad/s)$ eq(1)

$1/R_2C_2=\omega_2=2\pi f_2=2(3.14)(8000)=50{,}240(rad/s)$ eq(2)

We need two more equations to determine.

Applying Thevenin's method to the part B (LPF),



$Zth_B= ZR_2 \parallel ZC_2=(ZR_2ZC_2/(ZR_2+ZC_2)=R_2/j\omega C_2/(R_2+1/j\omega C_2)$

$=R_2/(1+j\omega R_2C_2)$ $\therefore$ $|Zth_B|=R_2/\sqrt{\{1+(\omega R_2C_2)^2\}} \leq R_2$

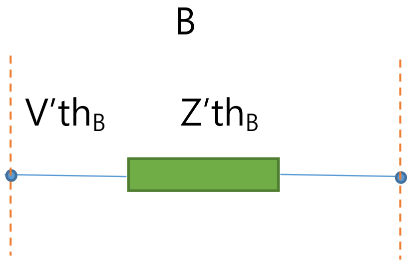
Zth_B should drive Z_L

If $R_2=Z_L/10=100K\Omega$

then $Zth_B \ll Z_L$, where the 10X rule is satisfied.

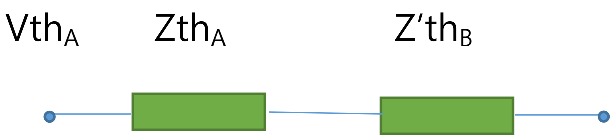
Thus $R_2=100K\Omega$ and $C_2=1.99E-10 \fallingdotseq 0.2nF$ (from commercial availability)

Now the part B can be further simplified by the Thevenin method,



$Z'th_B = Zth_B \parallel Z_L \fallingdotseq Zth_B$ ($\because Zth_B \ll Z_L$)

Now applying to the part A (HPF),



$Zth_A = ZR_1 \parallel ZC_1 = ZR_1 ZC_1/(ZR_1+ZC_1)$

$=R_1/j\omega C_1/(R_1+1/j\omega C_1)=R_1/(1+j\omega R_1C_1)$ and $|Zth_A|=R_1/\sqrt{\{1+(\omega R_1C_1)^2\}} \leq R_1$

Zth_A should drive $Z'th_B$ ($\fallingdotseq Zth_B \leq R_2$)

and if $R_1=R_2/10=10K\Omega$

then $Zth_A \ll Zth_B$ thus the 10X rule is satisfied.

Thus $R_1=10K\Omega$ and $C_1=0.01\mu F$

Verification

For a signal of 5KHz ($\omega=2\pi(5000)=31400$ rad/s);

$\omega R_1C_1=\omega/\omega_1=5/1.6=3.125$ and $\omega R_2C_2=\omega/\omega_2=5/8=0.625$

Thus $|H|= \omega R_1C_1/\sqrt{[\{1+(\omega R_1C_1)^2\}\{1+(\omega R_2C_2)^2\}]}$

$=(3.125)/\sqrt{[\{1+3.125^2\}\{1+0.625^2\}]}=0.808$

$\angle H=90^\circ-\tan^{-1}(\omega R_1C_1)-\tan^{-1}(\omega R_2C_2)$

$=90-72.3-32.0=-14.3^\circ$

For a noise of 50KHz ($\omega=2\pi(50000)=314000$ rad/s)

$\omega R_1C_1=\omega/\omega_1=50/1.6=31.25$ and $\omega R_2C_2=\omega/\omega_2=50/8=6.25$

Thus $|H|= \omega R_1C_1/\sqrt{[\{1+(\omega R_1C_1)^2\}\{1+(\omega R_2C_2)^2\}]}$

$=31.25/\sqrt{[\{1+31.25^2\}\{1+6.25^2\}]}=0.1579$

$\angle H=90^\circ-\tan^{-1}(\omega R_1C_1)-\tan^{-1}(\omega R_2C_2)$

$=90-88.2-80.9=-79.1^\circ$

Thus S/N=1.0 without BPF, and S/N=0.808/0.1579=5.12 with BPF

And phase $=-14.3^\circ$ for 5KHz signal and -79.1° for 50KHz noise.

Thus the BPF designed shows also good performance.

Q: Once a BPF design is completed,

Is it OK to change the sequence of serial connection? Answer is NO.

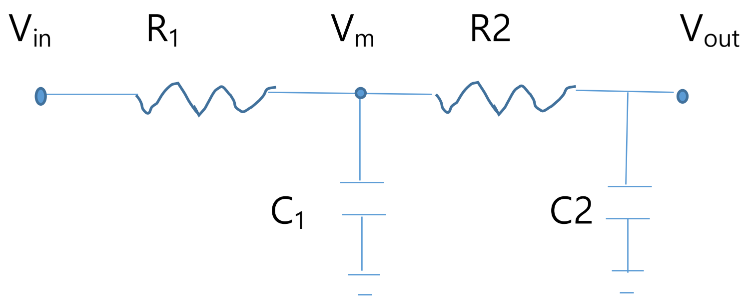
(The sequence of serial connection can be changed, but R,C components should be recalculated due to the 10X rule application.)

Q: What happen if the 10X rule is not satisfied?

A: This would be the case of LPF+HPF, both not satisfying 10X rule.

How then?

Two-stage or Cascaded LPF



$V_m/V_{in}=ZC_1/(ZR_1+ZC_1)=1/j\omega C_1/(R_1+1/j\omega C_1)=1/(1+j\omega R_1C_1)$

$V_{out}/V_m= ZC_2/(ZR_2+ZC_2)=1/j\omega C_2/(R_2+1/j\omega C_2)=1/(1+j\omega R_2C_2)$

$\therefore V_{out}/V_{in}=(V_m/V_{in})(V_{out}/V_m)=1/\{(1+j\omega R_1C_1)(1+j\omega R_2C_2)\}$

Magnitude=$|H|=1/\sqrt{\{(1+(\omega R_1C_1)^2)(1+(\omega R_2C_2)^2)\}}$

Phase=$\angle H=0 - \angle(1+ j\omega R_1C_1) - \angle(1+j\omega R_2C_2)$

$=\tan^{-1}(\omega R_1C_1) - \tan^{-1}(\omega R_2C_2)$

If $R_1C_1=R_2C_2=RC$ then

Magnitude=$|H|=1/\{(1+(\omega RC)^2\}$=Magnitude^2 of single-stage LPF

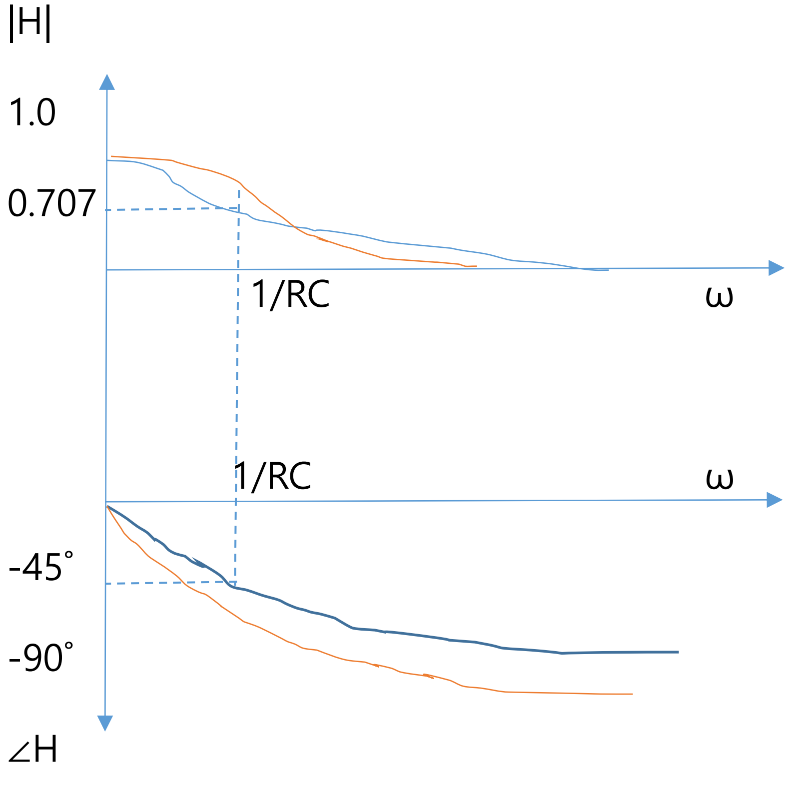
Phase=$\angle H=-2\tan^{-1}(\omega RC)$=Twice of phase of single-stage LPF

Note that the 10X rule $Z_1 \ll Z_2$ should be also satisfied.

(Q: what happen if $R_1=R_2=R$ and $C_1=C_2=C$?)

The two-stage or cascaded LPF can show higher performance of LPF when compared to the single one. Practically it may lead to excessive attenuation at the ω_{3dB} such as $|H|=(1/\sqrt{2})(1/\sqrt{2})=1/2$ and $\angle H=-90^{\circ}$.

Practically it is recommended to adjust ω_{3dB} in order to avoid that situation, by choosing new $\omega_1{}'_{3dB}$ and $\omega_2{}'_{3dB}$ such as $\omega_1{}'_{3dB}=\omega_2{}'_{3dB}=2\omega_{3dB}$; or $\omega_1{}'_{3dB}=3\omega_{3dB}$, $\omega_2{}'_{3dB}=\omega_{3dB}$. Then at ω_{3dB}, $|H|$ changes into 0.8 or 0.671, and $\angle H$ changes into -53.1° or -63.4°, respectively.



Blue: Original LPF for ω_{3dB} , Red: Cascaded LPF for $\omega'_{3dB}=2\omega_{3dB}$

Similarly, two-stage or cascaded HPF can be considered.

Q: How to prevent excessive attenuation for HPF?

(Answer: $\omega_1{}'_{3dB} = \omega_2{}'_{3dB} = \omega_{3dB}/2$; or $\omega_1{}'_{3dB} = \omega_{3dB}/3$, $\omega_2{}'_{3dB} = \omega_{3dB}$ Why?)

HW4)

(1) For the design of the above mentioned bandpass filter, please make an alternative design such as LPF+HPF instead of HPF+LPF

(2) Design the cascaded HPF for the second problem of HW3, considering strategy to prevent excessive attenuation