Lecture 8

Ch. 6 Compressive and tensile failures in vertical wells (part 1)
(27 April 2020)

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Compressive and tensile failures in vertical wells
Introduction

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- Importance
  - Wellbore stability
  - Observation from the wellbore provide much information – including in situ stress
  - Borehole breakout
    - Stress orientation, and (to some extent) magnitude
  - Drilling induced tensile fracture
    - Shmin is the minimum principal stress
    - Large difference between SHmax and Shmin
    - High mud weight
    - Cooling effect
  - Hydraulic fracturing
    - different from drilling induced tensile fracture
    - Away from the borehole

Stresses concentration around a cylindrical hole and wellbore failure

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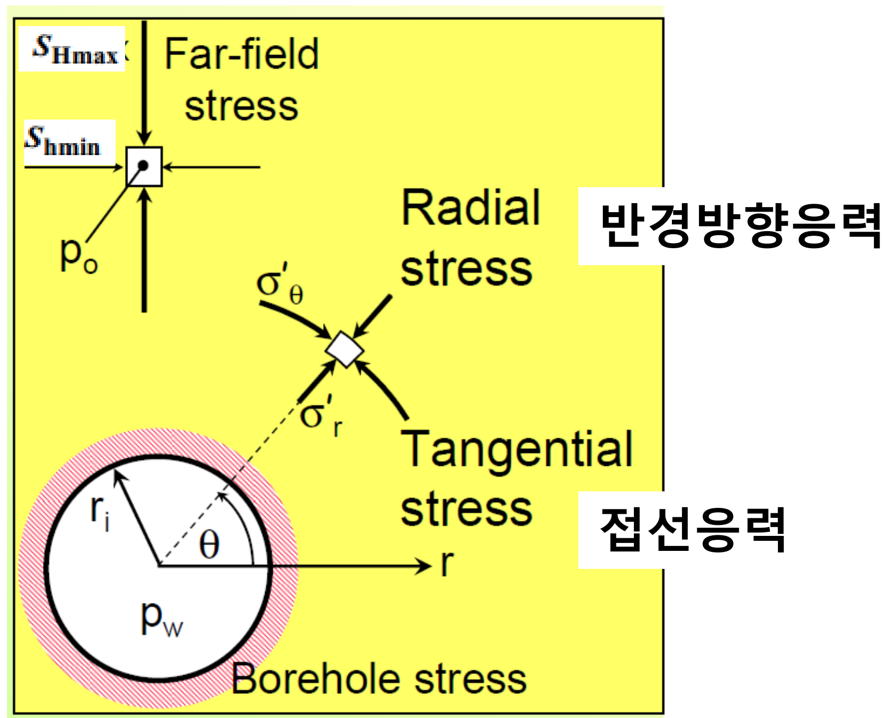
- Stresses in cylindrical coordinates



Dusseault, 2012

Stresses concentration around a cylindrical hole and wellbore failure

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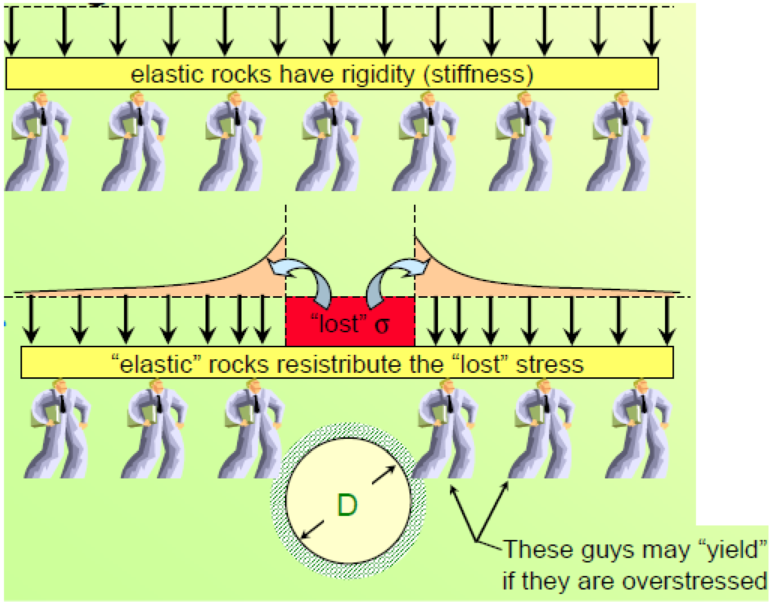
- Concept of "Stress Redistribution"



Dusseault, 2012

Stress concentration
Loading condition in (reservoir) geomechanics

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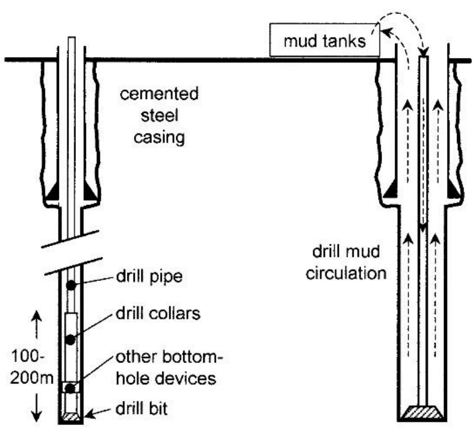
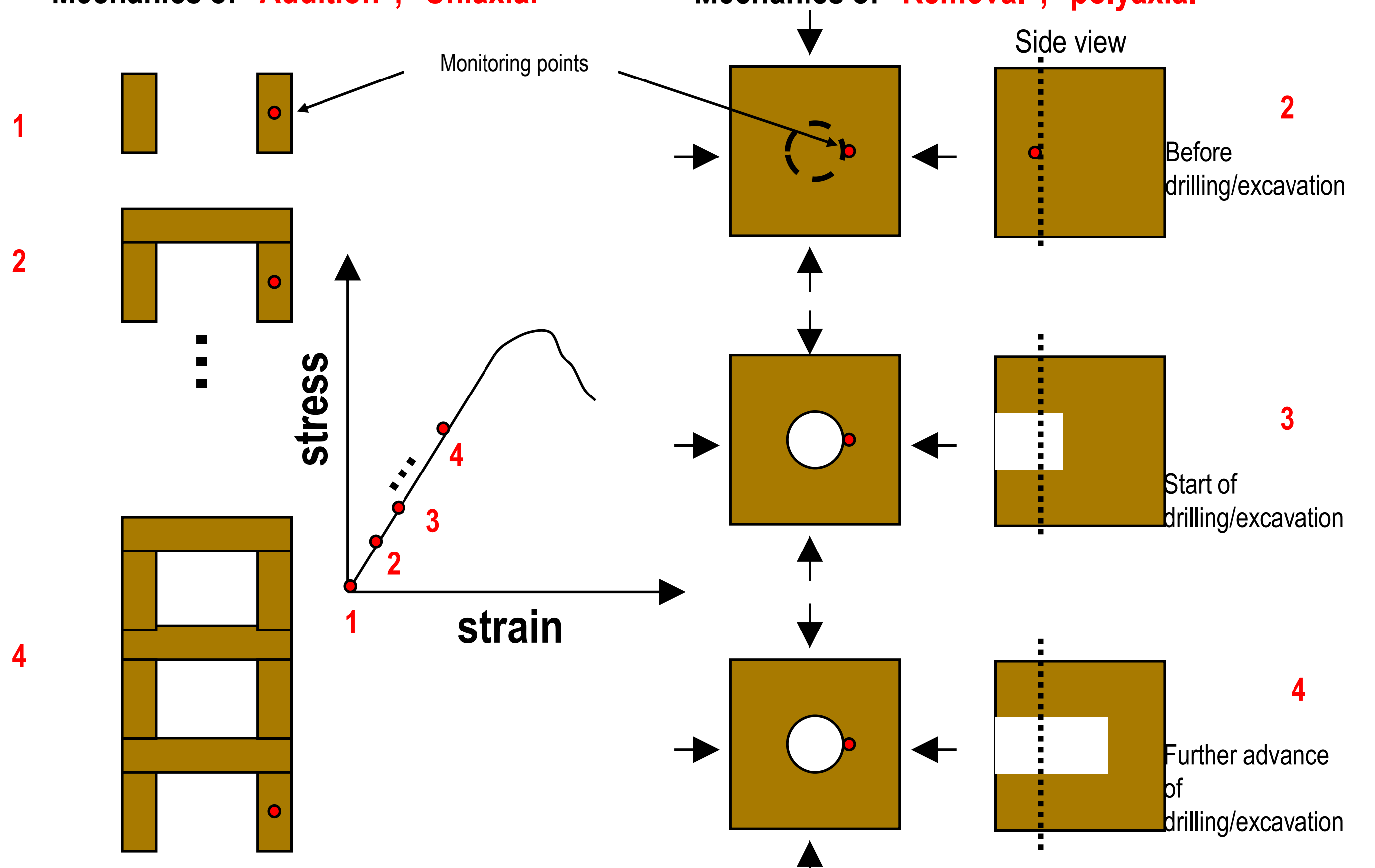


Nature of Underground Geomechanics

Civil structural problems:
Mechanics of "Addition", "Uniaxial"

Underground Geomechanics problems:
Mechanics of "Removal", "polyaxial"



Stresses concentration around a cylindrical hole and wellbore failure

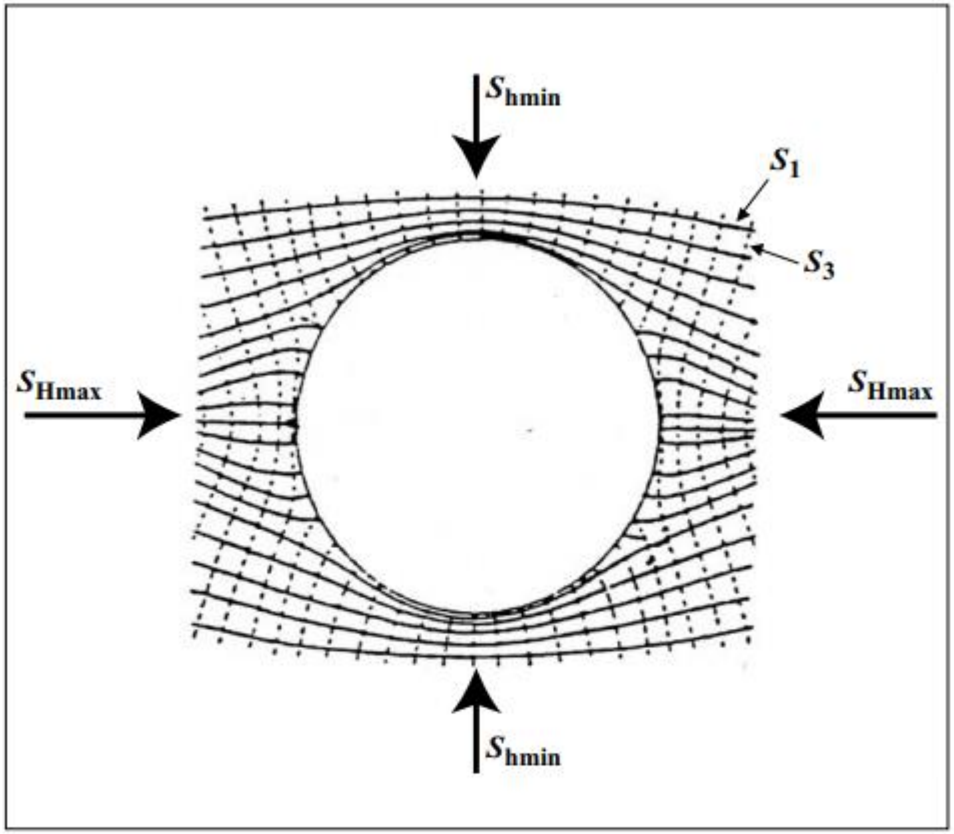
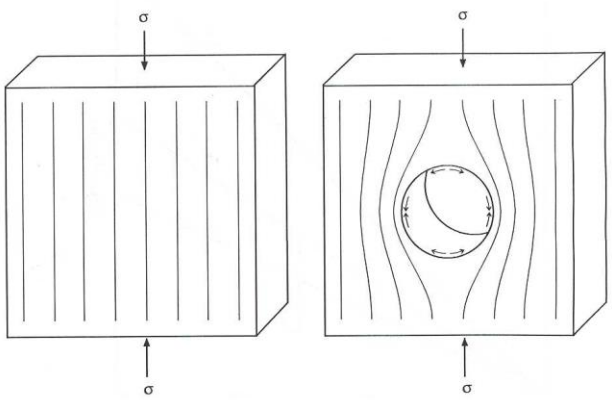


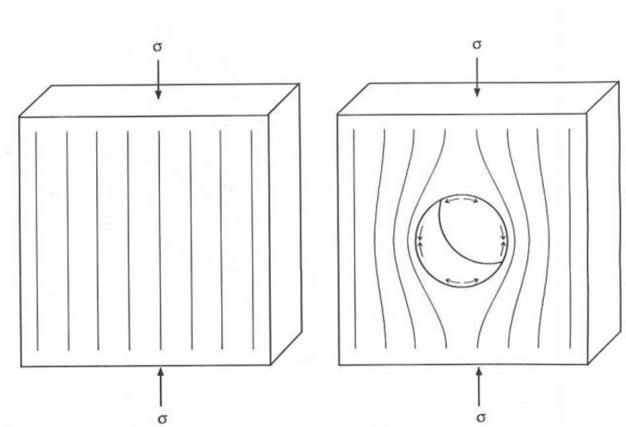
Figure 6.1. Principal stress trajectories around a cylindrical opening in a bi-axial stress field based on the Kirsch equations (Kirsch 1898). Note that as the wellbore wall is a free surface, the principal stress trajectories are parallel and perpendicular to it. Where the trajectories of maximum compressive stress converge, stresses are more compressive (at the azimuth of S_{hmin} in case of a vertical well). Where the trajectories diverge, the stresses are less compressive (at the azimuth of S_{Hmax}).

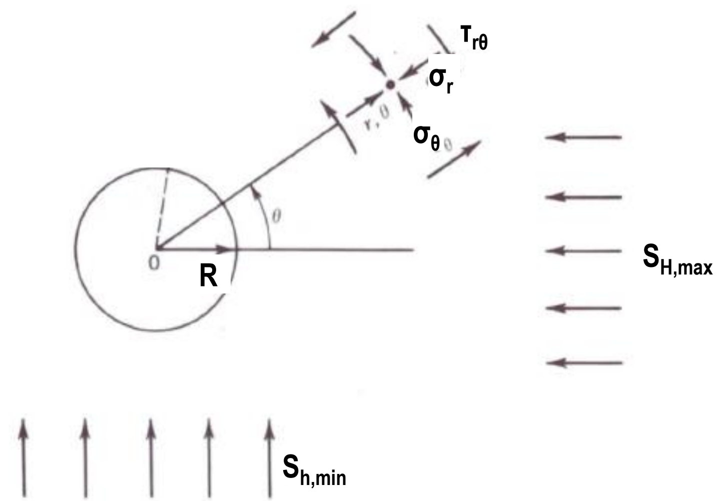


Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Stresses concentration around a cylindrical hole and wellbore failure

- Kirsch solution
  - Problem of stress distribution around a circular borehole was solved by Kirsch (1898)
  - Homogeneous, isotropic and elastic rock under plane strain condition within elastic range





Stresses concentration around a cylindrical hole and wellbore failure

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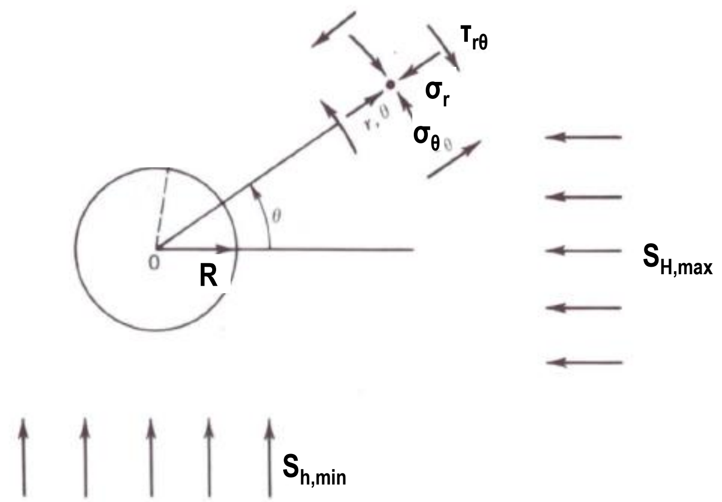
Kirsch solution

$$\sigma_r = \frac{S_{H\max} + S_{h\min}}{2}\left(1 - \frac{R^2}{r^2}\right) + \frac{S_{H\max} - S_{h\min}}{2}\left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{S_{H\max} + S_{h\min}}{2}\left(1 + \frac{R^2}{r^2}\right) - \frac{S_{H\max} - S_{h\min}}{2}\left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = \frac{S_{H\max} - S_{h\min}}{2}\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\theta$$



R: radius of well

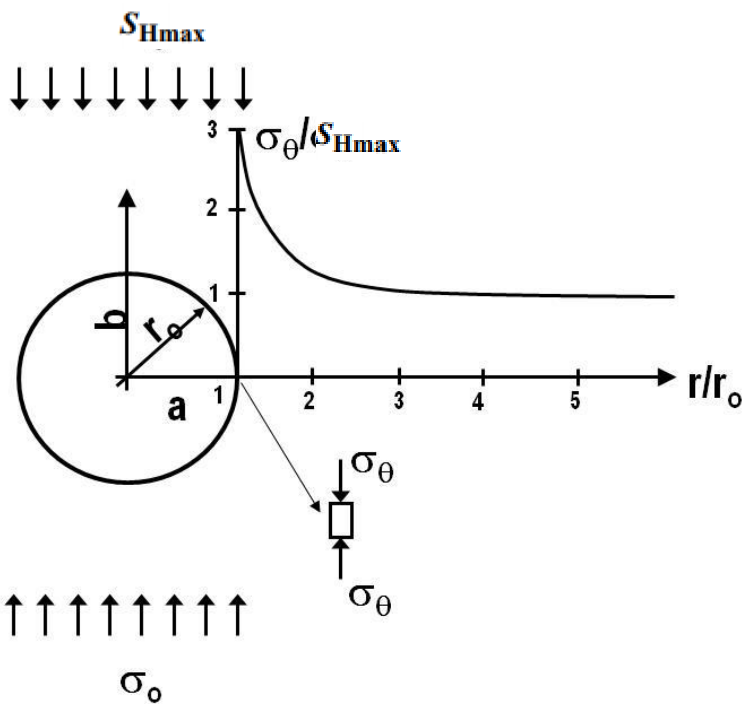
r: radial distance from the center of the well

θ: measured from $S_{H,max}$

$S_{H,max}$ and $S_{H,max}$: maximum and minimum horizontal insitu stress

Stresses concentration around a cylindrical hole and wellbore failure

- Kirsch solution - Uniaxial boundary stress
  - Stress concentration factor due to uniaxial stress is ‘3’.
  - Influence of borehole is within 2~3 times of radius



Stresses concentration around a cylindrical hole and wellbore failure

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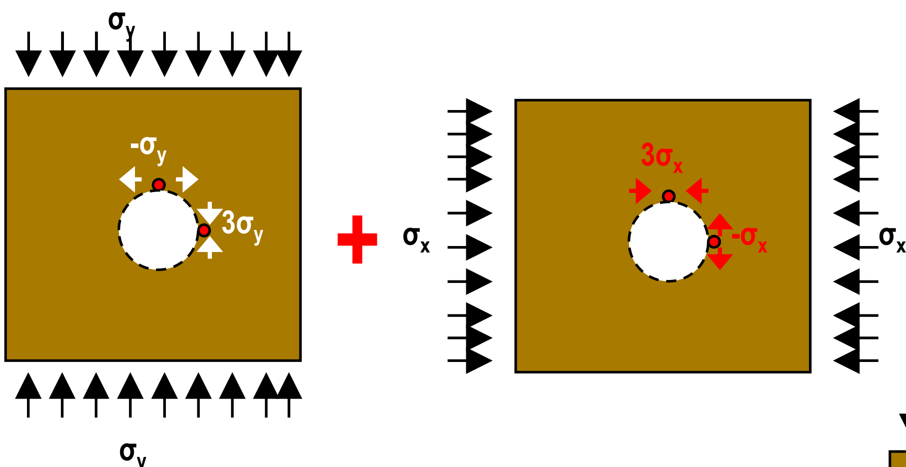
- Kirsch solution - Biaxial boundary stress



By superimposing two cases, we can obtain stress concentration under biaxial stress field.

In a uniaxial stress field, the maximum stress concentration is 3 (compressive) and minimum is -1 (tensile)

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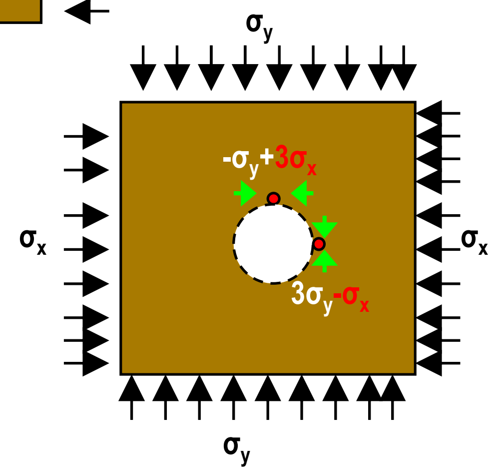


Stresses concentration around a cylindrical hole and wellbore failure

- Kirsch solution - Uniaxial boundary stress
  - At boundaries (by putting R=r)

$$\sigma_\theta = \frac{S_{H\max} + S_{h\min}}{2}\left(1 + \frac{R^2}{r^2}\right) - \frac{S_{H\max} - S_{h\min}}{2}\left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta$$

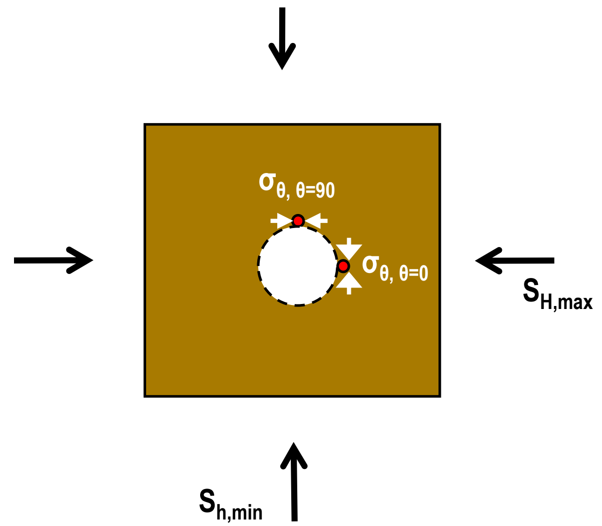
$$\sigma_r = 0$$

$$\boxed{\sigma_\theta = S_{H\max} + S_{h\min} - 2\left(S_{H\max} - S_{h\min}\right)\cos 2\theta}$$

$$\tau_{r\theta} = 0$$

$$\theta = 0, \quad \sigma_\theta = -S_{H\max} + 3S_{h\min}$$

$$\theta = 90, \quad \sigma_\theta = 3S_{H\max} - S_{h\min}$$



Stresses concentration around a cylindrical hole and wellbore failure

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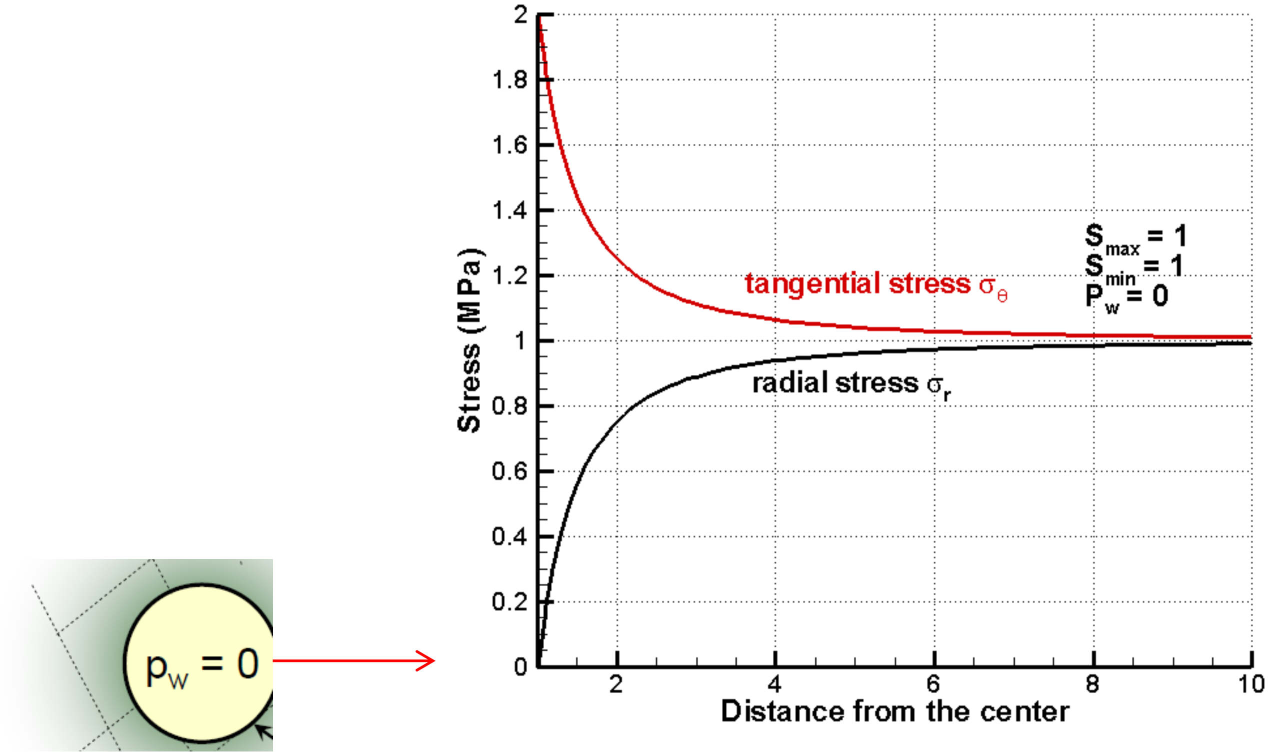
- Isotropic boundary stress



Stresses concentration around a cylindrical hole and wellbore failure

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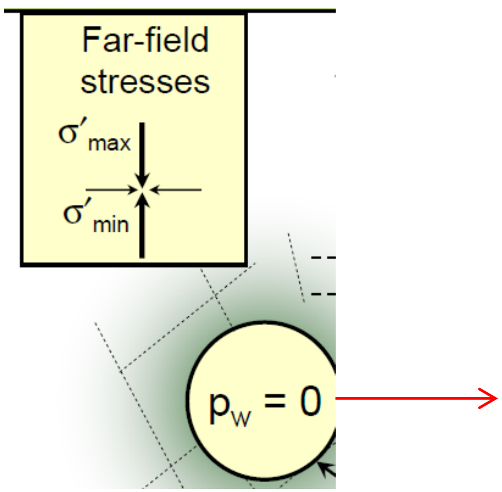
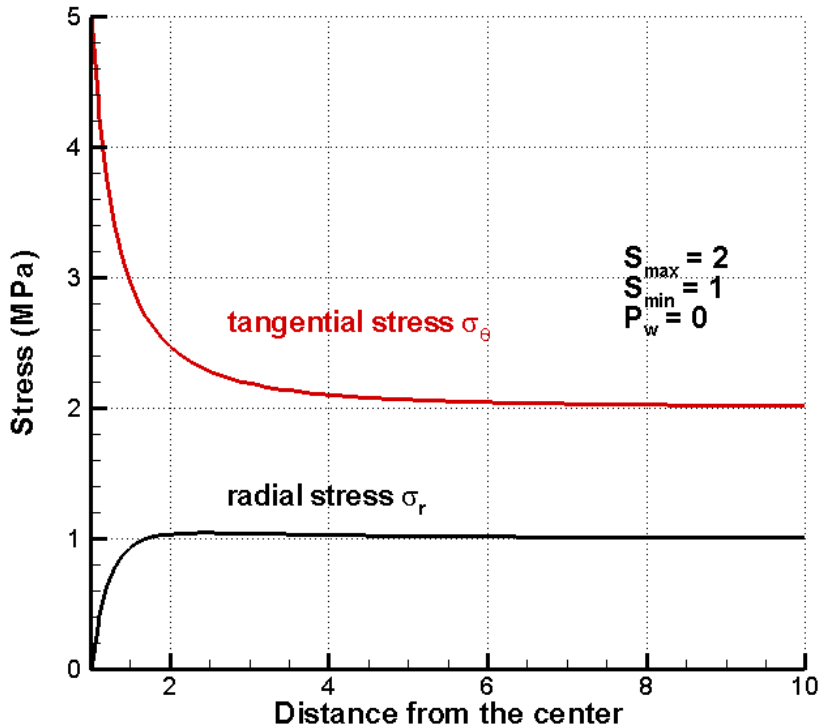
- Anisotropic boundary stress





Stresses concentration around a cylindrical hole and wellbore failure

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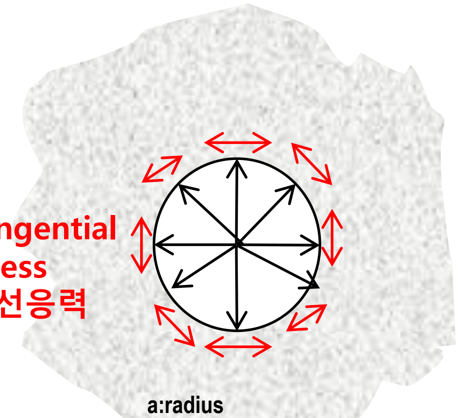
- Increase of internal mud/hydraulic pressure
  - Water pressure
  - Mud pressure
  - Injection pressure

Tangential stress
접선응력



a:radius
r: distance from the center

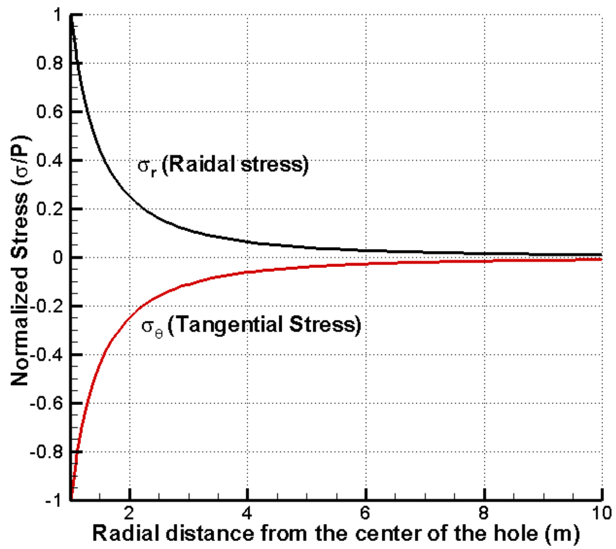
$$\sigma_r = P_w \frac{a^2}{r^2}$$

$$\sigma_\theta = -P_w \frac{a^2}{r^2}$$

$$\tau_{r\theta} = 0$$

$$u_r = -\frac{P_w}{2G}\frac{a^2}{r}$$

$$v_\theta = 0$$



Stresses concentration around a cylindrical hole and wellbore failure

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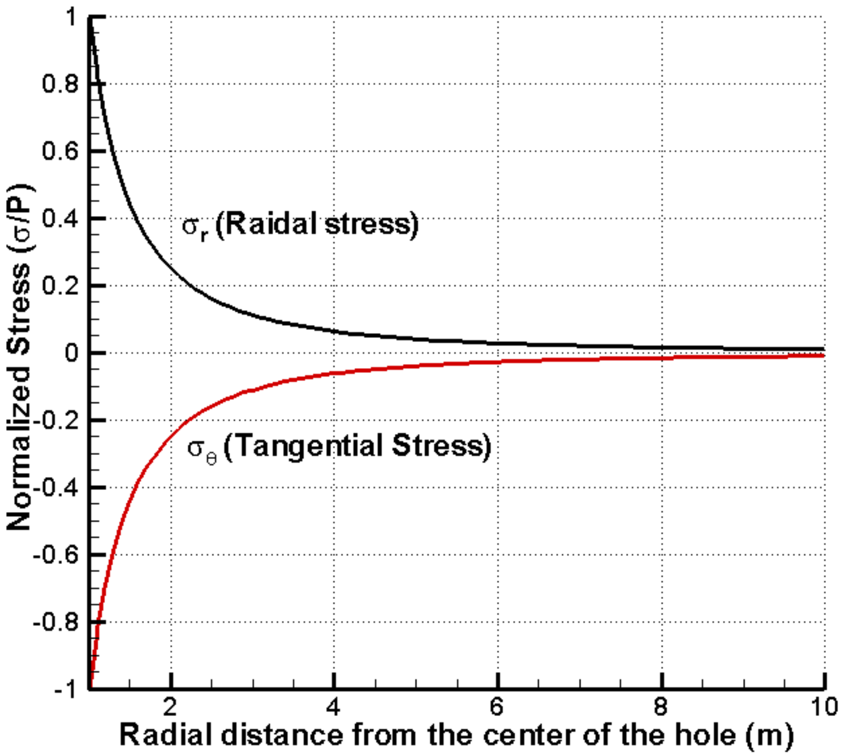
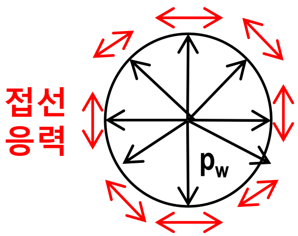
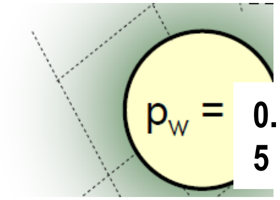
- Increase of internal mud/hydraulic pressure

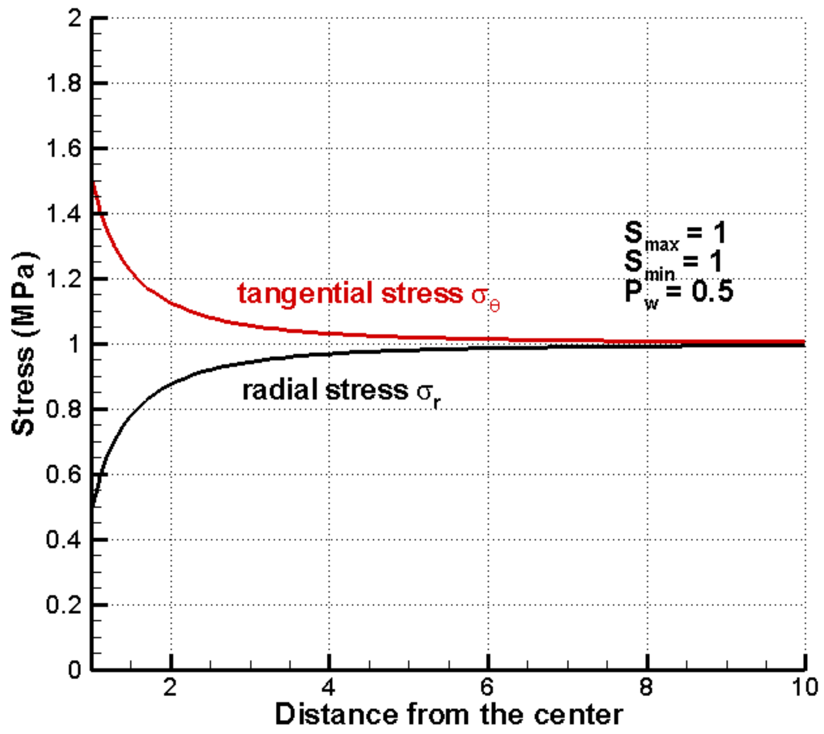




Stresses concentration around a cylindrical hole and wellbore failure

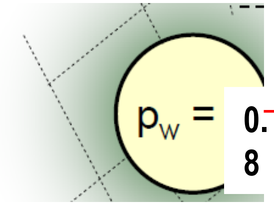
- Isotropic boundary stress + internal pressure (0.5)

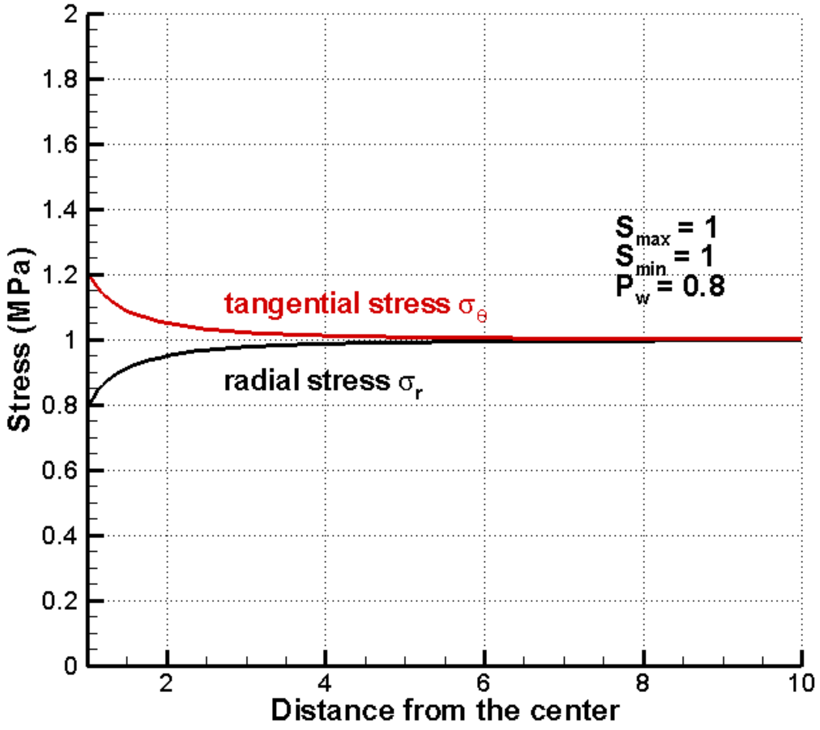




Stresses concentration around a cylindrical hole and wellbore failure

- Isotropic boundary stress + internal pressure (0.8)





Stresses concentration around a cylindrical hole and wellbore failure

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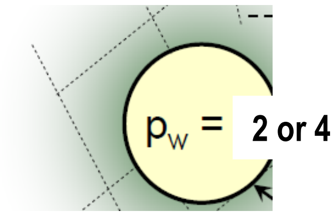
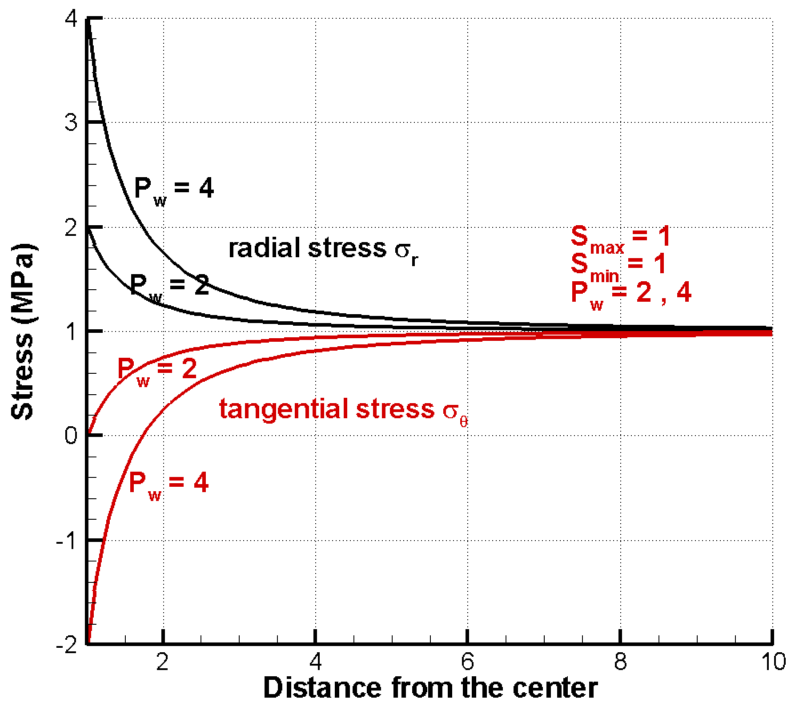
- **Isotropic boundary stress + internal pressure (2 or 4)**

  - Hydraulic fracturing occur when internal pressure is large.
  - The location of the initiation of fracture can be decided depending on the boundary stress





Stresses concentration around a cylindrical hole and wellbore failure

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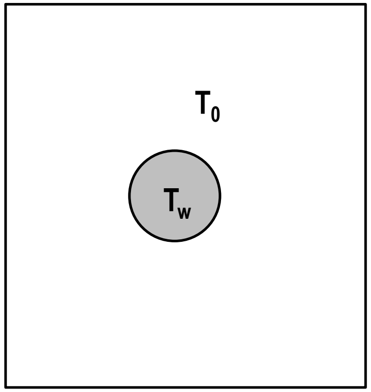
- Effect of temperature change - thermal stress
  - Thermally induced hoop stress at the wall of cylindrical borehole

$$\sigma_r = 0$$

$$\sigma_\theta = \frac{E}{1-\nu}\alpha\left(T_w - T_0\right)$$

$$\tau_{r\theta} = 0$$

α: linear thermal expansion coefficient
E: Elastic Modulus
T_w: well temperature
T_0: reservoir temperature



$$\sigma_{\theta\theta}^{\Delta T} = \frac{\alpha_t E \Delta T}{1-\nu}$$

Effect of temperature change - thermal stress

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- Thermal stress due to cold mud/water injection is important for geothermal project
  - Ex) Injecting water T = 25°C, reservoir T = 75°C, Elastic modulus = 50 GPa, $v=0.25$, $\alpha = 1\times10^{-5}/°C$ → hoop stress = 33 MPa ← big influence!!!

Stresses concentration around a cylindrical hole and wellbore failure

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- Stresses distribution around a borehole with;
  - Principal in situ stress boundary
  - Internal pore pressure (could be mud pressure)
  - Temperature change

$$\sigma_r = \frac{S_{H\max} + S_{h\min}}{2}\left(1 - \frac{R^2}{r^2}\right) + \frac{S_{H\max} - S_{h\min}}{2}\left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta + P_w \frac{R^2}{r^2}$$

$$\sigma_\theta = \frac{S_{H\max} + S_{h\min}}{2}\left(1 + \frac{R^2}{r^2}\right) - \frac{S_{H\max} - S_{h\min}}{2}\left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta - P_w \frac{R^2}{r^2} + \frac{E}{1-\nu}\alpha\left(T_w - T_0\right)$$

$$\tau_{r\theta} = \frac{S_{H\max} - S_{h\min}}{2}\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\theta$$

Stresses concentration around a cylindrical hole and wellbore failure

- Some insights
- Stresses distribution around a borehole is
  - independent of size of radius
  - independent of Elastic modulus of rocks
  - Poisson's ratio has some influence on vertical stress distribution (in vertical hole)

Stresses concentration around a cylindrical hole and wellbore failure

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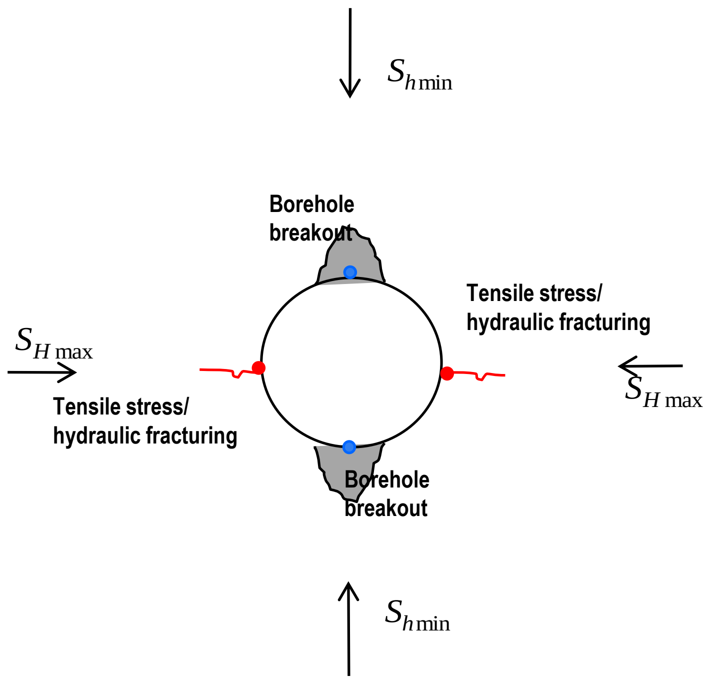
borehole breakout vs. hydraulic fracturing



Stresses concentration around a cylindrical hole and wellbore failure

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- Kirsch solution with <u>pore pressure in porous medium</u>

$$\sigma_{rr} = \frac{1}{2}\left(S_{\mathrm{Hmax}} + S_{\mathrm{hmin}} \boxed{-\,2P_0)}\left(1 - \frac{R^2}{r^2}\right) + \frac{1}{2}\left(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}}\right)\right.$$
$$\times\left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta + \frac{\Delta P R^2}{r^2}$$

$$\sigma_{\theta\theta} = \frac{1}{2}\left(S_{H\max} + S_{h\min} \boxed{-\,2P_0)}\left(1 + \frac{R^2}{r^2}\right) - \frac{1}{2}\left(S_{H\max} - S_{h\min}\right)\right.$$
$$\times\left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta - \frac{\Delta P R^2}{r^2} - \sigma^{\Delta T}$$

$$\tau_{r\theta} = \frac{1}{2}\left(S_{H\max} - S_{h\min}\right)\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\theta$$

$$\sigma_{zz} = S_v - 2\nu\left(S_{H\max} - S_{h\min}\right)\frac{r^2}{R^2}\cos 2\theta \;-\; P_0$$

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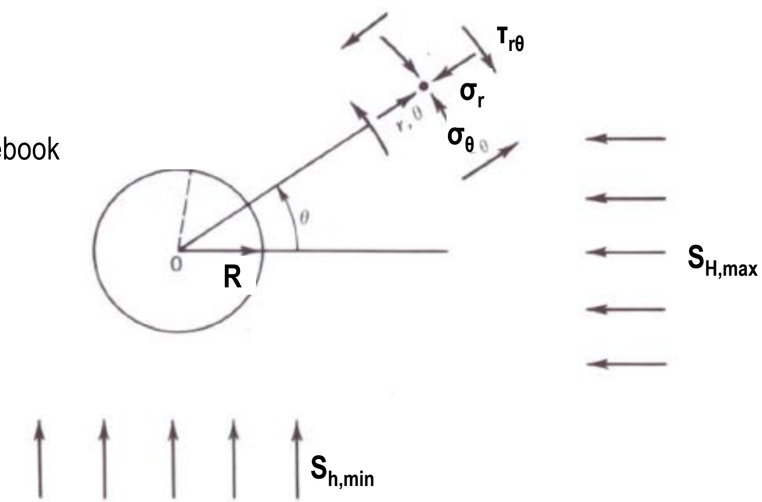

where θ is measured from the azimuth of S_{Hmax} and ΔP is the difference betwee mud weight in the wellbore and the pore pressure, P_0. $\sigma^{\Delta T}$ represents thermal str arising from the difference between the mud temperature and formation temper (ΔT). This will be ignored for the moment but is considered below. It can be sh

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Stresses concentration around a cylindrical hole and wellbore failure

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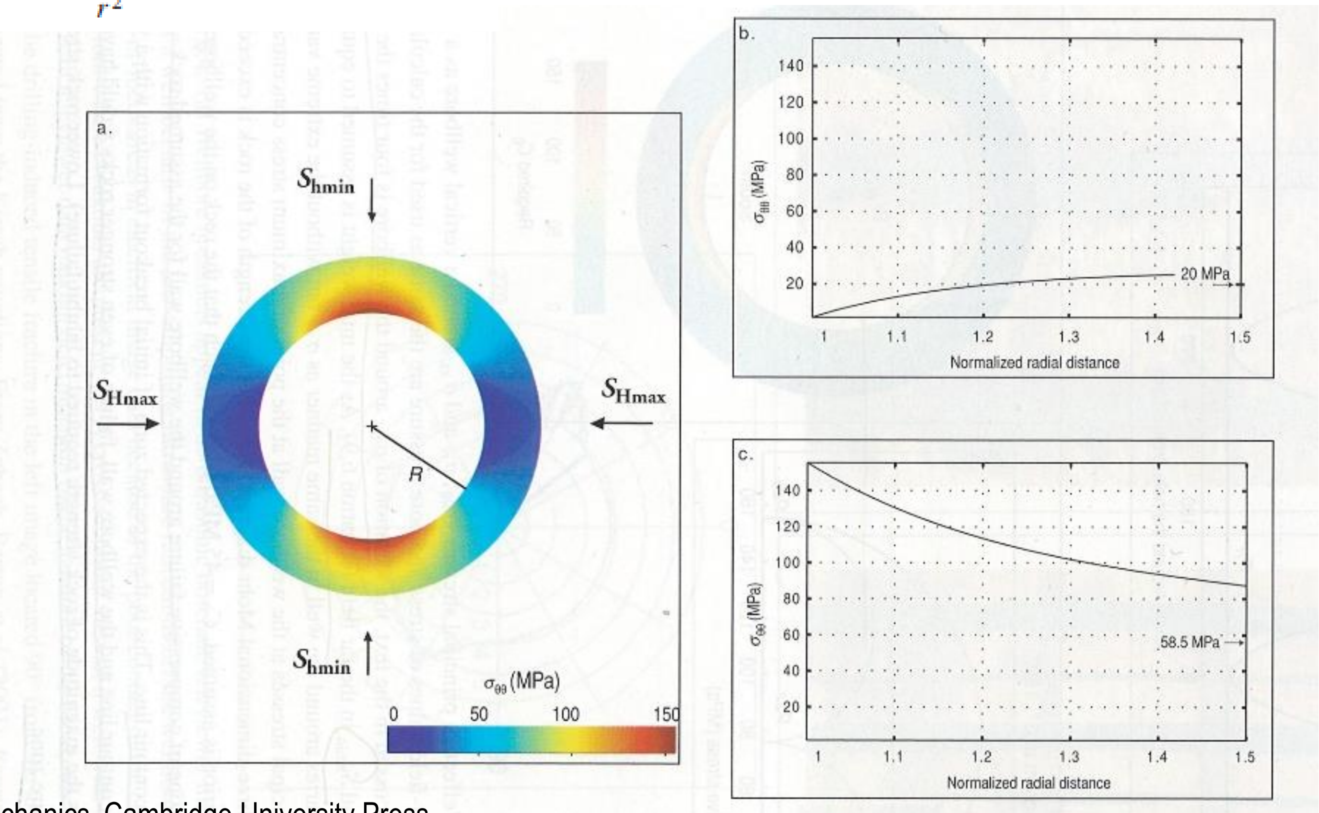
$$\sigma_{rr} = \frac{1}{2}(S_{\mathrm{Hmax}} + S_{\mathrm{hmin}} - 2P_0)\left(1 - \frac{R^2}{r^2}\right) + \frac{1}{2}(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}}) \times \left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta + \frac{\Delta P R^2}{r^2}$$

$$\sigma_{\theta\theta} = \frac{1}{2}(S_{H\mathrm{max}} + S_{h\mathrm{min}} - 2P_0)\left(1 + \frac{R^2}{r^2}\right) - \frac{1}{2}(S_{H\mathrm{max}} - S_{h\mathrm{min}}) \times \left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta - \frac{\Delta P R^2}{r^2} - \sigma^{\Delta T}$$

- $S_{\mathrm{Hmax}} = 90$ MPa
- S_{Hmax} orientation is N90°E (east–west)
- $S_{\mathrm{v}} = 88.2$ MPa (depth 3213m)
- $S_{\mathrm{hmin}} = 51.5$ MPa
- $P_{\mathrm{p}} = P_{\mathrm{mud}} = 31.5$ MPa



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Stresses concentration around a cylindrical hole and wellbore failure

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$$\sigma_{rr} = \frac{1}{2}(S_{\mathrm{Hmax}} + S_{\mathrm{hmin}} - 2P_0)\left(1 - \frac{R^2}{r^2}\right) + \frac{1}{2}(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}}) \times \left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta + \frac{\Delta P R^2}{r^2}$$

$$\sigma_{\theta\theta} = \frac{1}{2}(S_{H\mathrm{max}} + S_{h\mathrm{min}} - 2P_0)\left(1 + \frac{R^2}{r^2}\right) - \frac{1}{2}(S_{H\mathrm{max}} - S_{h\mathrm{min}}) \times \left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta - \frac{\Delta P R^2}{r^2} - \sigma^{\Delta T}$$

$$\tau_{r\theta} = \frac{1}{2}(S_{H\mathrm{max}} - S_{\mathrm{hmin}})\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\theta$$

for the stresses acting right at the wellbore wall by substituting $r = R$. In this case, the effective hoop stress and radial stress at the wellbore wall are given by the following equation:

$$\sigma_{\theta\theta} = S_{\mathrm{hmin}} + S_{\mathrm{Hmax}} - 2(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}})\cos 2\theta - 2P_0 - \Delta P - \sigma^{\Delta T} \quad (6.4)$$

$$\sigma_{rr} = \Delta P \quad (6.5)$$

where ΔP is the difference between the wellbore pressure (mud weight, P_{m}) and the pore pressure. The effective stress acting parallel to the wellbore axis is:

$$\sigma_{zz} = S_{\mathrm{v}} - 2\nu(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}})\cos 2\theta - P_0 - \sigma^{\Delta T} \quad (6.6)$$

where ν is Poisson's ratio. At the point of minimum compression around the wellbore (i.e. parallel to S_{hmin}) at $\theta = 0°, 180°$, equation (6.4) reduces to

$$\sigma_{\theta\theta}^{\mathrm{min}} = 3S_{\mathrm{hmin}} - S_{\mathrm{Hmax}} - 2P_0 - \Delta P - \sigma^{\Delta T} \quad (6.7)$$

whereas at the point of maximum stress concentration around the wellbore (i.e. parallel to S_{Hmax}) at $\theta = 90°, 270°$,

$$\sigma_{\theta\theta}^{\mathrm{max}} = 3S_{\mathrm{Hmax}} - S_{\mathrm{hmin}} - 2P_0 - \Delta P - \sigma^{\Delta T} \quad (6.8)$$

such that the difference between the two is

$$\sigma_{\theta\theta}^{\mathrm{max}} - \sigma_{\theta\theta}^{\mathrm{min}} = 4(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}}) \quad (6.9)$$

which corresponds to the amplitude of the sinusoidal variation of hoop stress around the wellbore shown in Figure 6.3a and helps explain why observations of wellbore failures so effectively indicate far-field stress directions. Fundamentally, the variation of stress around the wellbore wall amplifies the far-field stress concentration by a factor of 4.

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Stresses concentration around a cylindrical hole and wellbore failure

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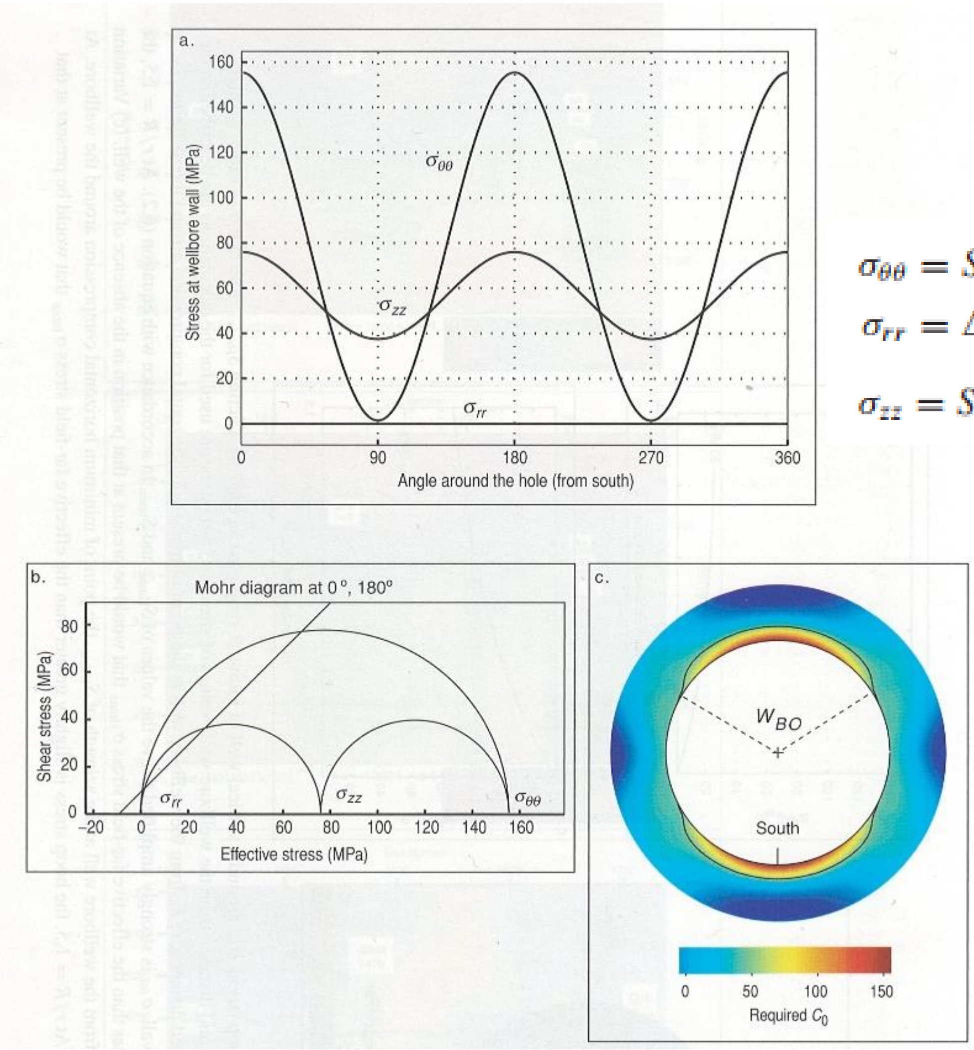
- $S_{\mathrm{Hmax}} = 90$ MPa
- S_{Hmax} orientation is N90°E (east–west)
- $S_{\mathrm{v}} = 88.2$ MPa (depth 3213m)
- $S_{\mathrm{hmin}} = 51.5$ MPa
- $P_{\mathrm{p}} = P_{\mathrm{mud}}$=31.5 MPa



$$\sigma_{\theta\theta} = S_{\mathrm{hmin}} + S_{\mathrm{Hmax}} - 2(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}})\cos 2\theta - 2P_0 - \Delta P - \sigma^{\Delta T}$$

$$\sigma_{rr} = \Delta P$$

$$\sigma_{zz} = S_{\mathrm{v}} - 2\nu(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}})\cos 2\theta - P_0 - \sigma^{\Delta T}$$

$$\sigma_{\theta\theta}^{\min} = 3S_{\mathrm{hmin}} - S_{\mathrm{Hmax}} - 2P_0 - \Delta P - \sigma^{\Delta T}$$

$$\sigma_{\theta\theta}^{\max} = 3S_{\mathrm{Hmax}} - S_{\mathrm{hmin}} - 2P_0 - \Delta P - \sigma^{\Delta T}$$

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Borehole Breakout

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- Borehole breakout
  - When stress exceed the rock strength near the borehole

$$\sigma_{rr} = \frac{1}{2}(S_{\mathrm{Hmax}} + S_{\mathrm{hmin}} - 2P_0)\left(1 - \frac{R^2}{r^2}\right) + \frac{1}{2}(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}}) \times \left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta + \frac{\Delta P R^2}{r^2}$$

$$\sigma_{\theta\theta} = \frac{1}{2}(S_{H\max} + S_{h\min} - 2P_0)\left(1 + \frac{R^2}{r^2}\right) - \frac{1}{2}(S_{H\max} - S_{h\min}) \times \left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta - \frac{\Delta P R^2}{r^2} - \sigma^{\Delta T}$$

$$\tau_{r\theta} = \frac{1}{2}(S_{H\max} - S_{h\min})\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\theta$$

$$\sigma_{zz} = S_v - 2\nu(S_{H\max} - S_{h\min})\frac{r^2}{R^2}\cos 2\theta - P_0$$

at r=R

$$\sigma_{\theta\theta} = S_{\mathrm{hmin}} + S_{\mathrm{Hmax}} - 2(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}})\cos 2\theta - 2P_0 - \Delta P - \sigma^{\Delta T}$$

$$\sigma_{rr} = \Delta P$$

$$\sigma_{zz} = S_{\mathrm{v}} - 2\nu(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}})\cos 2\theta - P_0 - \sigma^{\Delta T}$$

$\longleftrightarrow$ compare $\longleftrightarrow$

$$\tau = S_0 + \sigma_n \mu_i$$

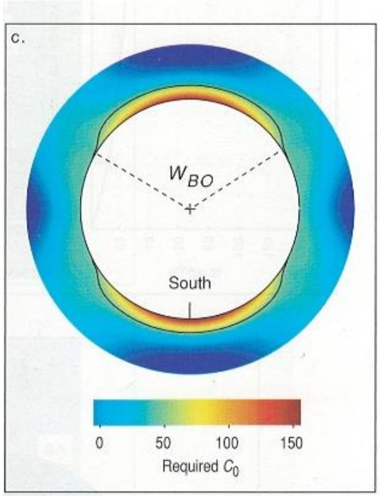
or

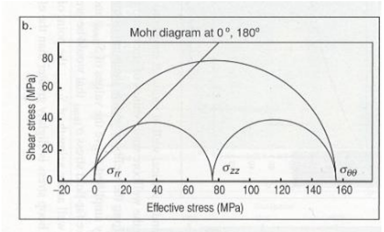
$$\sigma_1 = \sigma_3 + C_0\sqrt{m\frac{\sigma_3}{C_0} + s}$$

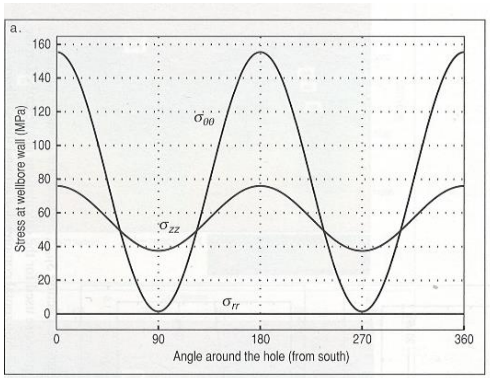
or

…

$\Rightarrow$







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Borehole Breakout

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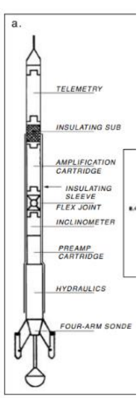
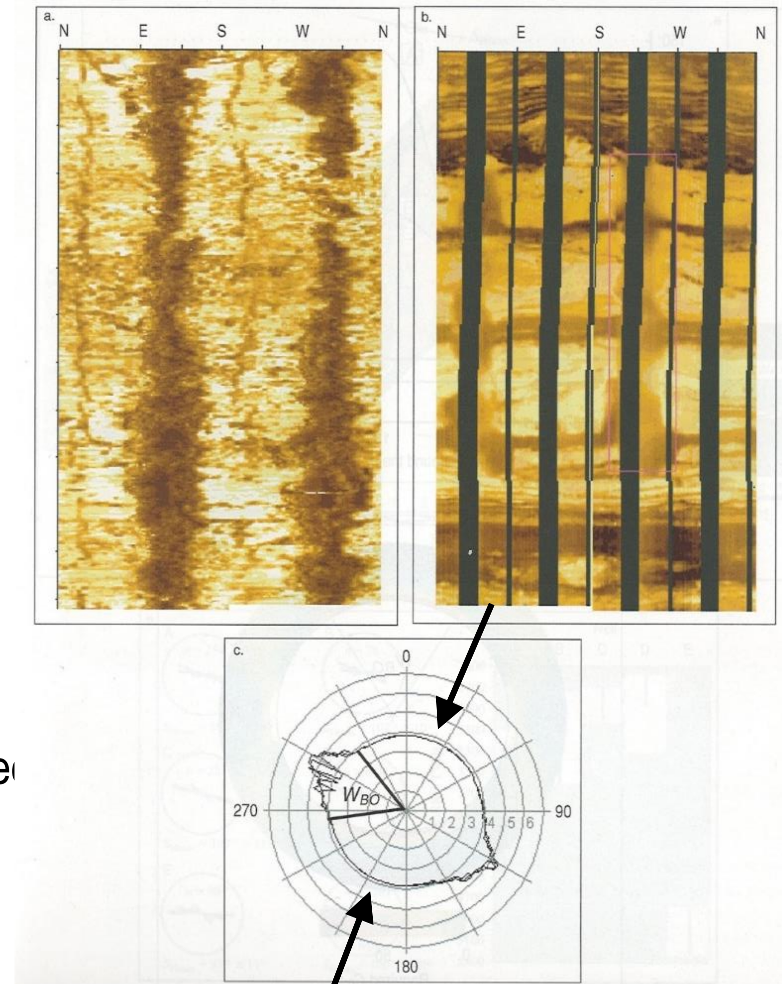
- Location
  - Perpendicular to maximum principal stress
    - Occur in pairs in opposite direction
  - Opening angle of Borehole Breakout: W_{BO}
  - Comprehensive analysis of borehole breakout yields profiles of stress orientation
- Characterization through logging:
  - Ultrasonic image log:
    - Fracture in dark area (low amplitude)
    - Travel time increase when radius is increase
  - Electrical image log:(e.g., FMI)
    - out of focus area (due to poor contact of electrode at breakout)



Ultrasonic televiewer image

Electrical image



maximum principal stress

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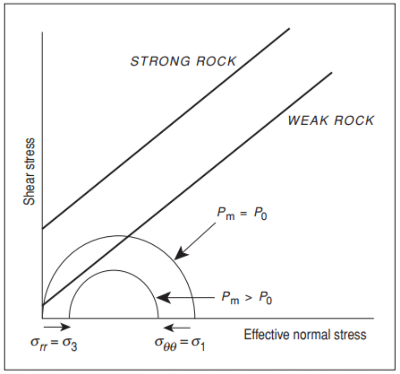
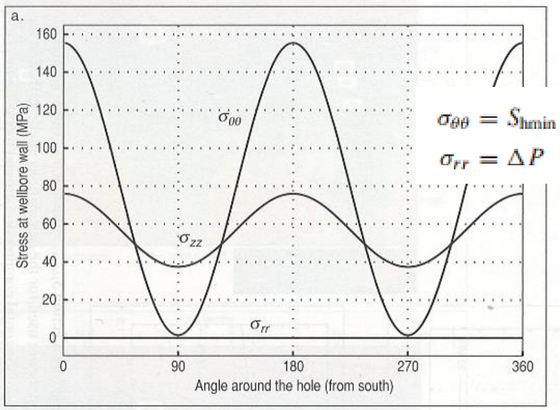
Borehole Breakout



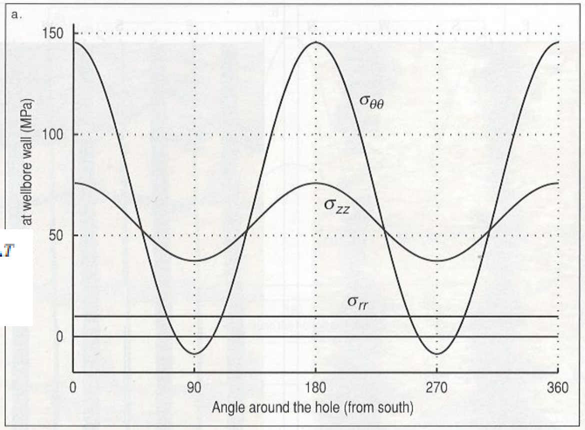
- Borehole stabilize with mud weight increase
  - Zone of failure gets smaller
  - Principle of mud weigtht control

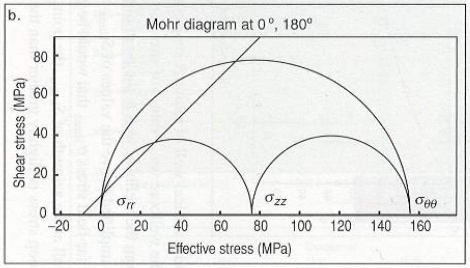


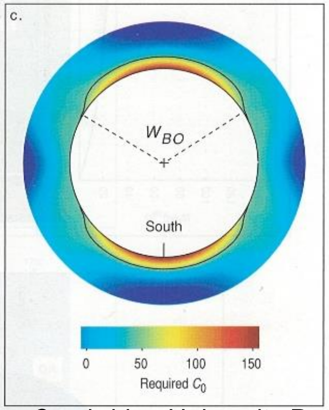
$$\sigma_{\theta\theta} = S_{\mathrm{hmin}} + S_{\mathrm{Hmax}} - 2(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}})\cos 2\theta - 2P_0 - \Delta P - \sigma^{\Delta T}$$

$$\sigma_{rr} = \Delta P$$

ΔP=0 MPa → ΔP=10 MPa







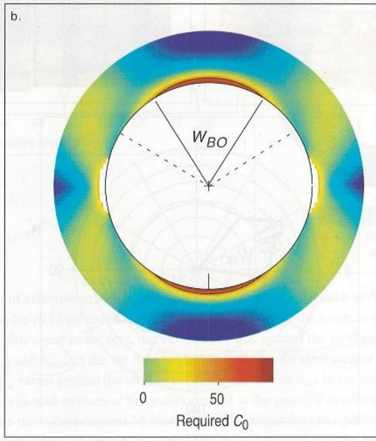


Figure 6.5. (a) Stress concentration at the wellbore wall and (b) zone of compressive failure around the wellbore (similar to Figure 6.3) when the mud weight has been raised 10 MPa above the mudweight. Figure 6.3b compares the width of breakouts for the two cases. Note that raising the mud weight decreases the size of the breakouts considerably. The area in white shows the region where tensile stresses exist at the wellbore wall.

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Borehole Breakout
progression of failure

$$\sigma_{\theta\theta} = S_{\mathrm{hmin}} + S_{\mathrm{Hmax}} - 2(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}})\cos 2\theta - 2P_0 - \Delta P - \sigma^{\Delta T}$$
$$\sigma_{rr} = \Delta P$$

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- Some observations:
  - BB tends to deepen (instead of widening)
  - Good correlation with theoretical prediction
  - Cooling make less BB
  - Effect of rock strength anisotropy
  - Effect of mud chemistry
  - Penetration of mud into fractured rock

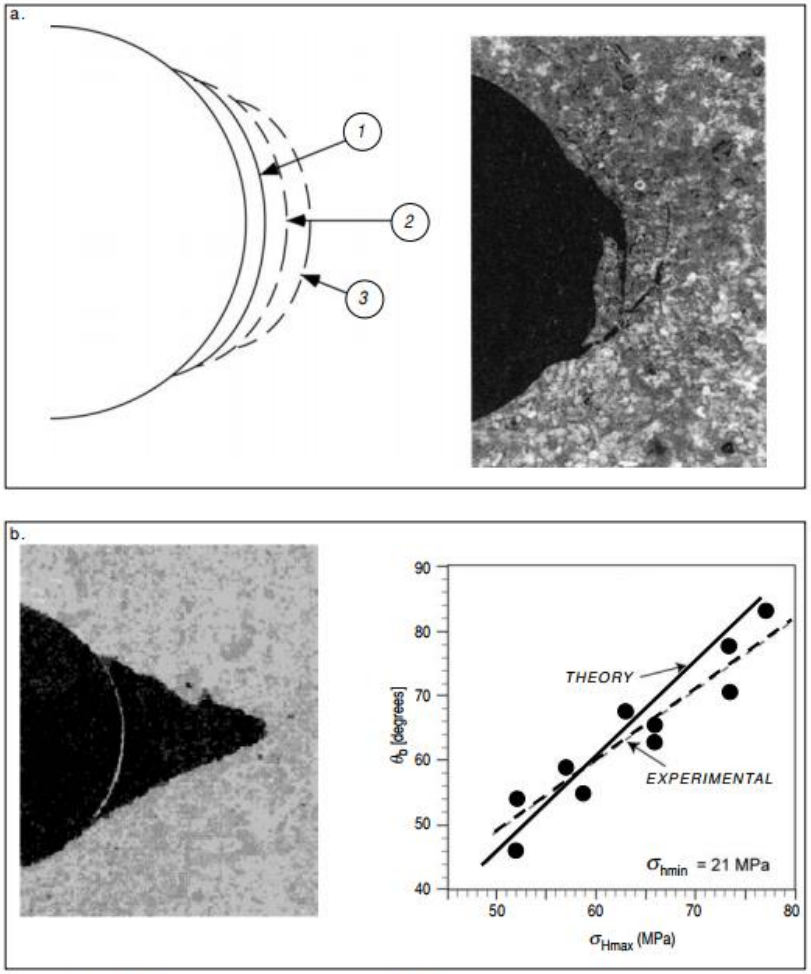

Figure 6.15. After the formation of wellbore breakouts, they are expected to increase in depth, but not width. This is as shown theoretically in (a) after Zoback, Moos *et al.* (1985) and confirmed by laboratory studies (Haimson and Herrick 1989). It can be seen photographically that breakouts in laboratory experiments deepen but do not widen after formation. A shown in (b), measured breakout widths compare very well with those predicted by the simple thoery presented in Zoback, Moos *et al.* (1985) which form the basic for the breakout shapes illustrated in Figures 6.2 and 6.3.

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Borehole Breakout
Rock Anisotropy

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- Rock strength anisotropy can affect the shape of BB
  - Due to the weak planes – double lobes are observed
- Situation of importance
  - Vertical drilling through steep bedding plane
  - Highly deviated well through near-horizontal bedding

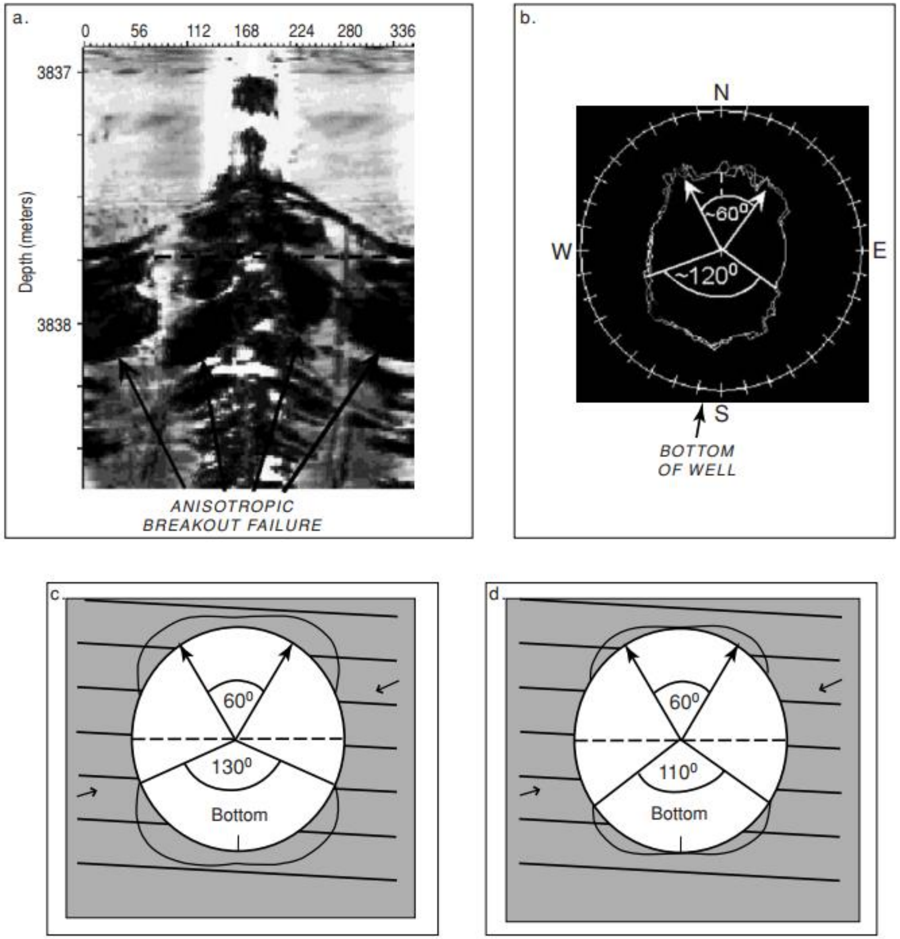

Figure 6.16. (a) Ultrasonic televiewer image of breakouts influenced by rock strength anisotropy associated with the presence of weak bedding planes cutting across a wellbore at a high angle. Note that there are four vertical bands of low reflectivity rather than two as shown in Figure 6.4. (b) Cross-sectional view of a breakout influenced by the presence of weak bedding planes shows a distinctive four-lobed shape. (c) This can be modeled by slip on bedding planes as the stress trajectories bend around the well. (d) When mud weight is increased, the size of the breakouts decreases.

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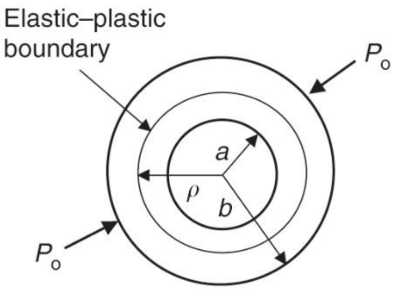
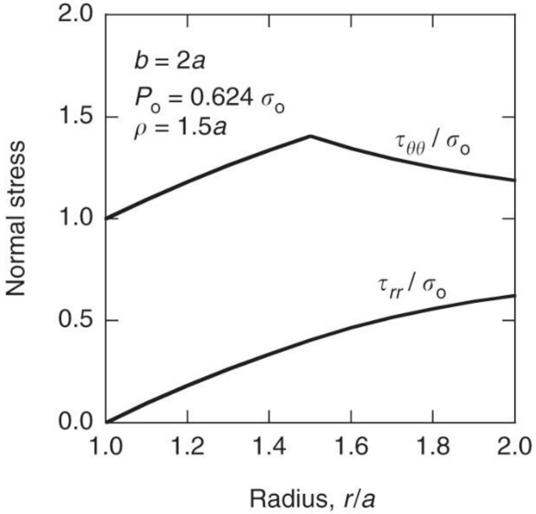
Borehole Breakout
Elastic vs. Elasto-plastic analysis

- Elastic analysis with borehole breakout is a simplification
  - Elasto-plastic analysis is more ideal
  - Nevertheless, elastic analysis is still fairly good - simple theory is still powerful





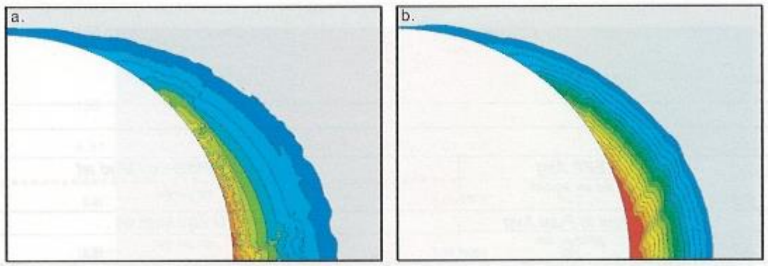


Figure 6.17. The area in which wellbore breakouts form around a cylindrical well can be modeled using a total plastic strain criterion rather than a stress criterion. These finite element calculations indicate the zone of expected breakouts assuming a critical strain level at which failure occurs (courtesy S. Willson). (a) Strain around a wellbore assuming a strain softening model of rock deformation (red indicates high strain). (b) Failure zone predicted using a strength of materials approach and Mohr–Coulomb failure criterion.

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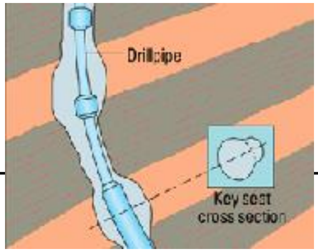
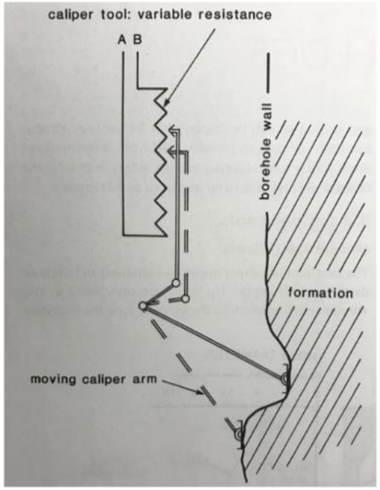
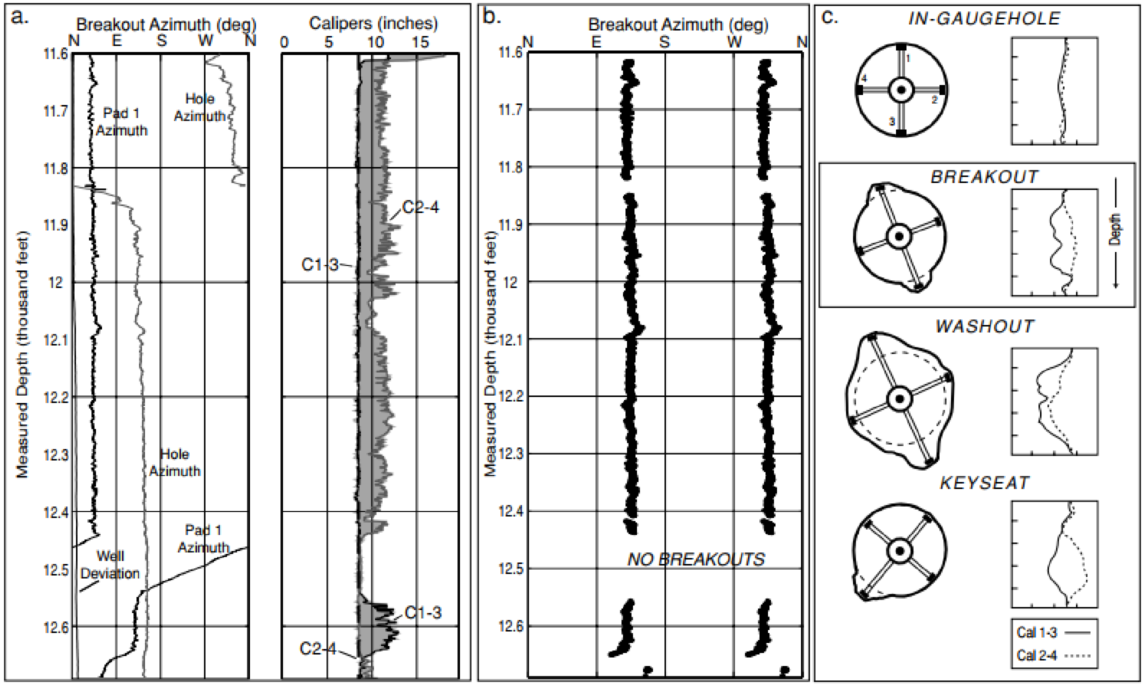
Determination of breakout orientation from caliper log



https://www.glossary.oilfield.slb.com

- Caliper log
  - continuous measurement of the size and shape of a borehole along its depth
  - Borehole break has to be distinguished from keyseat.
    - Keyset: grooves in the side of the well caused by the rubbing of pipe (asymmetrical). When there is rapid turn of well trajectory
    - Washout: enlargement of the entire wellbore (complete failure)





Rider M and Kennedy M, 2011, The geological interpretation of well logs, 3rd ed., Rider French

Determination of breakout orientation from caliper log

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- Example: Consistent stress direction mapped from caliper log

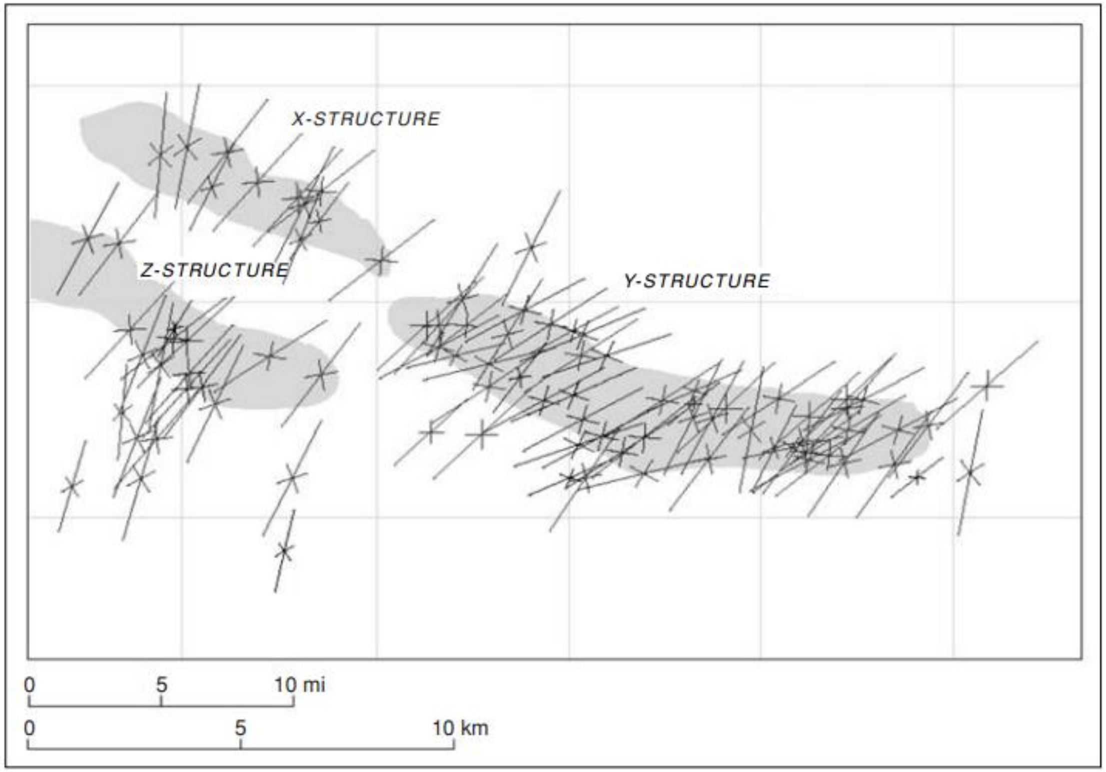

Figure 6.10. A detailed stress map of an oil field in an area of active faulting as determined with four-arm caliper data. The length of the arrow corresponds to the quality of the data as explained in the text. Note that while the stress field shows many local variations due to the processes associated with active faulting and folding, these variations are straightforwardly mappable (courtesy D. Castillo).

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Determination of breakout orientation from caliper log

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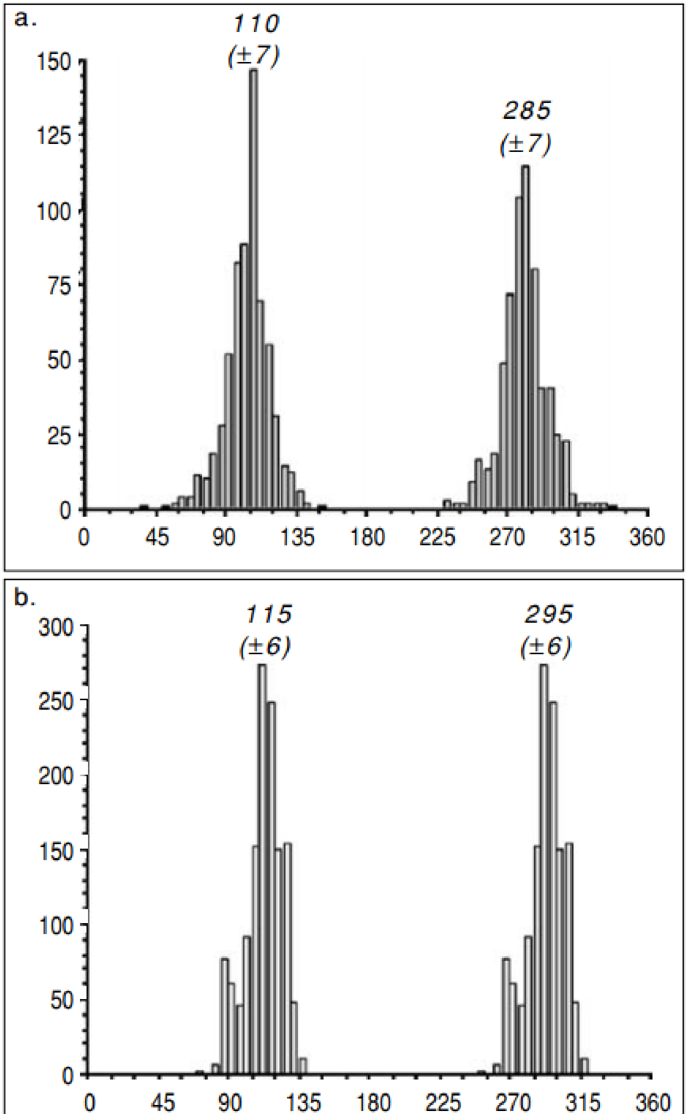
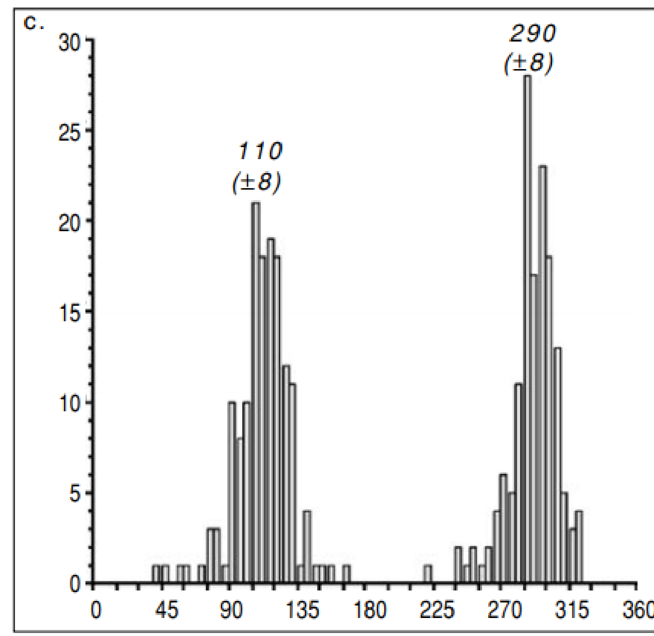


Comparison of different methods: Borehole televiewer, six arm caliper and electrical imaging



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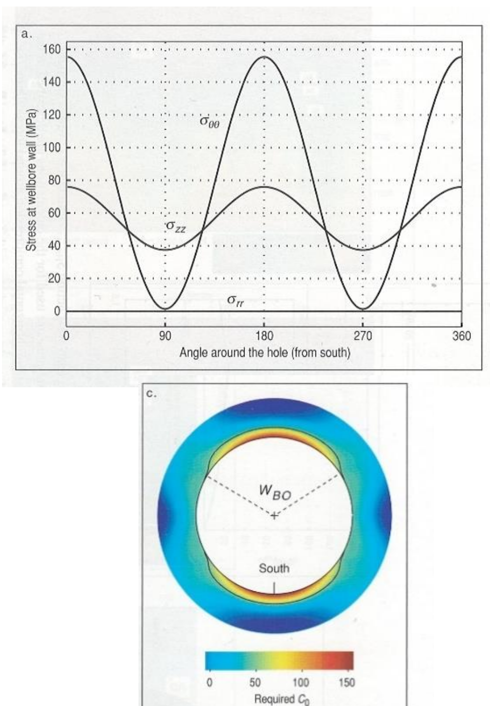
Drilling induced tensile fracture

- Drilling induced tensile fracture

$$\sigma_{\theta\theta}^{\min} = 3S_{\mathrm{hmin}} - S_{\mathrm{Hmax}} - 2P_0 - \Delta P - \sigma^{\Delta T}$$

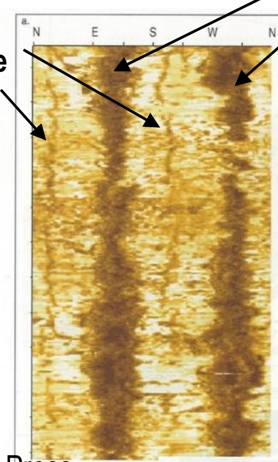
  - Local tension at the wellbore wall during drilling
  - Will not lead to hydraulic fracturing unless mud weights exceed the least principal stress
  - Wellbore image log is the only way to identify



ΔP=0 MPa → ΔP=10 MPa

Drilling induced Tensile fracture

Borehole breakout



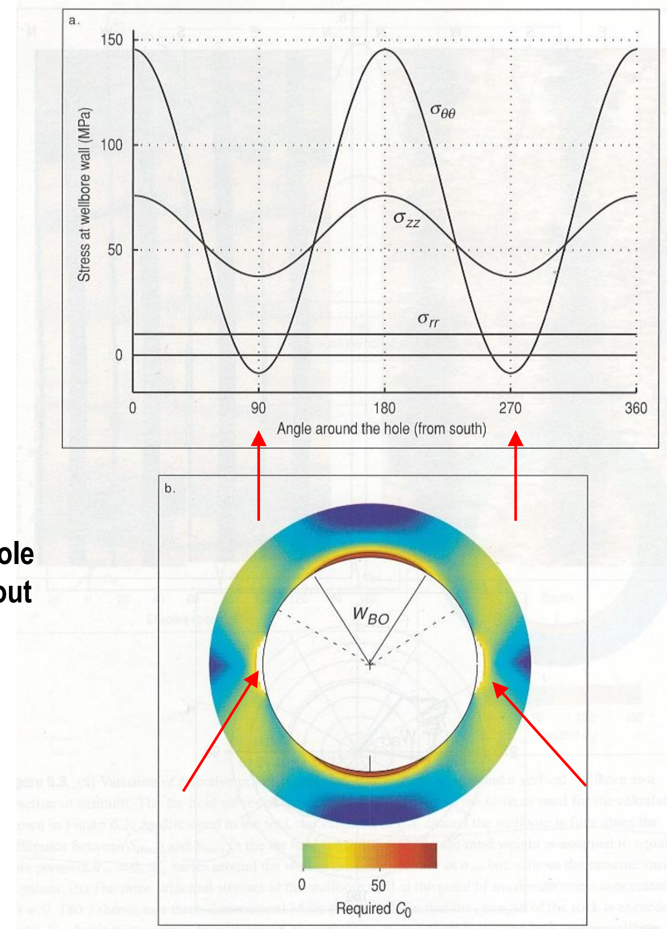


Figure 6.5. (a) Stress concentration at the wellbore wall and (b) zone of compressive failure around the wellbore (similar to Figure 6.3) when the mud weight has been raised 10 MPa above the mudweight. Figure 6.3b compares the width of breakouts for the two cases. Note that raising the mud weight decreases the size of the breakouts considerably. The area in white shows the region where tensile stresses exist at the wellbore wall.

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Drilling induced tensile fracture

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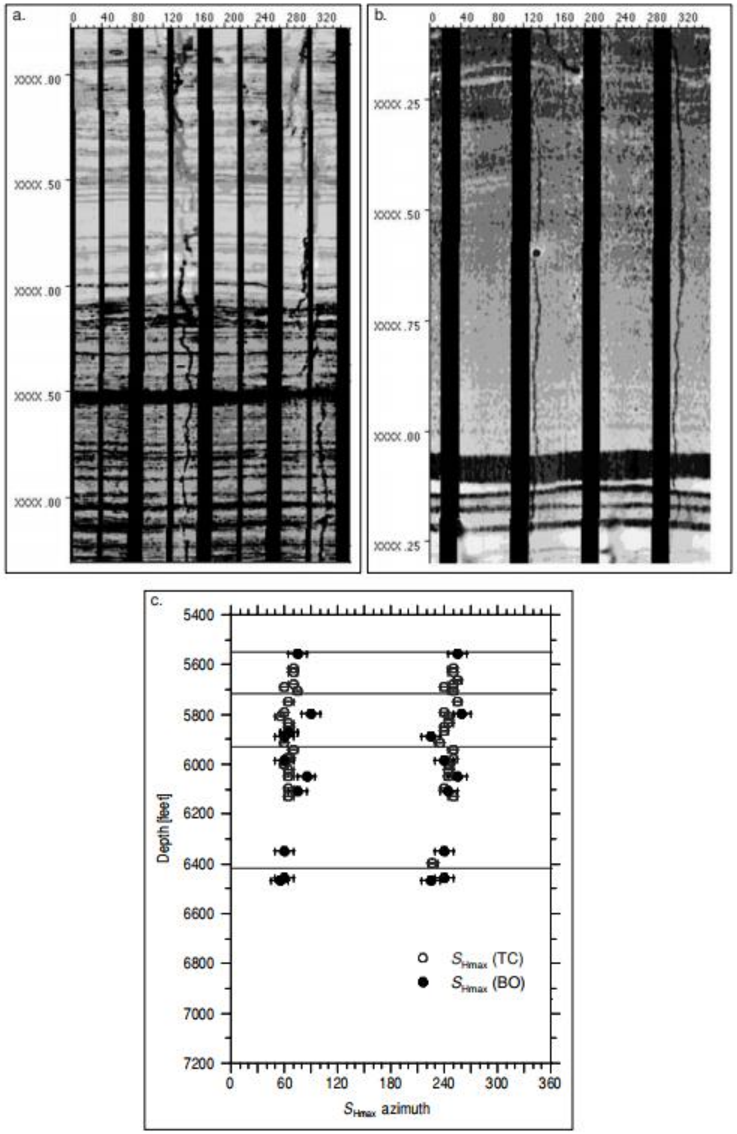
- Examples
  - Drilling induced tensile fractures by electrical image log
  - S_{Hmax} measured by drilling induced tensile frature matches with S_{Hmax} by borehole break



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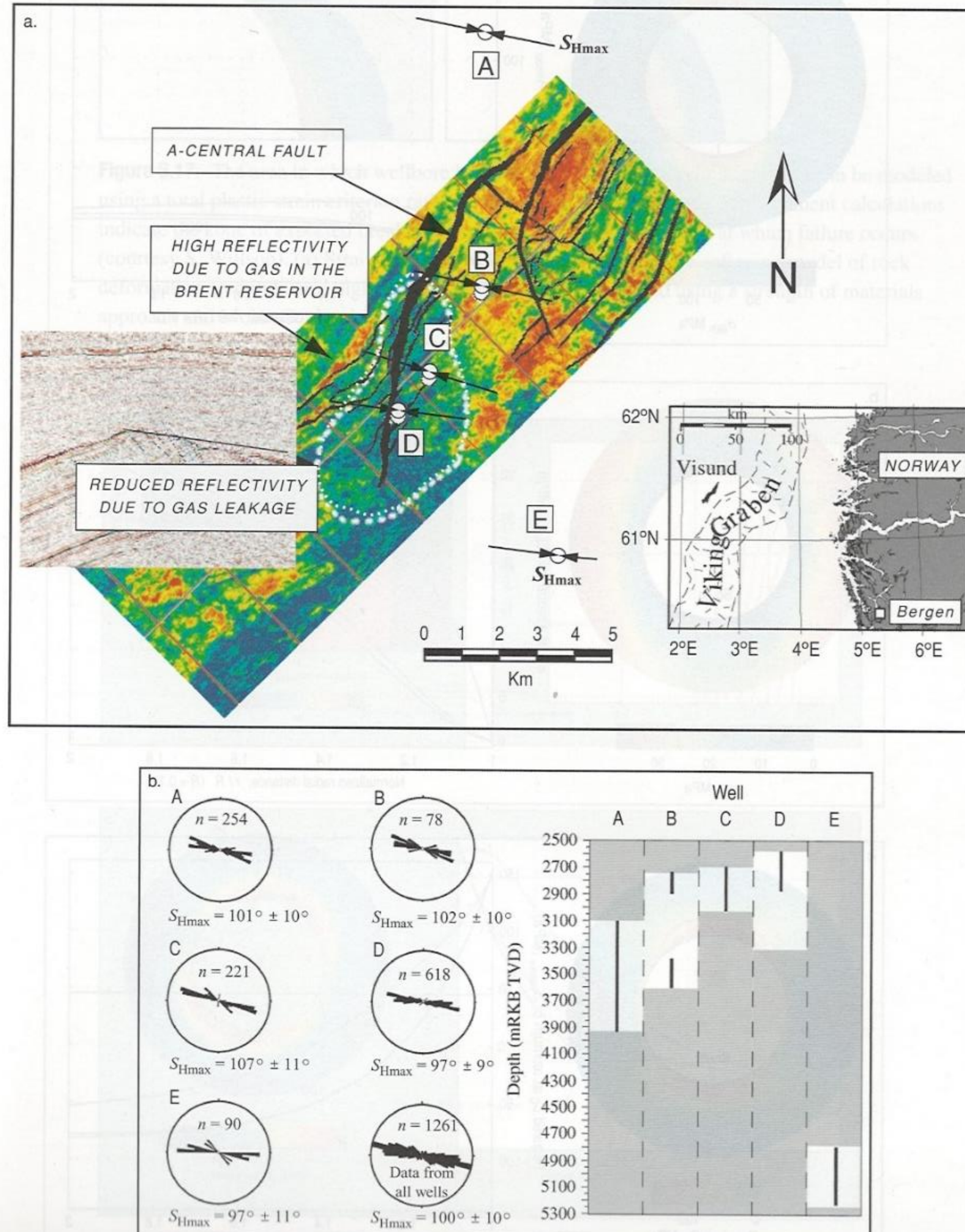
Drilling induced tensile fracture

- Examples North Sea
  - 1261 observations at five wells (depth 2,500 m – 5,200 m), up to ~ 20 km apart
  - Standard deviation: ~ 10°
  - Uniform stress fields are determined from drilling induced tensile failure



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Drilling induced tensile fracture
Observation at strike-slip faulting regime

- Tensile fracture and strike-slip faulting
  - At μ = 0.6

$$\frac{S_{\mathrm{Hmax}} - P_{\mathrm{p}}}{S_{\mathrm{hmin}} - P_{\mathrm{p}}} = (\sqrt{\mu^2 + 1} + \mu)^2 = 3.1 \qquad (6.16)$$

which can be simplified to

$$S_{\mathrm{hmax}} = 3.1 S_{\mathrm{Hmin}} - 2.1 P_{\mathrm{p}}$$

and, for reasons that will soon be evident, rewritten as

$$S_{\mathrm{Hmax}} = 3S_{\mathrm{hmin}} - 2P_{\mathrm{p}} + 0.1(S_{\mathrm{hmin}} - P_{\mathrm{p}}) \qquad (6.17)$$

  - Minimum hoop stress

$$\sigma_{\theta\theta}^{\mathrm{min}} = 3S_{\mathrm{hmin}} - S_{\mathrm{Hmax}} - 2P_0 - \Delta P - \sigma^{\Delta T}$$

    - When cooling, and Δp are negligible

$$\sigma_{\theta\theta}^{\mathrm{min}} = 3S_{\mathrm{hmin}} - S_{\mathrm{Hmax}} - 2P_{\mathrm{p}} = 0 \qquad (6.18)$$

or

$$S_{\mathrm{Hmax}} = 3S_{\mathrm{hmin}} - 2P_{\mathrm{p}} \qquad (6.19)$$

  - In, strike-slip faulting stress regime, wellbore wall will go into tension at the azimuth of SHmax even without high wellbore pressure or cooling effect

Drilling induced tensile fracture

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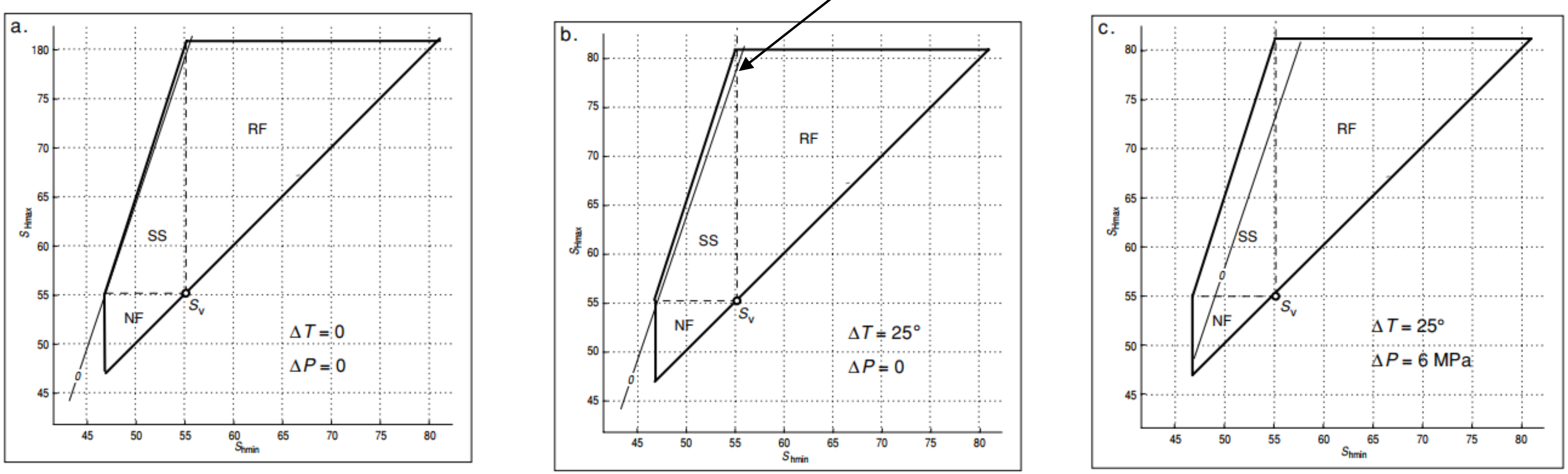
- Condition of tensile failure in relation to stress regime

$$\sigma_{\theta\theta}^{\min} = 3S_{\mathrm{hmin}} - S_{\mathrm{Hmax}} - 2P_0 - \Delta P - \sigma^{\Delta T}$$



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Quality ranking system for stress indicators

- Four types of stress indicators (indicated at depths)
  - Earthquake focal mechanism: 3~15 km
  - Stress-induced wellbore breakout: 1~4 km
  - Drilling-induced tensile fracture
  - Open hole hydraulic fracturing – usually at shallow depth (not in petroleum/geothermal)
- Quality of stress indicators: A, B, C & D (D is not acceptable)
  - The greater the depth interval where wellbore observations are made
  - The larger the number of observation
  - The smaller the standard deviation

Quality ranking system for stress indicators

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Table 6.1. *Quality ranking system*

	A	B	C	D
Earthquake focal mechanisms	Average P-axis or formal inversion of four or more single-event solutions in close geographic proximity(at least one event $M \geq 4.0$, other events $M \geq 3.0$)	Well-constrained single-event solution ($M \geq 4.5$) or average of two well-constrained single-event solutions ($M \geq 3.5$) determined from first motions and other methods (e.g. moment tensor wave-form modeling, or inversion)	Single-event solution (constrained by first motions only, often based on author'squality assignment) ($M \geq 2.5$). Average of several well-constrained composites ($M \geq 2.0$)	Single composite solution. Poorly constrained single-event solution. Single-event solution for $M < 2.5$ event
Wellbore breakouts	Ten or more distinct breakout zones in a single well with sd $\leq 12°$ and/or combined length >300 m. Average of breakouts in two or more wells in close geographic proximity with combined length >300 m and sd $\leq 12°$	At least six distinct breakout zones in a single well with sd $\leq 20°$ and/or combined length > 100 m	At least four distinct breakouts with sd $< 25°$ and/or combined length > 30 m	Less than four consistently oriented breakout or >30 m combined length in a single well. Breakouts in a single well with sd $\geq 25°$
Drilling-induced tensile fractures	Ten or more distinct tensile fractures in a single well with sd $\leq 12°$ and encompassing a vertical depth of 300 m, or more	At least six distinct tensile fractures in a single well with sd $\leq 20°$ and encompassing a combined length > 100 m	At least four distinct tensile fractures with sd $< 25°$ and encompassing a combined length > 30 m	Less than four consistently oriented tensile fractures with <30 m combined length in a single well. Tensile fracture orientations in a single well with sd $\geq 25°$
Hydraulic fractures	Four or more hydrostatic orientations in a single well with sd $\leq 12°$ depth >300 m. Average of hydrofrac orientations for two or more wells in close geographic proximity, sd $\leq 12°$	Three or more hydrofrac orientations in a single well with sd $< 20°$. Hydrofrac orientations in a single well with $20° <$ sd $< 25°$	Hydrofac orientations in a single well with $20° <$ sd $< 25°$. Distinct hydrofrac orientation change with depth, deepest measurements assumed valid. One or two hydrofrac orientations in a single well	Single hydrofrac measurements at <100 m depth

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Quality ranking system for stress indicators

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- Mean borehole breakout direction
  - Use cosine & sine (instead of direction directly, why?)

To calculate the mean breakout direction, θ_{m}, and the standard deviation, sd, of a set of breakouts on a given side of a well, we utilize Fisher statistics and let θ_i ($i = 1, \ldots, N$) denote the observed breakout directions in the range 0–360°. First, we define

$$l_i = \cos\theta_i \quad \text{and} \quad m_i = \sin\theta_i \tag{6.10}$$

$$l = \frac{\sum_{i=1}^{n} l_i}{R} \quad \text{and} \quad m = \frac{\sum_{i=1}^{n} m_i}{R} \tag{6.11}$$

$$R^2 = \sum_{i=1}^{n} l_i + \sum_{i=1}^{n} m_i \tag{6.12}$$

The mean breakout direction is given by

$$\theta_m = \tan^{-1}\left(\frac{m}{l}\right) \tag{6.13}$$

We define

$$k = \frac{N-1}{N-R} \tag{6.14}$$

such that the standard deviation is given by

$$\mathrm{sd} = \frac{81°}{\sqrt{k}} \tag{6.15}$$

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Thermal effect

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- Can be important when wellbore fluid is at a significantly different than the rock
- Cooler mud pressure generate tensile stress at the wall

Transient solution

$$\sigma_{\theta\theta} = \left[\frac{\alpha_t E \Delta T}{1-\nu}\right]\left[\left(\frac{1}{2\rho} - \frac{1}{2} - \ln\rho\right) I_0^{-1} - \left(\frac{1}{2} + \frac{1}{2\rho}\right)\right] \tag{6.20}$$

$$\sigma_{rr} = \left[\frac{\alpha_t E \Delta T}{1-\nu}\right]\left[\left(-\frac{1}{2\rho} + \frac{1}{2} - \ln\rho\right) I_0^{-1} - \left(\frac{1}{2} - \frac{1}{2\rho}\right)\right] \tag{6.21}$$

$$I_0^{-1} = \frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{0+} \frac{\mathrm{e}^{[4\tau_z/\sigma_2]^z}}{z \ln z} \mathrm{d}z$$

Once steady state has been reached, the change in the hoop stress is given by

$$\sigma_{\theta\theta}^{\Delta T} = \frac{\alpha_t E \Delta T}{1-\nu} \tag{6.22}$$

$\longleftarrow$ Steady state solution

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Thermal effect

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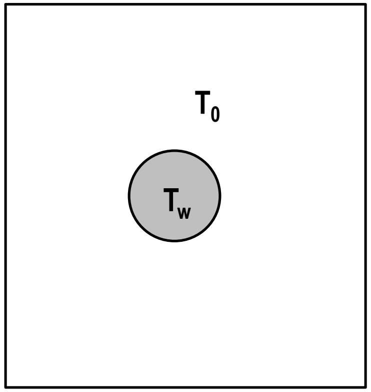
- Effect of temperature change - thermal stress
  - Thermally induced hoop stress at the wall of cylindrical borehole

$$\sigma_r = 0$$

$$\sigma_\theta = \frac{E}{1-\nu}\alpha\left(T_w - T_0\right)$$

$$\tau_{r\theta} = 0$$



α: linear thermal expansion coefficient
E: Elastic Modulus
T_w: well temperature
T_0: reservoir temperature

$$\sigma_{\theta\theta}^{\Delta T} = \frac{\alpha_t E \Delta T}{1-\nu}$$

Silica content is critical for thermal expansion coefficient

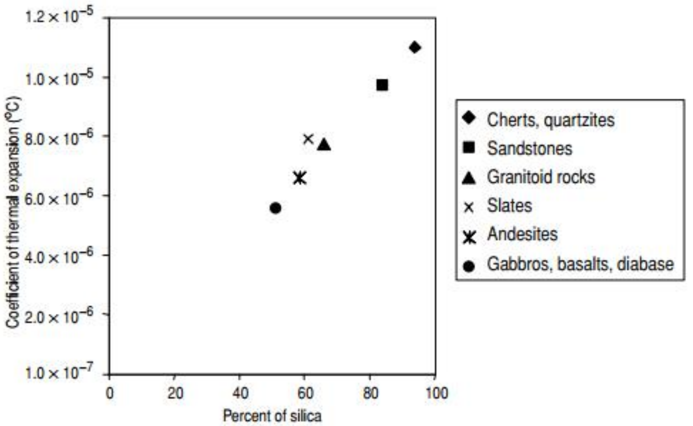


Figure 3.14. Measurements of the coefficient of linear thermal expansion for a variety of rocks as a function of the percentage of silica (data from Griffith 1936). As the coefficient of thermal expansion of silica ($\sim 10^{-5}$ °C^{-1}) is an order of magnitude higher than that of most other rock forming minerals ($\sim 10^{-6}$ °C^{-1}), the coefficient of thermal expansion ranges between those two amounts, depending on the percentage of silica.

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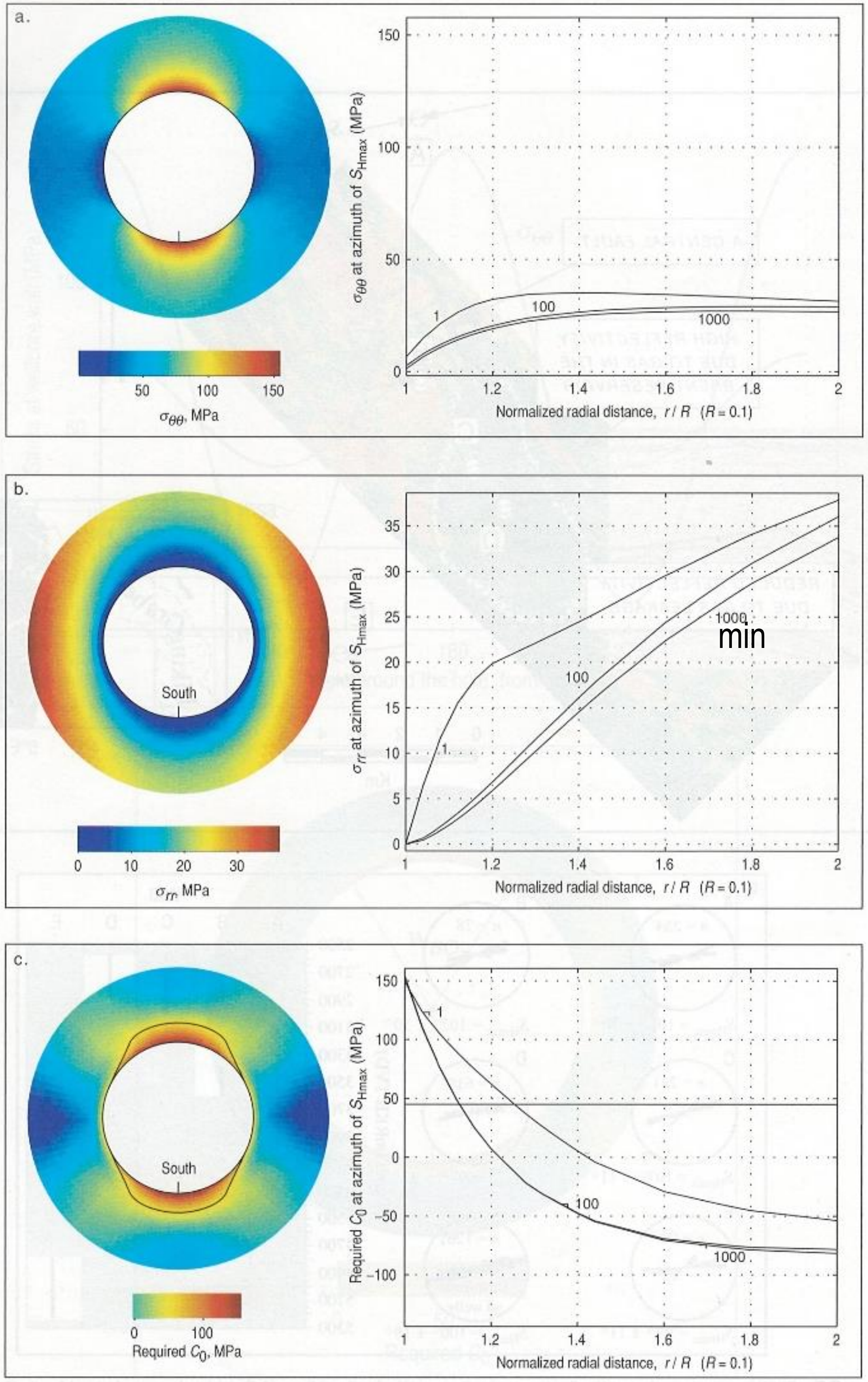
Thermal effect

- Expected time to reach steady state



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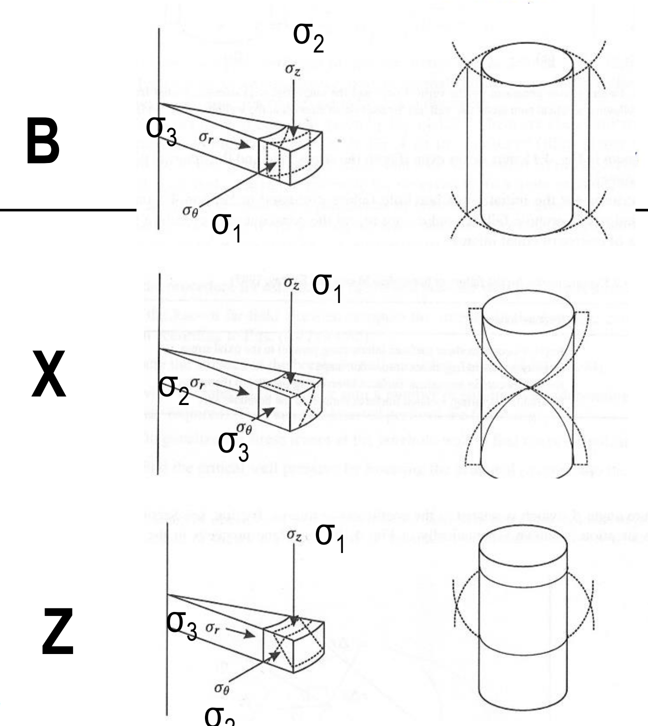
Multiple Mode of compressive wellbore failure

- Assumption in wellbore breakout about the principal stress:
  - (hoop stress is the maximum principal stress, radial stress is the least principal stress).
- Other modes can cause compressive wellbore failure



Fjaer E et al., 2008

Table 6.2. *Multiple modes of compressive wellbore failure*

Mode	σ_1	σ_2	σ_3	Comment
B	$\sigma_{\theta\theta}$	σ_{zz}	σ_{rr}	Conventional breakout
X	σ_{zz}	σ_{rr}	$\sigma_{\theta\theta}$	Forms on opposite side of well as a conventional breakout but the failed rock will not fall into the wellbore as $\sigma_{rr} \equiv \sigma_2$
Z	σ_{zz}	$\sigma_{\theta\theta}$	σ_{rr}	Results in failure all the way around the wellbore → washout
X2	$\sigma_{\theta\theta}$	σ_{rr}	σ_{zz}	Requires high mud weights. Failed rock will not fall into the wellbore as $\sigma_{rr} \equiv \sigma_2$
R1	σ_{rr}	σ_{zz}	$\sigma_{\theta\theta}$	Requires unreasonably high mud weights
R2	σ_{rr}	$\sigma_{\theta\theta}$	σ_{zz}	Requires unreasonably high mud weights

Fjaer E et al., 2008, Petroleum-related Rock Mechanics, 2nd Ed., Elsevier