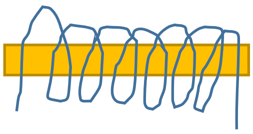
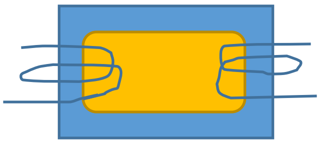
Lecture 8: Inductor and RLC filter

Inductor

Coil wound on various core materials such as iron, iron alloys, ferrite

to multiply the inductance of a given coil by permeability of core material. It is to store the electric energy into the inductor's magnetic field





Symbol L[H]

V 

Law

V=Ldi/dt or i=∫Vdt/L or L=V/(di/dt)

Where V=Across voltage, i=current flow, L=Inductance [H]

Let alternating voltage signal, $V=V_o\exp(j\omega t)$,

then $i=\int Vdt/L=V_o\exp(j\omega t)/j\omega L=V/j\omega L$

Thus Impedance, $Z=V/i=j\omega L$ [Ω] for alternating voltage or current

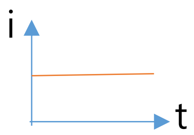
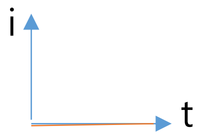
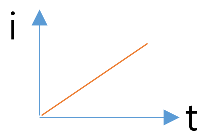
Application: Transformer (primary coil->magnetic field->secondary coil)

Filters, RF circuit, etc

Comparison between Resistor/Capacitor/Inductor

Resistor(R)	Capacitor(C)	Inductor(L)
		
Energy into	Energy into	Energy into
Heat dissi.	Electric field	Magnetic field
$i=V/R$	$i=CdV/dt$	$i=\int Vdt/L$

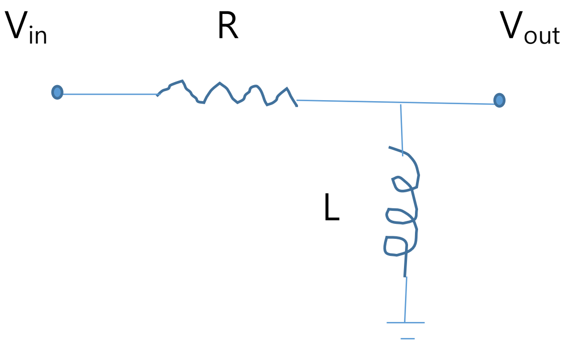
(When V=const.)

		
(Proportional)	(Derivative)	(Integral)
Impedance	Impedance	Impedance
$Z=R$	$Z=1/j\omega C$	$Z=j\omega L$

The above three elements are complimentary each other, thus can be used for variety of application.

RL filter

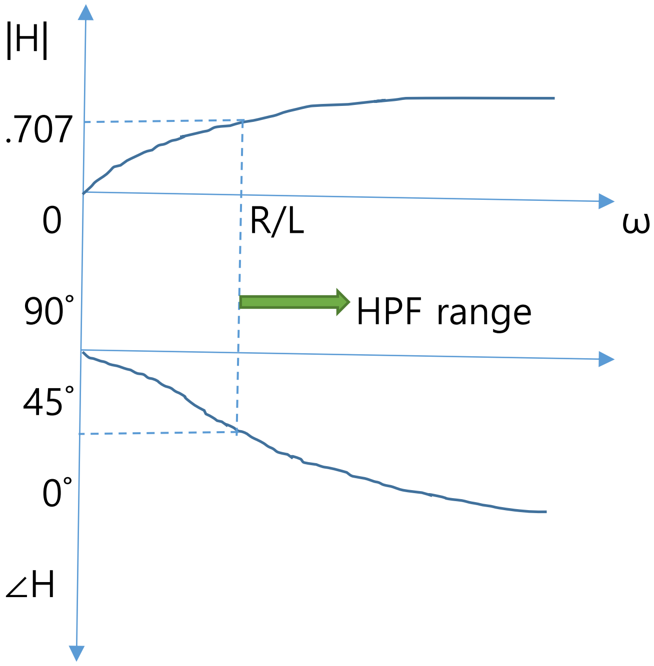
High Pass Filter



$H=V_{out}/V_{in}=Z_L/(Z_R+Z_L)=j\omega L/(R+j\omega L)$

Magnitude$=|H|=\omega L/\sqrt{R^2+(\omega L)^2}$

Phase$=\angle H=90^\circ - \tan^{-1}(\omega L/R)$

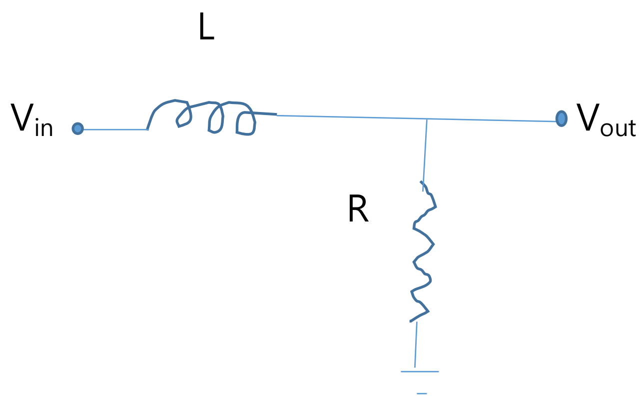


∴ This is a HPF with $\omega_{3dB}=R/L$

Thevenin's equivalent impedance, $Z_{th}=Z_R \parallel Z_L=j\omega LR/(j\omega L+R)$,

and $|Z_{th}|=\omega LR/\sqrt{(\omega L)^2+R^2}\leq R$ (at $\omega=\infty$)

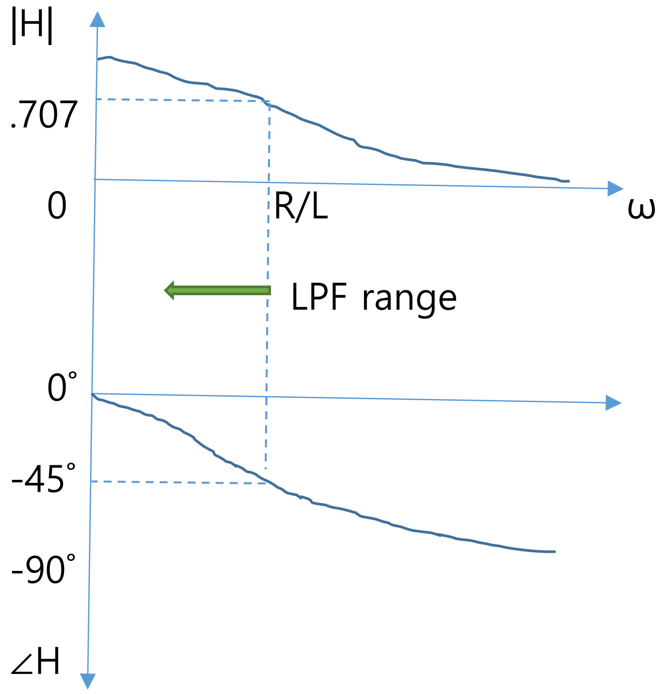
Low Pass Filter



$H=V_{out}/V_{in}=Z_R/(Z_R+Z_L)=R/(R+j\omega L)$

Magnitude$=|H|=R/\sqrt{R^2+(\omega L)^2}$

Phase$=\angle H=-\tan^{-1}(\omega L/R)$



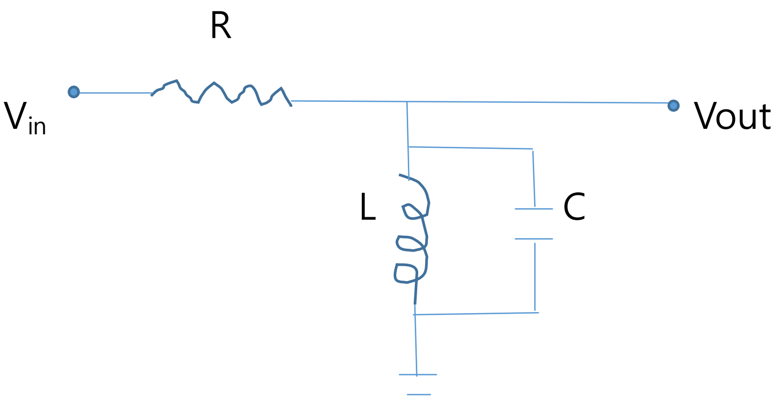
∴ This is a LPF with $\omega_{3dB}=R/L$

Thevenin's equivalent impedance, $Zth=Z_L \| Z_R=j\omega LR/(j\omega L+R)$,

and $|Zth|=\omega LR/\sqrt{\{(\omega L)^2+R^2\}}\leq R$ (at $\omega=\infty$)

RLC filter

:Narrow Bandpass Filter



The impedance of LC parallel connection becomes $Z_L \| Z_C$, and

$Z_L \| Z_C = Z_L Z_C/(Z_L+Z_C) = j\omega L/j\omega C/(j\omega L+1/j\omega C)$

$= j\omega L/(1-\omega^2 LC)$

$H = V_{out}/V_{in} = Z_L \| Z_C/(Z_R + Z_L \| Z_C)$

$= j\omega L/(1 - \omega^2 LC)/\{R + j\omega L/(1 - \omega^2 LC)\}$

$= j\omega L/\{R(1 - \omega^2 LC)+j\omega L\}$

Magnitude, $|H| = \omega L/\sqrt{\{ R^2(1 - \omega^2 LC)^2+(\omega L)^2\}}$

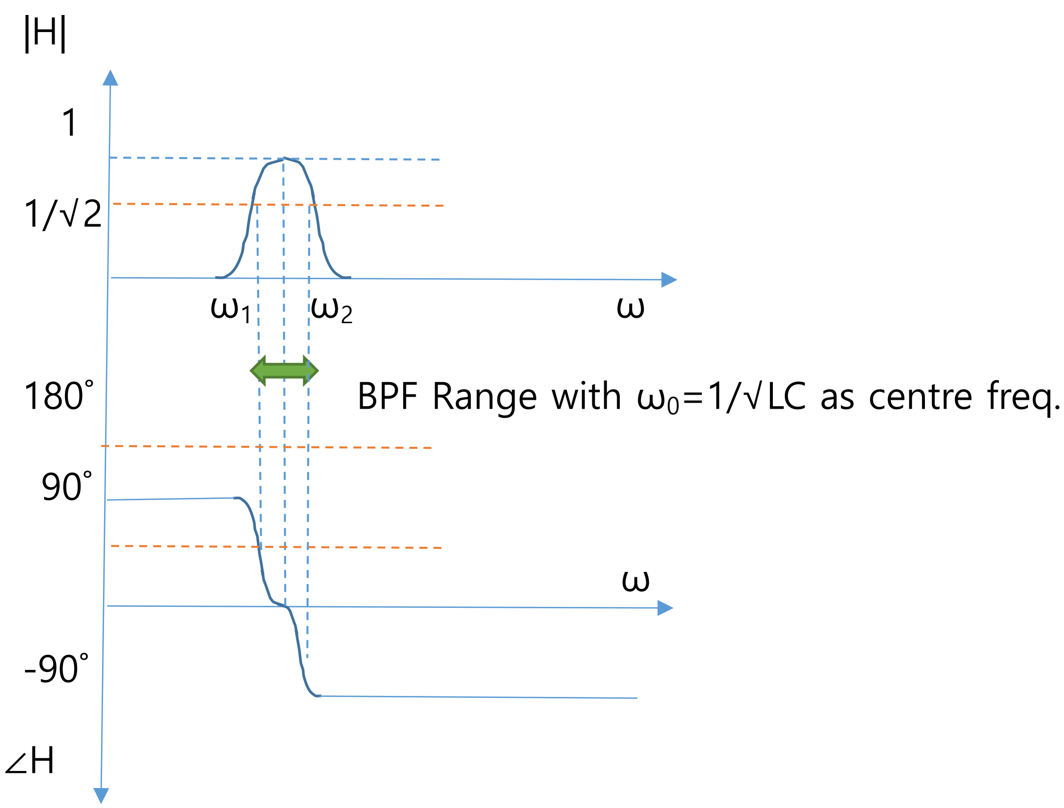
Phase, $\angle H = \angle(j\omega L) - \angle\{R(1 - \omega^2 LC)+j\omega L\}$

Plot for performance

If $\omega \ll 1/\sqrt{LC}$ then $|H| \fallingdotseq 0$ and $\angle H \fallingdotseq 90^\circ - 0^\circ = 90^\circ$

If $\omega = 1/\sqrt{LC}$ then $|H| = 1$ and $\angle H = 90^\circ - 90^\circ = 0^\circ$

If $\omega \gg 1/\sqrt{LC}$ then $|H| \fallingdotseq 0$ and $\angle H \fallingdotseq 90^\circ - 180^\circ = -90^\circ$



This is the <u>Narrow Band Pass Filter</u> that gives much narrower BPF with centre frequency $\omega_0=1/\sqrt{LC}$

The *3dB frequency* ω_1 and ω_2 can be evaluated as the freqs that gives the height of $1/\sqrt{2}$, by solving the equation

$|H|=\omega L/\sqrt{\{R^2(1 - \omega^2 LC)^2+(\omega L)^2\}}=1/\sqrt{2}$, and if $0<\omega_1<\omega_2$ then

$\omega_1=\{-L+\sqrt{(L^2+4R^2LC)}\}/2RLC$ and $\omega_2=\{L+\sqrt{(L^2+4R^2LC)}\}/2RLC$

Thus $\Delta\omega=\omega_2-\omega_1=2L/2RLC=1/RC\equiv\Delta\omega_{3dB}$

And Quality Factor, Q is defined as follows;

$Q\equiv\omega_0/\Delta\omega_{3dB}=\omega_0 RC=$ Measure of Profile Sharpness=10~50, typically,

and it is one of meaningful design parameters

Thevenin's equivalent impedance, Zth, for the narrow BPF is,

$Zth=Z_R \parallel Z_L \parallel Z_C=Z_R \parallel (Z_L \parallel Z_C)=Z_R(Z_L \parallel Z_C)/(Z_R+Z_L \parallel Z_C)$

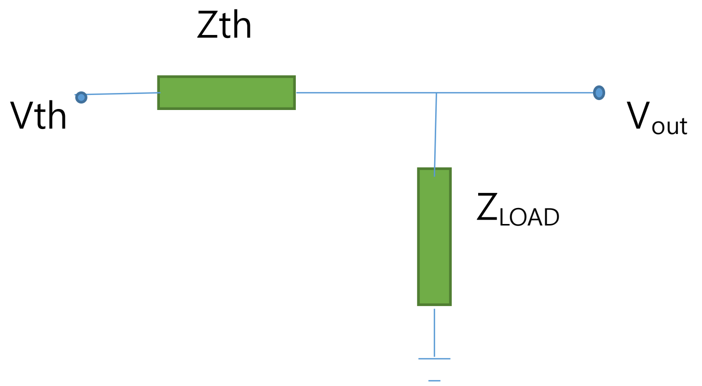
and $Z_L \parallel Z_C=Z_LZ_C/(Z_L+Z_C)= j\omega L/(1-\omega^2LC)$, thus

$Zth=\{j\omega RL/(1-\omega^2LC)\}/\{R+ j\omega L/(1-\omega^2LC)\}$

$=j\omega RL/\{R(1-\omega^2LC)+j\omega L\}$

∴ Magnitude $|Zth|=\omega RL/\sqrt{\{R^2(1-\omega^2LC)^2+(\omega L)^2\}} \leq R$ (at $\omega=1/\sqrt{LC}$)

Thevenin's equivalent circuit is as follows when Z_{LOAD} is applied,



Zth should drive Z_{LOAD}, and $Zth \ll Z_{LOAD}$ is to be satisfied. Therefore,

$R=R_{LOAD}/10$ from the 10X rule eq(1)

ω_0=centre frequency$=1/\sqrt{LC}$ eq(2)

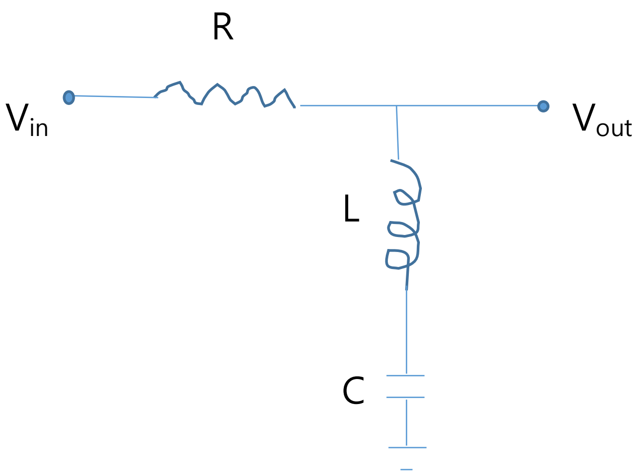
$Q=\omega_0/\Delta\omega_{3dB}=\omega_0RC$=Profile Sharpness eq(3)

=10 to 50 in practice (Design parameter)

With the above 3 equations, the R,L,C components can be determined.

Note that bigger R gives sharper profile, when L,C are fixed.

RLC notch filter



Serial connection of L and C gives the impedance of Z_L+Z_C

$=j\omega L+1/j\omega C$

$H=V_{out}/V_{in}=(Z_L+Z_C)/(Z_R+Z_L+Z_C)$

$=(j\omega L+1/j\omega C)/(R+j\omega L+1/j\omega C)$

$=(1-\omega^2 LC)/\{(1-\omega^2 LC)+j\omega RC\}$

Magnitude$=|H|=(1-\omega^2 LC)/\surd\{(1-\omega^2 LC)^2+(\omega RC)^2\}$

Phase$=\angle H=\angle(1-\omega^2 LC)-\angle\{(1-\omega^2 LC)+j\omega RC\}$

For plot of performance,

If $\omega \ll 1/\surd LC$ then $|H|\fallingdotseq 1$ and $\angle H\fallingdotseq 0^\circ-0^\circ=0^\circ$

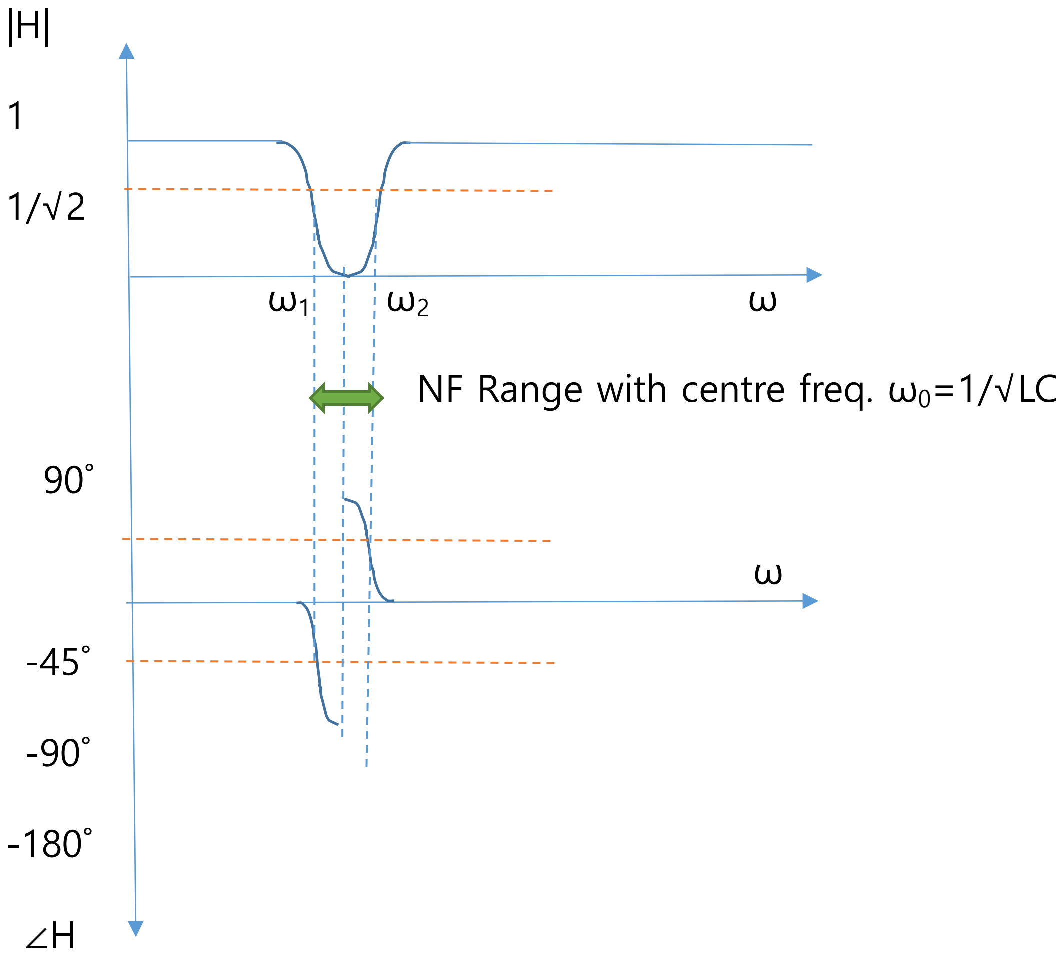
If $\omega={}^{-}1/\surd LC$ then $|H|=0$ and $\angle H=0^\circ-90^\circ=-90^\circ$

If $\omega={}^{+}1/\surd LC$ then $|H|=0$ and $\angle H=180^\circ-90^\circ=90^\circ$

If $\omega \gg 1/\surd LC$ then $|H|\fallingdotseq 1$ and $\angle H\fallingdotseq 180^\circ-180^\circ=0^\circ$

($^{-}$ indicates the slightly less, $^{+}$ indicates the slightly bigger)

∴ This is the notch filter or to trap with the centre frequency ω_0



The 3dB frequency ω_1 and ω_2 can be evaluated as the freqs that gives the height of $1/\sqrt{2}$, by solving the equation

$|H|=(1-\omega^2LC)/\sqrt{\{(1-\omega^2LC)^2+(\omega RC)^2\}}=1/\sqrt{2}$, and if $0<\omega_1<\omega_2$ then

$\omega_1=\{-RC+\sqrt{(R^2C^2+4LC)}\}/2LC$ and $\omega_2=\{+RC+\sqrt{(R^2C^2+4LC)}\}/2LC$

Thus $\Delta\omega=\omega_2-\omega_1=2RC/2LC=R/L\equiv\Delta\omega_{3dB}$

And Quality Factor, Q is defined as follows;

$Q \equiv \omega_0/\Delta\omega_{3dB} = \omega_0 L/R$ = Measure of Profile Sharpness, 10~50, typically and it is one of meaningful design parameters.

Thevenin's equivalent impedance Zth for the Notch filter is,

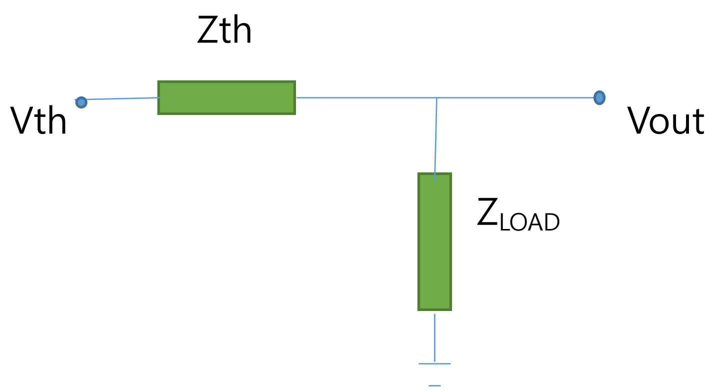
$Zth = Z_R \parallel (Z_L + Z_C) = Z_R(Z_L + Z_C)/(Z_R + Z_L + Z_C)$

$= R(j\omega L + 1/j\omega C)/\{R + j\omega L + 1/j\omega C\}$

$= R(1-\omega^2 LC)/\{(1-\omega^2 LC) + j\omega RC\}$

$\therefore$Magnitude, $|Zth| = R(1-\omega^2 LC)/\sqrt{\{(1-\omega^2 LC)^2 + (\omega RC)^2\}} \le R$ (at $\omega = 0$ or ∞)

Thevenin's equivalent circuit is as follows when Z_{LOAD} is applied,



Zth should drive Z_{LOAD}, and Zth ≪ Z_{LOAD} is to be satisfied. Therefore,

$R = R_{LOAD}/10$ from the 10X rule eq(4)

ω_0=centre frequency=$1/\sqrt{LC}$ eq(5)

$Q = \omega_0/\Delta\omega_{3dB} = \omega_0 L/R$=Profile Sharpness eq(6)

=10 to 50 in practice (Design parameter)

With the above 3 equations, the R,L,C components can be determined.

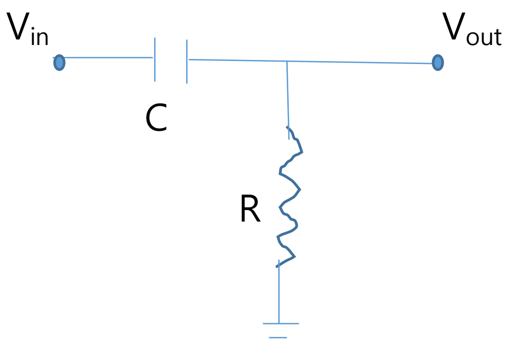
Note that smaller R gives sharper profile, when L,C are fixed.

HW5) Design a Narrow Band Pass Filter as follows;

Centre freq=5KHz, Quality factor=20, R_{LOAD}=200KΩ

<u>RC Circuits revisited</u>

1) <u>HPF as Differentiator</u>



$V_{out}/V_{in}=R/(1/j\omega C+R)=j\omega RC/(1+j\omega RC)$

If RC≪1 then jωRC is very small

From Taylor's expansion formula,

$1/(1+j\omega RC) \fallingdotseq 1-j\omega RC+(j\omega RC)^2-(jwRC)^3\ldots$

Thus $V_{out}/V_{in}=j\omega RC(1-j\omega RC+(j\omega RC)^2\ldots)$

$=j\omega RC-(j\omega RC)^2+(j\omega RC)^3-\ldots$

$\fallingdotseq j\omega RC$ (∵ Higher order term can be very small)

Thus $V_{out} \fallingdotseq j\omega RCV_{in}$

As V_{in} can be generally expressed as $V\exp(j\omega t)$,

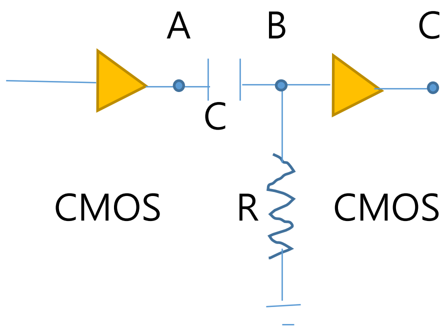
$dV_{in}/dt=j\omega V\exp(j\omega t)=j\omega V_{in}$, therefore,

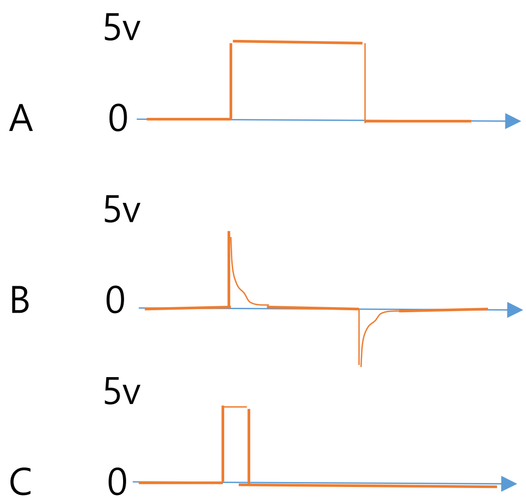
$V_{out} \fallingdotseq j\omega RCV_{in}=RCdV_{in}/dt$ if RC≪ 1 or 1/RC is very big.

∴ <u>HPF can be a differentiator if ω_{3dB}=1/RC is very high</u>

For a practical example, R=10KΩ and C=0.01μF in the above HPF design gives RC=1.0E-4 (i.e. f_{3dB}≒1600Hz) as quite small and can be used as a differentiator. This observation is really wonderful as the differentiator can be easily implemented with the HPF.

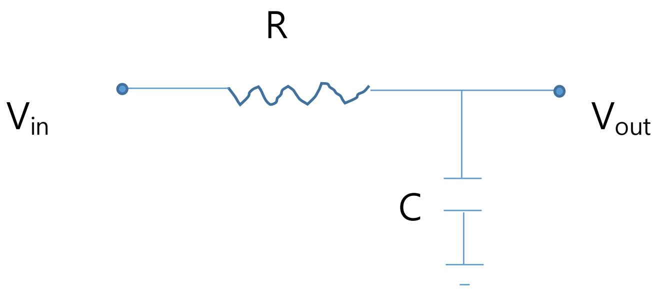
The following demonstrates a good example for the leading edge detector in CMOS circuit with C=100pF and R=10KΩ (or RC=1.0E-6)





(Q: What about trailing edge detector?)

2) <u>LPF as Integrator</u>



$V_{out}/V_{in}=1/j\omega C/(R+1/j\omega C)=1/(1+j\omega RC)$

If RC is quite big such as $RC \gg 1$, then $1/(1+j\omega RC) \fallingdotseq 1/j\omega RC$

Thus $V_{out}/V_{in} \fallingdotseq 1/j\omega RC$, and $V_{out} \fallingdotseq V_{in}/j\omega RC$

Remembering $V_{in}=V\exp(j\omega t)$, then $\int V_{in}dt=V\exp(j\omega t)/j\omega=V_{in}/j\omega$

Thus $V_{out} \fallingdotseq V_{in}/j\omega RC=\int V_{in}dt/RC$ if $RC \gg 1$ or $\omega_{3dB}=1/RC \ll 1$

∴ <u>LPF can be a Integrator if $\omega_{3dB}=1/RC$ is quite low</u>

For a LPF with R=100K, C=100μF, then RC=10, and $f_{3dB} \fallingdotseq 0.016$Hz. Thus it can be used as Integrator.

HPF/LPF based Integrator/Differentiator is quite simple to implement, but it needs some care for the assumption.

->OP amp based Differentiator/Integrator will give wider application.