SEOUL NATIONAL UNIVERSITY

2상유동 열전달 공학

Two-phase flow and heat transfer Engineering

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Two-phase flow models

❖ Void fraction

Models	Assumptions	Application
Homogeneous model	• Equal vapor & liquid velocities • Thermodynamic equil. between phases	• Dispersed flow • High-speed flow
Separated flow model	• Different velocities for both phases • Thermodynamic non-equil. between phases	• Annular flow • Horizontal separated flow
Drift flux model	• Mixture momentum equation • Relative velocity + concentration & velocity profiles	• Bubbly, slug & churn flow • Counter-current flow
Two-fluid model	• Two sets of balance equations • Thermodynamic equilibrium within phases • Detailed model for interface interactions	• General
Three-fluid model	• Different treatment of droplet fluid from the liquid film	• General (in particular annular dispersed flow)

One-Dimensional Homogeneous-Equilibrium Model

- Heuristic derivation

- Simplest of all two-phase flow models

- Assumptions
  - ✓ The two phases are assumed to be
    - Well mixed
    - Same velocities
    - At thermal equilibrium
      - – For steam-water flow: saturated mixture

$$U_G = U_L$$

$$T_G = T_L = T_{sat}$$

  - ✓ Mixture is treated as a single fluid.

  - ✓ Useful for high-pressure and high flow rate conditions
    - Very small diameter of dispersed phase
    - Evenly distributed along the radius

One-Dimensional Homogeneous-Equilibrium Model

- All variables are averaged!

$$G = \langle G \rangle \qquad j =< j > \qquad U_G =< U_G >_G \qquad \rho_G =< \rho_G >_G$$

- Mixture density

$$j = j_G + j_L \qquad G = G_G + G_L \qquad G_G = \rho_G j_G \qquad G_L = \rho_L j_L$$

$$\bar{\rho} = G / j = \frac{\rho_G \alpha_G U_G + \rho_L \alpha_L U_L}{\alpha_G U_G + \alpha_L U_L} = \frac{\rho_G \alpha_G + \rho_L \alpha_L}{\alpha_G + \alpha_L} = \rho_G \alpha + \rho_L (1 - \alpha)$$

$$\bar{v} = \left[(1 - x) v_L + x v_G \right] = 1 / \bar{\rho} \qquad \Rightarrow \qquad \bar{\rho} = \left[v_L + x (v_G - v_L) \right]^{-1}$$

One-Dimensional Homogeneous-Equilibrium Model

❖ Quality vs. void fraction

$$G_G = x \cdot G = \rho_G j_G = \rho_G \alpha_G U_G$$

$$G_L = (1-x) \cdot G = \rho_L j_L = \rho_L (1-\alpha) U_L$$

$$\frac{x}{1-x} = \frac{\rho_G \alpha}{\rho_L (1-\alpha)}$$

$$x = \frac{G_G}{G} \qquad 1-x = \frac{G_L}{G}$$

$$\alpha = \frac{1}{1 + \left(\frac{1-x}{x}\right)\frac{\rho_G}{\rho_L}}$$

Void-quality relation for homogeneous flow

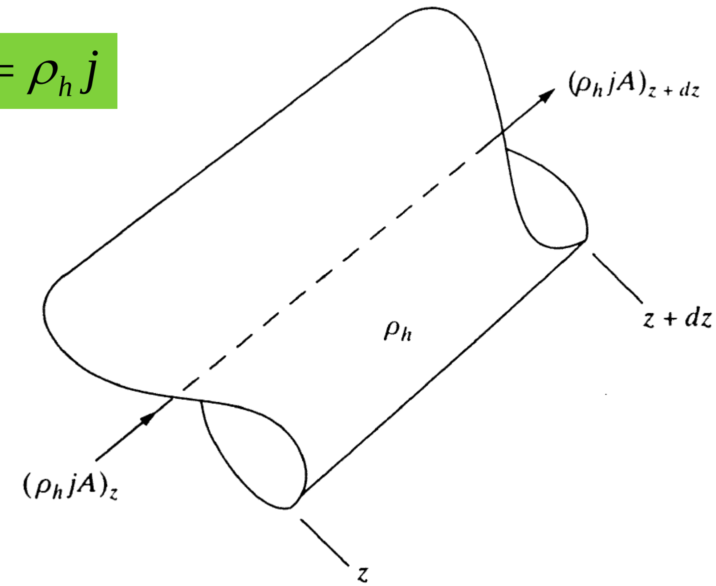
One-Dimensional Homogeneous-Equilibrium Model

❖ Mass conservation

$$\frac{\partial m}{\partial t} = \dot{m}_{in} - \dot{m}_{out} = (GA)_z - (GA)_{z+\delta z}$$

$$\overline{\rho} = G / j = \rho_h \;\Rightarrow G = \rho_h j$$

$$\frac{\partial (A\delta z \rho_h)}{\partial t} = (\rho_h jA)_z - \left[(\rho_h jA)_z + \frac{\partial (\rho_h jA)}{\partial z}\delta z \right]$$



Control volume
for mass conservation

For a fixed geometry

$$\frac{\partial (A\delta z \rho_h)}{\partial t} = \delta z A \frac{\partial \rho_h}{\partial t} + \rho_h \frac{\partial \delta z A}{\partial t} = \delta z A \frac{\partial \rho_h}{\partial t}$$

$$\frac{\partial \rho_h}{\partial t} + \frac{1}{A}\frac{\partial (\rho_h jA)}{\partial z} = 0$$

One-Dimensional Homogeneous-Equilibrium Model

❖ Momentum conservation

$$\frac{\partial GA\delta z}{\partial t} = (GAj)_z - (GAj)_{z+\delta z} + (PA)_z - (PA)_{z+\delta z} - A\delta z F_w + F_A - A\delta z \rho_h g \sin\theta$$

$$G = \rho_h j \qquad F_w = \tau_w p_f \delta z / A\delta z$$

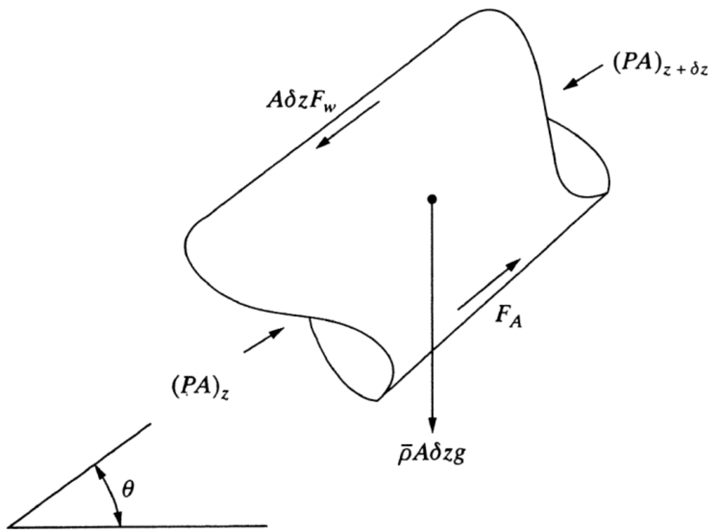
Frictional force per unit volume

$$F_A = P\frac{\partial A}{\partial z}\delta z = \frac{\partial PA}{\partial z}\delta z - A\frac{\partial P}{\partial z}\delta z = (PA)_{z+\delta z} - (PA)_z - A\frac{\partial P}{\partial z}\delta z$$

Net force exerted by the channel wall on the fluid

$$\frac{\partial \rho_h j A\delta z}{\partial t} = \left(\rho_h j^2 A\right)_z - \left(\rho_h j^2 A\right)_{z+\delta z} - A\frac{\partial P}{\partial z}\delta z - A\delta z F_w - A\delta z \rho_h g \sin\theta$$

Control volume for momentum conservation



$$\frac{\partial \rho_h j}{\partial t} + \frac{1}{A}\frac{\partial\left(\rho_h j^2 A\right)}{\partial z} = -\frac{\partial P}{\partial z} - F_w - \rho_h g \sin\theta$$

One-Dimensional Homogeneous-Equilibrium Model

❖ Momentum conservation

$$\frac{\partial \rho_h j}{\partial t} + \frac{1}{A}\frac{\partial\left(\rho_h j^2 A\right)}{\partial z} = -\frac{\partial P}{\partial z} - F_w - \rho_h g \sin\theta$$

$$\left(\rho_h \frac{\partial j}{\partial t} + j\frac{\partial \rho_h}{\partial t}\right) + \left(\frac{\rho_h jA}{A}\frac{\partial j}{\partial z} + \frac{j}{A}\frac{\partial\left(\rho_h jA\right)}{\partial z}\right)$$

$$= \left[\rho_h \frac{\partial j}{\partial t} + \rho_h j\frac{\partial j}{\partial z}\right] + \left[j\left(\frac{\partial \rho_h}{\partial t} + \frac{1}{A}\frac{\partial\left(\rho_h jA\right)}{\partial z}\right)\right] = -\frac{\partial P}{\partial z} - F_w - \rho_h g \sin\theta$$

$$\frac{\partial \rho_h}{\partial t} + \frac{1}{A}\frac{\partial\left(\rho_h jA\right)}{\partial z} = 0$$

$$\rho_h \frac{\partial j}{\partial t} + \rho_h j\frac{\partial j}{\partial z} = -\frac{\partial P}{\partial z} - F_w - \rho_h g \sin\theta$$

Eq. (5.40)
Non-conservative form of momentum conservation

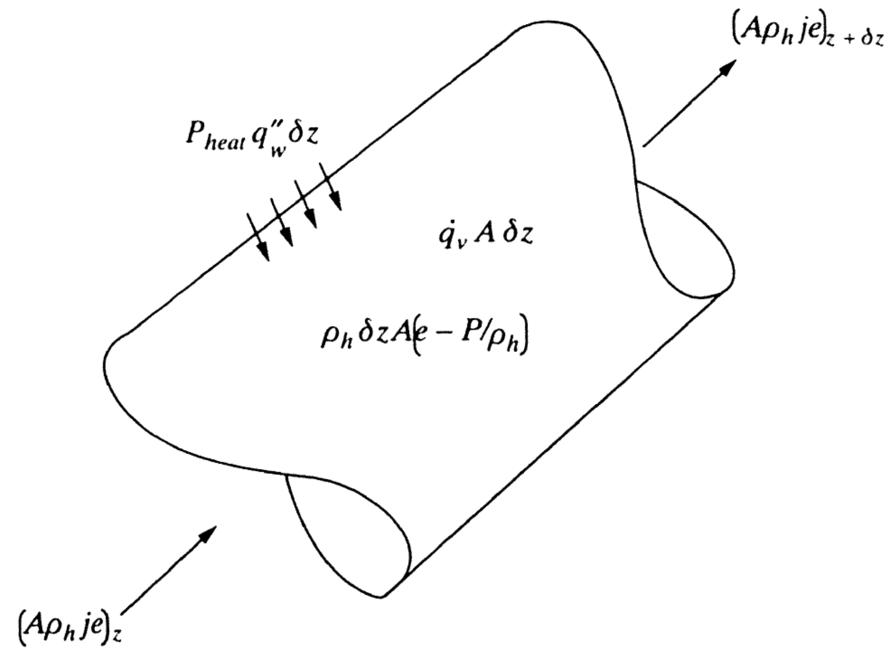
One-Dimensional Homogeneous-Equilibrium Model

❖ Energy conservation

$$\frac{\partial(\rho_h e - P)A\delta z}{\partial t} = (\rho_h jeA)_z - (\rho_h jeA)_{z+\delta z} + p_{heat} q_w'' \delta z + \dot{q}_v A \delta z$$

$$e = h + \frac{1}{2} j^2 + gz \sin\theta = u + \frac{P}{\rho} + \frac{1}{2} j^2 + gz \sin\theta$$ Total convected energy

Control volume for energy conservation



$$A\frac{\partial(\rho_h e - P)}{\partial t} + \frac{\partial}{\partial z}(\rho_h jeA) = p_{heat} q_w'' + \dot{q}_v A$$

$$A\frac{\partial \rho_h e}{\partial t} + \frac{\partial}{\partial z}(\rho_h jeA) - A\frac{\partial P}{\partial t} = p_{heat} q_w'' + \dot{q}_v A$$

One-Dimensional Homogeneous-Equilibrium Model

- Energy conservation

$$A\frac{\partial \rho_h e}{\partial t}+\frac{\partial}{\partial z}(\rho_h jeA)-A\frac{\partial P}{\partial t}=p_{heat}q_w^{''}+\dot{q}_v A$$

$$e=h+\frac{1}{2}j^2+gz\sin\theta$$

$$A\frac{\partial(\rho_h e)}{\partial t}=A\rho_h\frac{\partial e}{\partial t}+Ae\frac{\partial \rho_h}{\partial t}=A\rho_h\left[\frac{\partial h}{\partial t}+\frac{\partial}{\partial t}\left(\frac{1}{2}j^2\right)\right]+A\left(h+\frac{1}{2}j^2+gz\sin\theta\right)\frac{\partial \rho_h}{\partial t}$$

$$\frac{\partial}{\partial z}(\rho_h jeA)=\frac{\partial}{\partial z}\left[\rho_h jA\left(h+\frac{1}{2}j^2+gz\sin\theta\right)\right]$$

$$=\left(h+\frac{1}{2}j^2+gz\sin\theta\right)\frac{\partial}{\partial z}(\rho_h jA)+(\rho_h jA)\frac{\partial}{\partial z}\left(h+\frac{1}{2}j^2+gz\sin\theta\right)$$

One-Dimensional Homogeneous-Equilibrium Model

- Energy conservation

$$A\frac{\partial \rho_h e}{\partial t}+\frac{\partial}{\partial z}(\rho_h jeA)-A\frac{\partial P}{\partial t}=p_{heat}q_w^{''}+\dot{q}_v A \qquad e=h+\frac{1}{2}j^2+gz\sin\theta$$

$$A\frac{\partial(\rho_h e)}{\partial t}=A\rho_h\left[\frac{\partial h}{\partial t}+\frac{\partial}{\partial t}\left(\frac{1}{2}j^2\right)\right]+A\left(h+\frac{1}{2}j^2+gz\sin\theta\right)\frac{\partial \rho_h}{\partial t} \qquad \frac{\partial \rho_h}{\partial t}+\frac{1}{A}\frac{\partial(\rho_h jA)}{\partial z}=0$$

$$\frac{\partial}{\partial z}(\rho_h jeA)=\left(h+\frac{1}{2}j^2+gz\sin\theta\right)\frac{\partial}{\partial z}(\rho_h jA)+(\rho_h jA)\frac{\partial}{\partial z}\left(h+\frac{1}{2}j^2+gz\sin\theta\right)$$

$$A\frac{\partial \rho_h e}{\partial t}+\frac{\partial}{\partial z}(\rho_h jeA)=A\rho_h\left[\frac{\partial h}{\partial t}+\frac{\partial}{\partial t}\left(\frac{1}{2}j^2\right)\right]+(\rho_h jA)\frac{\partial}{\partial z}\left(h+\frac{1}{2}j^2+gz\sin\theta\right)$$

One-Dimensional Homogeneous-Equilibrium Model

- Energy conservation

$$A\rho_h\left[\frac{\partial h}{\partial t}+\frac{\partial}{\partial t}\left(\frac{1}{2}j^2\right)\right]+(\rho_h jA)\frac{\partial}{\partial z}\left(h+\frac{1}{2}j^2+gz\sin\theta\right)-\frac{\partial P}{\partial t}A=p_{heat}q_w^{"}+\dot{q}_v A$$

$$\rho_h\left[\frac{\partial h}{\partial t}+j\frac{\partial h}{\partial z}\right]+\rho_h\left[\frac{\partial}{\partial t}\left(\frac{1}{2}j^2\right)+j\frac{\partial}{\partial z}\left(\frac{1}{2}j^2\right)\right]+\rho_h jg\sin\theta=\frac{p_{heat}q_w^{"}}{A}+\dot{q}_v+\frac{\partial P}{\partial t}$$

One-Dimensional Homogeneous-Equilibrium Model

❖ Energy conservation

Remove the mechanical energy terms ⇒ multiply Eq. (5.40) by j

$$\rho_h \frac{\partial j}{\partial t} + \rho_h j \frac{\partial j}{\partial z} = -\frac{\partial P}{\partial z} - F_w - \rho_h g \sin\theta$$

$$\rho_h j \frac{\partial j}{\partial t} + \rho_h j^2 \frac{\partial j}{\partial z} = -\frac{\partial P}{\partial z} j - F_w j - \rho_h j g \sin\theta$$

Frictional or viscous dissipation

$$\frac{\tau_w p_f}{A} j$$

Irreversible transformation of mechanical energy into heat

$$\rho_h \frac{\partial}{\partial t}\left(\frac{j^2}{2}\right) + \rho_h j \frac{\partial}{\partial z}\left(\frac{j^2}{2}\right) = -\frac{\partial P}{\partial z} j - \frac{\tau_w p_f}{A} j - \rho_h j g \sin\theta$$

$$\rho_h\left[\frac{\partial h}{\partial t} + j\frac{\partial h}{\partial z}\right] + \rho_h\left[\frac{\partial}{\partial t}\left(\frac{1}{2} j^2\right) + j\frac{\partial}{\partial z}\left(\frac{1}{2} j^2\right)\right] + \rho_h j g \sin\theta = \frac{p_{heat} q_w''}{A} + \dot{q}_v + \frac{\partial P}{\partial t}$$

$$\rho_h\left[\frac{\partial h}{\partial t} + j\frac{\partial h}{\partial z}\right] = \frac{p_{heat} q_w''}{A} + \dot{q}_v + \frac{\partial P}{\partial t} + j\frac{\partial P}{\partial z} + \frac{\tau_w p_f}{A} j$$

Thermal energy equation