One-Dimensional Separated Flow Model

- Mass and momentum equations for each phase
  - ✓ Two mass and two momentum equations

- Energy equation
  - ✓ One equation
    - One of the phases can be assumed to be saturated
    - Liquid in bulk boiling/vapor in condensation
  - ✓ Two equations
    - Conditions involving a subcooled liquid and superheated vapor

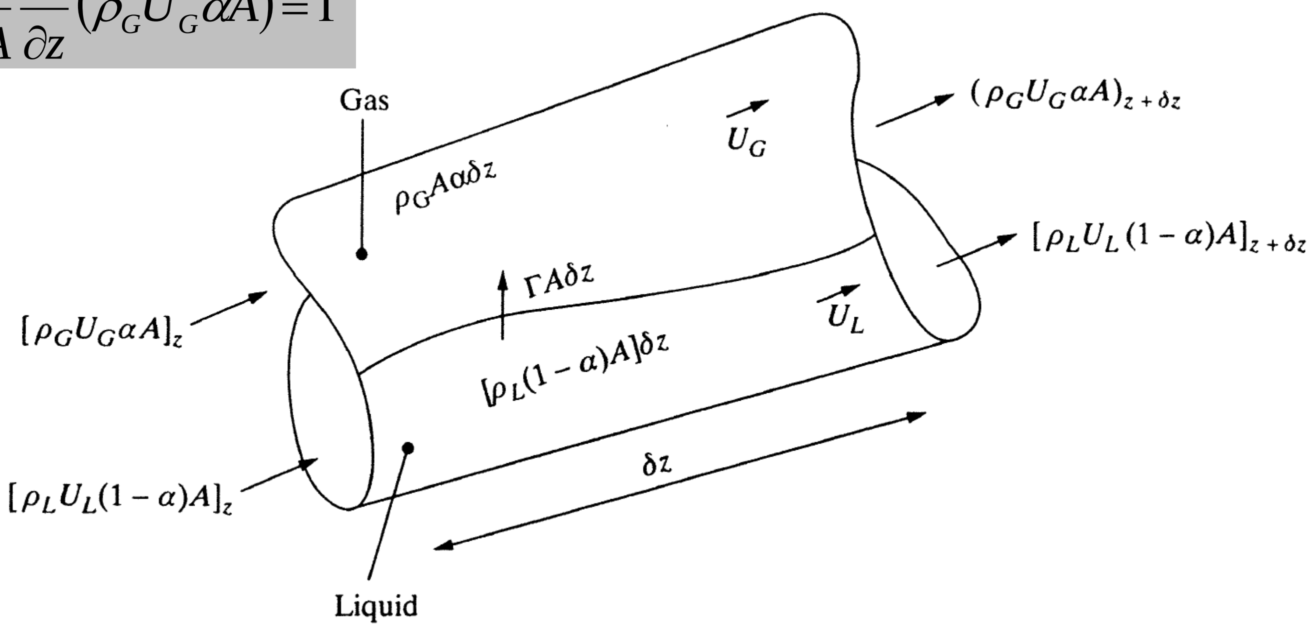
- Assumptions
  - ✓ A stratified or annular flow pattern

One-Dimensional Separated Flow Model

- Mass conservation equation
  - ✓ Γ: phase change rate per unit volume
    - Positive: evaporation, negative: condensation
  - ✓ For gas phase,

$$\frac{\partial \rho_G A \alpha \delta z}{\partial t} = [\rho_G U_G \alpha A]_z - [\rho_G U_G \alpha A]_{z+\delta z} + \Gamma A \delta z$$

$$\frac{\partial \rho_G \alpha}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}(\rho_G U_G \alpha A) = \Gamma$$

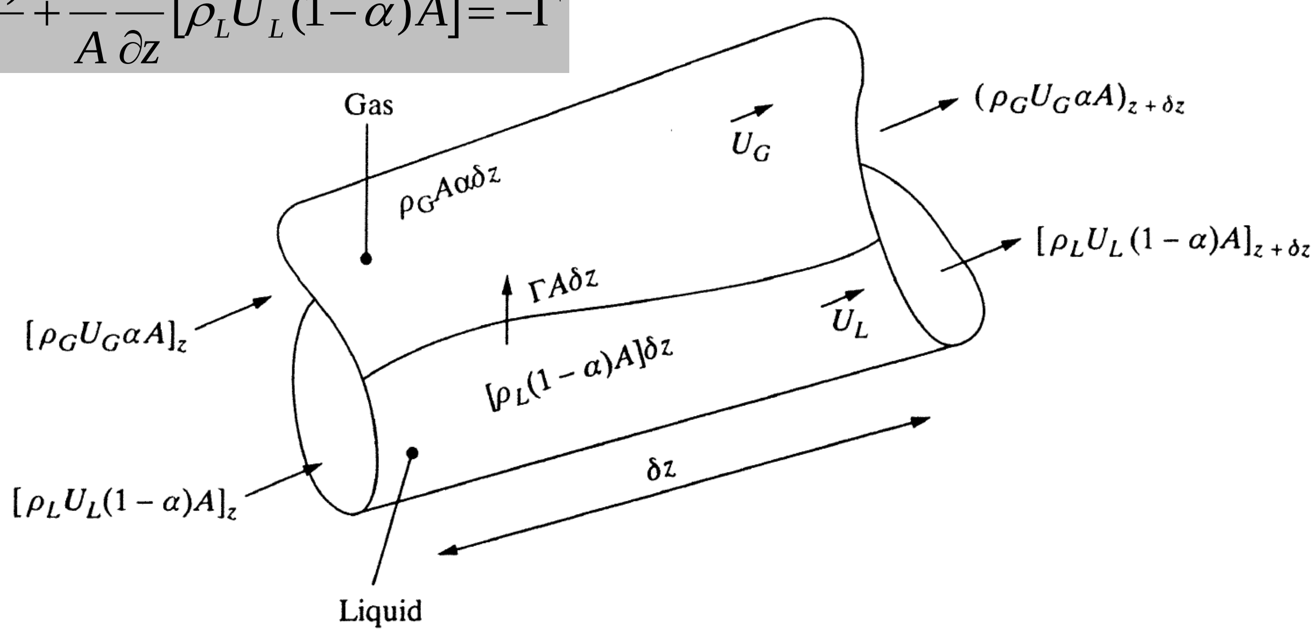


One-Dimensional Separated Flow Model

- Mass conservation equation
  - Γ: phase change rate per unit volume
    - Positive: evaporation, negative: condensation
  - For liquid phase,

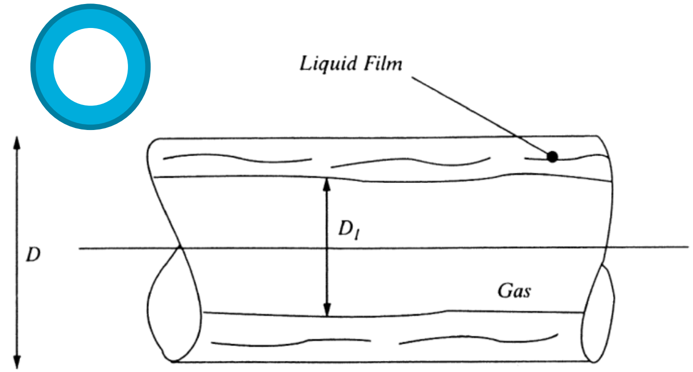
$$\frac{\partial \rho_L A(1-\alpha)\delta z}{\partial t} = \left[\rho_L U_L (1-\alpha)A\right]_z - \left[\rho_L U_L (1-\alpha)A\right]_{z+\delta z} - \Gamma A \delta z$$

$$\boxed{\frac{\partial \rho_L (1-\alpha)}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}\left[\rho_L U_L (1-\alpha)A\right] = -\Gamma}$$



One-Dimensional Separated Flow Model

- Mass conservation equation
  - ✓ Γ: phase change rate per unit volume
    - Positive: evaporation, negative: condensation



$$\Gamma = m_I'' a_I'' = \frac{[kg]}{[m^2 \cdot s]}\frac{[m^2]}{[m^3]} = \frac{[kg]}{[m^3 \cdot s]} = m_I'' \frac{p_I \delta z}{A \delta z} = \frac{m_I'' p_I}{A}$$

- a''_I : interfacial area concentration (IAC), interfacial surface area per unit volume
- p_I: interfacial perimeter, interfacial surface area per unit channel length

  - ✓ Interfacial perimeter
    - For ideal annular flow in a pipe

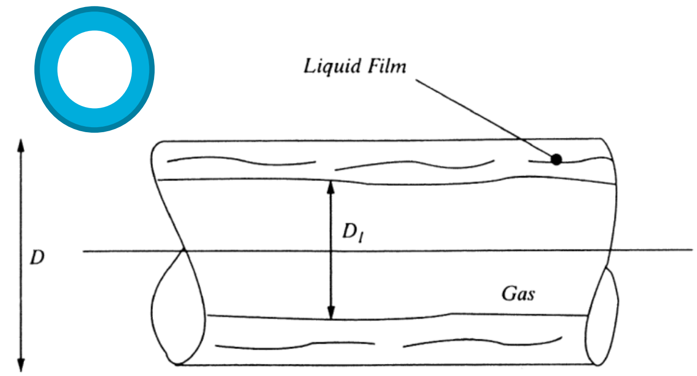
$$D_I / D = \sqrt{\alpha} \quad ; \quad p_I = \pi D \sqrt{\alpha} \quad ; \quad \Gamma = \frac{4 m_I'' \sqrt{\alpha}}{D}$$

$$\alpha = \frac{A_g}{A} = \frac{\pi D_I^2 / 4}{\pi D^2 / 4}$$

$$p_I = \pi D_I$$

One-Dimensional Separated Flow Model

- Mass conservation equation
  - ✓ Γ: phase change rate per unit volume
    - Positive: evaporation, negative: condensation



$$\Gamma = m_I'' a_I'' = \frac{[kg]}{[m^2 \cdot s]}\frac{[m^2]}{[m^3]} = \frac{[kg]}{[m^3 \cdot s]} = m_I'' \frac{p_I \delta z}{A \delta z} = \frac{m_I'' p_I}{A}$$

- a''_I : interfacial area concentration (IAC), interfacial surface area per unit volume
- p_I: interfacial perimeter, interfacial surface area per unit channel length

- ✓ Interfacial perimeter
  - For ideal bubbly flow

Bubble number density (#/m^3)
Number of bubbles in a unit volume

$$p_I = \frac{N_B \pi d_B^2 V}{\delta z} = N_B \pi d_B^2 A$$

$$N_B \left(\pi d_B^3 / 6\right) = \frac{\#}{V}\left(\pi d_B^3 / 6\right) = \frac{V_{bubble}}{V} = \alpha$$

$$N_B = \frac{\#}{V}$$

$$N_B = \frac{\alpha}{\left(\pi d_B^3 / 6\right)}$$

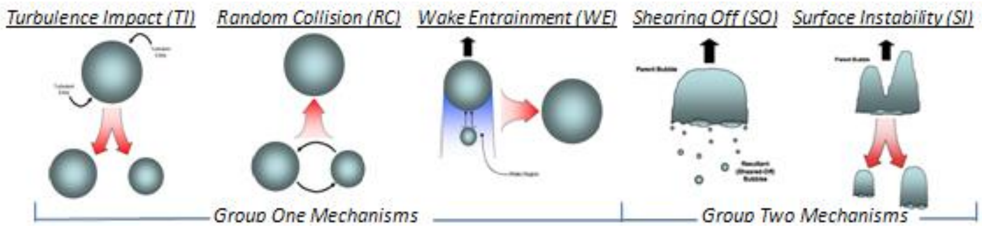
One-Dimensional Separated Flow Model

- Interfacial area concentration (IAC)
  - ✓ a''_I: interfacial surface area per unit volume

$$\Gamma = m_I^{''} a_I^{''}$$

    - In two-fluid model
      - Mass transfer, momentum transfer, energy transfer $\propto \quad IAC$
      - IAC varies depending on flow regime
      - Large uncertainty in developing flows

- Interfacial area transport equation
  - ✓ Transport equation of IAC
  - ✓ Dynamic flow regime map?
  - ✓ TRACE, CATHARE, SPACE
  - ✓ Can replace the conventional flow regime map?



One-Dimensional Separated Flow Model

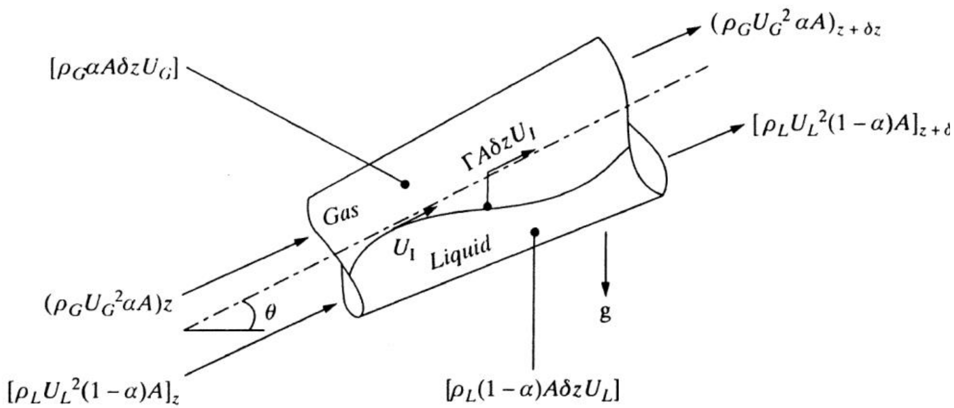
- Momentum conservation equation
  - ✓ U_I : interfacial velocity, the axial velocity at the interphase
  - ✓ For gas phase

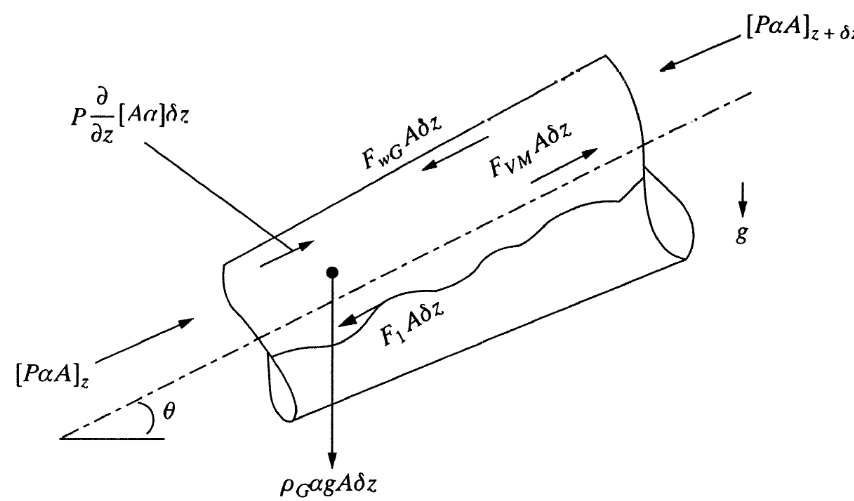
$$\frac{\partial \rho_G U_G \alpha A \delta z}{\partial t} = \left(\rho_G U_G^2 \alpha A\right)_z - \left(\rho_G U_G^2 \alpha A\right)_{z+\delta z} + (P\alpha A)_z - (P\alpha A)_{z+\delta z} + \Gamma U_I$$

$$- A\delta z F_{wG} + P\frac{\partial}{\partial z}[A\alpha]\delta z - A\delta z \rho_G \alpha g \sin\theta - F_I A\delta z + F_{VM} A\delta z$$

$$\frac{\partial \rho_G U_G \alpha}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}\left(\rho_G U_G^2 \alpha A\right) - \Gamma U_I = -\alpha\frac{\partial P}{\partial z} - \rho_G \alpha g \sin\theta - F_{wG} - F_I + F_{VM}$$

$$F_I = f\,(U_G - U_L)$$





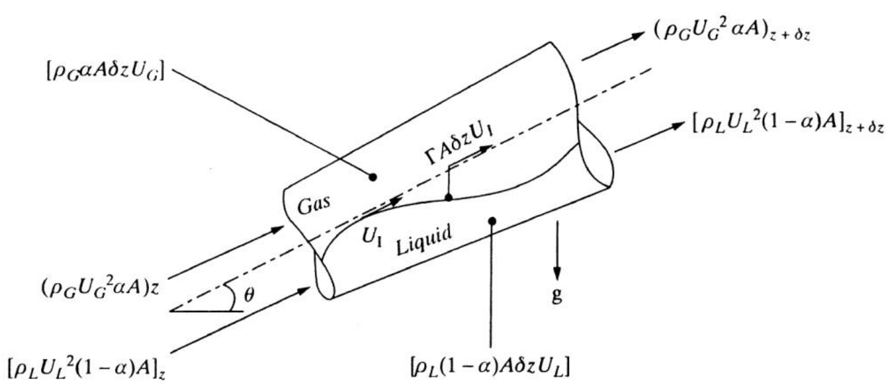
One-Dimensional Separated Flow Model

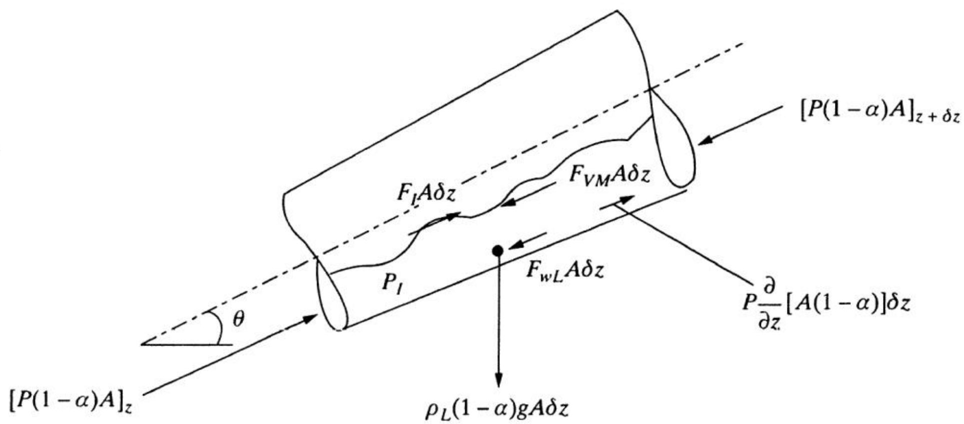
- Momentum conservation equation
  - ✓ U_I : interfacial velocity, the axial velocity at the interphase
  - ✓ For liquid phase

$$F_I = f(U_G - U_L)$$

$$\frac{\partial \rho_L U_L (1-\alpha) A\delta z}{\partial t} = \left[\rho_L U_L^2 (1-\alpha) A\right]_z - \left[\rho_L U_L^2 (1-\alpha) A\right]_{z+\delta z} + \left[P(1-\alpha)A\right]_z - \left[P(1-\alpha)A\right]_{z+\delta z} - \Gamma U_I$$
$$- A\delta z F_{wL} + P\frac{\partial}{\partial z}\left[A(1-\alpha)\right]\delta z - A\delta z \rho_L (1-\alpha) g \sin\theta + F_I A\delta z - F_{VM} A\delta z$$

$$\frac{\partial \rho_L U_L (1-\alpha)}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}\left[\rho_L U_L^2 (1-\alpha) A\right] + \Gamma U_I = -(1-\alpha)\frac{\partial P}{\partial z} - \rho_L (1-\alpha) g \sin\theta - F_{wL} + F_I - F_{VM}$$





One-Dimensional Separated Flow Model

- Momentum conservation equation
  - ✓ Interfacial momentum transfer terms

$$\frac{\partial \rho_G U_G \alpha}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}\left(\rho_G U_G^2 \alpha A\right) - \boxed{\Gamma U_I} = -\alpha \frac{\partial P}{\partial z} - \rho_G \alpha g \sin\theta - F_{wG} - \boxed{F_I} + \boxed{F_{VM}}$$

$$\frac{\partial \rho_L U_L (1-\alpha)}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}\left[\rho_L U_L^2 (1-\alpha) A\right] + \boxed{\Gamma U_I} = -(1-\alpha)\frac{\partial P}{\partial z} - \rho_L (1-\alpha) g \sin\theta - F_{wL} + \boxed{F_I} - \boxed{F_{VM}}$$

One-Dimensional Separated Flow Model

- Momentum conservation equation
  - ✓ Closure relations
    - Interface velocity

$$U_I = \frac{1}{2}(U_L + U_G) \quad or \quad U_I = U_{donor} \qquad U_{donor} = \begin{vmatrix} U_L & for\ evaporation \\ U_G & for\ condensation \end{vmatrix}$$

    - Interfacial friction force (for stratified or annular flow)

$$F_I = \tau_I a_I'' \qquad \tau_I = f_I \frac{1}{2}\rho_G |U_G - U_L|(U_G - U_L) \qquad f_I = 0.005[1 + 75(1-\alpha)]$$

    - Interfacial drag force (for bubbly flow)

$$F_I = F_D N \qquad N = \alpha / (\pi d_B^3 / 6) \qquad F_D = C_D \rho_L \frac{\pi d_B^2}{4} \frac{1}{2} |U_G - U_L|(U_G - U_L)$$

Bubble number density (#/m^3)

$$N(\pi d_B^3 / 6) = \frac{V_{bubble}}{V} = \alpha \qquad N = \frac{\alpha}{(\pi d_B^3 / 6)}$$

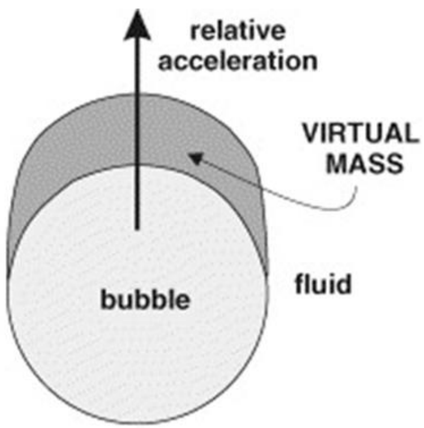
One-Dimensional Separated Flow Model

- Momentum conservation equation
  - Closure relations
    - Interfacial drag/friction force: caused by velocity difference
    - Virtual mass (added mass) force term: caused by acceleration difference

$$F_{VM} = -C_{VM}\left[\frac{\partial U_G}{\partial t} + U_G\frac{\partial U_G}{\partial z} - \frac{\partial U_L}{\partial t} - U_L\frac{\partial U_L}{\partial z}\right]$$

$$C_{VM} = C'\alpha(1-\alpha)\bar{\rho} \qquad \bar{\rho} = \alpha\rho_G + (1-\alpha)\rho_L \qquad C' \approx 1$$

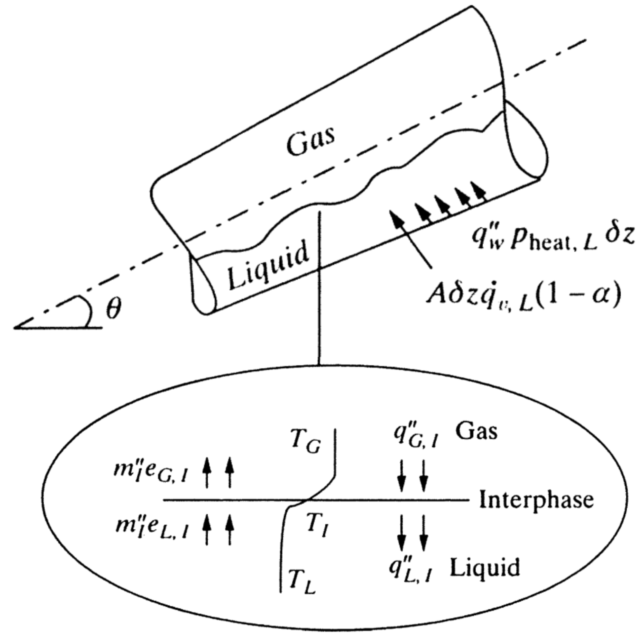
Watanabe et al. (1990)



An accelerating or decelerating body must move some volume of surrounding fluid as it moves through it. Added mass is a common issue because the object and surrounding fluid cannot occupy the same physical space simultaneously.

One-Dimensional Separated Flow Model

- Energy conservation equation
  - ✓ T_I : Temperature at the interphase
  - ✓ q''_{LI} , q''_{GI} : heat fluxes between liquid and ga and the interphase
  - ✓ For gas phase,



$$\frac{\partial}{\partial t}\left[\rho_G\alpha\left(e_G-\frac{P}{\rho_G}\right)A\delta z\right]=\left(\rho_G e_G U_G \alpha A\right)_z-\left(\rho_G e_G U_G \alpha A\right)_{z+\delta z}$$

$$+\Gamma e_{GI}A\delta z - P\frac{\partial\alpha}{\partial t}A\delta z$$

$$+\,p_{heat,G}q''_w\delta z - p_I q''_{GI}\delta z + \dot{q}_{v,G}\alpha A\delta z - (F_I - F_{VM})U_I A\delta z$$

$$\frac{\partial}{\partial t}\left[\rho_G\alpha\left(e_G-\frac{P}{\rho_G}\right)\right]+\frac{1}{A}\frac{\partial}{\partial z}\left[\rho_G e_G U_G \alpha A\right]-\Gamma e_{GI}+P\frac{\partial\alpha}{\partial t}$$

$$-\frac{p_{heat,G}q''_w}{A}+\frac{p_I q''_{GI}}{A}-\dot{q}_{v,G}\alpha+(F_I-F_{VM})U_I=0$$

One-Dimensional Separated Flow Model

- Energy conservation equation
  - For gas phase,

$$\frac{\partial}{\partial t}\left[\rho_G\alpha\left(e_G-\frac{P}{\rho_G}\right)\right]+\frac{1}{A}\frac{\partial}{\partial z}\left[\rho_G e_G U_G \alpha A\right]-\boxed{\Gamma e_{GI}+P\frac{\partial\alpha}{\partial t}}$$
$$-\frac{p_{heat,G}q_w''}{A}+\boxed{\frac{p_I q_{GI}''}{A}}-\dot{q}_{v,G}\alpha+(F_I-F_{VM})U_I=0$$

  - For liquid phase,

$$\frac{\partial}{\partial t}\left[\rho_L(1-\alpha)\left(e_L-\frac{P}{\rho_L}\right)\right]+\frac{1}{A}\frac{\partial}{\partial z}\left[\rho_L e_L U_L(1-\alpha)A\right]+\boxed{\Gamma e_{LI}-P\frac{\partial\alpha}{\partial t}}$$
$$-\frac{p_{heat,L}q_w''}{A}-\boxed{\frac{p_I q_{LI}''}{A}}-\dot{q}_{v,L}(1-\alpha)-(F_I-F_{VM})U_I=0$$

One-Dimensional Separated Flow Model

- Energy conservation equation
  - Closure relations
    - T_I : Temperature at the interphase

      $$T_I = T_{sat}(P)$$

    - Energy balance at the interphase

      $$m_I''\left(h_L + \frac{1}{2}U_{LI}^2\right)_I - q_{LI}'' = m_I''\left(h_G + \frac{1}{2}U_{GI}^2\right)_I - q_{GI}''$$

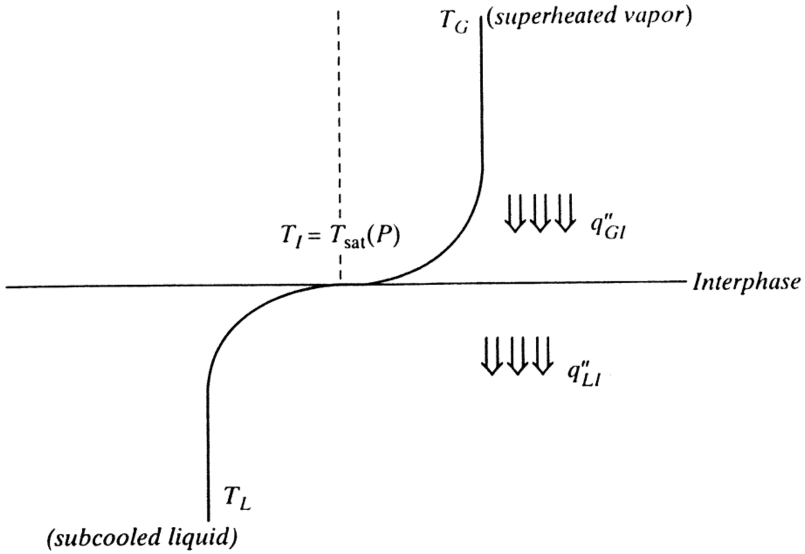
    - Interphase sensible heat transfer

      $$q_{GI}'' = H_{GI}(T_G - T_I) \qquad q_{LI}'' = H_{LI}(T_I - T_L)$$

      H_{kI} Interfacial heat transfer coefficient for k-phase

    - Then, the phase changer rate reduces to

      $$m_I'' = \frac{H_{GI}(T_G - T_I) - H_{LI}(T_I - T_L)}{h_{fg}}$$



One-Dimensional Separated Flow Model

Summary

- Unknowns: U_L, U_G, h_L, h_G, P, and α
- Six conservation equations $\Rightarrow$ six equation model (two-fluid model)
  - Nine equation model $\Rightarrow$ two-phase three-field model

Three equations for gas

$$\frac{\partial \rho_G \alpha}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}\left(\rho_G U_G \alpha A\right) = \Gamma$$

$$\frac{\partial \rho_G U_G \alpha}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}\left(\rho_G U_G^2 \alpha A\right) - \Gamma U_I = -\alpha \frac{\partial P}{\partial z} - \rho_G \alpha g \sin\theta - F_{wG} - F_I + F_{VM}$$

$$\frac{\partial}{\partial t}\left[\rho_G \alpha \left(e_G - \frac{P}{\rho_G}\right)\right] + \frac{1}{A}\frac{\partial}{\partial z}\left[\rho_G e_G U_G \alpha A\right] - \Gamma e_{GI} + P\frac{\partial \alpha}{\partial t}$$

$$-\frac{p_{heat,G} q_w^{"}}{A} + \frac{p_I q_{GI}^{"}}{A} - \dot{q}_{v,G}\alpha + (F_I - F_{VM})U_I = 0$$