
# 2상유동 열전달 공학

## Two-phase flow and heat transfer Engineering

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# Introduction

| Models | Assumptions | Application |
|---|---|---|
| Homogeneous model | • Equal vapor & liquid velocities<br>• Thermodynamic equil. between phases | • Dispersed flow<br>• High-speed flow |
| Separated flow model | • Constant but not equal velocities for both phases<br>• Thermodynamic equil. between phases | • Annular flow<br>• Horizontal separated flow |
| Drift flux model | • Mixture momentum equation<br>• Relative velocity + concentration & velocity profiles | • Bubbly, slug & churn flow<br>• Counter-current flow |
| Two-fluid model | • Two sets of balance equations<br>• Thermodynamic equilibrium within phases<br>• Detailed model for interface interactions | • General |
| Three-fluid model | • Different treatment of droplet fluid from the liquid film | • General (in particular annular dispersed flow) |

# Drift Flux Model

- History of diffusion model
  - ✓ Drift-flux model (Zuber & Findlay, 1965)
  - ✓ Mixture model (Ishii, 1975)
  - ✓ Algebraic-slip model (Pericleous & Drake, 1986)
  - ✓ Suspension model/approach (Verloop, 1995)
  - ✓ Diffusion model (Ungarish, 1993; Ishiii, 1975)
  - ✓ Local equilibrium model (Johansen et al., 1990)

- Drift flux model (DFM)
  - ✓ The most widely used diffusion model
  - ✓ Semi-empirical method for modeling the gas-liquid velocity slip
  - ✓ Accounting for the effects of lateral non-uniformity

  - ✓ One momentum equation
    - Velocity of one of the phases (or the mixture)
    - Slip velocity relation to find the other phasic velocity
    - Significant savings in computational cost
    - Major difficulties associated with the 2FM can be avoided
      - – Interfacial transport constitutive relations
      - – Flow regime dependent parameters
      - – Numerical difficulties

# Drift Flux Model

- Drift flux parameters

$$U_G = j + (U_G - j)$$

$$j = j_G + j_L = \alpha U_G + (1-\alpha) U_L$$ Velocity of mixture center of volume

$$(U_G - j)$$ Gas velocity with respect to the mixture center of volume

✓ Multiply α

$$j_G = \alpha j + \alpha (U_G - j)$$

✓ Area averaging

$$\langle j_G \rangle = \langle \alpha j \rangle + \langle \alpha (U_G - j) \rangle$$ Drift flux

# Drift Flux Model

- Drift flux parameters

$$\langle j_G \rangle = \langle \alpha j \rangle + \langle \alpha (U_G - j) \rangle$$

$\langle \alpha j \rangle$ $\quad$ $\langle \alpha (U_G - j) \rangle$ $\quad$ Difficult to handle

- ✓ Distribution parameter (concentration parameter, distribution coefficient)

$$C_0 = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle}$$

- ✓ Gas drift velocity

$$V_{gj} = \frac{\langle \alpha (U_G - j) \rangle}{\langle \alpha \rangle}$$

# Drift Flux Model

- Drift flux parameters

$$\langle j_G \rangle = \langle \alpha j \rangle + \langle \alpha (U_G - j) \rangle$$

$$C_0 = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle} \qquad V_{gj} = \frac{\langle \alpha (U_G - j) \rangle}{\langle \alpha \rangle}$$

$$\langle j_G \rangle = \langle \alpha j \rangle + \langle \alpha (U_G - j) \rangle = C_0 \langle \alpha \rangle \langle j \rangle + \langle \alpha \rangle V_{gj}$$

$$\langle \alpha \rangle = \frac{\langle j_G \rangle}{C_0 \langle j \rangle + V_{gj}}$$

# Drift Flux Model

- Drift flux parameters
  - ✓ Distribution parameter (concentration parameter, distribution coefficient)

If the void fraction is uniform across the duct

$$\alpha_w = \alpha_c = \langle\alpha\rangle \Rightarrow C_o = 1$$

If the void fraction at the center is lower than that at the wall

$$\alpha_c < \alpha_w \Rightarrow C_o < 1$$

If the void fraction at the center line is larger than that close to the wall

$$\alpha_c > \alpha_w \Rightarrow C_o > 1$$

$$C_0 = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle}$$

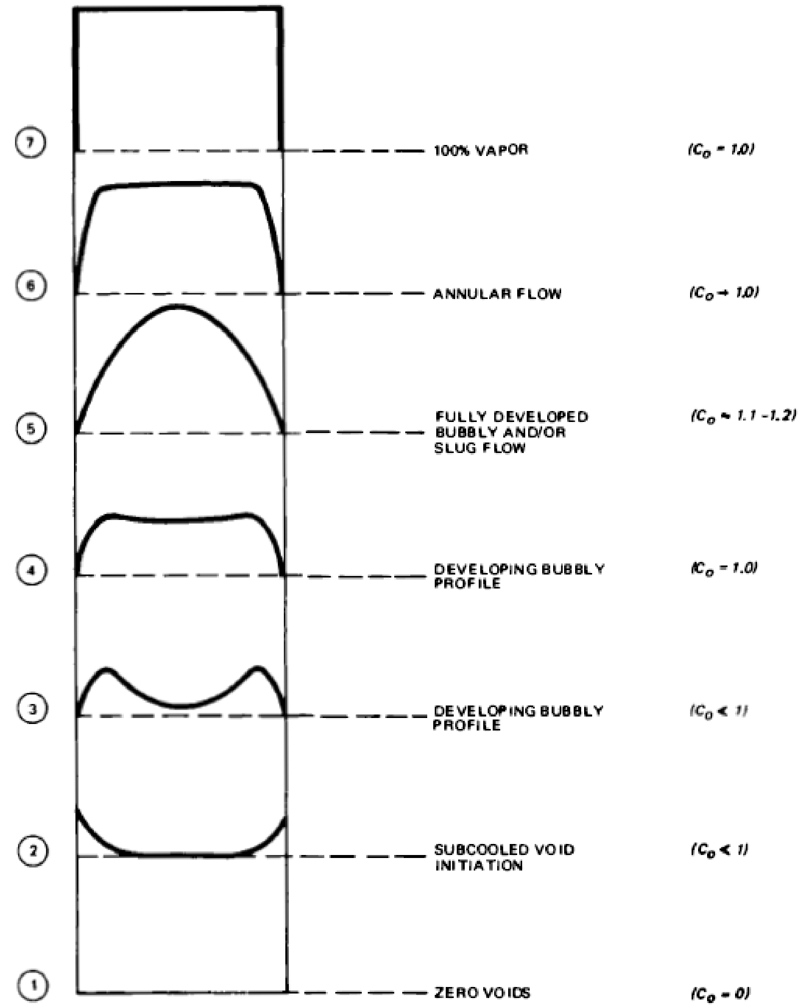



**Fig. 5-13** Diabatic void concentration profiles and flow regimes.

# Drift Flux Model

- Useful relations

$$\langle j_G \rangle = G\langle x \rangle / \rho_G \qquad \langle j_L \rangle = G(1-\langle x \rangle)/\rho_L$$

$$\langle \alpha \rangle = \frac{\langle j_G \rangle}{C_0 \langle j \rangle + V_{gj}} \quad \Rightarrow \quad \langle \alpha \rangle = \frac{\langle x \rangle}{C_0\left[\langle x \rangle + \frac{\rho_G}{\rho_L}(1-\langle x \rangle)\right] + \frac{\rho_G V_{gj}}{G}}$$

- ✓ Slip ratio and slip velocity

$$S_r = \frac{\langle U_G \rangle_G}{\langle U_L \rangle_L} = \frac{\langle j_G \rangle / \langle \alpha \rangle}{\langle j_L \rangle / (1-\langle \alpha \rangle)} = \frac{G\langle x \rangle / \rho_G}{G(1-\langle x \rangle)/\rho_L} = \frac{\langle x \rangle}{(1-\langle x \rangle)} \frac{\rho_L}{\rho_G} \frac{\langle \alpha \rangle}{(1-\langle \alpha \rangle)}$$

$$= C_0 + \frac{\langle x \rangle}{(1-\langle x \rangle)} \frac{\rho_L}{\rho_G}(C_0 - 1) + \frac{\rho_L V_{gj}}{G(1-\langle x \rangle)}$$

$$\langle j_G \rangle = \langle \alpha j \rangle + \langle \alpha (U_G - j) \rangle = C_0 \langle \alpha \rangle \langle j \rangle + \langle \alpha \rangle V_{gj}$$

$$U_r = \langle U_G \rangle_G - \langle U_L \rangle_L = \frac{\langle j_G \rangle}{\langle \alpha \rangle} - \frac{\langle j_L \rangle}{(1-\langle \alpha \rangle)} = \frac{\langle j_G \rangle}{\langle \alpha \rangle} - \frac{\langle j \rangle - \langle j_G \rangle}{(1-\langle \alpha \rangle)} = \frac{V_{gj} + (C_0 - 1)\langle j \rangle}{(1-\langle \alpha \rangle)}$$

# Drift Flux Model

❖ Useful relations

$$U_r = \langle U_G \rangle_G - \langle U_L \rangle_L = \frac{\langle j_G \rangle}{\langle \alpha \rangle} - \frac{\langle j_L \rangle}{(1-\langle \alpha \rangle)} = \frac{\langle j_G \rangle}{\langle \alpha \rangle} - \frac{\langle j \rangle - \langle j_G \rangle}{(1-\langle \alpha \rangle)} = \frac{V_{gj} + (C_0 - 1)\langle j \rangle}{(1-\langle \alpha \rangle)} = \frac{V_{gj}'}{(1-\langle \alpha \rangle)}$$

$$\langle U_G \rangle_G = \langle U_L \rangle_L + \frac{V_{gj}'}{(1-\langle \alpha \rangle)}$$

$$\downarrow$$

$$G = \langle \alpha \rangle \rho_G \langle U_G \rangle_G + (1-\langle \alpha \rangle)\rho_L \langle U_L \rangle_L$$

$$\longrightarrow \quad \langle U_L \rangle_L = \frac{G}{\langle \bar{\rho} \rangle} - \frac{\langle \alpha \rangle}{1-\langle \alpha \rangle}\frac{\rho_G}{\langle \bar{\rho} \rangle} V_{gj}'$$

$$\langle U_G \rangle_G = \frac{G}{\langle \bar{\rho} \rangle} + \frac{\rho_L}{\langle \bar{\rho} \rangle} V_{gj}'$$

Eqs. (6.14-15)

$$\langle j \rangle = \langle \alpha \rangle \langle U_G \rangle_G + (1-\langle \alpha \rangle)\langle U_L \rangle_L \quad \longrightarrow \quad \langle j \rangle = \frac{G}{\langle \bar{\rho} \rangle} + \frac{\langle \alpha \rangle (\rho_L - \rho_G)}{\langle \bar{\rho} \rangle} V_{gj}'$$

Eqs. (6.13)

# Drift Flux Model

- Useful relations

$$\langle \alpha \rangle = \frac{\langle j_G \rangle}{C_0 \langle j \rangle + V_{gj}}$$

$$\langle U_L \rangle_L = \frac{G}{\langle \bar{\rho} \rangle} - \frac{\langle \alpha \rangle}{1 - \langle \alpha \rangle} \frac{\rho_G}{\langle \bar{\rho} \rangle} V_{gj}^{'}$$

$$\langle U_G \rangle_G = \frac{G}{\langle \bar{\rho} \rangle} + \frac{\rho_L}{\langle \bar{\rho} \rangle} V_{gj}^{'}$$

$$\langle \bar{\rho} \rangle = \langle \alpha \rangle \rho_G + (1 - \langle \alpha \rangle) \rho_L$$

$$V_{gj}^{'} = V_{gj} + (C_0 - 1) \langle j \rangle$$

If $C_0$ and $V_{gj}$ are empirically known,

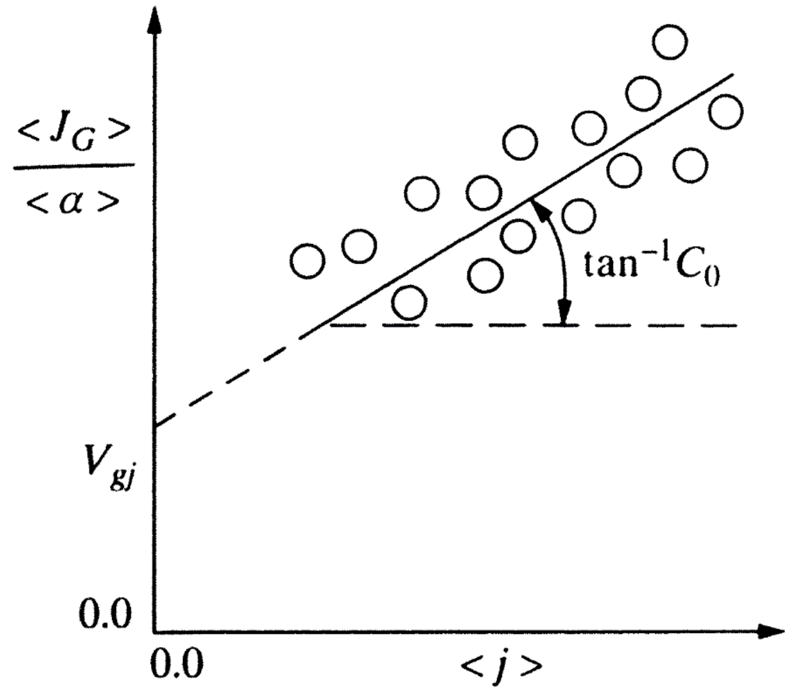
$<U_L>_L$, $<U_G>_G$, $<\alpha>$ can be estimated!

# Drift Flux Model

- Suitability of the DFM

$$\frac{\langle j_G \rangle}{\langle \alpha \rangle} = C_0 \langle j \rangle + V_{gj}$$

  - ✓ Ordinate intercept
  - ✓ Slope




- Limitations of the DFM
  - ✓ Best applicable to 1D flows
  - ✓ Not recommended for flow patterns with
    - Large slip velocities
    - Bubbly, slug churn flow patterns

# Drift Flux Model

- Empirical drift flux model parameters (Chap. 6.3~6.5)
  - ✓ Distribution parameter
  - ✓ Gas drift velocity

$$C_0 = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle} \qquad V_{gj} = \frac{\langle \alpha (U_G - j) \rangle}{\langle \alpha \rangle}$$

  - ✓ For pipe flow (proposed by Ishii)

$$C_0 = \left[1.2 - 0.2\sqrt{\rho_G / \rho_L}\right]\left[1 - \exp\left(-18\langle \alpha \rangle\right)\right]$$

- For bubbly flow

$$V_{gj} = \sqrt{2}\left(\frac{\sigma g \Delta\rho}{\rho_L^2}\right)^{1/4}\left(1 - \langle \alpha \rangle\right)^{1.75}$$

For churn flow

$$V_{gj} = \sqrt{2}\left(\frac{\sigma g \Delta\rho}{\rho_L^2}\right)^{1/4}$$

- For slug flow

$$V_{gj} = 0.35\sqrt{\frac{gD\Delta\rho}{\rho_L}}$$

For annular flow

$$V_{gj} = -(C_0 - 1)\langle j \rangle + \frac{1 - \langle \alpha \rangle}{\langle \alpha \rangle + \left[\frac{1+75(1-\langle \alpha \rangle)}{\sqrt{\langle \alpha \rangle}}\frac{\rho_G}{\rho_L}\right]^{1/2}} \cdot \left[\langle j \rangle + \sqrt{\frac{g\Delta\rho D(1-\langle \alpha \rangle)}{0.015\rho_L}}\right]$$

# Drift Flux Model

- For various angles
  - Woldesemayat and Ghajar (2007)

$$12.7 \leq D \leq 102.26 \text{ mm and } 0.0^\circ \leq \theta \leq 90^\circ$$

- θ: angle of inclination with respect to the horizontal plane
- Fluid: air–water, water–natural gas, and air–kerosene

$$C_0 = \frac{\langle j_G \rangle}{\langle j \rangle}\left[1 + \left(\frac{\langle j \rangle}{\langle j_G \rangle} - 1\right)^b\right] \qquad b = (\rho_G/\rho_L)^{0.1}$$

$$V_{gj} = 2.9\left[\frac{gD\sigma(1+\cos\theta)(\rho_L - \rho_G)}{\rho_L^2}\right]^{0.25}(1.22 + 1.22\sin\theta)^{1/a} \qquad a = (P/P_{atm})$$

- The coefficient 2.9 in this equation is in $m^{-0.25}$ units in the SI unit system

# Drift Flux Model

- For various angles
  - Bhagwat and Ghajar (2014)

    hydraulic diameters in the 0.5–305 mm range, $-90^\circ \le \theta \le 90^\circ$

    - θ: angle of inclination with respect to the horizontal plane
    - Fluid: air–water, argon–water, natural gas–water, air–kerosene, air–glycerin, argon–acetone, argon–ethanol, argon–alcohol, steam–water, air–oil
      - Liquid–vapor mixtures of various refrigerants
      - R-11, R-12, R-22, R-134a, R- 114, R-410A, R-290, R-1234yf
    - Liquid viscosity: $1.0 \times 10^{-4}$ kg/m·s $\le \mu_L \le 0.6$ kg/m·s
    - System pressure: 0.1 MPa $\le P \le$ 18.1 MPa
    - Two-phase Reynolds: $\mathrm{Re}_{TP} = \rho_L (j_L + j_G) D_H / \mu_L$): $10 \le \mathrm{Re}_{TP} \le 5 \times 10^6$

# Drift Flux Model

- For various angles
  - ✓ Bhagwat and Ghajar (2014)

$$C_0 = \frac{2-(\rho_G/\rho_L)^2}{1+(\mathrm{Re}_{TP}/1000)^2} + \frac{\left\{\left[\sqrt{[1+(\rho_G/\rho_L)^2\cos\theta]/(1+\cos\theta)}\right]^{(1-\langle\alpha\rangle)}\right\}^{0.4} + C_{0,1}}{1+(1000/\mathrm{Re}_{TP})^2} \tag{6.30}$$

where

$$C_{0,1} = \begin{cases} 0 & \text{for } -50^\circ \le \theta \le 0 \text{ and Fr } \le 0.1 \\ C_1\left(1-\sqrt{\rho_G/\rho_L}\right)\left[(2.6-\beta)^{0.15}-\sqrt{f_{TP}}\right](1-\langle x\rangle)^{1.5} & \text{otherwise} \end{cases} \tag{6.31}$$

$$C_1 = \begin{cases} 0.2 & \text{for circular and annular cross section} \\ 0.4 & \text{for rectangular cross section.} \end{cases} \tag{6.32}$$

$$\mathrm{Fr} = \sqrt{\frac{\rho_G}{\rho_L-\rho_G}}\frac{\langle j_G\rangle}{\sqrt{gD\cos\theta}}$$

$$\mathrm{Re}_{TP} = \frac{\rho_L D_H\langle j\rangle}{\mu_L}.$$

$$\frac{1}{\sqrt{f_{TP}}} = -4.0\log_{10}\left(\frac{\varepsilon_D/D_H}{3.7} + \frac{1.256}{\mathrm{Re}_{TP}\sqrt{f_{TP}}}\right)$$

# Drift Flux Model

- For various angles
  - ✓ Bhagwat and Ghajar (2014)

$$V_{gj} = C_2 C_3 C_4 \left(0.35 \sin\theta + 0.45 \cos\theta\right) \sqrt{\frac{g D_{\mathrm{H}} \Delta\rho}{\rho_{\mathrm{L}}}} \left(1 - \langle\alpha\rangle\right)^{0.5}$$

where

$$C_2 = \begin{cases} \left[\dfrac{0.434}{\log_{10}(\mu_{\mathrm{L}}/\mu_{\mathrm{ref}})}\right]^{0.15} & \text{for } (\mu_{\mathrm{L}}/\mu_{\mathrm{ref}}) > 10 \\ 1 & \text{for } (\mu_{\mathrm{L}}/\mu_{\mathrm{ref}}) \le 10 \end{cases}$$

$$C_3 = \begin{cases} \left[\dfrac{D_{\mathrm{H}}^*}{0.025}\right]^{0.9} & \text{for } D_{\mathrm{H}}^* < 0.025 \\ 1 & \text{for } D_{\mathrm{H}}^* \ge 0.025 \end{cases}$$

$$C_4 = \begin{cases} -1 & \text{for } -50^\circ \le \theta \le 0^\circ \text{ and Fr} \le 0.1 \\ +1 & \text{otherwise} \end{cases}$$

$$D_{\mathrm{H}}^* = \sqrt{\frac{\sigma}{g\left(\rho_{\mathrm{L}} - \rho_{\mathrm{G}}\right)}} \Big/ D_{\mathrm{H}}$$

# Drift Flux Model

| Author | Distribution coefficient | Drift velocity | Comments |
|---|---|---|---|
| Wallis (1969) | $C_0 = 1.0$ | $V_{gj} = 1.53\left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4}$ | Isolated bubbles without coalescence |
| Zuber and Findlay (1965) | $C_0 = 1.2$ | $V_{gj} = 1.53\left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4}$ | Churn-turbulent flow regime in a vertical tube |
| Dix (1971) | Eqs. (6.27) and (6.28) | $V_{gj} = 1.18\,(1 - \langle x\rangle)\left[\frac{\sigma g \Delta\rho}{\rho_f^2}\right]^{1/4}$ | Low-flow boiling in vertical rod bundles |
| Bonnecaze *et al.* (1971) | $C_0 = 1.2$ | $V_{gj} = 0.35\left[\frac{gD(\rho_L - \rho_G)}{\rho_L}\right]^{1/2}$ | Slug flow regime in a vertical tube |
| Rouhani and Axelsson (1970) | $C_0 = 1 + 0.12\,(1 - \langle x\rangle)$ (version I)<br>$C_0 = 1 + 0.2\,(1 - \langle x\rangle)\,(gD)^{1/4}\left(\frac{\rho_L}{G}\right)^{1/2}$ (version II) | $V_{gj} = 1.18\left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right]^{0.25}$ (vertical) | Subcooled and saturated boiling in tubes; valid for $\langle\alpha\rangle > 0.1$ |
| Ishii (1977) | Eq. (6.22) | Eqs. (6.23)–(6.26) | Boiling in vertical tubes |
| Sun *et al.* (1980) | $C_0 = [0.82 + 0.18\,(P/P_{cr})]^{-1}$ | $V_{gj} = 1.41\left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right]^{0.25}$ | Low-flow boiling of water in rod bundles |
| Shipley (1982) | $C_0 = 1.2$ ($V_{gj}$ in m/s) | $V_{gj} = 0.24 + 0.35\beta^2\sqrt{gD\,\langle\alpha\rangle}$ | Two-phase flow in large diameter tubes |
| Pearson *et al.* (1984) | $C_0 = 1 + 0.796\exp\left(-0.061\sqrt{\rho_L/\rho_G}\right)$ | $V_{gj} = 0.034\left[\sqrt{\rho_L/\rho_G} - 1\right]$ | Level swell |
| Kataoka and Ishii (1987); Hibiki and Ishii (2003a) | $C_0 = 1 - 0.2\sqrt{\rho_G/\rho_L}$ | Low viscosity: $N_{\mu_L} \le 2.25 \times 10^{-3}$<br>$V_{gj}^* = 0.0019 D_H^{*0.809}(\rho_G/\rho_L)^{-0.157} N_{\mu_L}^{-0.562}$ for $D_H^* \le 30$<br>$V_{gj}^* = 0.030(\rho_G/\rho_L)^{-0.157} N_{\mu_L}^{-0.562}$ for $D_H^* > 30$<br>High viscosity: $N_{\mu_L} > 2.25 \times 10^{-3}$<br>$V_{gj}^* = 0.92(\rho_G/\rho_L)^{-0.157}$ for $D_H^* > 30$<br>$D_H^* = D_H/\{\sigma/[g(\rho_L - \rho_G)]\}^{1/2}$;<br>$V_{gj}^* = V_{gj}/[\sigma g(\rho_L - \rho_G)/\rho_L^2]^{1/4}$<br>$N_{\mu_L} = \mu_L/\left\{\rho_L\sigma\sqrt{\sigma/[g(\rho_L - \rho_G)]}\right\}^{1/2}$ | Large-diameter pipes and bubbling or boiling pools |
| Steiner (1993) | $C_0 = 1 + 0.12\,(1 - \langle x\rangle)$ | $V_{gj} = 1.18\,(1 - \langle x\rangle)\left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right]^{0.25}$ | Horizontal tube |
| Gomez (2000) | $C_0 = 1.15$ | $V_{gj} = 1.53\left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right]^{0.25}\sqrt{1 - \langle\alpha\rangle}\sin\theta$ | Vertical and inclined tubes |
| Kataoka and Ishii (1987); Hibiki and Ishii (2003b) | $C_0 = \left[1 - 0.2\sqrt{\rho_G/\rho_L}\right] \times [1 - \exp(-18\langle\alpha\rangle)]$ (bubbly)<br>$C_0 = 1 - 0.2\sqrt{\rho_G/\rho_L}$ (slug and churn)<br>$C_0 = 1 + \frac{1 - \langle\alpha\rangle}{\langle\alpha\rangle + \left[\frac{1 + 75(1 - \langle\alpha\rangle)}{\sqrt{\langle\alpha\rangle}}\frac{\rho_G}{\rho_L}\right]^{\frac{1}{2}}} \times \left[1 + \frac{\sqrt{\frac{gD(\rho_L - \rho_G)(1 - \langle\alpha\rangle)}{0.015\rho_L}}}{j}\right]$ (annular) | $V_{gj} = 1.41[\sigma g(\rho_L - \rho_G)/\rho_L^2]^{1/4} \times (1 - \langle\alpha\rangle)^{1.75}$ (bubbly)<br>$V_{gj} = 0.35[gD(\rho_L - \rho_G)/\rho_L]^{1/2}$ (slug)<br>$V_{gj} = 1.41[\sigma g(\rho_L - \rho_G)/\rho_L^2]^{1/4}$ (churn)<br>$V_{gj} = \frac{1 - \langle\alpha\rangle}{\langle\alpha\rangle + \left[\frac{1 + 75(1 - \langle\alpha\rangle)}{\sqrt{\langle\alpha\rangle}}\frac{\rho_G}{\rho_L}\right]^{\frac{1}{2}}} \times \left[j + \sqrt{\frac{gD(\rho_L - \rho_G)(1 - \langle\alpha\rangle)}{0.015\rho_L}}\right]$ (annular) | Large-diameter vertical pipes and pools |
| Woldesemayat and Ghajar (2007) | Eqs. (6.27) and (6.28) | Eq. (6.29) | See discussion preceding Eq. (6.27) |

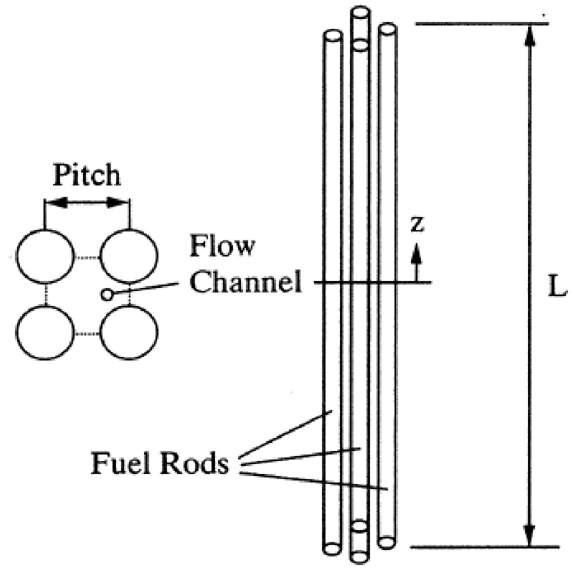
(*cont.*)

# Drift Flux Model

| Author | Distribution coefficient | Drift velocity | Comments |
|---|---|---|---|
| Choi *et al.* (2012) | $C_0 = \frac{2}{1+(\mathrm{Re}/1{,}000)^2} + \frac{1.2-0.2\sqrt{\rho_G/\rho_L}[1-\exp(-18\langle\alpha\rangle)]}{1+(1{,}000/\mathrm{Re})^2}$ <br> $\mathrm{Re} = \rho_L jD/\mu_L$ | $V_{gj} = 0.0246\cos\theta + 1.606\left[\sigma g(\rho_L-\rho_G)/\rho_L^2\right]^{1/4}\sin\theta$ | 0.05–0.15 m pipe diameters, $-10° < \theta < 10°$ |
| Bhagwat and Ghajar (2014) | Eqs. (6.30)–(6.32) | Eqs. (6.36)–(6.39) | Extensive data base, $-90° \le \theta \le 90°$, various fluids |
| Takeuchi *et al.* (1992) | Eqs. (6.42) | Eqs. (6.43) | Based on data representing PWR rod bundles |
| Bestion (1990) | $C_0 = 1.0$ | $V_{gj} = 0.188[g(\rho_L-\rho_G)D_H/\rho_G]^{1/2}$ | Bubbly, slug and churn-turbulent regimes in PWR rod bundles and secondary sides of steam generators during boil off |
| Svetlov *et al.* (1999) | $C_0 = \max\left[\frac{0.675(\rho_L/\rho_G)^{0.1}}{1-0.6\exp(-18\langle\alpha\rangle)}, 1.0\right]$ | $V_{gj}/\{\sigma g(\rho_L-\rho_G)/\rho_L^2\}^{1/4} = 2.19(\rho_L-\rho_G)^{0.25}Bd^{-0.25} \times [1-0.6\exp(-18\langle\alpha\rangle)]$ | Rod bundles, $\langle\alpha\rangle \le 0.75$ |
| Julia *et al.* (2009) | $C_0 = \begin{cases}(1.03-0.03\sqrt{\rho_G/\rho_L}) \times [1-\exp(-26.3\langle\alpha\rangle^{0.78})] & \times D/P_0 = 0.3 \\ (1.04-0.04\sqrt{\rho_G/\rho_L}) \times [1-\exp(-21.2\langle\alpha\rangle^{0.762})] & \times D/P_0 = 0.5 \\ (1.05-0.05\sqrt{\rho_G/\rho_L}) \times [1-\exp(-34.1\langle\alpha\rangle^{0.925})] & \times D/P_0 = 0.7\end{cases}$ <br> $P_0 =$ pitch <br> $D =$ rod diameter | $V_{gj}/\{\sigma g(\rho_L-\rho_G)/\rho_L^2\}^{1/4} = \sqrt{2}(1-\langle\alpha\rangle)^{1.75}B_{sf}$ <br> $B_{sf} = \begin{cases}1-d_B/(0.9L_{max}) & \text{for } d_B/L_{max} < 0.6 \\ 0.12(d_B/L_{max})^{-2} & \text{for } d_B/L_{max} \ge 0.6\end{cases}$ <br> $L_{max} = \sqrt{2}.P_0 - D$ | Bubbly flow in rod bundles, $\langle\alpha\rangle \le 0.20$ |
| Chen *et al.* (2012) | $C_0 = 4.79j_G^* + 1$ for $j_G^* \le 0.5$ <br> $C_0 = C_\infty - (C_\infty - 1)\sqrt{\rho_G/\rho_L}$ for $j_G^* > 0.5$ <br> $C_\infty = 3.45\exp(-0.52j_G^{*0.51}) + 1$ <br> $j_G^* = \langle j_G\rangle/\{\sigma g(\rho_L-\rho_G)/\rho_L^2\}^{1/4}$ | $V_{gj}^* = V_{gj,B}^*.\exp(-1.39j_G^*) + V_{gj,C}^*[1-\exp(-1.39j_G^*)]$ <br> $V_{gj,B}^* = \sqrt{2}(1-\langle\alpha\rangle)^{1.75}$ <br> For $N_{\mu L} \le 2.25\times10^{-3}$: <br> $V_{gj,C}^* = 0019D_C^{*0.809}(\rho_G/\rho_L)^{-0.157} N_{\mu L}^{-0.562}$ for $D_C^* \le 30$ <br> $V_{gj,C}^* = 0.030(\rho_G/\rho_L)^{-0.157} N_{\mu L}^{-0.562}$ for $D_C^* > 30$ <br> For $N_{\mu L} > 2.25\times10^{-3}$: <br> $V_{gj,C}^* = 0.92(\rho_G/\rho_L)^{-0.157}$ for $D_C^* \ge 30$ <br> $V_{gj}^* = V_{gj}/\{\sigma g(\rho_L-\rho_G)/\rho_L^2\}^{1/4}$ <br> $D_C^* = D_C/\sqrt{\sigma/[g(\rho_L-\rho_G)]}$ <br> $D_C =$ Casing diameter or width of the rectangular casing | Adiabatic (air-water) and steam-water boiling in rod bundles; experimental rod bundles with 8×8, 4×4, 2×2, 6×22 rods. |

# Drift Flux Model

**EXAMPLE 6.1.** A large fuel rod bundle that simulates the core of a PWR is made of 1.1-cm–diameter rods that are 3.66 m long. The rods are arranged in a square lattice, as shown in Fig. P4.4 (Problem 4.4), with a pitch-to-diameter ratio of 1.33. The tubes are uniformly heated. During an experiment, the rod bundle remains at 40 bar pressure, while saturated liquid enters the bottom of the bundle with a mass flux of 52 kg/m$^2$·s. The heat flux at the surface of the simulated fuel rods is $5 \times 10^4$ W/m$^2$. Calculate the equilibrium quality and the void fraction at the center of the rod bundle.




**SOLUTION.** The properties that are needed are as follows: $\rho_f = 798.5\,\text{kg/m}^3$, $\rho_g = 20.1\,\text{kg/m}^3$, $h_f = 1.087 \times 10^6\,\text{J/kg}$, $h_{fg} = 1.713 \times 10^6\,\text{J/kg}$, and $T_{sat} = 523.5\,\text{K}$. Also, using Eq. (2.17) with $T_{cr} = 647.2\,\text{K}$, we get $\sigma = 0.0264\,\text{N/m}$. The flow area and the heated perimeter of a channel, as defined in Fig. P4.4 (Problem 4.4), are found by writing

$$A_c = (1.33D)^2 - \frac{\pi}{4}D^2 = 1.19 \times 10^{-4}\,\text{m}^2$$

and

$$p_{heat} = \pi D = 0.0346\,\text{m}.$$

In view of the high pressure, it is assumed that the properties remain constant along the flow channel. This assumption is reasonable since the pressure variations that can be expected will have a small effect on fluid properties. The quality at the center of the rod bundle can be estimated by writing

$$\langle x_{eq} \rangle = \langle x \rangle = \frac{p_{heat} q''_w L_{heat}/2}{A_c G h_{fg}} = 0.298,$$

$$\begin{aligned} q(z) &= \int_0^z P_H q''(z)dz \\ &= \pi D q''_w z \\ &= \dot{m}_g (h_g - h_f) \\ &= \dot{m} < x > (h_g - h_f) \\ &= G A_c h_{fg} < x > \end{aligned}$$

$$< x > = \frac{\pi D q''_w z}{G A_c h_{fg}}$$

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where we have assumed thermodynamic equilibrium between the vapor and liquid phases. We can now calculate the superficial velocities at the center of the bundle:

$$\langle j_g \rangle = G\langle x \rangle / \rho_g = 0.772\,\mathrm{m/s},$$

$$\langle j_f \rangle = G(1 - \langle x \rangle)/\rho_f = 0.046\,\mathrm{m/s},$$

$$\langle j \rangle = \langle j_g \rangle + \langle j_f \rangle = 0.818\,\mathrm{m/s}.$$

The correlation of Dix (1971), based on rod bundle water–steam data

$$\langle \alpha \rangle = \frac{\langle j_G \rangle}{C_0 \langle j \rangle + V_{gj}}$$

$$C_0 = \frac{\langle j_G \rangle_G}{\langle j \rangle}\left[1 + \left(\frac{\langle j \rangle}{\langle j_G \rangle_G} - 1\right)^b\right]$$

$$V_{gj} = 2.9\left(\frac{\sigma g \Delta\rho}{\rho_L^2}\right)^{\frac{1}{4}}$$

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We can estimate the void fraction at the bundle center based on the DFM model, using the correlation of Dix (1971). Accordingly,

$$b = \left(\rho_g / \rho_f\right)^{0.1} = 0.692$$

Using Eq. (6.32), we will then get $C_0 = 1.078$, and from Eq. (6.33) we get $V_{gj} = 0.387$ m/s. Equation (6.7) now gives

$$\langle \alpha \rangle = \frac{\langle j_g \rangle}{C_0 \langle j \rangle + V_{gj}} \approx 0.61.$$

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**EXAMPLE 6.2.** For a steady air–water two-phase flow in an upward, 7.37-cm-diameter tube, estimate the void fraction and phase velocities, using the DFM and the correlation of Woldesemayat and Ghajar (2007). The mixture mass flux is $G = 520\,\text{kg/m}^2\cdot\text{s}$, and air constitutes 2% of the total mass flow rate. Assume that the water–air mixture is under atmospheric pressure and at room temperature (25°C).

**SOLUTION.** The properties that are needed are $\rho_L = 997.1\,\text{kg/m}^3$, $\rho_G = 1.18\,\text{kg/m}^3$, and $\sigma = 0.071\,\text{N/m}$. Knowing $\langle x \rangle = 0.02$, we find the superficial velocities by writing

$$\langle j_G \rangle = G\langle x \rangle/\rho_G = 8.78\,\text{m/s},$$

$$\langle j_L \rangle = G(1 - \langle x \rangle)/\rho_L = 0.51\,\text{m/s},$$

$$\langle j \rangle = \langle j_G \rangle + \langle j_L \rangle = 9.29\,\text{m/s}.$$

The calculations then proceed as follows:

$$V_{gj} = 2.9\left[\frac{gD\sigma(1+\cos\theta)(\rho_L - \rho_G)}{\rho_L^2}\right]^{0.25}(1.22 + 1.22\sin\theta)^{1/a}$$

$$b = (\rho_G/\rho_L)^{0.1} = 0.51,$$

$$a = (P/P_{atm}) = 1.$$

$$C_0 = \frac{\langle j_G \rangle_G}{\langle j \rangle}\left[1 + \left(\frac{\langle j \rangle}{\langle j_G \rangle_G} - 1\right)^b\right]$$

Also, $\theta = \pi/2$; therefore Eq. (6.49) gives

$$V_{gj} = 2.9\left[\frac{gD\sigma(\rho_L - \rho_G)}{\rho_L^2}\right]^{0.25}(1.22 + 1.22)^1 = 0.60\,\text{m/s}.$$

Equation (6.32) leads to $C_0 = 1.17$. Finally, Eq. (6.7) gives

$$\langle \alpha \rangle = \frac{\langle j_G \rangle}{C_0\langle j \rangle + V_{gj}} = 0.77.$$

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- Homework-1
  - ✓ Void fraction estimation in NEOUL-R experiment
  - ✓ Vertical and inclined conditions
  - ✓ Using Bhagwat and Ghajar (2014)
  - ✓ Check the OSV point
  - ✓ If $x_{eq,exit} < x_{OSV}$, this approach is not available.
  - ✓ If $x_{eq,exit} > x_{OSV}$, calculate the flow quality using the Ahmad (1970) model.
  - ✓ Then, you can calculate the void fraction

$$Pe_L = GD_H C_{PL} / k_L$$

$$If\ Pe_L < 70{,}000$$

$$(h_f - h_{OSV}) \leq 0.0022 q_w^{"} D_H C_{PL} / k_L$$

$$If\ Pe_L \geq 70{,}000$$

$$(h_f - h_{OSV}) \leq 154 q_w^{"} / G$$

$$x_{eq,OSV} = (h_{OSV} - h_f)/h_{fg}$$

$$x = \frac{x_{\mathrm{eq}} - x_{\mathrm{eq,OSV}} \exp\left(\frac{x_{\mathrm{eq}}}{x_{\mathrm{eq,OSV}}} - 1\right)}{1 - x_{\mathrm{eq,OSV}} \exp\left(\frac{x_e}{x_{\mathrm{eq,OSV}}} - 1\right)}$$

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- Homework-1