Two-Fluid Model

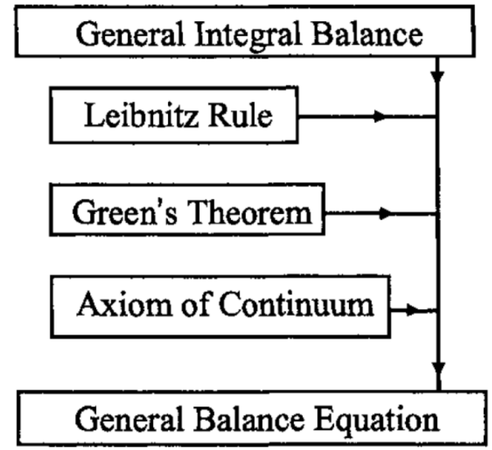
- Derivation procedure
  - In the previous class: separated flow model
    - Heuristic derivation
  - Reference
    - Ishii and Hibiki, "Thermo-Fluid Dynamics of Two-Phase Flow," Springer, 2nd edition, 2010.

Two-Fluid Model

❖ Local Instantaneous Balance Equation

✓ General integral balance equation

$$\frac{d}{dt}\int_{V_m}\rho_k\psi_k dV = -\oint_{A_m} n_k\cdot J_k dA + \int_{V_m}\rho_k\phi_k dV$$



✓ Leibnitz rule or Reynolds transport theorem

$$\frac{d}{dt}\int_{V_m}F_k dV = \int_{V_m}\frac{\partial F_k}{\partial t}dV + \oint_{A_m}F_k v_k\cdot n_k dA$$

Integration for control mass

✓ Green's theorem

$$\oint_{A_m}F_k v_k\cdot n_k dA = \int_V \nabla\cdot(v_k F_k)dV$$

$$\frac{d}{dt}\int_{V_m}F_k dV = \int_{V_m}\left[\frac{\partial F_k}{\partial t} + \nabla\cdot(v_k F_k)\right]dV$$

$$\oint_{A_m} n_k\cdot J_k dA = \int_{V_m}\nabla\cdot J_k dV$$

✓ General balance equation

$$\frac{d}{dt}\int_{V_m}\rho_k\psi_k dV = \int_{V_m}\left[\frac{\partial\rho_k\psi_k}{\partial t} + \nabla\cdot(v_k\rho_k\psi_k)\right]dV = \int_{V_m}\nabla\cdot J_k dV + \int_{V_m}\rho_k\phi_k dV$$

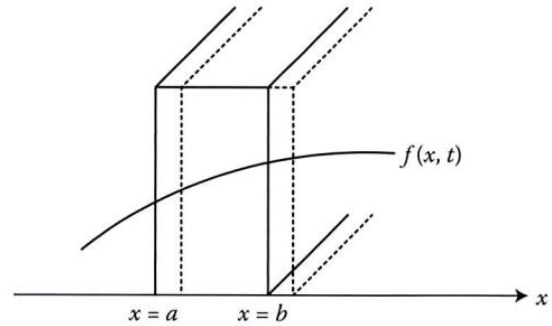
$$\frac{\partial\rho_k\psi_k}{\partial t} + \nabla\cdot(v_k\rho_k\psi_k) = -\nabla\cdot J_k + \rho_k\phi_k$$

No spatial and time averaging
→ local and instantaneous !

Two-Fluid Model

- Leibnitz's rules

$$\frac{d}{d\lambda}\int_{a(\lambda)}^{b(\lambda)} f(x,\lambda)\,dx = \int_{a(\lambda)}^{b(\lambda)} \frac{\partial f(x,\lambda)}{\partial \lambda}\,dx + f(b,\lambda)\frac{db}{d\lambda} - f(a,\lambda)\frac{da}{d\lambda}$$



  - ✓ For 1D

$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(x,t)A\,dx = \int_{a(t)}^{b(t)} \frac{\partial f(x,t)}{\partial t}A\,dx + f(b,t)\,A\frac{db}{dt} - f(a,t)\,A\frac{da}{dt}$$

  - ✓ For 3D

$$\frac{d}{dt}\iiint_V f(\vec{r},t)\,dV = \iiint_V \frac{\partial f(\vec{r},t)}{\partial t}\,dV + \oint\!\!\oint_S f(\vec{r},t)\vec{\upsilon}_s\cdot\vec{n}\,dS$$

$\vec{\upsilon}_s$ = local and instantaneous velocity of the surface S.

- For a deformable volume, whether the center the volume is stationary or moving, v_s is not equal to zero

Two-Fluid Model

❖ General transport theorem

$$\frac{\mathrm{d}}{\mathrm{d}t}\iiint_V f(\vec{r},t)\,\mathrm{d}V = \iiint_V \frac{\partial f(\vec{r},t)}{\partial t}\,\mathrm{d}V + \oiint_S f(\vec{r},t)\vec{\upsilon}_s \cdot \vec{n}\,\mathrm{d}S$$

- ✓ A special case of interest is when the volume under consideration is a material volume V_m (i.e., a volume encompassing a certain mass) within the surface S_m
- ✓ In that case the surface velocity represents the fluid velocity
  - The total derivative becomes identical to the substantial derivative when $v_s = v$.

$$\frac{\mathrm{D}}{\mathrm{D}t}\iiint_{V_m} f(\vec{r},t)\,\mathrm{d}V = \iiint_{V_m} \frac{\partial f(\vec{r},t)}{\partial t}\,\mathrm{d}V + \oiint_{S_m} f(\vec{r},t)\vec{\upsilon} \cdot \vec{n}\,\mathrm{d}S$$

  - Reynolds transport theorem
    - Useful for transforming the derivatives integrals from material-based coordinates (Lagrangian) into spatial coordinates (Eulerian).

Two-Fluid Model

- A stationary and nondeformable volume

$$\frac{\mathrm{d}}{\mathrm{d}t}\iiint_V f(\vec{r},t)\,\mathrm{d}V = \iiint_V \frac{\partial f(\vec{r},t)}{\partial t}\,\mathrm{d}V = \frac{\partial}{\partial t}\iiint_V f(\vec{r},t)\,\mathrm{d}V$$

- The total rate of change of the integral of the function f can be related to the material derivative at a particular instant when the volume boundaries V and V_m are the same

$$\frac{\mathrm{d}}{\mathrm{d}t}\iiint_V f(\vec{r},t)\mathrm{d}V = \iiint_V \frac{\partial f(\vec{r},t)}{\partial t}\,\mathrm{d}V + \oiint_S f(\vec{r},t)\vec{\upsilon}_s \cdot \vec{n}\,\mathrm{d}S$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\iiint_{V_m} f(\vec{r},t)\mathrm{d}V = \iiint_{V_m} \frac{\partial f(\vec{r},t)}{\partial t}\,\mathrm{d}V + \oiint_{S_m} f(\vec{r},t)\vec{\upsilon}\cdot \vec{n}\,\mathrm{d}S$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\iiint_V f(\vec{r},t)\mathrm{d}V = \frac{\mathrm{d}}{\mathrm{d}t}\iiint_V f(\vec{r},t)\mathrm{d}V + \oiint_S f(\vec{r},t)(\vec{\upsilon}-\vec{\upsilon}_s)\cdot \vec{n}\,\mathrm{d}S$$

$\vec{\upsilon}_r = \vec{\upsilon} - \vec{\upsilon}_s =$ relative velocity of the material with respect to the surface of the control volume:

Two-Fluid Model

- 3D Local Instantaneous Balance Equations

$$\frac{\partial \rho_k \psi_k}{\partial t} + \nabla \cdot (\vec{v}_k \rho_k \psi_k) = -\nabla \cdot \vec{J}_k + \rho_k \phi_k$$

- ✓ Continuity equation

$$\psi_k = 1, \quad \phi_k = 0, \quad \vec{J}_k = 0 \quad \Rightarrow \quad \frac{\partial \rho_k}{\partial t} + \nabla \cdot (\vec{v}_k \rho_k) = 0$$

- ✓ Momentum equation

$$\psi_k = \vec{v}, \quad \phi_k = \vec{g}, \quad \vec{J}_k = -\overline{\overline{T}}_k = P_k \vec{I} - \overline{\overline{\tau}}_k \quad \Rightarrow \quad \frac{\partial \rho_k \vec{v}_k}{\partial t} + \nabla \cdot (\rho_k \vec{v}_k \vec{v}_k) = -\nabla P_k + \nabla \cdot \overline{\overline{\tau}}_k + \rho_k \vec{g}_k$$

- ✓ Energy equation

$$\psi_k = u_k + \frac{v_k^2}{2}, \quad \phi_k = \vec{g}_k \cdot \vec{v}_k + \frac{\dot{q}_k}{\rho_k}, \quad \vec{J}_k = \vec{q}_k - \overline{\overline{T}}_k \cdot \vec{v}_k \quad \Rightarrow$$

$$\frac{\partial \rho_k \left(u_k + \frac{v_k^2}{2} \right)}{\partial t} + \nabla \cdot \left[\left(\rho_k u_k + \frac{v_k^2}{2} \right) \vec{v}_k \right] = -\nabla \cdot \vec{q}_k + \nabla \cdot \left(\overline{\overline{T}}_k \cdot \vec{v}_k \right) + \rho_k \vec{g}_k \cdot \vec{v}_k + \dot{q}_k$$

Two-Fluid Model

- Local instantaneous equation

$$\frac{\partial \rho_k \psi_k}{\partial t} + \nabla \cdot (\boldsymbol{v}_k \rho_k \psi_k) = -\nabla \cdot \mathcal{J}_k + \rho_k \phi_k$$

Volume average

Integration for k-phase volume

$$\int_{V_k} \frac{\partial \rho_k \Psi_k}{\partial t} dV + \int_{V_k} \nabla \cdot (\overrightarrow{u_k} \rho_k \Psi_k + J_k) dV - \int_{V_k} \rho_k \Phi_k = 0$$

- Leibniz rule

$$\frac{d}{dt} \int_{V_k} F dV = \int_{V_k} \frac{\partial F}{\partial t} dV + \int_{A_k} F \vec{v}_s \cdot \vec{n} dA \quad \Rightarrow \quad \int_{V_k} \frac{\partial F}{\partial t} dV = \frac{d}{dt} \int_{V_k} F dV - \int_{A_k} F \vec{v}_s \cdot \vec{n} dA$$

$$\int_{V_k} \frac{\partial \rho_k \psi_k}{\partial t} dV = \frac{d}{dt} \int_{V_k} \rho_k \psi_k dV - \int_{A_k} \rho_k \psi_k \vec{v}_s \cdot \vec{n} dA$$

$$\int_{V_k} \nabla \cdot (\vec{v}_k \rho_k \psi_k + \vec{J}_k) dV = \int_{A_k} (\vec{v}_k \rho_k \psi_k + \vec{J}_k) \cdot \vec{n} dA$$

$$\int_{V_k} \rho_k \varphi_k dV = \frac{V_k}{V_k} \int_{V_k} \rho_k \varphi_k dV = V_k \langle\langle \rho_k \varphi_k \rangle\rangle_k$$

Two-Fluid Model

$$\int_{V_k} \frac{\partial \rho_k \Psi_k}{\partial t} dV + \int_{V_k} \nabla \cdot (\overrightarrow{u_k} \rho_k \Psi_k + J_k) dV - \int_{V_k} \rho_k \Phi_k = 0$$

$$\frac{d}{dt} \int_{V_k} \rho_k \psi_k dV - \int_{A_k} \rho_k \psi_k \vec{v}_s \cdot \vec{n} dA + \int_{A_k} (\vec{v}_k \rho_k \psi_k + \vec{J}_k) \cdot \vec{n} dA - V_k \langle\langle \rho_k \varphi_k \rangle\rangle_k = 0$$

$$\frac{d}{dt} \int_{V_k} \rho_k \psi_k dV - \int_{A_k} \rho_k \psi_{\vec{k}} \vec{v}_s \cdot \vec{n} dA + \int_{A_k} \rho_k \psi_{\vec{k}} \vec{v}_k \cdot \vec{n} dA + \int_{A_k} \vec{J}_k \cdot \vec{n} dA - V_k \langle\langle \rho_k \varphi_k \rangle\rangle_k = 0$$

$$\frac{d}{dt} \int_{V_k} \rho_k \psi_k dV + \int_{A_k} \rho_k \psi_k (\vec{v}_k - \vec{v}_s) \cdot \vec{n} dA + \int_{A_k} \vec{J}_k \cdot \vec{n} dA - V_k \langle\langle \rho_k \varphi_k \rangle\rangle_k = 0$$

$$A_k = A_{ki} + A_{kw} + A_{kj}$$

상간 면적 + 벽면에 닿은 면적 + 입출구에 닿은 면적

Two-Fluid Model

$$\frac{d}{dt}\int_{V_k}\rho_k\psi_k dV+\int_{A_k}\rho_k\psi_k(\vec{v}_k-\vec{v}_s)\cdot\vec{n}dA+\int_{A_k}\vec{J}_k\cdot\vec{n}dA-V_k\langle\langle\rho_k\varphi_k\rangle\rangle_k=0$$

$$A_k=A_{ki}+A_{kw}+A_{kj}$$

상간 면적 + 벽면에 닿은 면적 + 입출구에 닿은 면적

$v_s=0$ $v_s=0$

$$\int_{A_k}\rho_k\psi_k(\vec{v}_k-\vec{v}_s)\cdot\vec{n}dA=\int_{A_{ki}}\rho_k\psi_k(\vec{v}_k-\vec{v}_s)\cdot\vec{n}dA+\int_{A_{kw}}\rho_k\psi_k\vec{v}_k\cdot\vec{n}dA+\int_{A_{kj}}\rho_k\psi_k\vec{v}_k\cdot\vec{n}dA$$

$$=(\dot{m}_k\hat{\psi}_k)_i+(\dot{m}_k\hat{\psi}_k)_w+\sum_j\langle\rho_k\psi_k\vec{v}_k\rangle_k\langle\alpha_k\rangle A_j$$

$$\int_{A_k}\vec{J}_k\cdot\vec{n}dA=\int_{A_{ki}+A_{kw}+A_{kj}}\vec{J}_k\cdot\vec{n}dA$$

$$\frac{d}{dt}\int_{V_k}\rho_k\psi_k dV+\sum_j\langle\rho_k\psi_k v_k\rangle_k\langle\alpha_k\rangle A_j=-(\dot{m}_k\hat{\psi}_k)_i-(\dot{m}_k\hat{\psi}_k)_w-\int_{A_{ki}+A_{kw}+A_{kj}}\vec{J}_k\cdot\vec{n}dA+V_k\langle\langle\rho_k\psi_k\rangle\rangle_k$$

Two-Fluid Model

$$\frac{d}{dt}\int_{V_k}\rho_k\psi_k dV+\sum_j\langle\rho_k\psi_k v_k\rangle_k\langle\alpha_k\rangle A_j=-\left(\dot{m}_k\hat{\psi}_k\right)_i-\left(\dot{m}_k\hat{\psi}_k\right)_w-\int_{A_{ki}+A_{kw}+A_{kj}}\vec{J}_k\cdot\vec{n}dA+V_k\langle\langle\rho_k\varphi_k\rangle\rangle_k$$

- For a fixed volume (i.e., a stationary and non-deformable volume)

$$\frac{d}{dt}\int_{V_k}\rho_k\psi_k dV=\frac{d}{dt}\int_V\alpha_k\rho_k\psi_k dV=\frac{\partial}{\partial t}\left(\int_V\alpha_k\rho_k\psi_k dV\right)=\frac{\partial}{\partial t}\left(\frac{V_k}{V_k}\int_V\alpha_k\rho_k\psi_k dV\right)=\frac{\partial}{\partial t}\left(\frac{V_k}{V_k}\int_{V_k}\rho_k\psi_k dV\right)$$

$$=\frac{\partial}{\partial t}\left(V_k\langle\langle\rho_k\psi_k\rangle\rangle_k\right)=\frac{\partial}{\partial t}\left(\langle\langle\alpha_k\rangle\rangle V\langle\langle\rho_k\psi_k\rangle\rangle_k\right)=V\frac{\partial}{\partial t}\left(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\psi_k\rangle\rangle_k\right)$$

$$\int_{A_k}\vec{J}_k\cdot\vec{n}dA=\int_{A_{ki}}\vec{J}_k\cdot\vec{n}dA+\int_{A_{kw}}\vec{J}_k\cdot\vec{n}dA+\int_{A_{kj}}\vec{J}_k\cdot\vec{n}dA$$

$$=\int_{A_{ki}}\vec{J}_k\cdot\vec{n}dA+\int_{A_{kw}}\vec{J}_k\cdot\vec{n}dA+\sum_j\langle\alpha_k\rangle\vec{J}_k\cdot\vec{n}_jA_j$$

$$V\frac{\partial}{\partial t}\left(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\psi_k\rangle\rangle_k\right)+\sum_j\langle\rho_k\psi_k v_k\rangle_k\langle\alpha_k\rangle A_j$$

$$=-\left(\dot{m}_k\hat{\psi}_k\right)_i-\left(\dot{m}_k\hat{\psi}_k\right)_w-\int_{A_{ki}}\vec{J}_k\cdot\vec{n}dA-\int_{A_{kw}}\vec{J}_k\cdot\vec{n}dA-\sum_j\langle\alpha_k\rangle\vec{J}_k\cdot\vec{n}_jA_j+V_k\langle\langle\rho_k\varphi_k\rangle\rangle_k$$

Two-Fluid Model

$$V\frac{\partial}{\partial t}\left(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\psi_k\rangle\rangle_k\right)+\sum_j\langle\rho_k\psi_k v_k\rangle_k\langle\alpha_k\rangle A_j$$
$$=-\left(\dot{m}_k\hat{\psi}_k\right)_i-\left(\dot{m}_k\hat{\psi}_k\right)_w-\int_{A_{ki}}\vec{J}_k\cdot\vec{n}dA-\int_{A_{kw}}\vec{J}_k\cdot\vec{n}dA-\sum_j\langle\alpha_k\rangle\vec{J}_k\cdot\vec{n}_jA_j+V_k\langle\langle\rho_k\varphi_k\rangle\rangle_k$$

mass $\quad \psi_k=1, \vec{J}_k=0, \varphi_k=0$

$$V\frac{\partial}{\partial t}\left(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\rangle\rangle_k\right)+\sum_j\langle\rho_k v_k\rangle_k\langle\alpha_k\rangle A_j=-\left(\dot{m}_k\hat{\psi}_k\right)_i-\left(\dot{m}_k\hat{\psi}_k\right)_w$$

$$\frac{\partial}{\partial t}\left(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\rangle\rangle_k\right)+\frac{1}{V}\sum_j\langle\rho_k v_k\rangle_k\langle\alpha_k\rangle A_j=-\frac{1}{V}\left(\dot{m}_k\right)_i-\frac{1}{V}\left(\dot{m}_k\right)_w$$

$$\frac{\partial}{\partial t}\left(\alpha_k\rho_k\right)+\frac{1}{V}\sum_j\rho_k v_k\alpha_k A_j=-\frac{1}{V}\left(\dot{m}_k\right)_i-\frac{1}{V}\left(\dot{m}_k\right)_w$$

$$\frac{\partial\rho_G\alpha}{\partial t}+\frac{1}{A}\frac{\partial}{\partial z}\left(\rho_G U_G\alpha A\right)=\Gamma$$

$$\frac{\partial\rho_L(1-\alpha)}{\partial t}+\frac{1}{A}\frac{\partial}{\partial z}\left[\rho_L U_L(1-\alpha)A\right]=-\Gamma$$

Two-Fluid Model

$$V\frac{\partial}{\partial t}(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\psi_k\rangle\rangle_k)+\sum_j\langle\rho_k\psi_k v_k\rangle_k\langle\alpha_k\rangle A_j$$

$$=-(\dot{m}_k\hat{\psi}_k)_i-(\dot{m}_k\hat{\psi}_k)_w-\int_{A_{ki}}\vec{J}_k\cdot\vec{n}dA-\int_{A_{kw}}\vec{J}_k\cdot\vec{n}dA-\sum_j\langle\alpha_k\rangle\vec{J}_k\cdot\vec{n}_jA_j+V_k\langle\langle\rho_k\varphi_k\rangle\rangle_k$$

momentum $\psi_k=\vec{v}_k,\vec{J}_k=p_k\bar{\bar{I}}-\bar{\bar{\tau}}_k,\varphi_k=\vec{g}$

$\hat{\vec{v}}_k$: Interface 에서 k 상의 평균속도

$$V\frac{\partial}{\partial t}(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\vec{v}_k\rangle\rangle_k)+\sum_j\langle\rho_k v_k\vec{v}_k\rangle_k\langle\alpha_k\rangle A_j$$

$$=-\left(\dot{m}_k\hat{\vec{v}}_k\right)_i-\left(\dot{m}_k\hat{\vec{v}}_k\right)_w-\int_{A_{ki}}\vec{J}_k\cdot\vec{n}dA-\int_{A_{kw}}\vec{J}_k\cdot\vec{n}dA-\sum_j\langle\alpha_k\rangle\vec{J}_k\cdot\vec{n}_jA_j+V_k\langle\langle\rho_k\vec{g}\rangle\rangle_k$$

$$\int_{A_{ki}}\vec{J}_k\cdot\vec{n}dA=\int_{A_{ki}}\left(p_k\bar{\bar{I}}-\bar{\bar{\tau}}_k\right)\cdot\vec{n}dA=F_{ik}$$

Interfacial drag

$$\int_{A_{kw}}\vec{J}_k\cdot\vec{n}dA=\int_{A_{kw}}\left(p_k\bar{\bar{I}}-\bar{\bar{\tau}}_k\right)\cdot\vec{n}dA=F_{wk}$$

Wall drag

$$\sum_j\langle\alpha_k\rangle\vec{J}_k\cdot\vec{n}_jA_j\approx\sum_j\langle\alpha_k\rangle p_kA_j$$

Two-Fluid Model

$$V\frac{\partial}{\partial t}\left(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\vec{v}_k\rangle\rangle_k\right)+\sum_j\langle\rho_k v_k\vec{v}_k\rangle_k\langle\alpha_k\rangle A_j$$
$$=-\left(\dot{m}_k\hat{\vec{v}}_k\right)_i-\left(\dot{m}_k\hat{\vec{v}}_k\right)_w-F_{ik}-F_{wk}-\sum_j\langle\alpha_k\rangle pA_j+V\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\vec{g}\rangle\rangle_k$$

$$\frac{\partial}{\partial t}\left(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\vec{v}_k\rangle\rangle_k\right)+\frac{1}{V}\sum_j\langle\rho_k v_k\vec{v}_k\rangle_k\langle\alpha_k\rangle A_j$$
$$=-\frac{1}{V}\left(\dot{m}_k\hat{\vec{v}}_k\right)_i-\frac{1}{V}\left(\dot{m}_k\hat{\vec{v}}_k\right)_w-\frac{1}{V}F_{ik}-\frac{1}{V}F_{wk}-\frac{1}{V}\sum_j\langle\alpha_k\rangle pA_j+\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\vec{g}\rangle\rangle_k$$

$$\frac{\partial}{\partial t}\left(\alpha_k\rho_k\vec{v}_k\right)+\frac{1}{V}\sum_j\alpha_k\rho_k v_k\vec{v}_k A_j$$
$$=-\frac{1}{V}\left(\dot{m}_k\hat{\vec{v}}_k\right)_i-\frac{1}{V}\left(\dot{m}_k\hat{\vec{v}}_k\right)_w-\frac{1}{V}F_{ik}-\frac{1}{V}F_{wk}-\frac{1}{V}\sum_j\alpha_k pA_j+\alpha_k\rho_k\vec{g}$$

$$\frac{\partial\rho_G U_G\alpha}{\partial t}+\frac{1}{A}\frac{\partial}{\partial z}\left(\rho_G U_G^2\alpha A\right)-\Gamma U_I=-\alpha\frac{\partial P}{\partial z}-\rho_G\alpha g\sin\theta-F_{wG}-F_I+F_{VM}$$

$$\frac{\partial\rho_L U_L(1-\alpha)}{\partial t}+\frac{1}{A}\frac{\partial}{\partial z}\left[\rho_L U_L^2(1-\alpha)A\right]+\Gamma U_I=-(1-\alpha)\frac{\partial P}{\partial z}-\rho_L(1-\alpha)g\sin\theta-F_{wL}+F_I-F_{VM}$$

Two-Fluid Model

$$V\frac{\partial}{\partial t}(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\psi_k\rangle\rangle_k)+\sum_j\langle\rho_k\psi_k v_k\rangle_k\langle\alpha_k\rangle A_j$$

$$=-(\dot{m}_k\hat{\psi}_k)_i-(\dot{m}_k\hat{\psi}_k)_w-\int_{A_{ki}}\vec{J}_k\cdot\vec{n}dA-\int_{A_{kw}}\vec{J}_k\cdot\vec{n}dA-\sum_j\langle\alpha_k\rangle\vec{J}_k\cdot\vec{n}_jA_j+V_k\langle\langle\rho_k\varphi_k\rangle\rangle_k$$

energy

$$\psi_k=u_k+0.5v_k^2$$

$$\vec{J}_k=\vec{q}''+(p_k\bar{\bar{I}}-\bar{\bar{\tau}}_k)\cdot\vec{v}_k$$

$$\varphi_k=\vec{g}\cdot\vec{v}_k+q_k'''/\rho_k$$

$$V\frac{\partial}{\partial t}(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k u_k\rangle\rangle_k)+\sum_j\langle\rho_k u_k v_k\rangle_k\langle\alpha_k\rangle A_j+\left(\dot{m}_k\hat{h}_k\right)_i+\left(\dot{m}_k\hat{h}_k\right)_w$$

$$=\dot{W}_{diss,wk}+\dot{W}_{diss,pk}+\dot{W}_{diss,ik}-V\langle\langle p_k\rangle\rangle_k\frac{\partial\alpha_k}{\partial t}-\sum_j\langle\alpha_k\rangle\langle p_k v_k\rangle_k A_j$$

$$+\dot{q}_{wk}+\dot{q}_{ik}+\sum_j\dot{q}_k+V\langle\langle\alpha_k\rangle\rangle\langle\langle q_k'''\rangle\rangle_k$$

$$+V\langle\langle\alpha_k\rangle\rangle\langle\langle q_k'''\rangle\rangle_k+V\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\vec{g}\cdot\vec{v}_k\rangle\rangle_k$$

Two-Fluid Model

Pressure work
Due to phase change at interface

pressure work
due to phase change
at fixed wall

$$V\frac{\partial}{\partial t}(\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k u_k\rangle\rangle_k)+\sum_j\langle\rho_k u_k v_k\rangle_k\langle\alpha_k\rangle A_j+\left(\dot{m}_k\hat{h}_k\right)_i+\left(\dot{m}_k\hat{h}_k\right)_w$$

$p_k\bar{\bar{I}}-\bar{\bar{\tau}}_k$

Pump work
(at moving wall)

Pressure work
at interface

$$=\dot{W}_{diss,wk}+\dot{W}_{diss,pk}+\dot{W}_{diss,ik}-V\langle\langle p_k\rangle\rangle_k\frac{\partial\alpha_k}{\partial t}-\sum_j\langle\alpha_k\rangle\langle p_k v_k\rangle_k A_j$$

Dissipation work
due to phase change
at fixed wall

Shear work at interface

Heat between CV

$$+\dot{q}_{wk}+\dot{q}_{ik}+\sum_j\dot{q}_k+V\langle\langle\alpha_k\rangle\rangle\langle\langle q_k'''\rangle\rangle_k$$

Wall heat

Interfacial heat

$$+V\langle\langle\alpha_k\rangle\rangle\langle\langle\rho_k\vec{g}\cdot\vec{v}_k\rangle\rangle_k$$

$$-(\dot{m}_k u_k)_i-\int_{A_{ki}}p_k\vec{v}_k\cdot\vec{n}dA=-(\dot{m}_k u_k)_i-\int_{A_{ki}}p_k(\vec{v}_s-\vec{v}_k)\cdot\vec{n}dA-\int_{A_{ki}}p_k\vec{v}_s\cdot\vec{n}dA$$

$$=-(\dot{m}_k u_k)_i-\int_{A_{ki}}\frac{p_k}{\rho_k}\rho_k(\vec{v}_s-\vec{v}_k)\cdot\vec{n}dA-\int_{A_{ki}}p_k\vec{v}_s\cdot\vec{n}dA$$

$$=-\left(\dot{m}_k\hat{h}_k\right)_i-\langle\langle p_k\rangle\rangle_k V\frac{\partial\alpha_k}{\partial t}$$

Three-field Model

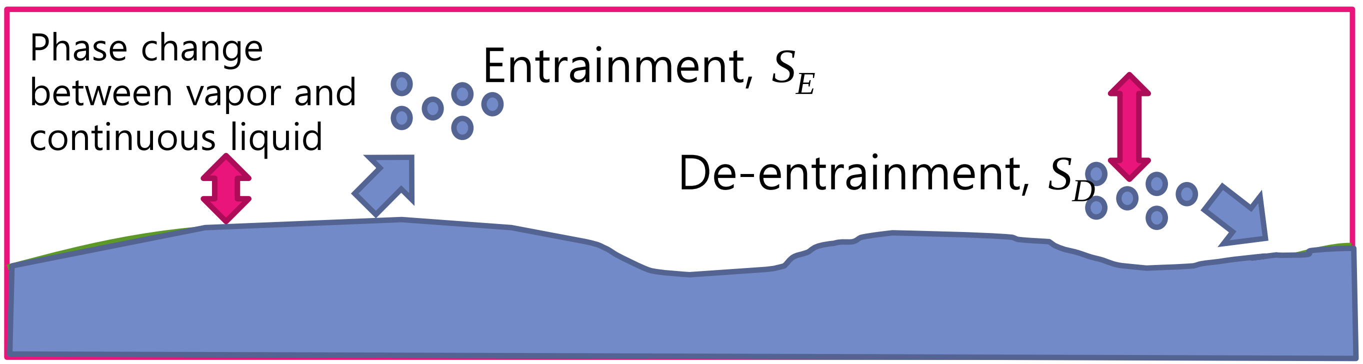
- One more field for droplet
  - Ex) Continuity equation

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla \cdot (\alpha_g \rho_g v_g) = \Gamma_{gl} + \Gamma_{gd}$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l) + \nabla \cdot (\alpha_l \rho_l v_l) = \Gamma_{\mathrm{lg}} - S_E + S_D$$

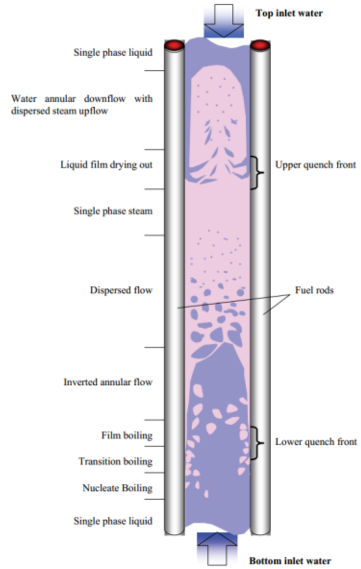
$$\frac{\partial}{\partial t}(\alpha_d \rho_d) + \nabla \cdot (\alpha_d \rho_d v_d) = \Gamma_{dg} + S_E - S_D$$

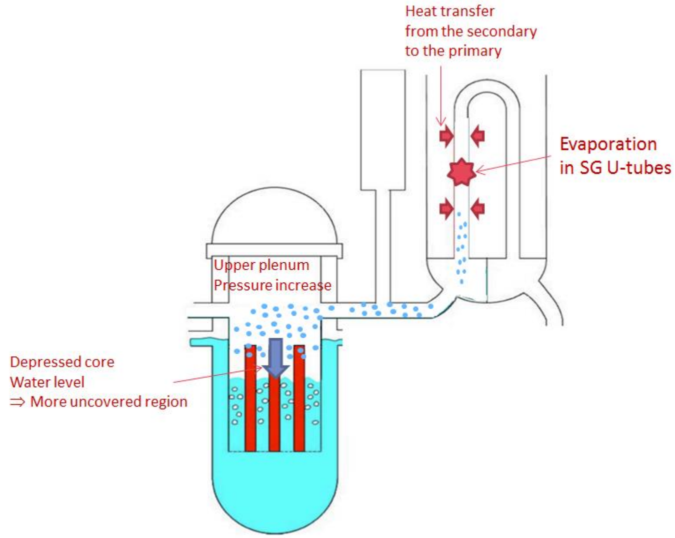
Phase change between vapor and droplet

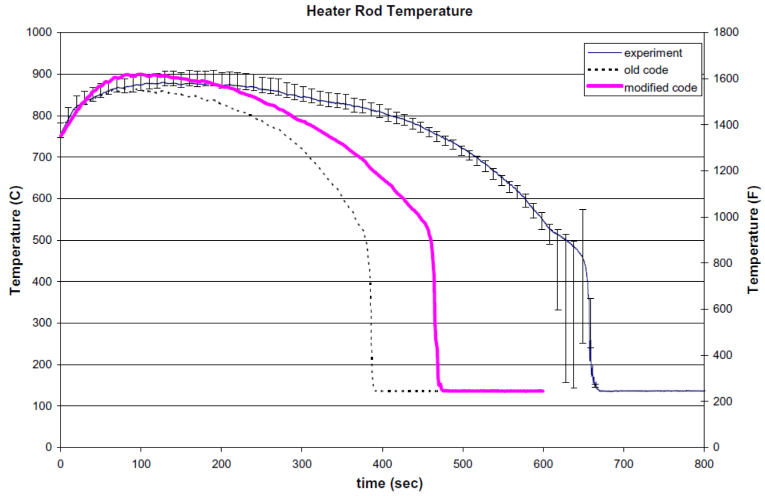


Three-field Model

- Three-field model code
  - ✓ COBRA-TF (sub-channel code)
    - MARS: RELAP5 + COBRA-TF
  - ✓ SPACE, CATHARE-3, TRACE
  - ✓ Necessity of three-field model
    - Reflood heat transfer
    - Steam binding
  - ✓ Problems
    - Droplet behaviors
      - Difficult to measure
      - Many interaction mechanisms with structure
      - Difficult to model
    - Robustness
    - Unphysical prediction of droplet behaviors
      - Droplet can exist in bubbly flow?







Three–field Model

❖ Three-field model code

Droplet-grid spacer interaction