2상유동 열전달 공학

Two-phase flow and heat transfer Engineering

2022년 1학기

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Introduction

- Objectives
  - ✓ Understanding of basic phenomena on the two-phase flow regimes which are important for the mass, momentum and energy transfers between phases.
  - ✓ Introduction of not only traditional empirical flow regime maps but also state-of-the-art dynamic flow regime model.

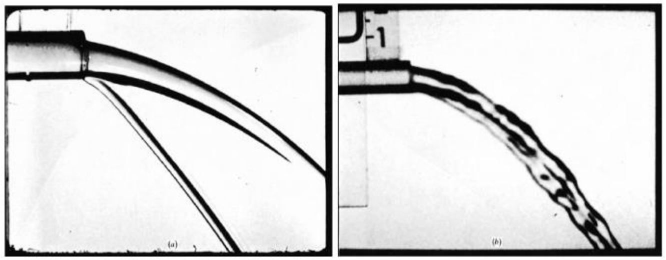
- Textbook
  - ✓ S. Mostafa Ghiaasiann,"Two-phase Flow, Boiling, and Condensation", Cambridge (2008).
    - Chap.4
    - Chap.7

Introduction

❖ Importance of flow regime

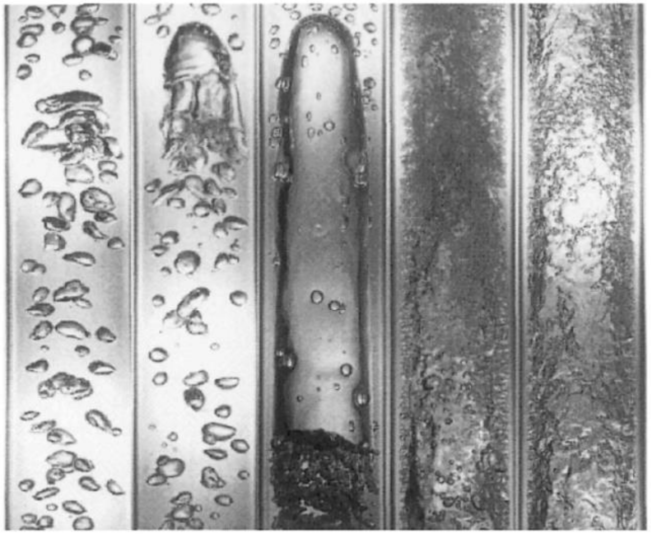
✓ Single-phase flow

- Laminar, transition, turbulent
- Change of flow regime : changes of phenomena governing the transport process of fluid.



✓ Two-phase flow

- Change of interface shapes between phases
- Constitutive models and correlations for conservation equations depend strongly on the flow regimes.
- Method for the prediction of flow regimes is required for the modeling and analysis of two-phase flow systems.



"the most intriguing and difficult aspects of two-phase flow"

Introduction

- Current methods for predicting the flow regimes
  - Far from perfect
  - The difficulty and challenge arise out of the extremely varied morphological configurations that a gas–liquid mixture can acquire.

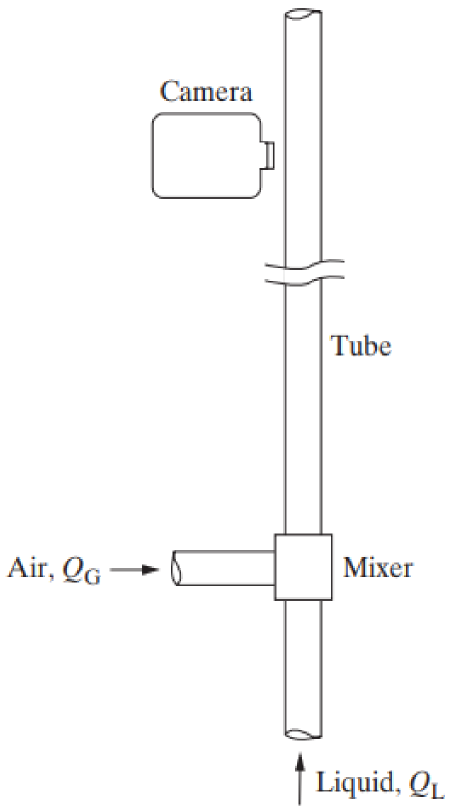
- Physical factors that lead to morphological variations
  - The density difference between the phases
    - As a result the two phases respond differently to forces such as gravity and centrifugal force
  - The deformability of the gas–liquid interphase
    - That often results in incessant coalescence and breakup processes
  - Surface tension forces
    - which tends to maintain one phase dispersal.

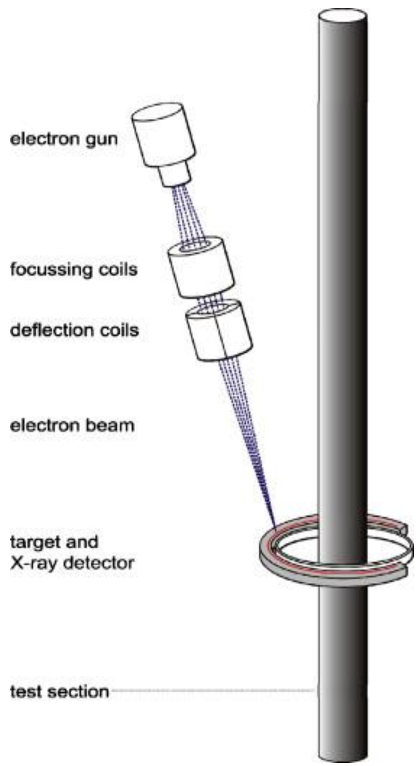
Introduction

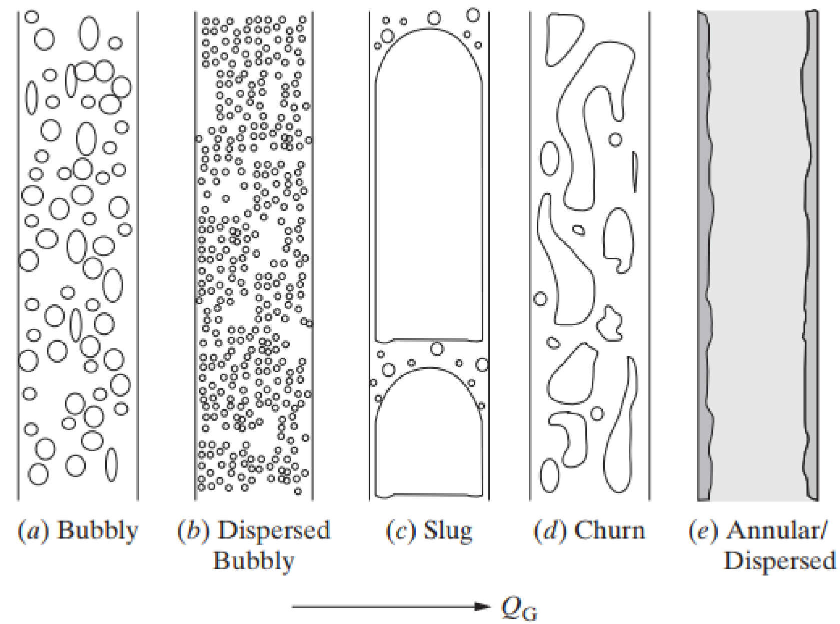
- Other factors
  - ✓ Fluid properties, system configuration/and orientation, size scale of the system, occurrence of phase change, etc.
- Nevertheless,
  - ✓ For the most widely used configurations and/or relatively well-defined conditions reasonably accurate predictive methods exist.
    - Steady-state and adiabatic air–water and steam–water flow in uniform-cross-section long vertical pipes, or large vertical rod bundles with uniform inlet conditions
  - ✓ The literature also contains data and correlations for a vast number of specific system configurations, fluid types, etc.
- Experiments are often needed when a new system configuration and/or fluid type is of interest

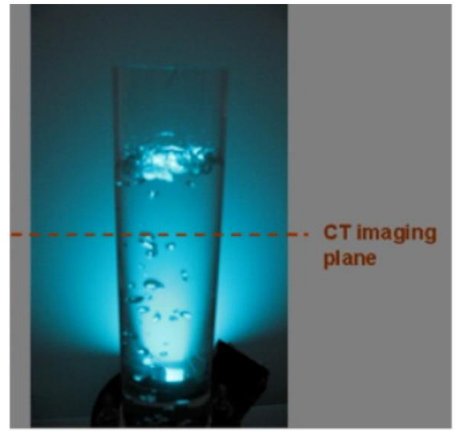
Two-Phase Flow Regimes in Adiabatic Pipe Flow

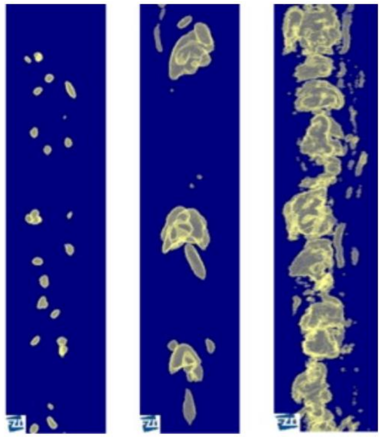
- Vertical, Co-current, Upward Flow





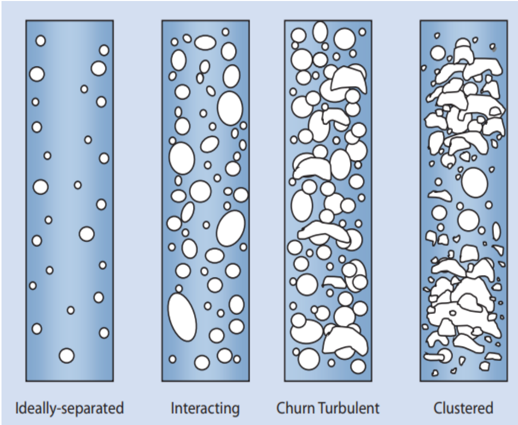




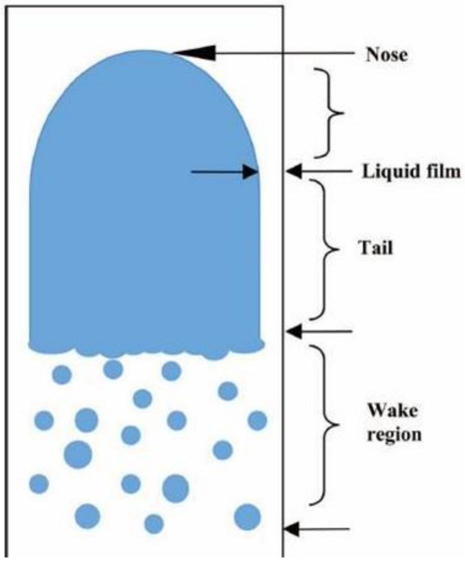


Two-Phase Flow Regimes in Adiabatic Pipe Flow

- Vertical, Co-current, Upward Flow
  - ✓ Bubbly flow
    - Spherical or distorted-spherical shaped bubbles
    - Little interaction at very low gas flows
    - At higher gas flows, bubbles interact, leading to their coalescence and breakup
    - Finely dispersed bubbly flow
    - Highly turbulent liquid flow &low void fraction
    - Increase of bubble number density due to break up

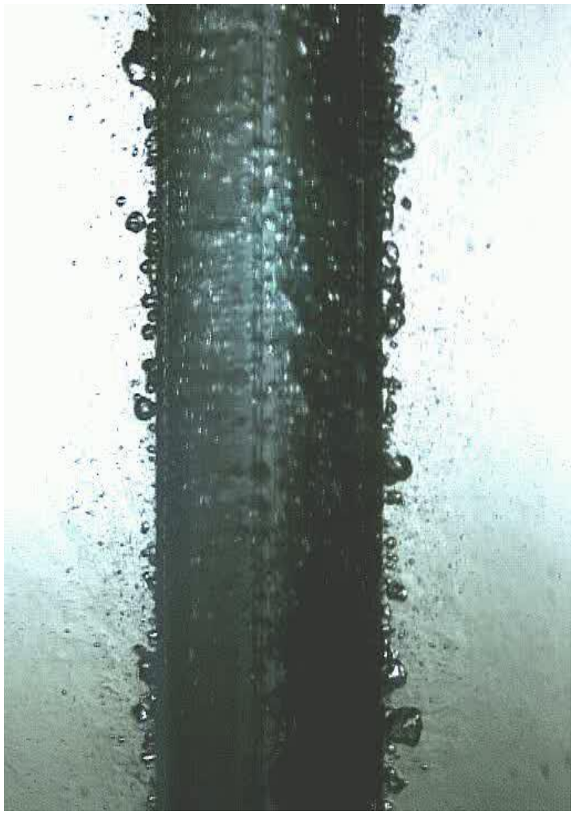


  - ✓ Slug flow
    - Bullet-shaped bubbles (Taylor bubbles) that have approximately hemispherical caps and are separated from one another by liquid slugs which often contains small bubbles.
    - A Taylor bubble approximately occupies the entire cross section and is separated from the wall by a thin liquid film.
    - Taylor bubbles coalesce and grow in length until a relative equilibrium liquid slug length (Ls /D ~ 16)



Two-Phase Flow Regimes in Adiabatic Pipe Flow

- Vertical, Co-current, Upward Flow



Wall voidage

(L/D_h~30)

Net vapor generation

(L/D_h~80)

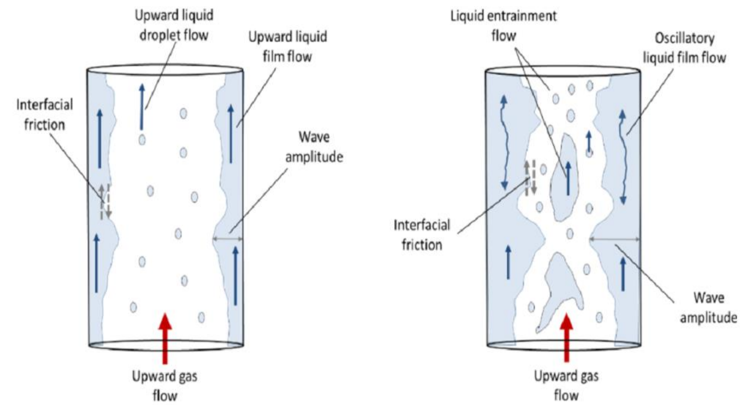
Condensation

(L/D_h~130)

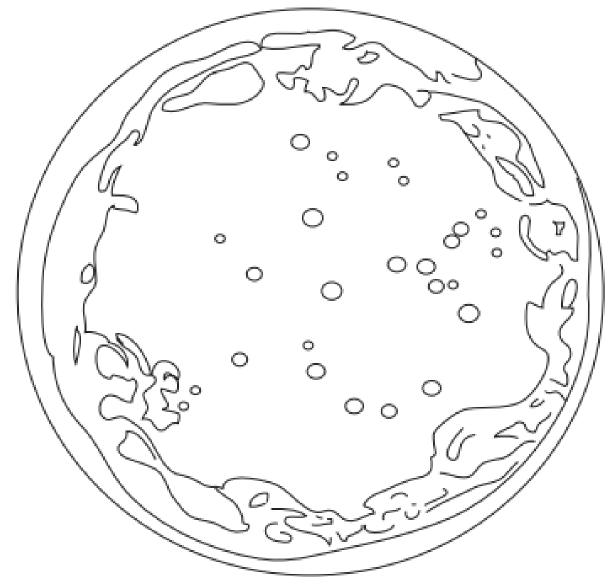
$$(q'' = 470.7kw/m^2, G = 1132.6kg/m^2 \cdot \sec, \Delta T_{sub,inlet} = 19.1K)$$

Two-Phase Flow Regimes in Adiabatic Pipe Flow

- Vertical, Co-current, Upward Flow
  - ✓ Churn (froth) flow: unstable slug flow
    - Chaotic motion of the irregular-shaped gas pockets
    - Actually, no discernable interfacial shape
    - Incessant churning and oscillatory backflow



  - ✓ Annular-dispersed (annular-mist) flow
    - A thin liquid film, often wavy, sticks to the wall while a gas-occupied core, often with entrained droplets (10~100 mm in diameter) is observed
    - Continuous impingement of droplets onto the liquid film and simultaneously an incessant process of entrainment of liquid droplets from liquid film surface.

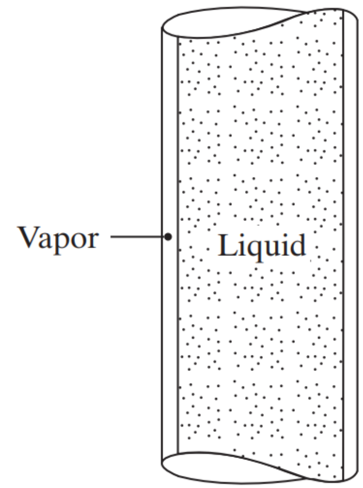


Two-Phase Flow Regimes in Adiabatic Pipe Flow

❖ Vertical, Co-current, Upward Flow

✓ Inverted-annular regime

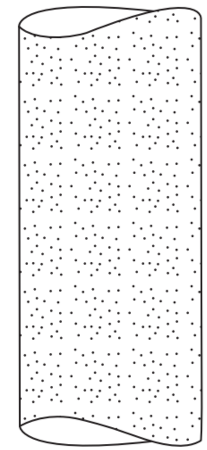
- Observed not in adiabatic gas-liquid flow but in boiling channels.
- A vapor film separates a predominantly liquid flow from the wall.
- The liquid film may contain entrained bubbles
- It takes place in channels subject to high wall heat fluxes and leads to an undesirable condition called the DNB(departure from nucleate boiling).



Inverted-annular

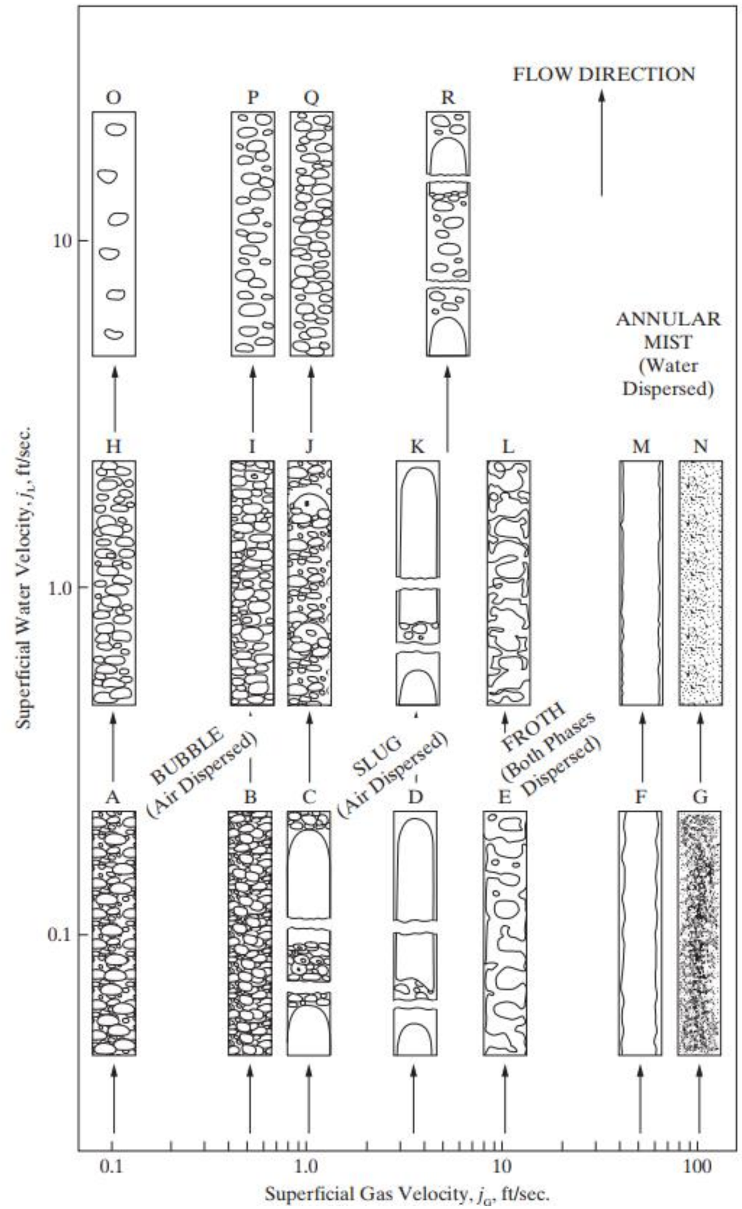
✓ Dispersed-droplet regime

- Often superheated vapor containing entrained droplets flows .
- It can occur when massive evaporation has already caused the depletion of most of the liquid.


Dispersed-droplet

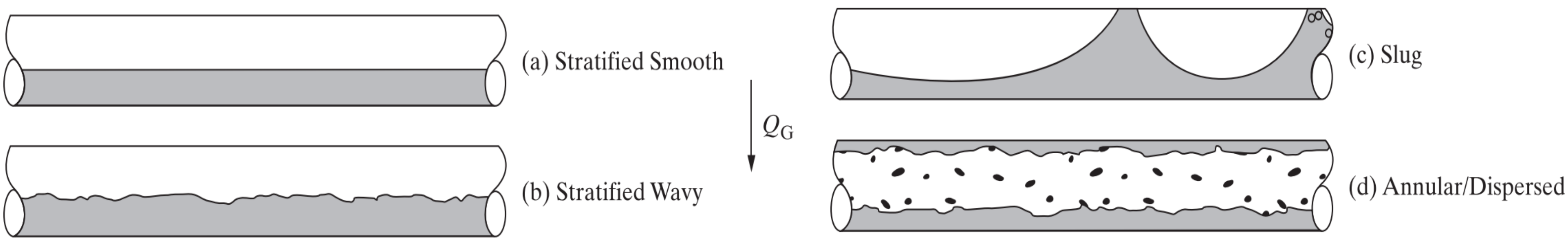
Two-Phase Flow Regimes in Adiabatic Pipe Flow

- Vertical, Co-current, Upward Flow
  - ✓ Schematics of flow regime in a 2.6cm ID vertical pipe
  - ✓ Govier and Aziz (1972)



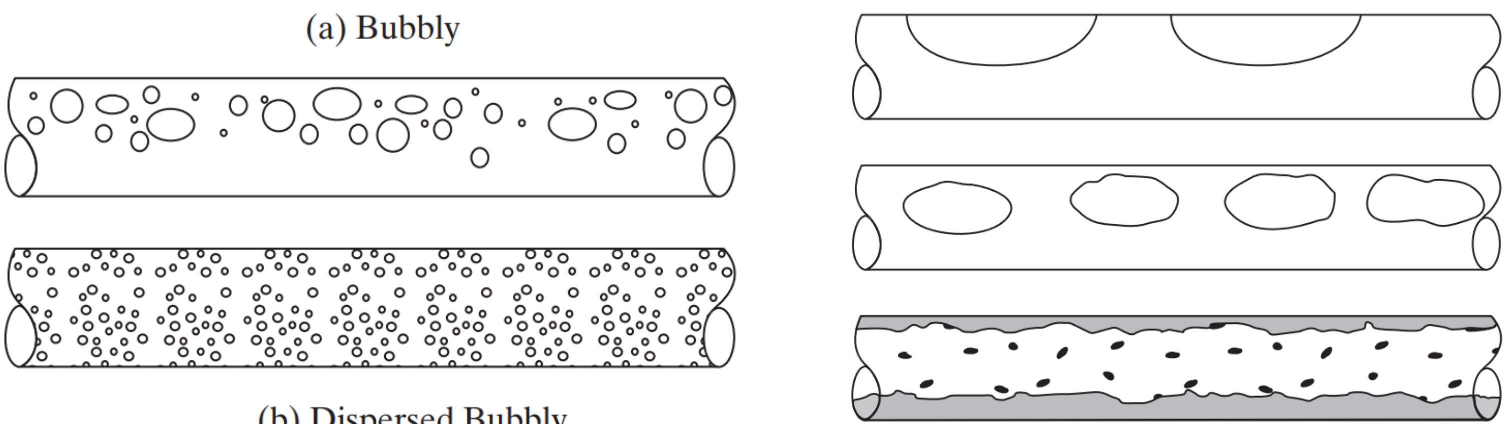
Two-Phase Flow Regimes in Adiabatic Pipe Flow

- Co-current Horizontal Flow
  - ✓ Low liquid flow
    - Stratified smooth flow
    - Stratified wavy flow
      - Formation of large-amplitude waves
    - Slug flow (Intermittent)
      - Large wave grows enough to bridge the entire channel
      - Each phase can contain entrained adverse phase particles
    - Annular-dispersed(Annular-mist) flow
      - Film at bottom is thicker than that that at top



Two-Phase Flow Regimes in Adiabatic Pipe Flow

- Co-current Horizontal Flow
  - ✓ High liquid flow
    - Bubbly flow
    - Finely dispersed bubbly flow
    - Plug or elongated bubbles flow (Intermittent)
      - Separated by elongated gas bubbles
      - Similar with slug flow in vertical pipe
    - Annular-dispersed(Annular-mist) flow



(a) Bubbly

(b) Dispersed Bubbly

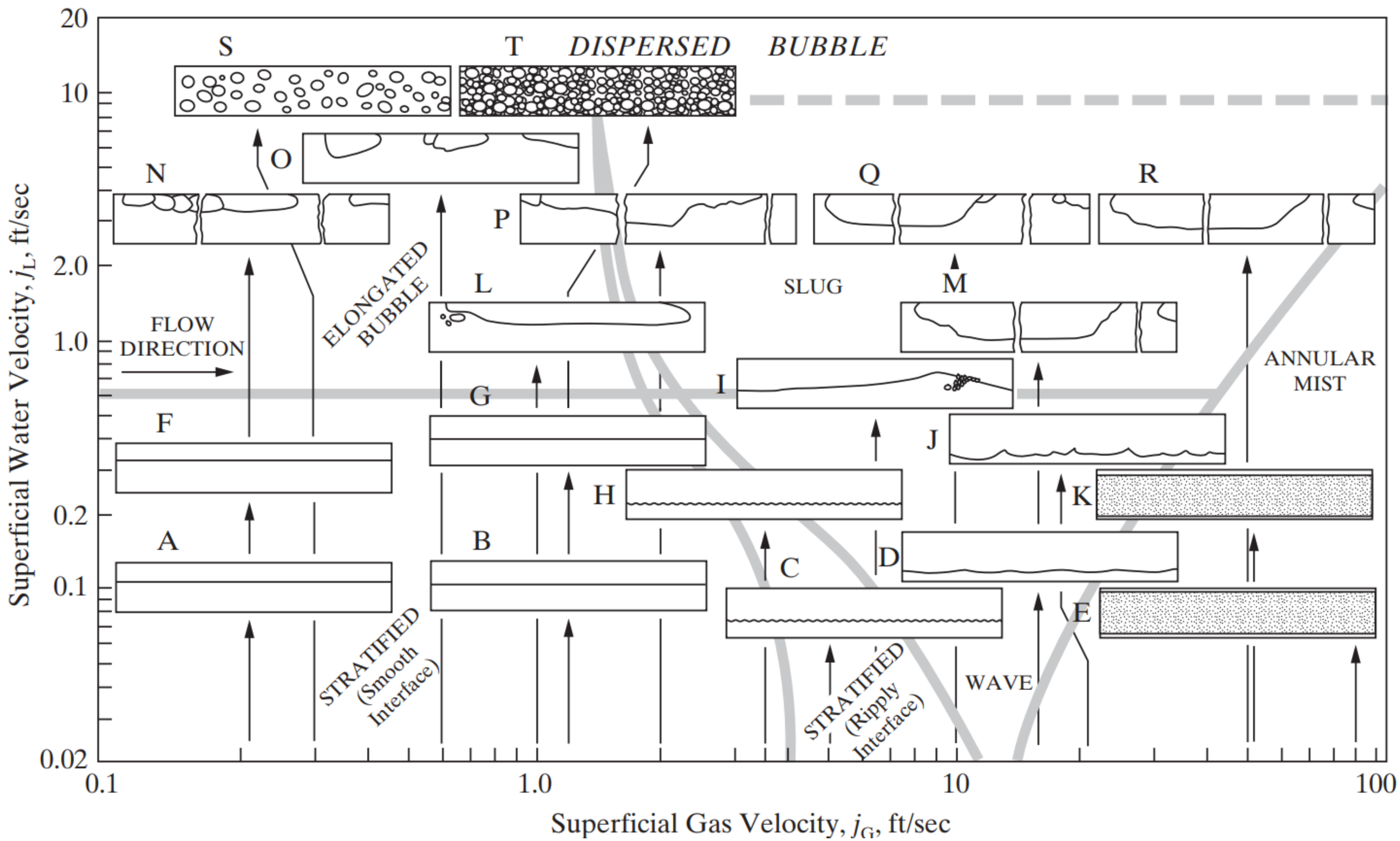
(c) Plug/Elongated Bubble

(d) Annular/Dispersed

Two-Phase Flow Regimes in Adiabatic Pipe Flow

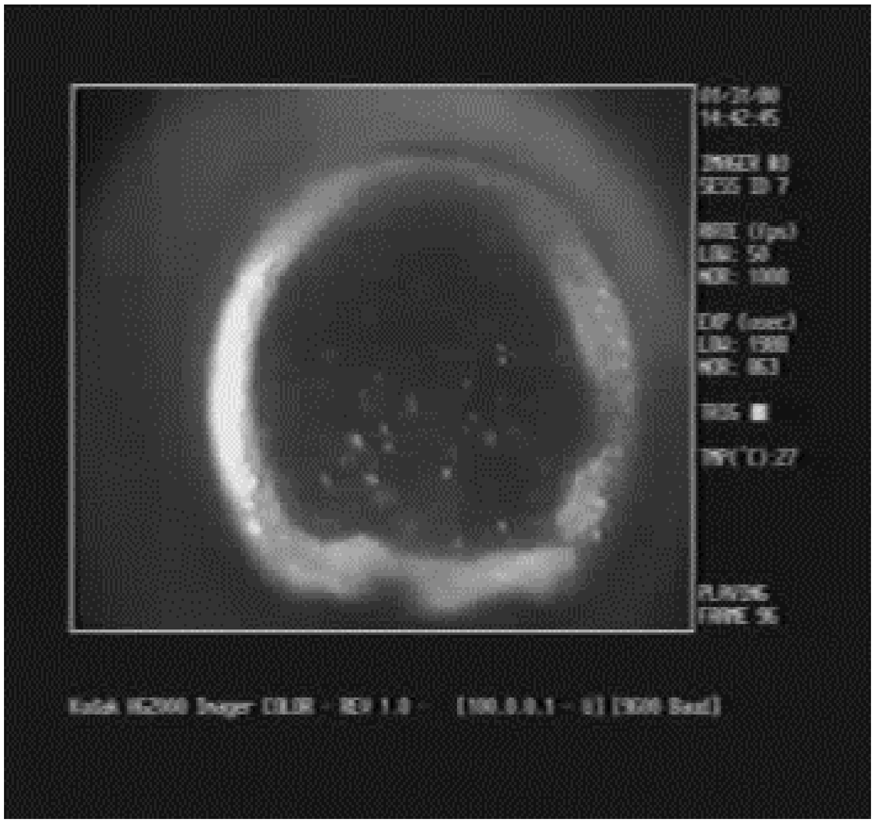
❖ Co-current Horizontal Flow

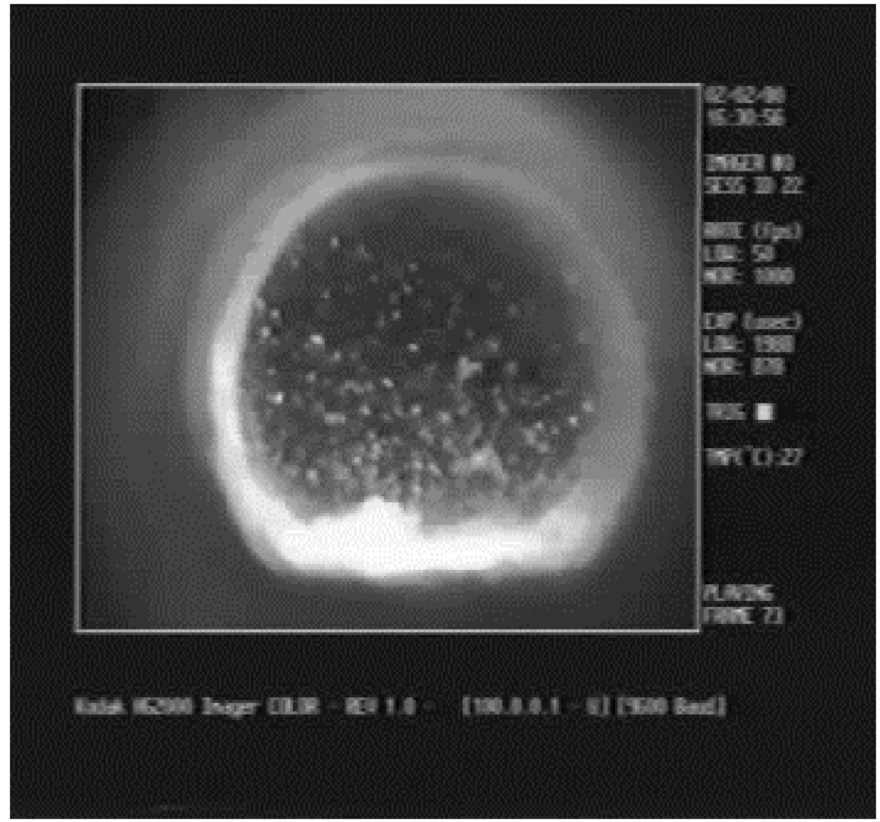
- ✓ Schematics of flow regime in a 2.6cm ID horizontal pipe
- ✓ Govier and Aziz (1972)



Two-Phase Flow Regimes in Adiabatic Pipe Flow

❖ Co-current Horizontal Flow





Two-Phase Flow Regimes in Adiabatic Pipe Flow

- With regard to the two-phase flow regimes, the following points should be borne in mind.
  - ✓ Flow regimes and regime transitions: geometry dependent and are sensitive to liquid properties
    - Surface tension, liquid viscosity, and liquid/gas density ratio
    - Orientation, the size and shape of the flow channel, the aspect ratio, etc.
  - ✓ The basic flow regimes such as bubbly, stratified, churn, and annular-dispersed occur in virtually all system configurations, such as slots, tubes, and rod bundles.
    - Details of the flow regimes of course vary according to channel geometry.
  - ✓ It is possible to define a multitude of subtle flow regimes.
    - However, flow regime maps based on the basic regimes presented here have achieved wide acceptance over time.
    - The regime change boundaries are generally difficult to define because of the occurrence of extensive “transitional” regimes.
  - ✓ In adiabatic, horizontal flow, often for simplicity
    - stratified (smooth and wavy), intermittent (plug, slug, and all subtle flow patterns between them), annular-dispersed, and bubbly

Two-Phase Flow Regimes in Adiabatic Pipe Flow

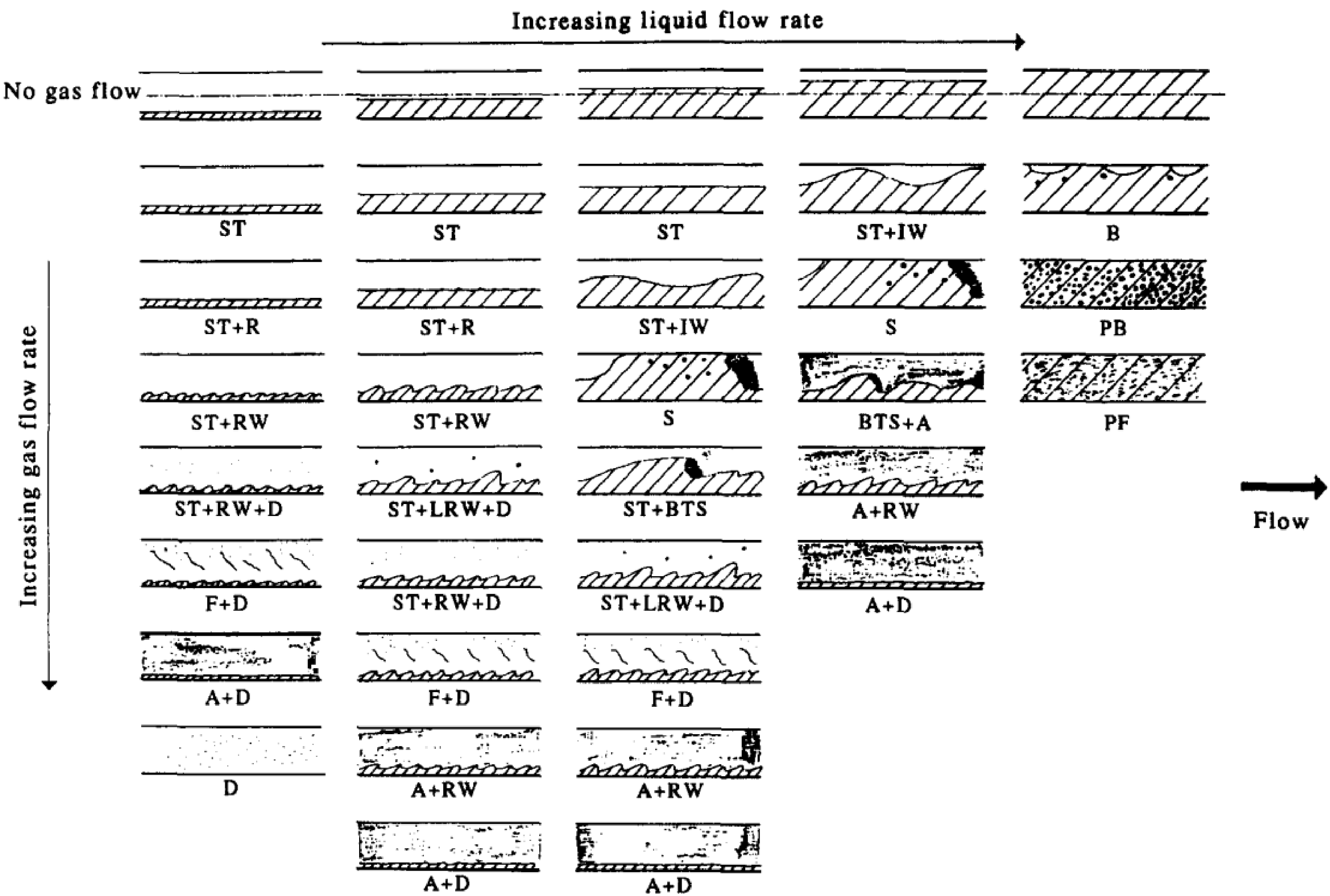
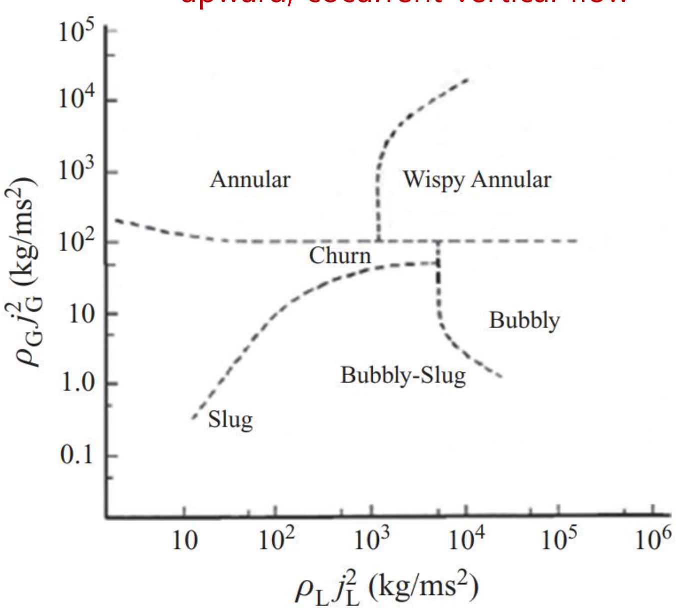


Figure 9. Observed flow patterns for a 0.0935 m i.d. pipe.

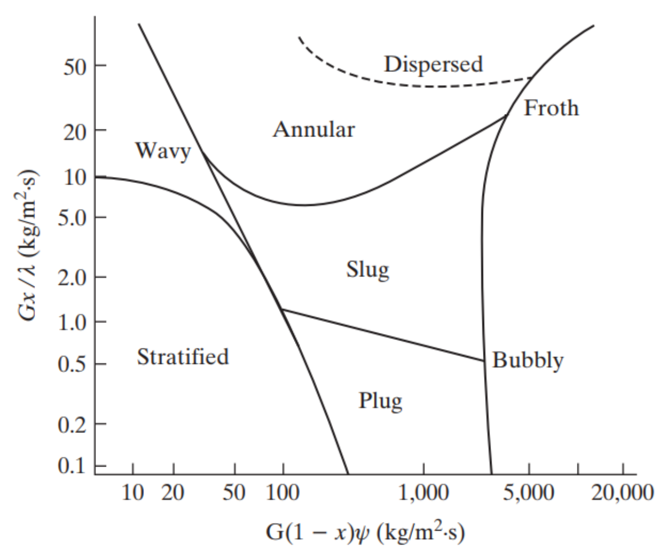
Flow Regime Maps for Pipe Flow

- Flow regime map
  - ✓ Most widely used predictive tools for two-phase flow regimes
  - ✓ Empirical 2D maps with coordinates representing easily quantifiable parameters
    - Phasic superficial velocities + others
  - ✓ Based on vertical or horizontal tubes with small and moderate D ($1 \le D \le 10\ cm$)
  - ✓ For developed conditions with minimal channel end effects

Hewitt and Roberts (1969) for upward, cocurrent vertical flow



Baker (1954) for co-current flow in horizontal pipes (air-water)



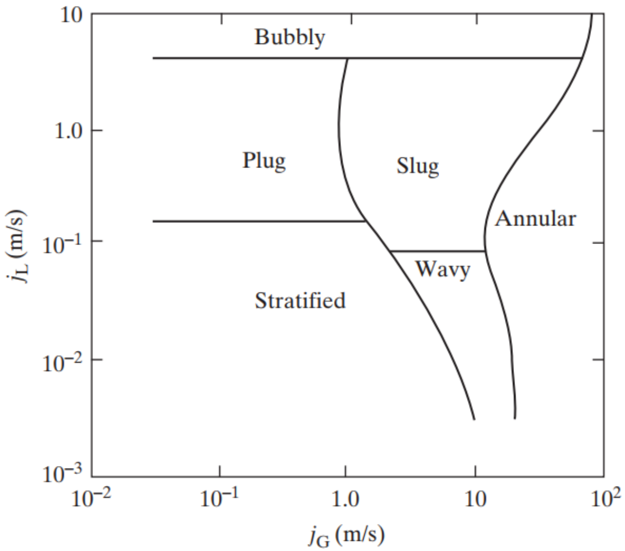
$$\lambda = \left[\frac{\rho_G}{\rho_a}\frac{\rho_L}{\rho_w}\right]^{1/2},$$

$$\psi = \left(\frac{\sigma_w}{\sigma}\right)\left[\left(\frac{\mu_L}{\mu_w}\right)\left(\frac{\rho_w}{\rho_L}\right)^2\right]^{1/3}$$

Flow Regime Maps for Pipe Flow

- Flow regime map
  - ✓ Mandhane et al. (1974)
    - Most widely accepted map for cocurrent flow in horizontal pipes

Mandhane et al. (1974) for co-current flow in horizontal pipes



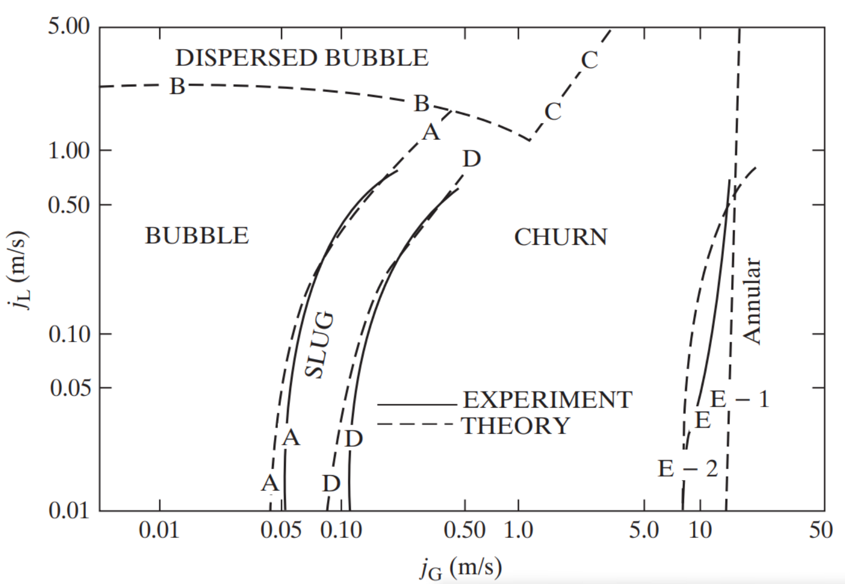
Pipe diameter	12.7–165.1 mm
Liquid density	705–1,009 kg/m^3
Gas density	0.80–50.5 kg/m^3
Liquid viscosity	3×10^{-4}–9×10^{-2} kg/m·s
Gas viscosity	10^{-5}–2.2×10^{-5} kg/m·s
Surface tension	0.024–0.103 N/m
Liquid superficial velocity	0.9×10^{-3}–7.31 m/s
Gas superficial velocity	0.04–171 m/s

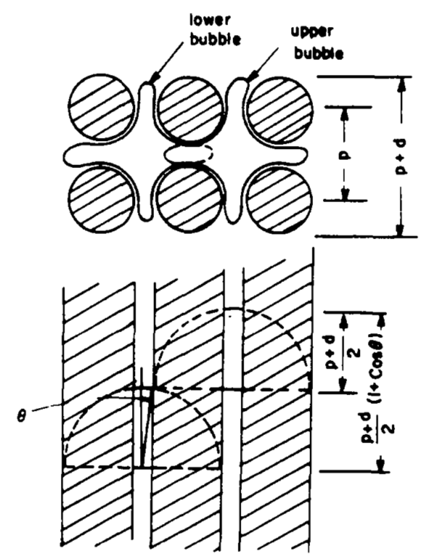
Two-Phase Flow Regimes in Rod Bundles

❖ Flow regime map

✓ Venkateswararao et al. (1982)

- Adiabatic experimental studies using a 20-rod bundle
- With near-prototypical bundle height, rod diameter, and pitch
- In bubbly flow: the bubbles are typically small enough to move within a subchannel
- In slug flow: three configurations
  - Taylor bubbles moving within subchannels (cell-type Taylor bubbles)
  - Large-cap bubbles occupying more than a subchannel
  - Taylor-like bubbles occupying the test section's entire flow (shroud-type Taylor bubbles)



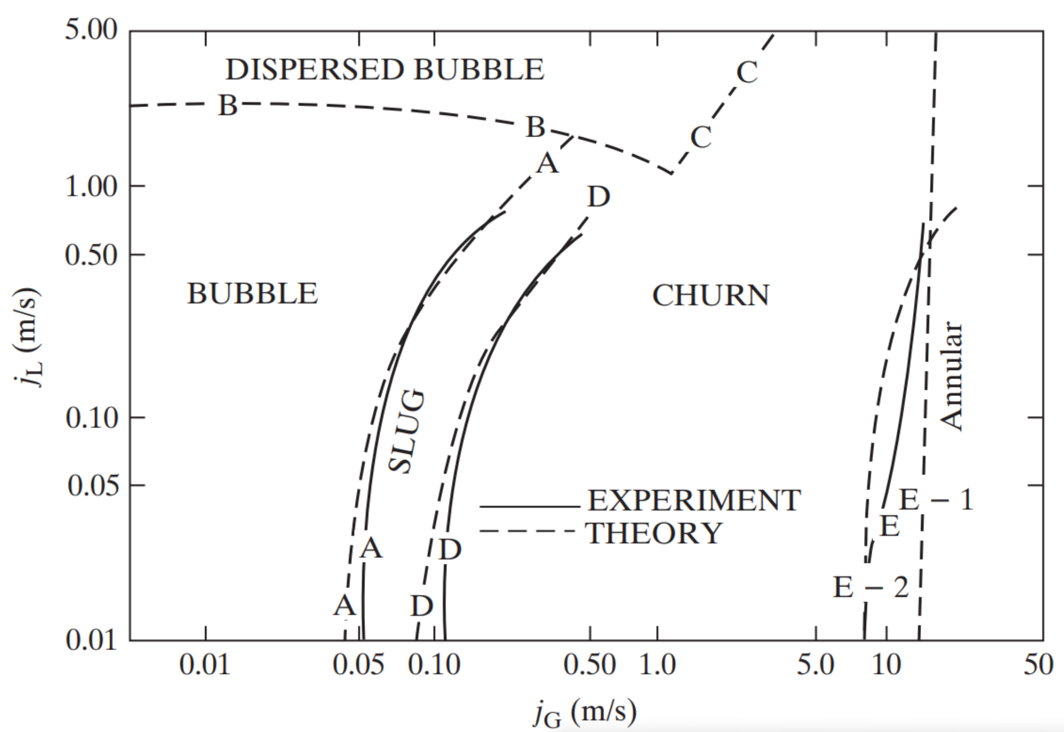


Two-Phase Flow Regimes in Rod Bundles

❖ Flow regime map

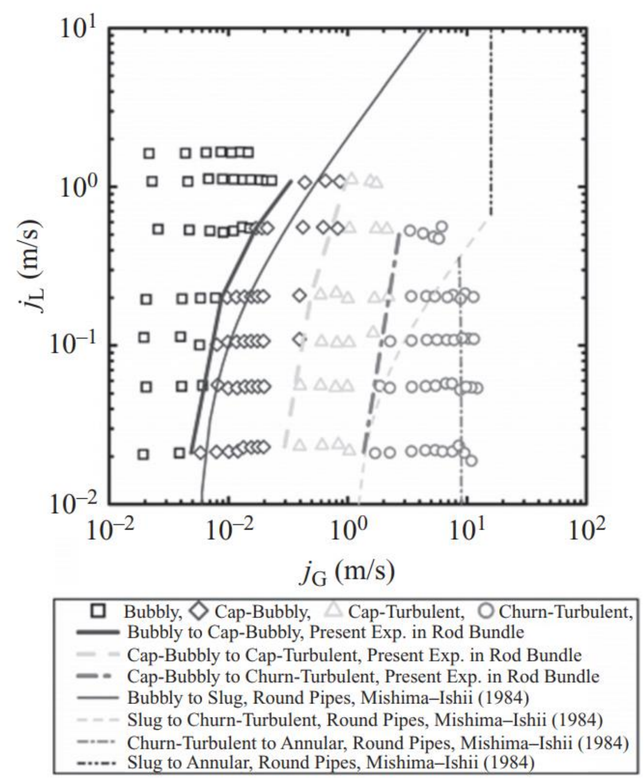
✓ Venkateswararao et al. (1982)

- The churn flow regime is characterized by irregular and alternating motion of liquid and can result from the instability of "cell-type" slug flow.
- Their data could be predicted by the flow regime transition models of Taitel et al. (1980) with modifications to account for the rod bundle geometric configuration.



Two-Phase Flow Regimes in Rod Bundles

- Flow regime map
  - ✓ Paranjape et al. (2011)
    - Air–water experiments in an 8 × 8 square-lattice rod bundle test section
      - – Prototypical BWR rod bundle (12.7-mm rod diameter, 16.7-mm pitch)
    - Used an artificial neural network-based technique for flow regime identification
    - Impedance void meter for the flow area averaged void fractions measurement
    - Bubbly Flow
      - – Dispersed spherical and distorted spherical bubbles throughout the test section (2~10 mm)
    - Cap-Bubbly Flow
      - – Cap-shaped bubbles
    - Cap-Turbulent Flow
      - – Cap bubbles grew sufficiently large to occupy more than two subchannels
    - Churn-Turbulent Flow
      - – Large bubbles spanning five to six subchannels
      - – With highly agitated motion
      - – Zigzag motion



Two-Phase Flow Regimes in Rod Bundles

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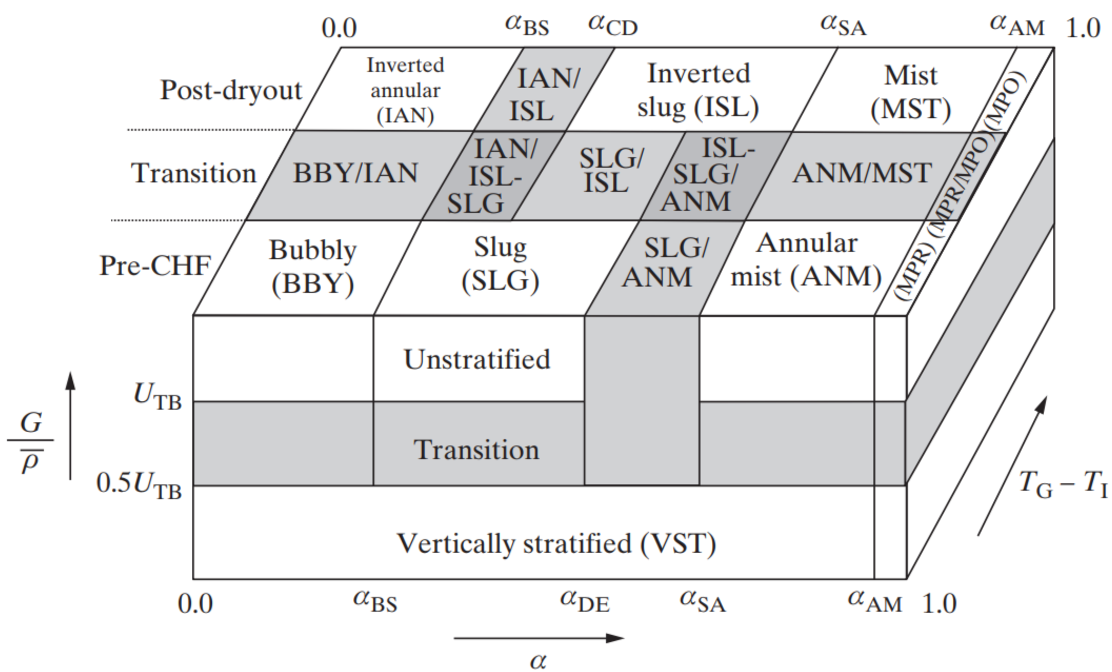
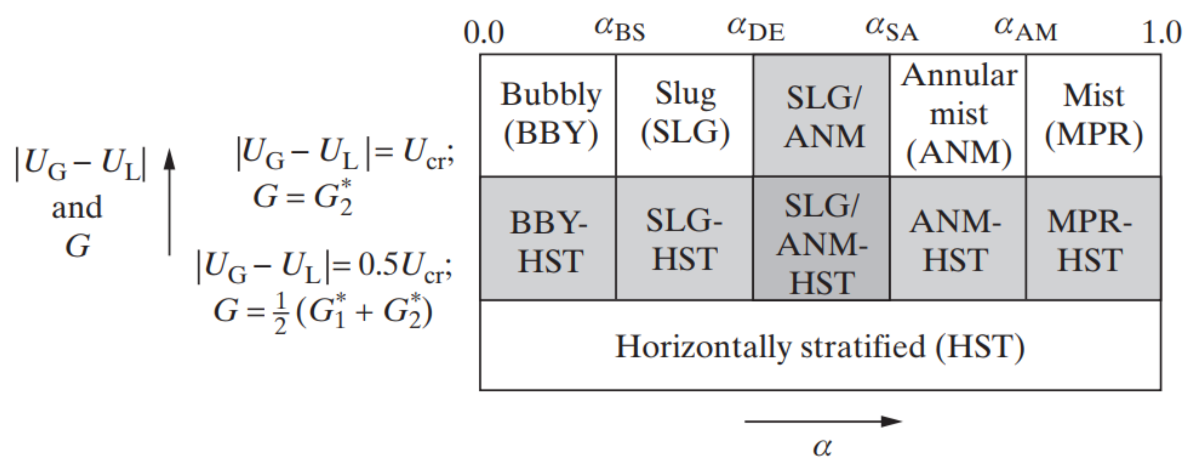
Two-Phase Flow Regimes in Rod Bundles

❖ Flow regime map in RELAP5





Flow Regime Map

- Mechanistic two-phase flow regime models
  - ✓ Rely on physically based models for each major regime transition process
  - ✓ Can be applied to new parameter ranges with better confidence than purely empirical methods
- Important investigations
  - ✓ Taitel and Dukler (1976a, b), Taitel et al. (1980), Weisman and co-workers (1979, 1981), Mishima and Ishii (1984), and Barnea and co- workers (1986, 1987)
- In this chapter (Chap. 7)
  - ✓ For conventional flow passages ($D \geq 3mm$)

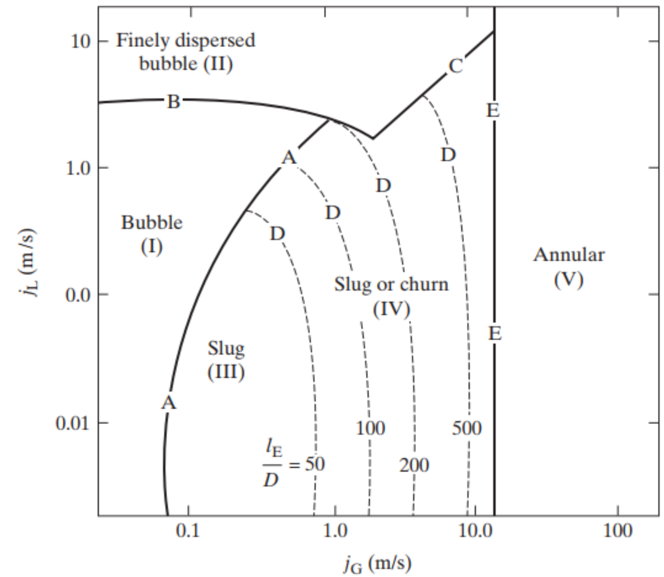
Flow Regime Map

❖ Upward, Co-current Flow in Vertical Tubes

✓ Taitel et al. (1980)

✓ Line A: transition from the bubbly to slug regime

- Transitions happen when bubble become so numerous that they can no longer avoid coalescing and forming larger bubbles
- Eventually forming Taylor bubbles
  - bullet-shaped bubble



✓ Criteria

$$\alpha \approx 0.25$$

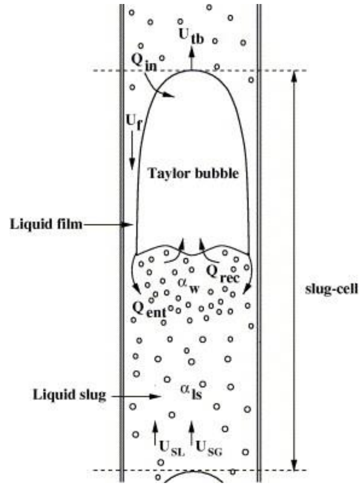
Bubble rise velocity

$$U_B = 1.53\left[\frac{g\Delta\rho\sigma}{\rho_L^2}\right]^{1/4}, \Delta\rho = \rho_L - \rho_G$$

$$U_G - U_L = U_B$$

$$U_G = j_G/\alpha \text{ and } U_L = j_L/(1-\alpha)$$

$$j_L = 3j_G - 1.15\left[\frac{\sigma g\Delta\rho}{\rho_L^2}\right]^{1/4}$$



Flow Regime Map

- Upward, Co-current Flow in Vertical Tubes
  - ✓ Bubble rise velocity (HARMATHY, 1960)

$$C = \frac{F}{\frac{1}{2}\rho_c u^2 A} \qquad C^* = \frac{F}{\frac{1}{2}\rho_c u^2 \frac{d^2\pi}{4}} \qquad \beta = \sqrt{\frac{A}{\frac{d^2\pi}{4}}} \qquad C^* = \beta^2 C$$

for oblate spheroids

$$\beta = \sqrt[3]{\frac{a}{b}}$$

$$F = (d^3\pi/6)\Delta\rho g$$

for spherical caps

$$\beta = \frac{\frac{a}{b}}{\sqrt[3]{1 + \frac{3}{4}\left(\frac{a}{b}\right)^2}}$$

$$u = K\sqrt{\frac{g\Delta\rho d}{\rho_c}} \qquad K = \sqrt{\frac{4}{3C^*}} \qquad K = \frac{1}{\beta}\sqrt{\frac{4}{3C}}$$

For spheres $\beta = 1$, and in infinite media $C_{s\infty} = 0.44$ and $K_{s\infty} = 1.74$

For non-spherical bubbles

$$\frac{u_\infty}{u_{s\infty}} = \frac{1}{\beta}\sqrt{\frac{C_{s\infty}}{C_\infty}} = \sqrt{\frac{C_{s\infty}}{C^*_\infty}} = \phi_1\,(\text{shape}) = \phi_2(N_{Eo})$$ Eotvos number $g\Delta\rho D^2/\sigma$

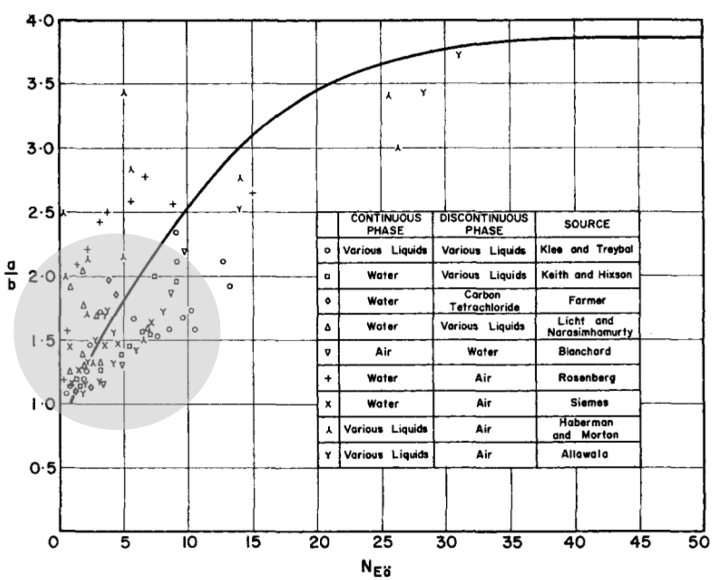
Flow Regime Map

- Upward, Co-current Flow in Vertical Tubes
  - ✓ Bubble rise velocity (HARMATHY, 1960)

For non-spherical bubbles

$$\frac{u_\infty}{u_{s\infty}} = \sqrt{\frac{C_{s\infty}}{C^*_\infty}} = \phi_1\,(\text{shape}) = \phi_2(N_{Eo})$$ Eotvos number $g\Delta\rho D^2/\sigma$

$$\frac{C^*_\infty}{C_{s\infty}} = 1.29\, N_{Eo}{}^{1/2} \quad (N_{Eo} < 13) \qquad \frac{u_\infty}{u_{s\infty}} = \frac{0.88}{N_{Eo}{}^{1/4}}$$



$$u = K\sqrt{\frac{g\Delta\rho d}{\rho_o}} \qquad K_{s\infty} = 1.74$$

$$u_\infty = 1.53\sqrt{\frac{g\Delta\rho d}{\rho_o}} \cdot \frac{1}{\left(\frac{g\Delta\rho d^2}{\sigma}\right)^{\frac{1}{4}}}$$

$$= 1.53\left(\frac{g\Delta\rho\sigma}{\rho_o^2}\right)^{\frac{1}{4}}$$

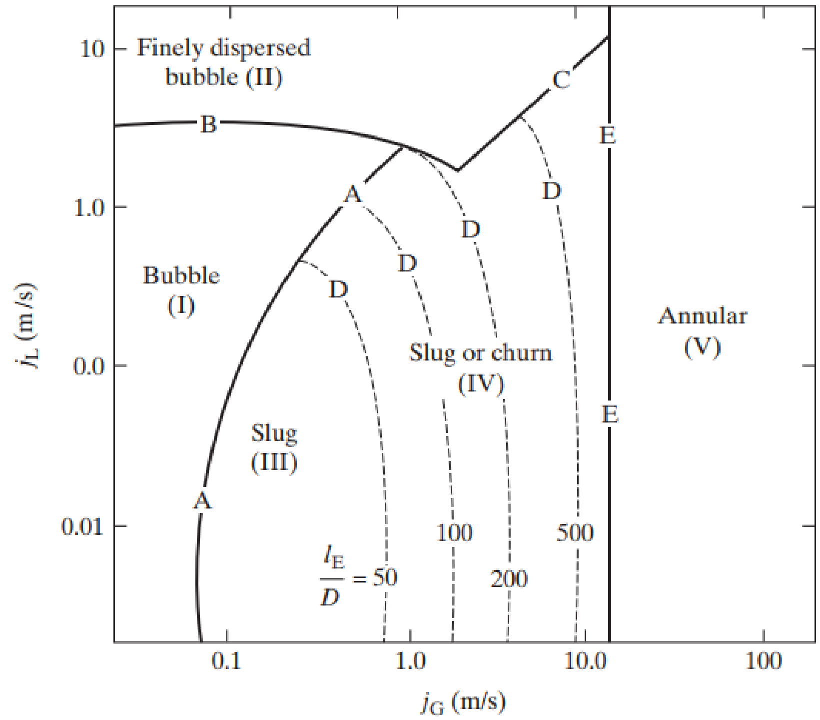
Flow Regime Map

- Upward, Co-current Flow in Vertical Tubes
  - Conditions necessary for the existence of bubbly flow
    - Taitel et al. (1980)
      - Bubbly flow becomes impossible when the rise velocity of a Taylor bubble is lower than the rise velocity of regular bubbles

$$0.35\sqrt{gD} \leq 1.53[g\Delta\rho\sigma/\rho_L^2]^{1/4}$$

$$Fr_D = \frac{V_T}{\sqrt{\Delta\rho g D/\rho_L}} = 0.35$$

$$\left[\frac{\rho_L^2 g D^2}{\sigma\Delta\rho}\right]^{1/4} \leq 4.36$$



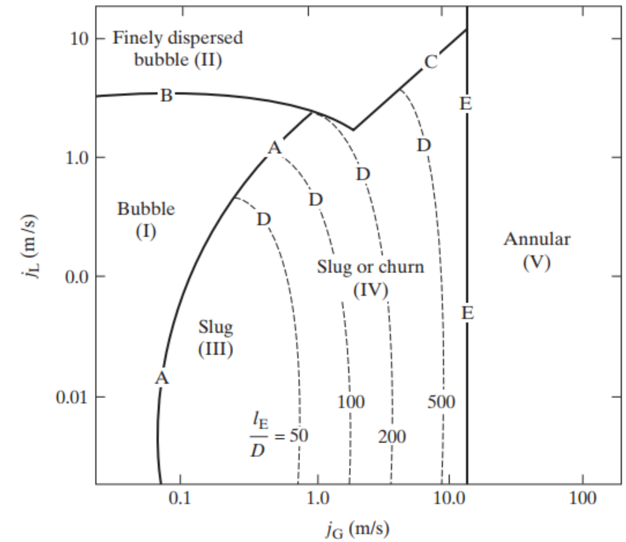
Flow Regime Map

- Upward, Co-current Flow in Vertical Tubes
  - ✓ Line B
    - Transition to finely dispersed bubble flow
    - Small, nearly spherical bubbles
    - Strong turbulence → bubble break up
    - Assumptions

    (a) The flow must be fully turbulent.

    (b) The size of the finely dispersed bubbles is within the inertial turbulent eddy size range

    (c) Dispersed bubbles must remain spherical since distorted bubbles have a higher chance of coalescence.



$$j_L + j_G = 4\left\{\frac{D^{0.429}\left(\frac{\sigma}{\rho_L}\right)^{0.089}}{\nu_L^{0.072}}\left(\frac{g\Delta\rho}{\rho_L}\right)^{0.446}\right\}$$

  - ✓ Line C
    - Maximum packing
    - In high velocity region, small velocity slip
    - The upper limit for the existence of the dispersed bubbly regime

$$\alpha_{max} = \frac{\pi}{6} d_B^3 / d_B^3 \approx 0.52$$

$$\alpha \approx \beta = j_G/(j_L + j_G)$$

$$\frac{j_G}{j_L + j_G} = 0.52$$

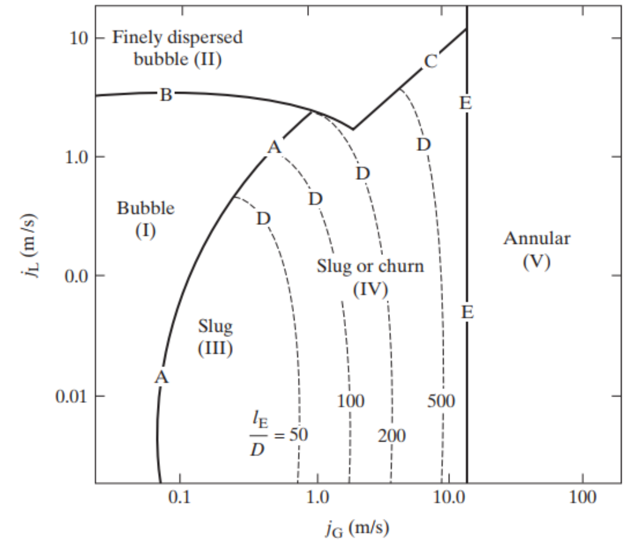
$$\boxed{j_G/(j_L + j_G) > 0.52}$$

would lead to slug flow

Flow Regime Map

- Upward, Co-current Flow in Vertical Tubes
  - ✓ Line D
    - Churn-to-slug flow regime transition
    - Churn defined by Taitel
      - – Entrance regime for the development of slug flow.
    - Developing length from entrance for the slug flow

$$\frac{l_E}{D} = 40.6\left(\frac{j}{\sqrt{gD}} + 0.22\right)$$



- – The flow regime will be slug only at distances from the inlet larger than l_E. Otherwise, the flow regime will be churn.

Flow Regime Map

- Upward, Co-current Flow in Vertical Tubes
  - Line E
    - Transition to the annular-dispersed flow regime
    - The gas velocity is sufficient to shatter the liquid core in the pipe into dispersed droplets
    - Drag force on the droplets overcomes their weight
    - Droplet diameter

$$\mathrm{We}_{\mathrm{cr}} = \rho_{\mathrm{G}} j_{\mathrm{G}}^2 d/\sigma = 30$$

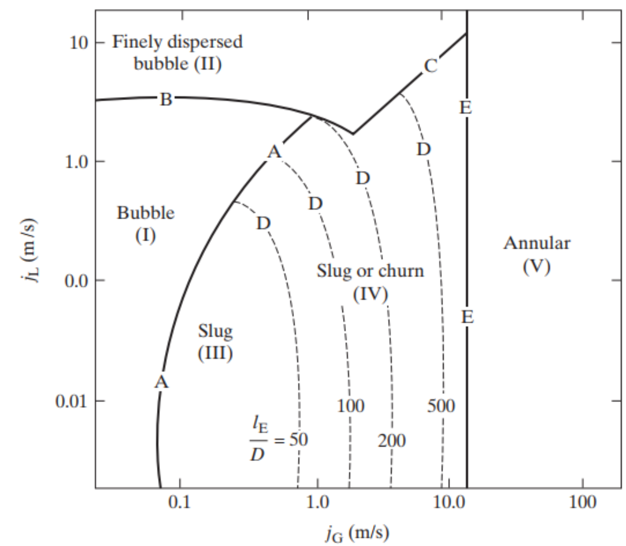
    - Force balance

$$C_{\mathrm{D}} \frac{\pi d^2}{4} \frac{1}{2} \rho_{\mathrm{G}} j_{\mathrm{G}}^2 = \frac{\pi}{6} d^3 g \Delta\rho$$

$$\frac{j_{\mathrm{G}} \rho_{\mathrm{G}}^{1/2}}{(\sigma g \Delta\rho)^{1/4}} = 3.1$$

    - Annular flow criterion

$$j_{\mathrm{G}} \rho_{\mathrm{G}}^{1/2} / (\sigma g \Delta\rho)^{1/4} > 3.1$$



Flow Regime Map

- Flow Regime Transition Models of Mishima and Ishii
  - ✓ The void fraction: the most important geometric parameter affecting flow regime transition
  - ✓ Four major flow regimes
    - Bubbly, slug, churn-turbulent, and annular
  - ✓ Transition criteria
    - Void fraction except for the churn-turbulent to annular
  - ✓ Void fraction
    - Estimated by DFM with parameters proposed by Ishii (1977)

$$C_0 = \begin{cases} 1.2 - 0.2\sqrt{\frac{\rho_G}{\rho_L}} & \text{for round tubes,} \\ 1.35 - 0.35\sqrt{\frac{\rho_G}{\rho_L}} & \text{for rectangular ducts.} \end{cases}$$

Flow Regime Map

- Flow Regime Transition Models of Mishima and Ishii
  - Transition to slug flow
    - $\alpha = 0.3$

$$C_0 = \begin{cases} 1.2 - 0.2\sqrt{\frac{\rho_G}{\rho_L}} & \text{for round tubes,} \\ & \text{angular ducts.} \end{cases}$$

$$V_{gj} = \sqrt{2}\left(\frac{\sigma g \Delta\rho}{\rho_L^2}\right)^{1/4}(1 - \langle\alpha\rangle)^{1.75}$$

$$D < \frac{\sqrt{\frac{\sigma}{g\Delta\rho}}N_{\mu L}^{-0.4}}{[(1 - 0.11C_0)/C_0]^2}$$

$$j_L = \left(\frac{3.33}{C_0} - 1\right) j_G - \frac{0.76}{C_0}\left(\frac{\sigma g \Delta\rho}{\rho_L^2}\right)^{1/4}$$

    - Transition occurs when $j_L < j_{L,transition}$, otherwise bubble flow

$$\langle\alpha\rangle = \frac{\langle j_G\rangle}{C_0\langle j\rangle + V_{gj}}$$

  - Transition to churn-turbulent flow
    - $\alpha \geq \alpha_B$

$$\alpha = \frac{j_G}{C_0 j + 0.35\sqrt{\frac{\Delta\rho g D}{\rho_L}}}$$

$$\alpha_B = 1 - 0.813\left\{\frac{(C_0 - 1)j + 0.35\sqrt{\frac{\Delta\rho g D}{\rho_L}}}{j + 0.75\sqrt{\frac{\Delta\rho g D}{\rho_L}}\left[\frac{\Delta\rho g D^3 \rho_L}{\mu_L^2}\right]^{1/18}}\right\}^{3/4}$$

Flow Regime Map

Flow Regime Transition Models of Mishima and Ishii

- Transition to annular flow (for small diameter)

$$j_G = \sqrt{\frac{\Delta\rho g D}{\rho_G}\alpha^{1.25}\left\{\frac{1-\alpha}{0.015[1+75(1-\alpha)]}\right\}^{1/2}} \qquad D < \frac{\sqrt{\frac{\sigma}{g\Delta\rho}}N_{\mu L}^{-0.4}}{[(1-0.11C_0)/C_0]^2}$$

- Transition to annular flow (for larger diameter)
  - Regime transition should not depend on the tube diameter.
  - It is assumed that the destruction of the liquid slug and the entrainment of generated droplets cause the flow regime transition.
  - Onset of liquid droplet entrainment in annular flow (Ishii, 1977)

$$j_G \geq \left(\frac{\sigma g \Delta\rho}{\rho_G^2}\right)^{1/4} N_{\mu L}^{-0.2} \qquad N_{\mu L} = \frac{\mu_L}{\left[\rho_L \sigma \sqrt{\frac{\sigma}{g\Delta\rho}}\right]^{1/2}}$$

  - Another model
    - Jayanti and Hewitt
    - Models based on flooding of the liquid film surrounding the Taylor bubbles

EXAMPLE 7.1. Based on the flow regime models of Taitel *et al.* (1980), for an air–water mixture upward flow in a 2-m-long tube with 5-cm inner diameter, under atmospheric pressure and at room temperature, determine the flow regime for the following conditions:

SOLUTION. With respect to the relevant properties, we have $\rho_L = 997\,\text{kg/m}^3$, $\rho_G = 1.185\,\text{kg/m}^3$, $\mu_L = 8.94 \times 10^{-4}\,\text{kg/m·s}$, $\mu_G = 10^{-5}\,\text{kg/m·s}$, and $\sigma = 0.071\,\text{N/m}$. Let us start with case (a), namely, $j_L = 0.9\,\text{m/s}$ and $j_G = 8\,\text{m/s}$. Given the relatively high gas superficial velocity, we should use Fig. 7.1 as a guide, and start from the right side of the map. First check Eq. (7.10). For the given conditions, we find

$$\frac{j_G \rho_G^{1/2}}{(\sigma g \Delta\rho)^{1/4}} = 1.697 < 3.1.$$

The annular-dispersed regime is therefore not applicable. We next will check the finely dispersed bubbly and slug/churn regimes. Let us check Eq. (7.6). Accordingly,

$$\frac{j_G}{j_G + j_L} = 0.899 > 0.52.$$

Thus, finely dispersed bubbly flow is also not possible. A check of Eq. (7.3) would show that

$$3\,j_G - 1.15\left[\frac{\sigma g \Delta\rho}{\rho_L^2}\right]^{0.25} = 23.8.$$

Clearly, then, the flow regime is not bubbly. It must therefore be slug or churn. To determine which one, let us use Eq. (7.8), according to which, for our case, $l_E = 3.06\,\text{m}$.

Flow Regime Map

Since the total length of our tube is smaller than l_E, our entire tube will remain in churn flow.

We will now consider case (b), namely, $j_L = 1.1\,\mathrm{m/s}$ and $j_G = 0.4\,\mathrm{m/s}$. Starting with Eq. (7.10), we find

$$\frac{j_G \rho_G^{1/2}}{(\sigma g \Delta\rho)^{1/4}} = 0.085 < 3.1.$$

Therefore annular-dispersed flow does not apply. (This was actually obvious, given that $j_G = 8\,\mathrm{m/s}$ in case (a), which was larger than j_G in case (b), did not lead to annular-dispersed flow.)

We now examine the finely dispersed flow. Accordingly, $j_G/(j_G + j_L) = 0.267$. The right side of Eq. (7.5) is found to be 252.3, which is clearly larger than $j_G + j_L$. The flow regime cannot be finely dispersed bubbly. We should now check Eq. (7.3). For the given conditions we find

$$3 j_G - 1.15\left[\frac{\sigma g \Delta\rho}{\rho_L^2}\right]^{0.25} = 1.01 < j_L.$$

The flow regime is therefore bubbly.

Flow Regime Map

EXAMPLE 7.2. Repeat the problem in Example 7.1, this time using the flow regime transition models of Mishima and Ishii (1984). Compare the results with those obtained in Example 7.1.

SOLUTION. The properties calculated in Example 7.1 apply. From Eq. (7.12), we get $C_0 = 1.193$; from Eq. (6.25) for churn flow we get $V_{gj} = \sqrt{2}[\sigma g \Delta\rho/\rho_L^2]^{0.25} = 0.23\,\text{m/s}$; and from Eq. (7.19), we get $N_{\mu L} = 2.04 \times 10^{-3}$. Also, with respect to the criterion of Eq. (7.17), we get

$$\frac{\sqrt{\frac{\sigma}{g\Delta\rho}}N_{\mu L}^{-0.4}}{[(1-0.11C_0)/C_0]^2} = 0.06\,\text{m} > D.$$

Let us now focus on the conditions of case (a), where $j_G = 8\,\text{m/s}$ and $j_L = 0.9\,\text{m/s}$. First, we will check the possibility of annular-dispersed flow, given the relatively high value of j_G. Since the criterion of Eq. (7.17) is satisfied, we must calculate α from $\alpha = j_G/(C_0 j + V_{gj})$ and then check Eq. (7.16). The expression for the void fraction gives $\alpha = 0.737$. With this value of α, the right side of Eq. (7.16) is found to be 12.76 m/s, which is evidently larger than j_G. The flow regime, therefore, is not annular-dispersed. In other words, conditions for the transition from churn to annular have not been met.

Flow Regime Map

Next, we will calculate α from Eq. (7.14) and α_B from Eq. (7.15). We will get $\alpha = 0.736$ and $\alpha_B = 0.77$. Since $\alpha < \alpha_B$, the flow regime cannot be churn. We are left with bubbly or slug. We should use Eq. (7.13) to decide which one of these two regimes applies. The right side of Eq. (7.13) is calculated to be 14.22 m/s, which is evidently larger than j_L. The flow regime is therefore slug.

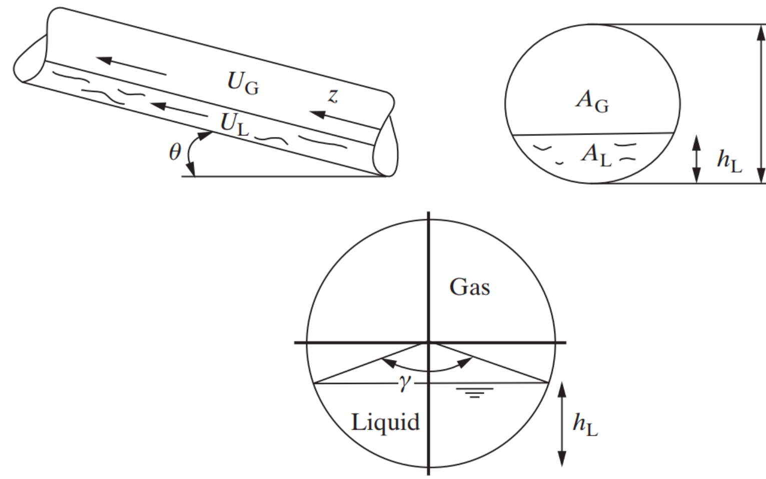
We should now consider case (b), namely, $j_L = 1.1$ m/s and $j_G = 0.4$ m/s. Repetition of the previous calculations will result in the elimination of annular-dispersed and churn flow regimes. The right side of Eq. (7.13) is found to be 0.613 m/s, which is actually smaller than j_L. The flow pattern is therefore bubbly.

Flow Regime Map

❖ Co-current Flow in a Near-Horizontal Tube

✓ Taitel and Dukler

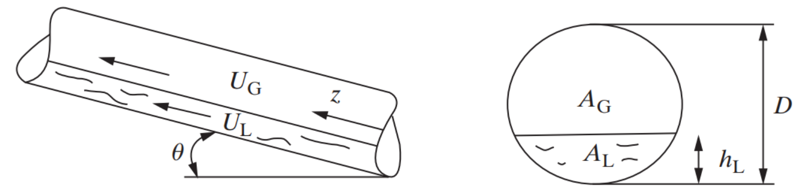
- Stratified (smooth and wavy)
- Intermittent (slug, plug/elongated bubbles),
- Dispersed bubbly
- Annular-dispersed

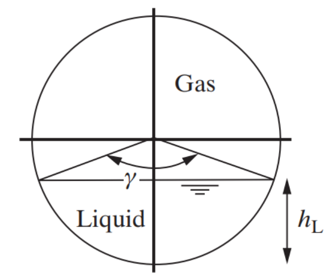


- Stratified-to-wavy & stratified-to-intermittent
  - Most successful models
  - Important for pipelines:
    - ➢ Intermittent flow regimes have a higher frictional pressure drop than stratified flow.
    - ➢ Intermittency also leads to countercurrent flow limitation (CCFL), or flooding, in channels with countercurrent gas–liquid flow

Flow Regime Map

- Co-current Flow in a Near-Horizontal Tube
  - ✓ Separated flow phasic momentum eqs.





$$-\frac{dP}{dz} - \frac{\tau_{\mathrm{wL}} p_{\mathrm{L}} - \tau_{\mathrm{I}} p_{\mathrm{I}}}{A(1-\alpha)} - \rho_{\mathrm{L}} g \sin\theta = 0,$$

$$-\frac{dP}{dz} - \frac{\tau_{\mathrm{wG}} p_{\mathrm{G}} + \tau_{\mathrm{I}} p_{\mathrm{I}}}{A\alpha} - \rho_{\mathrm{G}} g \sin\theta = 0.$$

$$\boxed{\frac{\tau_{\mathrm{wG}} p_{\mathrm{G}}}{A\alpha} - \frac{\tau_{\mathrm{wL}} p_{\mathrm{L}}}{A(1-\alpha)} + \frac{\tau_{\mathrm{I}} p_{\mathrm{I}}}{A}\left(\frac{1}{1-\alpha} + \frac{1}{\alpha}\right) - (\rho_{\mathrm{L}} - \rho_{\mathrm{G}}) g \sin\theta = 0}$$

$$\tau_{\mathrm{wL}} = f_{\mathrm{L}} \frac{1}{2} \rho_{\mathrm{L}} U_{\mathrm{L}}^2, \qquad \tau_{\mathrm{wG}} = f_{\mathrm{G}} \frac{1}{2} \rho_{\mathrm{G}} U_{\mathrm{G}}^2, \qquad \tau_{\mathrm{I}} = f_{\mathrm{I}} \frac{1}{2} \rho_{\mathrm{G}} |U_{\mathrm{G}} - U_{\mathrm{L}}| (U_{\mathrm{G}} - U_{\mathrm{L}}) \qquad f_{\mathrm{I}} = f_{\mathrm{G}}$$

- $f_G = \frac{C_G}{Re_G^{-m}}$, $Re_G = \frac{U_G D_G}{\nu_G}$, D_G: hydraulic diameter of the gas-occupied part
- For turbulent flow, $C_G = 0.046, m = 0.2$, for laminar flow, $C_G = 16, m = 1$
- For given j_G and j_L, the equation above can be solved to calculate α and h_L
- For circular pipes,

$$\gamma = 2\cos^{-1}\left(1 - 2\frac{h_{\mathrm{L}}}{D}\right) \qquad \alpha = 1 - \frac{1}{2\pi}(\gamma - \sin\gamma)$$

Flow Regime Map

❖ Co-current Flow in a Near-Horizontal Tube

✓ Transition from stratified-smooth to stratified wavy flow

$$U_G \geq \left[\frac{4\nu_L \Delta\rho g \cos\theta}{S \rho_L U_L}\right]^{1/2}, S = 0.01$$

✓ Extended Kelvin–Helmholtz instability

- Infinitesimally small waves at the interphase grow as a result of the aerodynamic force caused by the reduction in the gas-occupied flow area

$$\boxed{\mathrm{Fr}^2 \left[\frac{1}{c_2^2} \frac{d\tilde{A}_L/d\tilde{h}_L}{\alpha \tilde{A}_G}\right] \geq 1} \qquad \mathrm{Fr} = \sqrt{\frac{\rho_G}{\rho_L - \rho_G}} \frac{j_G}{\sqrt{gD\cos\theta}} \qquad c_2 = 1 - \frac{h_L}{D}$$

In dimensional form

$$\tilde{h}_L = h_L/D,\ \tilde{A}_G = A_G/D^2,\ \text{and}\ \tilde{A}_L = A_L/D^2 \qquad U_G > \left(1 - \frac{h_L}{D}\right)\left[\frac{\Delta\rho g \cos\theta A_G}{\rho_G dA_L/dh_L}\right]^{1/2}$$

- Annular-dispersed flow: eq. above + $\frac{h_L}{D} < 0.5$
- Intermittent flow: eq. above + $\frac{h_L}{D} > 0.5$

Flow Regime Map

- Co-current Flow in a Near-Horizontal Tube
  - Transition from stratified-smooth to stratified wavy flow

$$\boxed{\mathrm{Fr}^2\left[\frac{1}{c_2^2}\frac{d\tilde{A}_\mathrm{L}/d\tilde{h}_\mathrm{L}}{\alpha\tilde{A}_\mathrm{G}}\right]\geq 1} \qquad \mathrm{Fr}=\sqrt{\frac{\rho_\mathrm{G}}{\rho_\mathrm{L}-\rho_\mathrm{G}}}\frac{j_\mathrm{G}}{\sqrt{gD\cos\theta}}$$

    - Cheng et al. (1988)
      - For horizontal channel

$$\mathrm{Fr}=\left(\frac{1}{0.65+1.11X_\mathrm{tt}^{0.6}}\right)^2 \qquad X_\mathrm{tt}=[(1-x)/x]^{0.9}(\mu_\mathrm{L}/\mu_\mathrm{G})^{0.1}(\rho_\mathrm{G}/\rho_\mathrm{L})^{0.5}$$