2상유동 열전달 공학

Two-phase flow and heat transfer Engineering

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8.1 Introduction

❖ Mixture momentum equation

$$\frac{\partial G}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}\left(A\frac{G^2}{\rho'}\right) = -\frac{\partial P}{\partial z} - \bar{\rho} g \sin\theta - F_w$$

$$\rho' = \left[\frac{(1-x)^2}{\rho_L(1-\alpha)} + \frac{x^2}{\rho_G \alpha}\right]^{-1}$$

$$\bar{\rho} = \rho_L(1-\alpha) + \rho_G \alpha$$

✓ Alternative form

$$\left(-\frac{\partial P}{\partial z}\right) = \left(-\frac{\partial P}{\partial z}\right)_{ta} + \left(-\frac{\partial P}{\partial z}\right)_{sa} + \left(-\frac{\partial P}{\partial z}\right)_{g} + \left(-\frac{\partial P}{\partial z}\right)_{fr}$$

$\left(-\frac{\partial P}{\partial z}\right)$ = channel total pressure gradient,

$\left(-\frac{\partial P}{\partial z}\right)_{ta} = \frac{\partial G}{\partial t}$ = temporal mixture acceleration,

$\left(-\frac{\partial P}{\partial z}\right)_{sa} = \frac{1}{A}\frac{\partial}{\partial z}\left(A\frac{G^2}{\rho'}\right)$ = spatial mixture acceleration,

$\left(-\frac{\partial P}{\partial z}\right)_{g} = \rho g \sin\theta$ = hydrostatic pressure gradient,

$\left(-\frac{\partial P}{\partial z}\right)_{fr} = \tau_w P_f / A$ = frictional pressure gradient.

✓ The acceleration terms are often important in two-phase flows with phase change.

✓ In steady-state boiling or condensing flows, the magnitude of the spatial acceleration term is often larger than the frictional pressure gradient.

8.1 Introduction

- Integration of the momentum equation
  - Other terms appear that cannot be included in 1D equations.
  - Form (minor) pressure drops
    - Caused by abrupt changes in flow area
    - Result from complicated multi-dimensional processes

$$\left(-\frac{\partial P}{\partial z}\right) = \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{ta}} + \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{sa}} + \left(-\frac{\partial P}{\partial z}\right)_{g} + \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}}$$

total pressure drop due to flow disturbance

$$P_{\mathrm{I}} - P_{\mathrm{O}} = \int_{z_{\mathrm{I}}}^{z_{\mathrm{O}}} \left\{ \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{ta}} + \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{sa}} + \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{g}} + \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} \right\} dz + \sum_{i=1}^{N} \Delta P_i$$

- In the forthcoming sections
  - Frictional pressure drops
  - Minor pressure drops

8.2 2−ϕ frictional pressure drop in homogenous flow

- In homogeneous mixture (HM)
  - ✓ Two phases remain well mixed and move with identical velocities everywhere
  - ✓ Acts as a single-phase fluid that is compressible and has variable properties

- HM flow along a 1D conduit
  - ✓ For turbulent single-phase flow

Blasius's correlation, fanning friction factor – ¼ of Darcy friction factor

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = 4f\frac{1}{D_{\mathrm{H}}}\frac{G^2}{2\rho} \qquad f = 0.079\mathrm{Re}^{-0.25} \qquad \mathrm{Re} = GD/\mu$$

  - ✓ For homogeneous two-phase flow

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = 4f_{\mathrm{TP}}\frac{1}{D_{\mathrm{H}}}\frac{G^2}{2\rho_{\mathrm{TP}}} \qquad f_{\mathrm{TP}} = 0.079\mathrm{Re}_{\mathrm{TP}}^{-0.25} \qquad \rho_{\mathrm{TP}} = \rho_{\mathrm{h}} = \left(\frac{x}{\rho_{\mathrm{G}}} + \frac{1-x}{\rho_{\mathrm{L}}}\right)^{-1} \qquad \mathrm{Re}_{\mathrm{TP}} = \frac{GD_{\mathrm{H}}}{\mu_{\mathrm{TP}}}$$

    - Viscosity of a HM

$$\mu_{\mathrm{TP}} = \left(\frac{x}{\mu_{\mathrm{G}}} + \frac{1-x}{\mu_{\mathrm{L}}}\right)^{-1}$$

8.2 2−ϕ frictional pressure drop in homogenous flow

- Four different forms

Single-phase-flow based pressure gradient terms

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = \Phi^2_{\mathrm{L0}}\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,L0}},$$

L0, G0: frictional pressure gradient when all the mixture is liquid and gas

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = \Phi^2_{\mathrm{G0}}\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,G0}},$$

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = \Phi^2_{\mathrm{L}}\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,L}},$$

L: frictional pressure gradient when only pure liquid at a mass flux G(1 – x) flows in the channel

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = \Phi^2_{\mathrm{G}}\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,G}}.$$

G: frictional pressure gradient when only pure liquid at a mass flux Gx flows in the channel

Φ^2_{L0}, Φ^2_{G0}, Φ^2_{L}, and Φ^2_{G} are *two-phase multipliers*.

- ✓ For example,

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = 4 f_{\mathrm{TP}}\frac{1}{D_{\mathrm{H}}}\frac{G^2}{2\rho_{\mathrm{TP}}} \qquad \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = \Phi^2_{\mathrm{L0}}\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,L0}} \qquad \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,L0}} = f_{\mathrm{L0}}4\frac{1}{D_{\mathrm{H}}}\frac{G^2}{2\rho_{\mathrm{L}}}$$

$$\Phi^2_{\mathrm{L0}} = \left[1 + x\frac{\mu_{\mathrm{L}} - \mu_{\mathrm{G}}}{\mu_{\mathrm{G}}}\right]^{-1/4}\left[1 + x(\rho_{\mathrm{L}}/\rho_{\mathrm{G}} - 1)\right]$$

8.2 2−ϕ frictional pressure drop in homogenous flow

❖ Four different forms

✓ For example,

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = 4 f_{\mathrm{TP}} \frac{1}{D_{\mathrm{H}}} \frac{G^2}{2\rho_{\mathrm{TP}}} \qquad \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = \Phi_{\mathrm{G0}}^2 \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,G0}} \qquad \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,G}} = 4 f_{\mathrm{G}} \frac{1}{D_{\mathrm{H}}} \frac{(Gx)^2}{2\rho_{\mathrm{G}}}$$

$$\Phi_{\mathrm{G}}^2 = \left[1 + \frac{\rho_{\mathrm{G}}}{\rho_{\mathrm{L}}}(1-x)\right] x^{-7/4} \left[x + \frac{\mu_{\mathrm{G}}}{\mu_{\mathrm{L}}}(1-x)\right]^{-1/4}$$

✓ Also,

$$\Phi_{\mathrm{G0}}^2 = \left[x + \frac{\rho_{\mathrm{G}}}{\rho_{\mathrm{L}}}(1-x)\right]\left[x + \frac{\mu_{\mathrm{G}}}{\mu_{\mathrm{L}}}(1-x)\right]^{-1/4} \qquad \Phi_G^2 = \Phi_{G0}^2 \cdot x^{-7/4} \qquad \Phi_{\mathrm{L}}^2 = \Phi_{\mathrm{L0}}^2 (1-x)^{-7/4}$$

✓ This analysis also has introduced us to the concept of two-phase multipliers, which provides a good way for correlating two-phase frictional pressure losses.

✓ However, the idea of two-phase multipliers was developed based on an idealized annular flow (**Lockhart and Martinelli**, 1949)

8.2 2−ϕ frictional pressure drop in homogenous flow

❖ Correlations for the homogenous two-phase viscosity

Table 8.1. *Correlations for two-phase frictional pressure drop based on homogeneous flow assumption.*

Author	Correlation	Comments
Akers *et al.* (1959)	$\mu_{TP} = \frac{\mu_L}{[(1-x)+x(\rho_L/\rho_G)^{0.5}]}$	Based on condensing two-phase flow data
McAdams *et al.* (1942)	Eq. (8.8b)	Effective mixture viscosity
Cicchitti *et al.* (1960)	$\mu_{TP} = x\mu_G + (1-x)\mu_L$	Effective mixture viscosity
Dukler *et al.* (1964)	$\mu_{TP} = \rho_h \left[\frac{x\mu_G}{\rho_G} + \frac{(1-x)\mu_L}{\rho_L}\right]$	Effective mixture viscosity using density-weighted averaging
Beattie and Whalley (1982)	Eqs. (8.21)–(8.23)	Effective mixture viscosity
Lin *et al.* (1991)	$\mu_{TP} = \frac{\mu_L\mu_G}{\mu_G+x^{1.4}(\mu_L-\mu_G)}$	Based on R-12 vaporization data
Awad and Muzychka (2008)	$\mu_{TP} = \mu_L \frac{2\mu_L+\mu_G-2x(\mu_L-\mu_G)}{2\mu_L+\mu_G+x(\mu_L-\mu_G)}$ (Definition 3) $\mu_{TP} = \mu_G \frac{2\mu_G+\mu_L-2(1-x)(\mu_G-\mu_L)}{2\mu_G+\mu_L+(1-x)(\mu_G-\mu_L)}$ (Definition 4) Arithmetic average of the above two equations (Definition 6)	Based on analogy with thermal conductivity of a porous medium

8.2 2−ϕ frictional pressure drop in homogenous flow

❖ Beattie and Whalley (1982)

✓ For mini channels and narrow rectangular channels and annuli

✓ Modification of the homogeneous flow model

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = 4 f_{\mathrm{TP}} \frac{1}{D_{\mathrm{H}}} \frac{G^2}{2\rho_{\mathrm{h}}} \qquad \mu_{\mathrm{TP}} = \alpha_{\mathrm{h}}\mu_{\mathrm{G}} + \mu_{\mathrm{L}}(1-\alpha_{\mathrm{h}})(1+2.5\alpha_{\mathrm{h}}) \qquad f_{\mathrm{TP}} = f'/4$$

$$\frac{1}{\sqrt{f'}} = 1.14 - 2\log_{10}\left[\frac{\varepsilon_{\mathrm{D}}}{D} + \frac{9.35}{\mathrm{Re}_{\mathrm{TP}}\sqrt{f'}}\right]$$

❖ HM model

✓ Performs reasonably well when the two-phase flow pattern represents a well-mixed configuration (e.g., dispersed bubbly).

✓ In general, however, it deviates from experimental data for flow patterns such as annular, slug, and stratified flows

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

Empirical model

- Originally developed by Lockhart and Martinelli (1949)
- Based on a simple separated flow model
- Applicable to all flow regimes
- G effect is included.

$$\Phi^2 = f(G, x, \text{fluid properties})$$

Lockhart and Martinelli method

- Martinelli parameter (factor)
- Correlated Φ_G^2 and Φ_L^2 as functions of X
- For turbulent-turbulent flow combination
  - Using Blasius' correlation

Martinelli parameter

$$X^2 = \frac{\Phi_G^2}{\Phi_L^2} = \frac{\left(-\frac{\partial P}{\partial z}\right)_{\text{fr,L}}}{\left(-\frac{\partial P}{\partial z}\right)_{\text{fr,G}}}$$

$$X_{\text{tt}}^2 = \left(\frac{\mu_L}{\mu_G}\right)^{0.25}\left(\frac{1-x}{x}\right)^{1.75}\frac{\rho_G}{\rho_L}$$

$$\Phi_{G0}^2 = \left[x + \frac{\rho_G}{\rho_L}(1-x)\right]\left[x + \frac{\mu_G}{\mu_L}(1-x)\right]^{-1/4}$$

$$\Phi_{L0}^2 = \left[1 + x\frac{\mu_L - \mu_G}{\mu_G}\right]^{-1/4}\left[1 + x(\rho_L/\rho_G - 1)\right]$$

$$\Phi_L^2 = \Phi_{L0}^2(1-x)^{-7/4}$$

$$\Phi_G^2 = \Phi_{G0}^2 \cdot x^{-7/4}$$

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

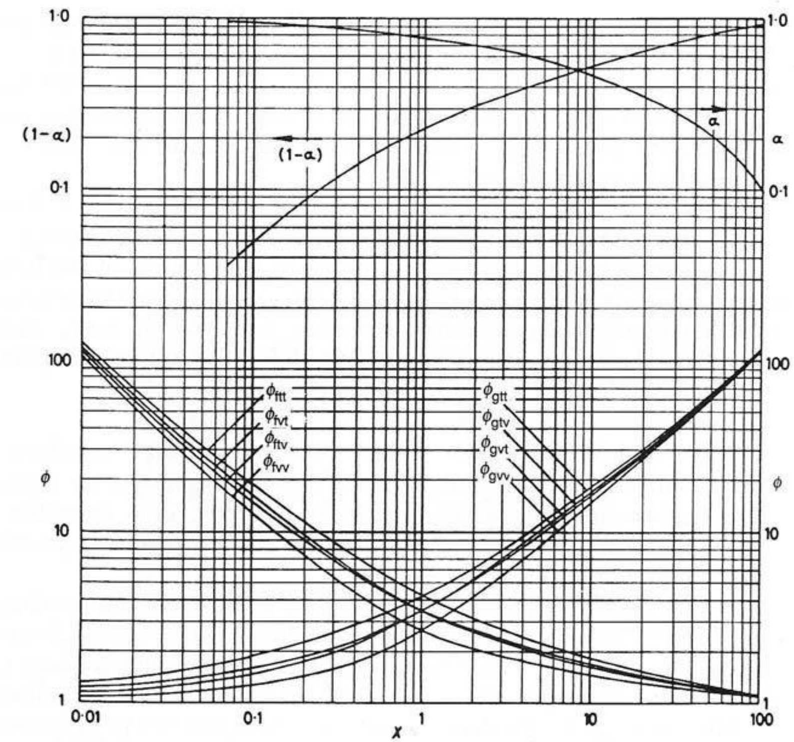
❖ Lockhart and Martinelli method

- ✓ Algebraic correlations have proposed based on the LM approach
- ✓ Widely used correlation (Chisholm and Laird, 1958; Chisholm, 1967)

$$\Phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \qquad \Phi_G^2 = 1 + CX + X^2 \qquad X_{tt} = \left(\frac{\rho_G}{\rho_L}\right)^{0.5}\left(\frac{\mu_L}{\mu_G}\right)^{0.1}\left(\frac{1-x}{x}\right)^{0.9}$$

water, oil, and hydrocarbons in flow passages with D = 1.49– 25.8 mm

Liquid	Gas	C
Turbulent	Turbulent	20
Viscous	Turbulent	12
Turbulent	Viscous	10
Viscous	Viscous	5



- ✓ It has been found, however,

  that C is not a constant and is sensitive to

  the size of the channel.

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

- Some proposed methods for the calculation of the constant C

Author	Correlation	Comments
Chisholm (1967)	See table below Eq. (8.28)	Based on experimental data of Lockhart and Martinelli (1949) with water, oil, and hydrocarbons
Mishima and Hibiki (1996)	$C = 21(1 - e^{-0.319D_H})$, D_H in mm (non-circular channel) $C = 21(1 - e^{-0.333D})$, D in mm (circular channel)	Based on minichannel data ($D_H = 1.4$ mm) with R-134a and R-236ea
Lee and Lee (2001a)	$C = A\left[\frac{\mu_L^2}{\rho_L \sigma D_H}\right]^q \left[\frac{\mu_L j}{\sigma}\right]^r \mathrm{Re}_{L0}^s$ (Eq. (10.18)) See Table 10.1 for the values of coefficients	Based on 350 data from several sources and data in horizontal rectangular channels with 0.4 to 4 mm gap size
Lee and Mudawar (2005a)	$C = \begin{cases} 2.16\mathrm{Re}_{L0}^{0.047}\mathrm{We}_{L0}^{0.6} & \text{viscous liquid and gas} \\ 1.45\mathrm{Re}_{L0}^{0.25}\mathrm{We}_{L0}^{0.33} & \text{viscous liquid turbulent gas} \end{cases}$ $\mathrm{We}_{L0} = (GD)/(\sigma \rho_L)$	Based on boiling data with R-134a in a 231 μm × 713 μm microchannel
Yue *et al.* (2007)	$C = 0.185X^{-0.0942}\mathrm{Re}_{L0}^{0.711}$	Horizontal 1000 μm × 500 μm rectangular channel, CO_2–aqueous salt solutions
Li and Wu (2010a)	$C = 11.9\mathrm{Bd}^{0.45}$ for $\mathrm{Bd} \le 1.5$ $C = 109.4\left(\mathrm{Bd}\mathrm{Re}_{L0}^{0.5}\right)^{-0.56}$ for $1.5 < \mathrm{Bd} \le 11$ $\mathrm{Bd} = \left[g(\rho_L - \rho_G)D_H^2\right]/\sigma$	Based on circular and rectangular channels; with R-12, R-22, R-32, R-134a, R-245fa, R236ea, R-404a, R-410a, R-422d, liquid nitrogen, $0.22 \le D_H \le 3.25$ mm
Pamitran *et al.* (2010)	$C = 3 \times 10^{-3}\mathrm{We}_{TP}^{-0.433}\mathrm{Re}_{TP}^{1.23}$ $\mathrm{Re}_{TP} = GD/\mu_{TP}$; $\mathrm{We}_{TP} = (G^2 D)/(\bar{\rho}\sigma)$ $\bar{\rho} = \alpha\rho_G + (1-\alpha)\rho_L$ Find μ_{TP} from Eq. (8.23) (Beattie and Whalley, 1982); find α from the DFM (Steiner, 1993), see Table 6.1	Based on data with R-22, R-134a, R-410A, R-290, and R-744 in horizontal heated tubes of 0.5, 1.5, and 3.0 mm diameter
Sun and Mishima (2009)	$C = 26\left(1 + \frac{\mathrm{Re}_L}{1{,}000}\right)\left[1 - \exp\left(\frac{-0.153}{0.27D_H^* + 0.8}\right)\right]$ for $\mathrm{Re}_L < 2{,}000$, $\mathrm{Re}_G < 2{,}000$ $C = 1.79X^{-0.19}\left(\frac{\mathrm{Re}_G}{\mathrm{Re}_L}\right)^{0.4}\left(\frac{1-x}{x}\right)^{0.5}$ for $\mathrm{Re}_L > 2{,}000$ or $\mathrm{Re}_G > 2{,}000$ X = Martinelli factor	Extensive data base with air–water, R-22, R-123, R-134a, R-236ea, R-245fa, R-404a,R-410a, R-407c, R-507, CO_2, circular, semi-triangular, and rectangular channels, $0.506 < D_H < 12$ mm
Zhang *et al.* (2010)	$C = \begin{cases} 21\left[1 - \exp\left(-0.358/D_H^*\right)\right] & \text{flow boiling} \\ 21\left[1 - \exp\left(-0.674/D_H^*\right)\right] & \text{adiabatic gas–liquid mixture} \\ 21\left[1 - \exp\left(-0.142/D_H^*\right)\right] & \text{adiabatic vapor–liquid mixture} \end{cases}$ $D_H^* = D_H / \sqrt{\frac{\sigma}{g(\rho_L - \rho_G)}}$	Data with water–air, water–N_2, ethanol–air, oil–air, ammonia; and liquid–vapor mixtures of ammonia, R-134a, R-22, R236ea, water–steam; $1.4 < D_H < 3.25$ mm, round and rectangular channels
Kim and Mudawar (2012c)	See Table 8.3	Based on 7115 data points with air/CO_2/N_2–water, N_2–ethanol; and liquid–vapor mixtures of water, R-12, R-22, R-134a, R-236ea, R-245fa, R-404a, R-410a, R-407c, propane, methane, ammonia, and CO_2; $0.0695 < D_H < 6.22$ mm, round and rectangular channels

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

- Kim and Mudawar (2012)
  - ✓ 7115 adiabatic and condensing two-phase channel flow data points, and covers the following parameter range

Working fluids: air/CO_2/N_2–water mixtures, N_2–ethanol mixture; and liquid–vapor mixtures of water, R-12, R-22, R-134a, R-236ea, R-245fa, R-404a, R-410a, R-407c, propane, methane, ammonia, and CO_2.

Channel geometry: circular, rectangular

$0.0695 < D_H < 6.22$ mm $\quad 3.9 < Re_L < 7.9 \times 10^4 \quad 0 < x < 1$

$4.0 < G < 8528$ kg/m^2 $\quad 0 < Re_G < 2.5 \times 10^5 \quad 0.0052 < P_r < 0.91$

Table 8.3. *The coefficients in the Chisholm–Laird correlation for two-phase pressure drop multiplier, as suggested by Kim and Mudawar (2012c).*

Liquid	Gas	C
Turbulent	Turbulent	$0.39 Re_{L0}^{0.03} Su_{G0}^{0.10} (\rho_L/\rho_G)^{0.35}$
Turbulent	Laminar	$8.7 \times 10^{-4} Re_{L0}^{0.17} Su_{G0}^{0.50} (\rho_L/\rho_G)^{0.14}$
Laminar	Turbulent	$1.5 \times 10^{-3} Re_{L0}^{0.59} Su_{G0}^{0.19} (\rho_L/\rho_G)^{0.36}$
Laminar	Laminar	$3.5 \times 10^{-5} Re_{L0}^{0.44} Su_{G0}^{0.50} (\rho_L/\rho_G)^{0.48}$

$$Su_{G0} = \frac{Re_{G0}^2}{We_{G0}} = Su_G = \frac{Re_G^2}{We_G} = \frac{\rho_G \sigma D_H}{\mu_G^2}$$

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

- Kim and Mudawar (2012)
  - For single-phase Fanning friction factors

Laminar flow, circular channels ($Re_{D_H,k} < 2000$),

$$f_k = 16 Re_k^{-1}.$$

Laminar flow, rectangular channels ($Re_{D_H,k} < 2000$) (Shah and London, 1978

$$f_k Re_{D_H,k} = 24(1 - 1.3553\alpha^* + 1.9467\alpha^{*2} - 1.7012\alpha^{*3} + 0.9564\alpha^{*4} - 0.2537\alpha^{*5}).$$

Turbulent flow:

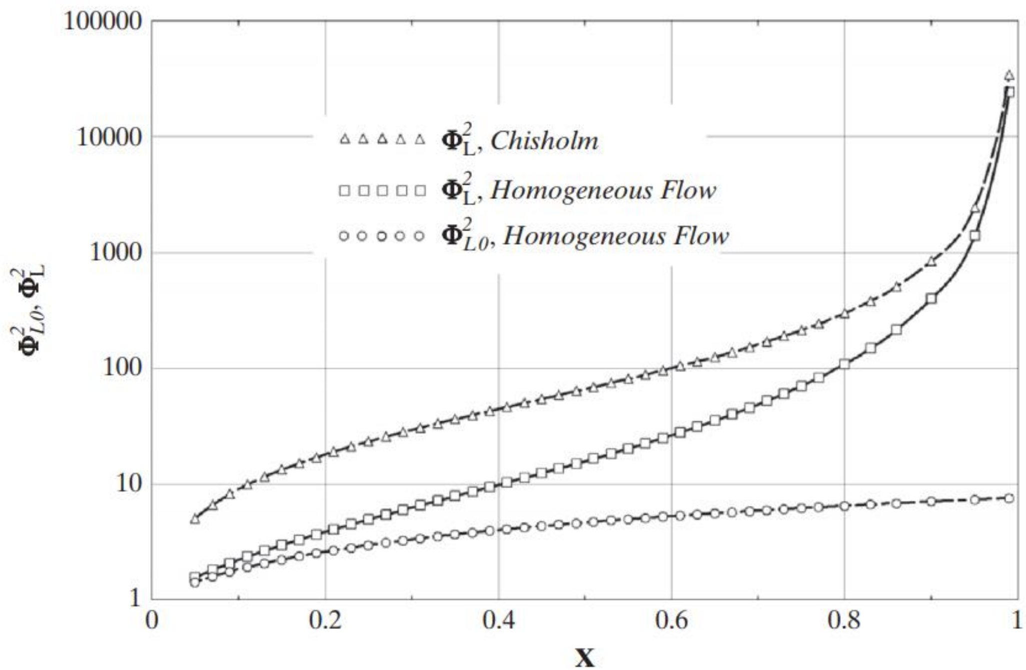
$$f_k = 0.079 Re_k^{-0.25} \quad 2 \times 10^3 < Re_{D_H,k} < 2 \times 10^4 \text{ (Blasius, 1913).}$$

$$f_k = 0.046 Re_k^{-0.2} \quad Re_{D_H,k} \geq 2 \times 10^4 \text{ (Kays and London, 1984).}$$

- α^*: aspect ratio of the rectangular cross section
- Maximum mean average error: 26.8 %

EXAMPLE 8.1. For saturated water–steam flow at 11-MPa pressure with a mixture mass flux of $G = 1500\ \text{kg/m}^2\text{s}$ in a 1-cm-inner-diameter pipe, calculate and plot Φ^2_{L0} using the HM model, and calculate and plot Φ^2_L using the HM model and Chisholm's method for the range $0.01 < x < 0.97$.

SOLUTION. The important properties are $\rho_f = 672\ \text{kg/m}^3$, $\rho_g = 62.56\ \text{kg/m}^3$, $\mu_f = 7.92 \times 10^{-5}\ \text{kg/m·s}$, and $\mu_g = 2.07 \times 10^{-5}\ \text{kg/m·s}$. Equations (8.15), (8.20), (8.27), and (8.28) can now be applied. For $x \leq 0.97$, Re_g and Re_f are both larger than 2300, implying that Eq. (8.28) applies, and $C = 20$. The calculated Φ^2_L and Φ^2_{L0} are plotted in the figure below.



Example Calculation: Using the homogeneous flow pressure drop method, calculate the two-phase pressure drop for up-flow in a vertical tube of 10 mm internal diameter that is 2 m long. The flow is adiabatic, the mass flow rate is 0.02 kg/s and the vapor quality is 0.05. The fluid is R-123 at a saturation temperature of 3°C and saturation pressure of 0.37 bar, whose physical properties are: liquid density = 1518 kg/m3, vapor density = 2.60 kg/m3, liquid dynamic viscosity = 0.0005856 kg/m s, vapor dynamic viscosity = 0.0000126 kg/m s.

Solution: The homogeneous void fraction εH is determined from the quality x using Eq. 13.1.4 where uG/uL = 1:

$$\varepsilon_H = \frac{1}{1+\left(\left(\frac{u_G}{u_L}\right)\frac{(1-x)}{x}\frac{\rho_G}{\rho_L}\right)} = \frac{1}{1+\left((1)\frac{(1-0.05)}{0.05}\frac{2.60}{1518}\right)} = 0.9685$$

The homogeneous density ρH is obtained using Eq. 13.1.3:

$$\rho H = \rho L\ (1 - \varepsilon H) + \rho G\ \varepsilon H = 1518(1 - 0.9685) + 2.60(0.9685) = 50.3\ \text{kg / m3}$$

The static pressure drop for a homogeneous two-phase fluid with H = 2 m and θ = 90° is obtained using Eq. 13.1.2:

$$\Delta p\text{static} = \rho H\ g\ H \sin\theta = 50.3(9.81)(2) \sin 90° = 987\ \text{N / m2}$$

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

The momentum pressure drop is Δpmom = 0 since the vapor quality is constant from inlet to outlet. The viscosity for calculating the Reynolds number choosing the quality averaged viscosity μtp: is obtained with Eq. 13.1.9:

$$\mu tp = x\ \mu G + (1 - x)\ \mu L = 0.05(0.0000126) + (1 - 0.05)(0.0005856) = 0.000557 \text{ kg / m s}$$

The mass velocity is calculated by dividing the mass flow rate by the cross-sectional area of the tube and is 254.6 kg/m2s. The Reynolds number is then obtained with Eq. 13.1.8:

$$\mathrm{Re} = \frac{\dot{m}_{total} d_i}{\mu_{tp}} = \frac{254.6(0.01)}{0.000557} = 4571$$

The friction factor is obtained from Eq. 13.1.7:

$$f_{tp} = \frac{0.079}{\mathrm{Re}^{0.25}} = \frac{0.079}{4571^{0.25}} = 0.00961$$

The frictional pressure drop is then obtained with Eq. 13.1.6:

$$\Delta p_{frict} = \frac{2 f_{tp} L \dot{m}_{total}^2}{d_i \rho_{tp}} = \frac{2(0.00961)(2)(254.6^2)}{0.01(50.3)} = 4953 \text{ N / m}^2$$

Thus, the total pressure drop is obtained with Eq. 13.1.1:

$$\Delta ptotal = \Delta pstatic + \Delta pmom + \Delta pfrict = 987 + 0 + 4953 = 5940 \text{ N/m2} = 5.94 \text{ kPa } (0.86\text{psi})$$

$$\left(-\frac{\partial P}{\partial z}\right)_{fr} = 4 f_{TP} \frac{1}{D_H} \frac{G^2}{2\rho_{TP}} \qquad f_{TP} = 0.079 \mathrm{Re}_{TP}^{-0.25} \qquad \rho_{TP} = \rho_h = \left(\frac{x}{\rho_G} + \frac{1-x}{\rho_L}\right)^{-1} \qquad \mathrm{Re}_{TP} = \frac{G D_H}{\mu_{TP}} \qquad \mu_{TP} = \left(\frac{x}{\mu_G} + \frac{1-x}{\mu_L}\right)^{-1}$$

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

- Example of two-phase pressure drop calculation
  - A mixture of gas and oil flow through a pipeline.
  - Find the two-phase pressure gradient using the Lockhart-Martinelli correlation.

  - Physical parameters
    - Pipe relative roughness $e = 0.0001$
    - Pipe diameter $D = 150\ \text{mm}$
    - Liquid flowrate $W_L = 20\ \text{kg}\cdot\text{s}^{-1}$
    - Gas flowrate $W_G = 2\ \text{kg}\cdot\text{s}^{-1}$
    - Liquid viscosity $\mu_L = 0.005\ \text{Pa}\cdot\text{s}$
    - Gas viscosity $\mu_G = 1.35\times10^{-5}\ \text{Pa}\cdot\text{s}$
    - Liquid density $\rho_L = 710\ \text{kg}\cdot\text{m}^{-3}$
    - Gas density $\rho_G = 2.73\ \text{kg}\cdot\text{m}^{-3}$

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

- Example of two-phase pressure drop calculation
  - A mixture of gas and oil flow through a pipeline.
  - Find the two-phase pressure gradient using the Lockhart-Martinelli correlation.
  - Mass fluxes
    - Cross-sectional area of pipe $A=\pi D^2/4$ = 0.018 m^2
    - Liquid mass flux $G_L=W_L/A$ = 1131.8 $kg/(m^2 \cdot S)$
    - Gas mass flux $G_G=W_G/A$ = 113.2 $kg/(m^2 \cdot S)$
  - Reynolds numbers
    - Liquid Reynolds number $Re_L=G_L \cdot D/\mu_L$ = $3.395 \cdot 10^4$
    - Gas Reynolds number $Re_G=G_G \cdot D/\mu_G$ = $1.258 \cdot 10^6$
  - Friction factors: numerical solution method

$$\frac{1}{\sqrt{f_{turb}}} = -2 \cdot \log\left(\frac{e}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f_{turb}}}\right) \qquad f_L = 0.023, \quad f_G = 0.013$$

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

- Example of two-phase pressure drop calculation
  - A mixture of gas and oil flow through a pipeline.
  - Find the two-phase pressure gradient using the Lockhart-Martinelli correlation.
  - Individual pressure gradient
    - Liquid phase pressure gradient $$\left(\frac{dp}{dx}\right)_L = \frac{f_L}{2}\frac{G_L^2}{\rho_L D} = 138.932\left[\frac{Pa}{m}\right]$$ $$\left(-\frac{\partial P}{\partial z}\right)_{fr} = 4 f_{TP}\frac{1}{D_H}\frac{G^2}{2\rho_{TP}}$$
    - Gas phase pressure gradient $$\left(\frac{dp}{dx}\right)_G = \frac{f_G}{2}\frac{G_G^2}{\rho_G D} = 206.384\left[\frac{Pa}{m}\right]$$
  - Lockhart-Martinelli factor and the total pressure gradient

$$\chi = \sqrt{\frac{\left(\frac{dP}{dx}\right)_f}{\left(\frac{dP}{dx}\right)_g}} = 0.82$$

$$\phi_L = \left(1 + \frac{18}{\chi} + \frac{1}{\chi^2}\right)^{0.5} = 4.942$$

$$\left(\frac{dP}{dx}\right)_{2\phi} = \phi_L^2\left(\frac{dP}{dx}\right)_L = 3.393\cdot 10^3\ [Pa/m]$$

$$\phi_G = \sqrt{\chi^2 + 18\chi + 1} = 4.055$$

$$\left(\frac{dP}{dx}\right)_{2\phi} = \phi_G^2\left(\frac{dP}{dx}\right)_G = 3.393\cdot 10^3\ [Pa/m]$$

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

- Martinelli-Nelson method (1948)
  - Frictional pressure drop in boiling channels, assuming a saturated mixture
  - Pressure: 1 bar~221 bars (water's critical pressure)
  - For a uniformly heated boiling channel with uniform cross section

$$\Delta P_{\mathrm{fr}} = \int_0^{Z_{\mathrm{out}}} \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} dz \qquad \Delta P_{\mathrm{fr}} = \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,f0}} \int_0^{x \;\leftarrow L} \Phi_{\mathrm{f0}}^2(x)dz = \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,f0}} \frac{L}{x_{\leftarrow x_{out}}} \int_0^{x \;\leftarrow x_{out}} \Phi_{\mathrm{f0}}^2(x)dx$$

$$\overline{\Phi}_{\mathrm{f0}}^2 = \frac{1}{x}\int_0^x \Phi_{\mathrm{f0}}^2(x)dx$$

  - Approximation to Martinelli-Nelson curve
    - Soliman et al. (1968) $\Phi_{\mathrm{G}} = 1 + 2.85X_{\mathrm{tt}}^{0.523}$
    - Friedel (1979) $A = (1-x)^2 + x^2\rho_{\mathrm{L}} f_{\mathrm{G0}}(\rho_{\mathrm{G}} f_{\mathrm{L0}})^{-1}$

horizontal and vertical upward flow

$$\Phi_{\mathrm{L0}}^2 = A + 3.24x^{0.78}(1-x)^{0.24}\left(\frac{\rho_{\mathrm{L}}}{\rho_{\mathrm{L}}}\right)^{0.91}\left(\frac{\mu_{\mathrm{G}}}{\mu_{\mathrm{L}}}\right)^{0.19}\left(1-\frac{\mu_{\mathrm{G}}}{\mu_{\mathrm{L}}}\right)^{0.7}\mathrm{Fr}^{-0.0454}\mathrm{We}^{-0.035}$$

vertical downward flow

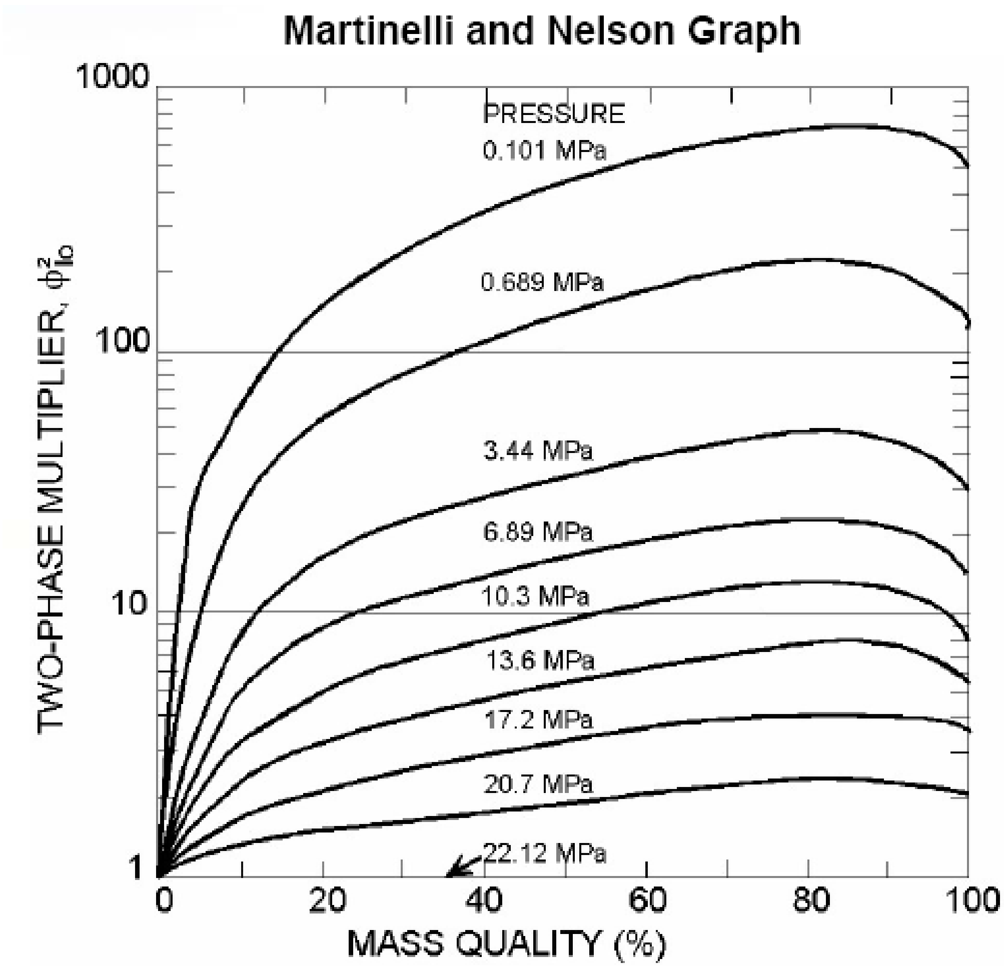
$$\Phi_{\mathrm{L0}}^2 = A + 48.6x^{0.8}(1-x)^{0.29}\left(\frac{\rho_{\mathrm{L}}}{\rho_{\mathrm{G}}}\right)^{0.90}\left(\frac{\mu_{\mathrm{G}}}{\mu_{\mathrm{L}}}\right)^{0.73}\left(1-\frac{\mu_{\mathrm{G}}}{\mu_{\mathrm{L}}}\right)^{7.4}\mathrm{Fr}^{0.03}\mathrm{We}^{-0.12}$$

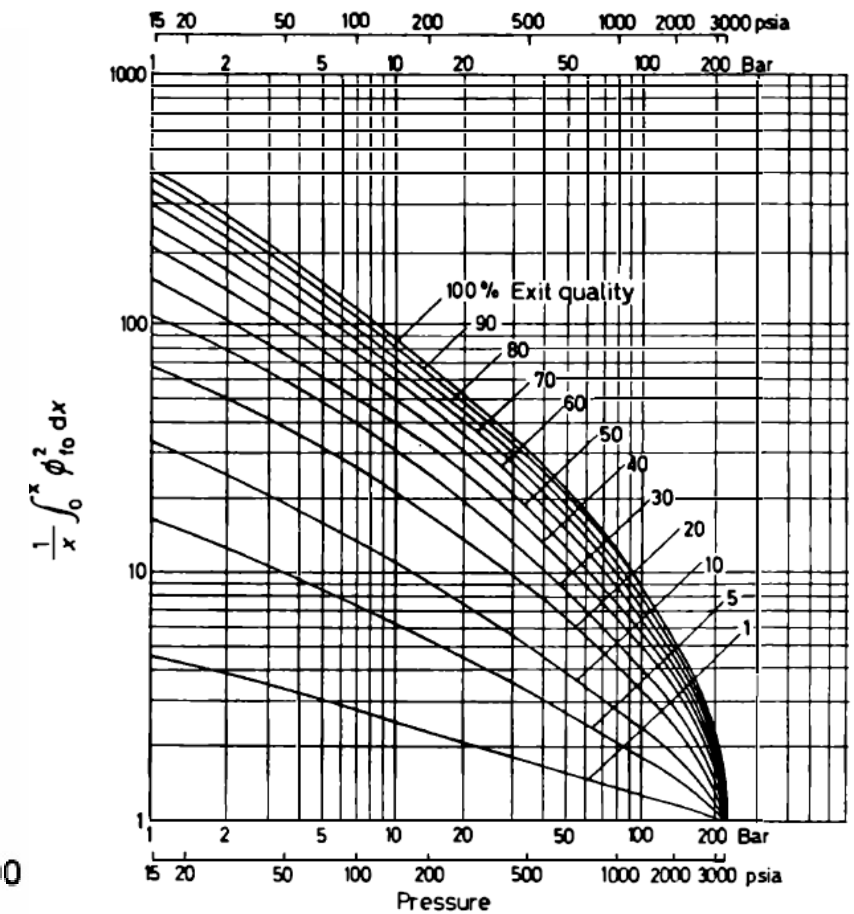
$We = G^2D/\rho_h\sigma$

$Fr = G^2D/gD\rho_h^2$

8.3 Empirical Two-Phase Frictional Pressure Drop Methods

- Martinelli-Nelson method (1948)





8.3 Empirical Two-Phase Frictional Pressure Drop Methods

❖ Martinelli-Nelson method (1948)

Table P8.2a. *Selected values of* $\overline{\Phi}^{2}_{L0}$ *from Thom (1964)*

x_{eq} (%)	$P = 17.2$ bars (250 psia)	$P = 41$ bars (600 psia)	$P = 8.6$ MPa (1250 psia)	$P = 14.48$ MPa (2100 psia)	$P = 20.68$ MPa (3000 psia)
1	1.49	1.11	1.03	–	–
5	3.71	2.09	1.31	1.10	–
10	6.30	3.11	1.71	1.21	1.06
20	11.4	5.08	2.47	1.46	1.12
30	16.2	7.00	3.20	1.72	1.18
40	21.0	8.80	3.89	2.01	1.26
50	25.9	10.6	4.55	2.32	1.33
60	30.5	12.4	5.25	2.62	1.41
70	35.2	14.2	6.00	2.93	1.50
80	40.1	16.0	6.75	3.23	1.58
90	45.0	17.8	7.50	3.53	1.66
100	49.93	19.65	8.165	3.832	1.74

HW-3

- Frictional pressure drop in NEOUL-R
  - ✓ Before OSV, single-phase pressure drop
  - ✓ After OSV, Martinelli-Nelson

$$\Delta P_{\mathrm{fr}} = \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,f0}} \int_0^{x \,\leftarrow L} \Phi_{\mathrm{f0}}^2(x)\,dz = \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,f0}} \frac{L}{x_{\uparrow x_{out}}} \int_0^{x \,\leftarrow x_{out}} \Phi_{\mathrm{f0}}^2(x)\,dx \qquad \overline{\Phi}_{\mathrm{f0}}^2 = \frac{1}{x}\int_0^x \Phi_{\mathrm{f0}}^2(x)\,dx$$

- Interpolation for pressure conditions

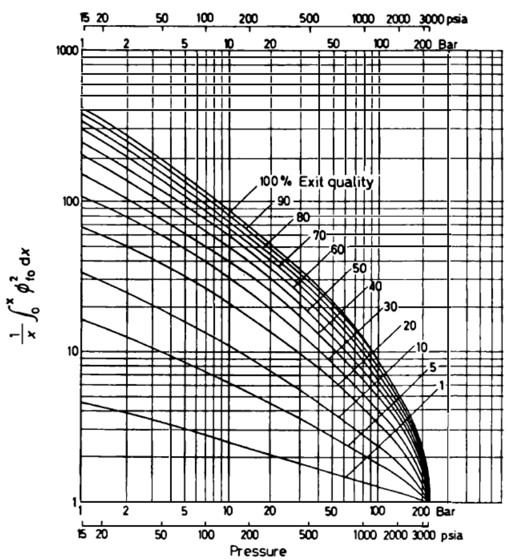


Table P8.2a. *Selected values of* $\overline{\Phi}_{L0}^2$ *from Thom (1964)*

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