8.4 General Remarks about Local Pressure Drops

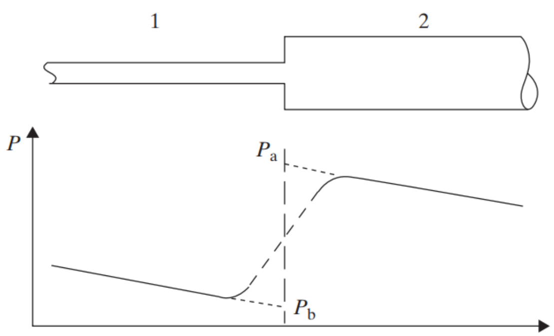
- Flow disturbances: irreversible loss of fluid mechanical energy into heat
  - ✓ Bends, orifices, valves, flow area changes
  - ✓ Dissipation processes: complicated and multi-dimensional
  - ✓ In 1D modelling, local and sudden pressure drops

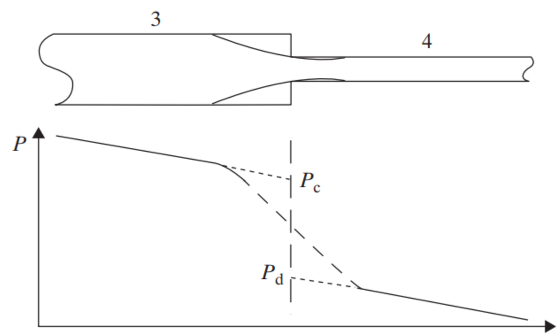
$$P_{\text{in}} - P_{\text{out}} = \int \left[\left(-\frac{\partial P}{\partial z}\right)_{\text{ta}} + \left(-\frac{\partial P}{\partial z}\right)_{\text{sa}} + \left(-\frac{\partial P}{\partial z}\right)_{\text{fr}} + \left(-\frac{\partial P}{\partial z}\right)_{\text{g}} \right] dz + \sum_{i=1}^{N} \Delta P_i$$

ΔP_i is the *total* pressure drop across flow disturbance i

$$P_1 - P_2 = \int_{z_1}^{z_i} \left(-\frac{\partial P}{\partial z}\right)_{\text{ta}} + \left(-\frac{\partial P}{\partial z}\right)_{\text{sa}} + \left(-\frac{\partial P}{\partial z}\right)_{\text{fr}} + \left(-\frac{\partial P}{\partial z}\right)_{\text{g}} dz + \Delta P_i + \int_{z_i}^{z_2} \left[\left(-\frac{\partial P}{\partial z}\right)_{\text{ta}} + \left(-\frac{\partial P}{\partial z}\right)_{\text{sa}} + \left(-\frac{\partial P}{\partial z}\right)_{\text{fr}} + \left(-\frac{\partial P}{\partial z}\right)_{\text{g}} \right] dz$$

$\Delta P_i = P_b - P_a$ (for expansion) $\quad P_c - P_d$ (for contraction)





$\Delta P_i = \Delta P_{i,\text{R}} + \Delta P_{i,\text{I}}$

The reversible component, $\Delta P_{i,\text{R}}$

Can be positive or negative

the *pressure loss*, $\Delta P_{i,\text{I}}$

Irreversible component: always positive
Second law of thermodynamics
Transformation of mechanical energy into heat

8.5 Single-Phase Flow Pressure Drops Caused by Flow Disturbances

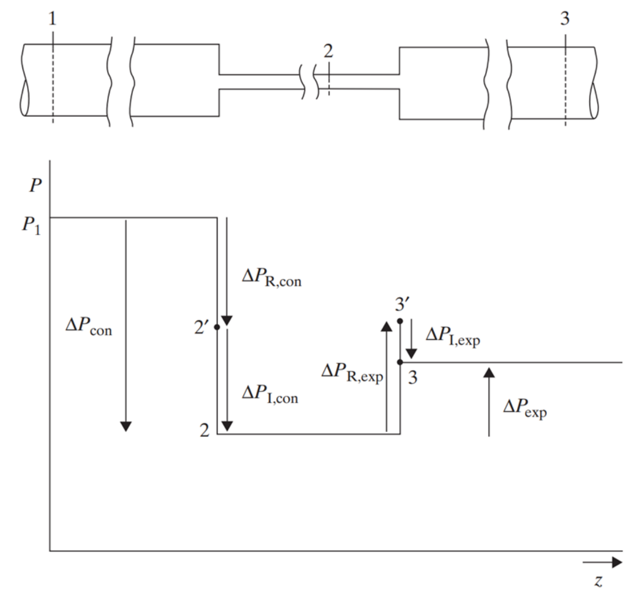
- Flow-area contraction followed by an expansion
  - ✓ Horizontal configuration, incompressible flow, no frictional loss, 1D flow, etc.
  - ✓ Area ratio (ratio between smaller and larger flow areas): $\sigma'_A = A_2/A_1$ and $\sigma_A = A_2/A_3$
  - ✓ Mass continuity: $U_1/U_2 = \sigma'_A$
  - ✓ Bernoulli equation
    - Ideal, reversible flow, no loss in the vicinity of the flow area change

$$P_1 + \frac{1}{2}\rho U_1^2 = P_{2'} + \frac{1}{2}\rho U_2^2 \qquad (P_1 - P_{2'}) = \Delta P_{\mathrm{R,con}} = \frac{1}{2}\rho U_2^2(1 - \sigma_A'^2)$$

$$\Delta P_{\mathrm{con}} = \Delta P_{\mathrm{R,con}} + \Delta P_{\mathrm{I,con}}$$ Real pressure drop

$$P_2 + \frac{1}{2}\rho U_2^2 = P_{3'} + \frac{1}{2}\rho U_3^2 \qquad P_2 - P_{3'} = \Delta P_{\mathrm{R,exp}} = -\frac{1}{2}\rho U_2^2(1 - \sigma_A^2)$$

$$\Delta P_{\mathrm{exp}} = \Delta P_{\mathrm{R,exp}} + \Delta P_{\mathrm{I,exp}}$$ Real pressure drop

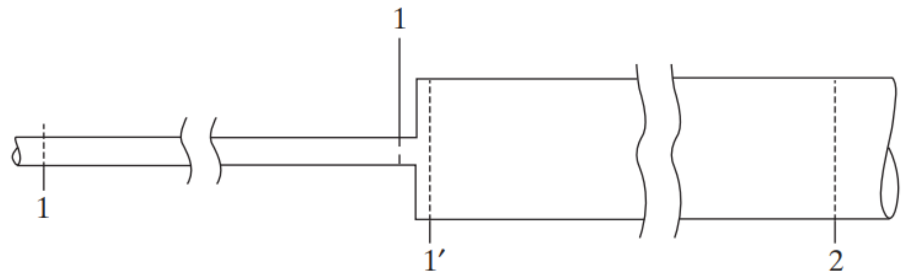


The irreversible pressure drop is difficult to find from theory.
We often rely on empirical correlations for its calculation.

8.5 Single-Phase Flow Pressure Drops Caused by Flow Disturbances

❖ Single-Phase Flow Pressure Drop across a Sudden Expansion

✓ Irreversible pressure loss for a simple expansion



✓ Momentum conservation

$$P_1 A_2 - P_2 A_2 = \rho A_2 U_2 (U_2 - U_1) \qquad (P_1 - P_2) = \Delta P_{\mathrm{ex}} = \rho U_1^2 \sigma_A (\sigma_A - 1)$$

✓ Reversible mechanical energy equation

$$P_1 + \frac{1}{2}\rho U_1^2 = P_{2'} + \frac{1}{2}\rho U_2^2 \qquad (P_1 - P_2)_{\mathrm{R}} = \Delta P_{\mathrm{R,ex}} = \frac{1}{2}\rho U_1^2 (\sigma_A^2 - 1)$$

✓ Irreversible pressure loss and loss coefficient

$$(P_1 - P_2)_{\mathrm{I}} = \Delta P_{\mathrm{I,ex}} = (1 - \sigma_A)^2 \frac{1}{2}\rho U_1^2 \qquad \Delta P_{\mathrm{I}} = K \frac{1}{2}\rho U_{\mathrm{ref}}^2$$

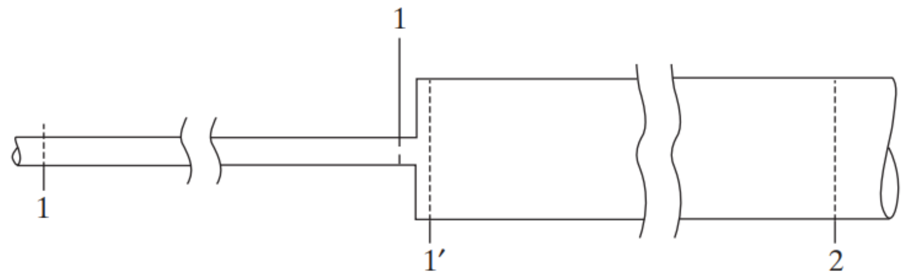
K: Loss coefficient

$$K_{\mathrm{ex}} = (1 - \sigma_A)^2$$

U_{ref}: average velocity in the smallest channel

8.5 Single-Phase Flow Pressure Drops Caused by Flow Disturbances

- Single-Phase Flow Pressure Drop across a Sudden Expansion
  - ✓ Irreversible pressure loss for a simple expansion



  - ✓ Momentum conservation

$$P_1A_2 - P_2A_2 = \rho A_2U_2(U_2 - U_1) \qquad (P_1 - P_2) = \Delta P_{ex} = \rho U_1^2\sigma_A(\sigma_A - 1)$$

  - ✓ Reversible mechanical energy equation

$$P_1 + \frac{1}{2}\rho U_1^2 = P_{2'} + \frac{1}{2}\rho U_2^2 \qquad (P_1 - P_2)_R = \Delta P_{R,ex} = \frac{1}{2}\rho U_1^2(\sigma_A^2 - 1)$$

  - ✓ Irreversible pressure loss and loss coefficient

$$(P_1 - P_2)_I = \Delta P_{I,ex} = (1 - \sigma_A)^2\frac{1}{2}\rho U_1^2 \qquad \Delta P_I = K\frac{1}{2}\rho U_{ref}^2$$

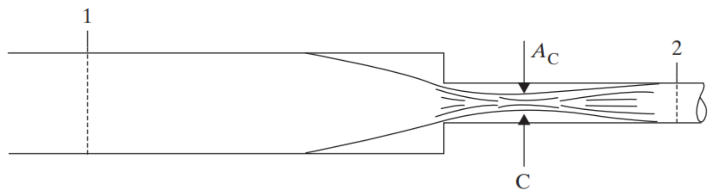
K: Loss coefficient

$$K_{ex} = (1 - \sigma_A)^2$$

U_{ref}: average velocity in the smallest channel

8.5 Single-Phase Flow Pressure Drops Caused by Flow Disturbances

- Single-Phase Flow Pressure Drop across a Sudden contraction
  - ✓ Irreversible pressure loss for a simple contraction



- Vena-contracta phenomena (point C)
  - – Irreversible losses between point C and 2 by sudden expansion

$$(P_1 - P_2)_R = \Delta P_{R,ex} = \frac{1}{2}\rho U_1^2(\sigma_A^2 - 1)$$ For expansion $$\Delta P_I = K\frac{1}{2}\rho U_{ref}^2$$ $$K_{ex} = (1-\sigma_A)^2$$

$$\Delta P_{R,con} = \frac{1}{2}\rho U_2^2(1-\sigma_A^2)$$ For contraction $$\Delta P_{I,con} = K_{con}\frac{1}{2}\rho U_2^2$$ $$K_{con} = \left(\frac{1}{C_C} - 1\right)^2$$

- Contraction coefficient, C_C

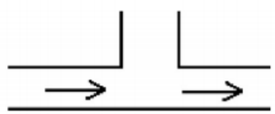
$$C_C = 1 - \frac{1-\sigma_A}{2.08(1-\sigma_A) + 0.5371}$$ $$K_{con} \approx \begin{cases} 0.42(1-\sigma_A) & \text{for } \sigma_A \le 0.58 \\ (1-\sigma_A)^2 & \text{for } \sigma_A > 0.58 \end{cases}$$

- For other pressure drops $$\Delta P_I = K\frac{1}{2}\rho U_{ref}^2$$

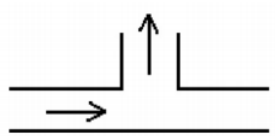
8.5 Single-Phase Flow Pressure Drops Caused by Flow Disturbances

❖ Single-Phase Flow Pressure Drop across a Sudden contraction

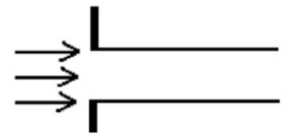
Flow disturbance	K
45° bend	0.35 to 0.45
90° bend	0.50 to 0.75
Regular 90° elbow	$K = 1.49\ \mathrm{Re}^{-0145}$
45° standard elbow	0.17 to 0.45
180° return bend, flanged	0.2
180° return bend, threaded	1.5



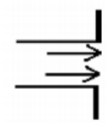
Line flow, flanged tee	0.2
Line flow, threaded tee	0.9



Branch flow, flanged tee	1.0
Branch flow, threaded tee	2.0
Fully open gate valves	0.15
$\frac{1}{4}$-closed gate valve	0.26
Half-closed gate valve	2.1
$\frac{3}{4}$-closed gate valve	17
Open check valves	3.0
Fully open globe valve	6.4
Half-closed globe valve	9.5
Fully open ball valve	0.05
$\frac{1}{3}$-closed ball valve	5.5
$\frac{2}{3}$-closed ball valve	210



Entrance from a plenum into a pipe	
Sharp edged	0.5
Slightly rounded	0.23
Well-rounded	0.04
Projecting pipe	0.78
Exit from pipe into a plenum	1.0



[a] U_{ref} is the mean velocity in the pipe.
[b] From various sources, including White (1999) and Munson *et al.* (1998).

8.6 Two-Phase Flow Local Pressure Drops

- Two-phase multiplier

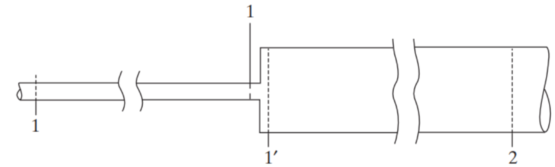
$$\Delta P = \Delta P_{\mathrm{L0}}\Phi_{\mathrm{L0}} = \Delta P_{\mathrm{G0}}\Phi_{\mathrm{G0}} = \Delta P_{\mathrm{L}}\Phi_{\mathrm{L}} = \Delta P_{\mathrm{G}}\Phi_{\mathrm{G}}$$

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = \Phi_{\mathrm{G}}^2\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,G}} \qquad \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,G}} = 4 f_{\mathrm{G}}\frac{1}{D_{\mathrm{H}}}\frac{(Gx)^2}{2\rho_{\mathrm{G}}}$$

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = \Phi_{\mathrm{L0}}^2\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,L0}} \qquad \left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr,L0}} = f_{\mathrm{L0}}4\frac{1}{D_{\mathrm{H}}}\frac{G^2}{2\rho_{\mathrm{L}}}$$

- Sudden expansion
  - Steady-state, uniform phasic velocity
  - Mass and momentum conservation

$$P_1A_2 - P_2A_2 = \rho A_2 U_2(U_2 - U_1)$$



$$G_1A_1 = G_2A_2 \qquad \Delta P_{\mathrm{ex}} = P_{1'} - P_2 = P_1 - P_2 = G_2^2\left(\frac{1}{\rho_2'} - \frac{1}{\sigma_A\rho_1'}\right)$$

  - Momentum density

$$\rho' = \left[\frac{1-x^2}{\rho_{\mathrm{L}}(1-\alpha)} + \frac{x^2}{\rho_{\mathrm{G}}\alpha}\right]^{-1}$$

$$\sigma_A = A_1/A_2$$

$$(P_1 - P_2) = \Delta P_{\mathrm{ex}} = \rho U_1^2\sigma_A(\sigma_A - 1)$$

  - For incompressible fluids and for a two-component mixture: $x_1 = x_2$, and $\alpha_1 = \alpha_2$

$$\Delta P_{\mathrm{ex}} = \Delta P_{\mathrm{L0,ex}}\Phi_{\mathrm{L0,ex}} \qquad \Delta P_{\mathrm{L0}} = -\frac{G_1^2}{\rho_{\mathrm{L}}}\sigma_A(1-\sigma_A) \qquad \Phi_{\mathrm{L0,ex}} = \frac{\rho_{\mathrm{L}}}{\rho'} = \left[\frac{(1-x)^2}{(1-\alpha)} + \frac{\rho_{\mathrm{L}}}{\rho_{\mathrm{G}}}\frac{x^2}{\alpha}\right]$$

    - For homogeneous flow, $\alpha = \dfrac{1}{1+\left(\dfrac{1-x}{x}\right)\dfrac{\rho_G}{\rho_L}}$ $\qquad \Phi_{\mathrm{L0,ex}} = (1-x) + \left(\dfrac{\rho_{\mathrm{L}}}{\rho_{\mathrm{G}}}\right)x$

8.6 Two-Phase Flow Local Pressure Drops

- ✓ Reversible and irreversible pressure drop components
  - Mechanical energy conservation for reversible flow

$$P_1\left\{[\alpha U_G + (1-\alpha)U_L]A\right\}_1 + \frac{1}{2}\left\{[\rho_L U_L^3(1-\alpha) + \rho_G U_G^3 \alpha]A\right\}_1$$

$$= P_2\left\{[\alpha U_G + (1-\alpha)U_L]A\right\}_2 + \frac{1}{2}\left\{[\rho_L U_L^3(1-\alpha) + \rho_G U_G^3 \alpha]A\right\}_2$$

  - $\rho_L[U_L(1-\alpha)]_1 = G_1(1-x_1) \quad \rho_G(U_G\alpha)_1 = G_1 x_1,\ G_1 A_1 = G_2 A_2 \qquad \Delta P_{R,ex} = P_1 - P_2$

$$\Delta P_{R,ex} = -\frac{G_1^2}{2}(1-\sigma_A^2)\left[\frac{x^3}{\alpha^2\rho_G^2} + \frac{(1-x)^3}{(1-\alpha)^2\rho_L^2}\right]\left(\frac{x}{\rho_G} + \frac{1-x}{\rho_L}\right)^{-1}$$

  - For homogeneous flow,

$$\Delta P_{I,ex} = \frac{G_1^2}{2\rho_L}(1-\sigma_A^2)[1 + x(\rho_L/\rho_G - 1)]$$

- ✓ Wang et al. (2010)

$$\Delta P_{ex} = \Delta P_{ex,h}\,(1 + \Omega_1 - \Omega_2)\,(1 + \Omega_3) \qquad \mathrm{Bd} = \frac{(\rho_L - \rho_G) g D^2}{\sigma}$$

$$\Omega_1 = \left(\frac{\mathrm{WeBd}}{\mathrm{Re}_{L0}}\right)^2 \left(\frac{1-x}{x}\right)^{0.3} \frac{1}{\mathrm{Fr}^{0.8}}$$

$$\Omega_2 = 0.2\left(\frac{\mu_G}{\mu_L}\right)^{0.4}$$

$$\Omega_3 = 0.4\left(\frac{x}{1-x}\right)^{0.3} + 0.3 e^{\frac{1.6}{\mathrm{Re}_{L0}^{0.1}}} - 0.4\left(\frac{\rho_L}{\rho_G}\right)^{0.2}$$

$$\mathrm{We} = \frac{G^2 D}{\sigma \rho_h}$$

$$\mathrm{Fr} = \frac{G^2}{\rho_h^2 g D}$$

$$\rho_h = \left[\frac{x}{\rho_G} + \frac{1-x}{\rho_L}\right]^{-1}$$

$506 < G < 5642\ \mathrm{kg/m^2 \cdot s};\ 0.002 < x < 0.99;\ 0.057 < \sigma_A < 0.607$
$0.84 < D < 19\ \mathrm{mm};\ 0.095 < \mathrm{Bd} < 92;\ 10.3 < \mathrm{Fr} < 9.19 \times 10^5$
$100 < \mathrm{We} < 8.3 \times 10^4;\ 4.35 \times 10^2 < \mathrm{Re}_{L0} < 4.95 \times 10^5.$

8.6 Two-Phase Flow Local Pressure Drops

❖ Sudden contraction

$$\Delta P_{\text{con}} = P_1 - P_2 = G_2^2 \left\{ \frac{\rho_{\text{h}}}{2\rho'''^2} \left(\frac{1}{C_{\text{C}}^2} - \sigma_A^2 \right) + \frac{1}{\rho'} (1 - C_{\text{C}}) \right\}$$

$$\Delta P_{\text{I,con}} = \frac{G_2^2}{C_{\text{C}}^2} (1 - C_{\text{C}}) \left\{ -\frac{C_{\text{C}}}{\rho'} + \frac{1 + C_{\text{C}}}{2} \frac{\rho_{\text{h}}}{\rho'''^2} \right\}.$$

$$\rho' = \left[\frac{(1-x)^2}{\rho_{\text{L}}(1-\alpha)} + \frac{x^2}{\rho_{\text{G}}\alpha} \right]^{-1}$$

$$\rho'''^2 = \left[\frac{(1-x)^3}{\rho_{\text{L}}^2(1-\alpha)^2} + \frac{x^3}{\rho_{\text{G}}^2\alpha^2} \right]^{-1}$$

✓ Strong mixing is caused by the contraction → homogeneous flow assumption

$$\Delta P_{\text{I,con}} = \frac{1}{2} \frac{G_2^2}{\rho_{\text{L}}} \left(\frac{1}{C_{\text{C}}} - 1 \right)^2 [1 + x(\rho_{\text{L}}/\rho_{\text{G}} - 1)],$$

$$\Delta P_{\text{con}} = P_1 - P_2 = \frac{1}{2} \frac{G_2^2}{\rho_{\text{L}}} \left[\left(\frac{1}{C_{\text{C}}} - 1 \right)^2 + (1 - \sigma_A^2) \right] [1 + x(\rho_{\text{L}}/\rho_{\text{G}} - 1)].$$

✓ Total pressure drop

$$\Delta P_{\text{con}} = \Delta P_{\text{L0,con}} \Phi_{\text{L0,con}},$$

$$\Delta P_{\text{L0,con}} = \frac{1}{2} \frac{G^2}{\rho_{\text{L}}} \left[\left(\frac{1}{C_{\text{C}}} - 1 \right)^2 + (1 - \sigma_A^2) \right]$$

$$\Phi_{\text{L0}} = 1 + x(\rho_{\text{L}}/\rho_{\text{G}} - 1).$$

8.6 Two-Phase Flow Local Pressure Drops

- For various geometry
  - Orifices

$$\Phi_{L0} = [1 + x(\rho_L/\rho_G - 1)]^{0.8}\left[1 + x\left(\frac{\rho_L \mu_G}{\rho_G \mu_L} - 1\right)\right]^{0.2}$$

  - Spacer grids in rod bundles

$$\Phi_{L0} = [1 + x(\rho_L/\rho_G - 1)]^{0.8}\left[1 + x\left(\frac{3.5\rho_L}{\rho_G} - 1\right)\right]^{0.2}$$

  - Return bends

$$\Phi_{L0} = (1 - x^2)\left(1 + \frac{C}{X} + \frac{1}{X^2}\right) \qquad X = [(\Delta P_L)/(\Delta P_G)]_{\text{bend}}^{1/2}$$

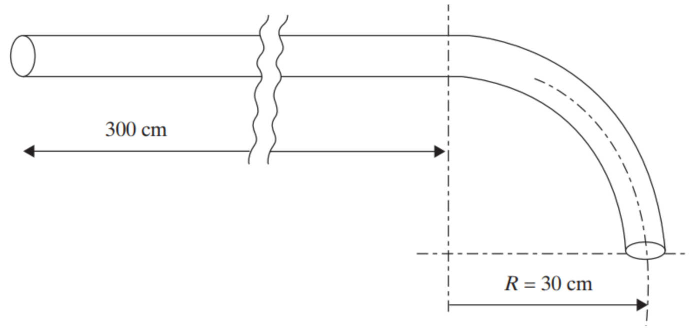
$$C = \left[1 + (C_2 - 1)\left(\frac{\rho_L - \rho_G}{\rho_L}\right)^{0.5}\right]\left[\sqrt{\frac{\rho_L}{\rho_G}} + \sqrt{\frac{\rho_G}{\rho_L}}\right] \qquad C_2 = 1 + \frac{2.2}{K_{L0}(2 + R/D)}$$

$$K_{L0} = f_{L0}\frac{L_{\text{bend}}}{D} + 0.294\left(\frac{R}{D}\right)^{1/2}$$

EXAMPLE 8.2. Calculate the total pressure drop in the system shown in Fig. 8.8 , for air–water mixture flow with the following specifications: pipe diameter $D = 3.7$ cm, liquid mass flux $G_L = 1500$ kg/m^2·s, gas mass flux $G_G = 130$ kg/m^2 s, temperature $T = 25\,°$C, and average pressure $P = 10$ bars. Assume that the piping system lies in a horizontal plane.

SOLUTION. For properties we get $\rho_L = 997.5\,\text{kg/m}^3$, $\rho_G = 11.7\,\text{kg/m}^3$, $\mu_L = 8.93 \times 10^{-4}\,\text{kg/m·s}$, and $\mu_G = 1.85 \times 10^{-5}\,\text{kg/m·s}$.

For the bend, we can use the correlation of Chisholm, Eqs. (8.96)–(8.98). For the 90° bend, let us use $K_0 = 0.75$. Noting that $G_L = 1500\,\text{kg/m}^2\cdot\text{s}$ and $G_G = 130\,\text{kg/m}^2\cdot\text{s}$, we get



$$\Delta P_{\text{L,bend}} = K_0 \frac{1}{2} \frac{G_L^2}{\rho_L} = 845.9\,\text{N/m}^2,$$

$$\Delta P_{\text{G,bend}} = K_0 \frac{1}{2} \frac{G_G^2}{\rho_G} = 542.1\,\text{N/m}^2,$$

$$X = \left[\frac{\Delta P_{\text{L,bend}}}{\Delta P_{\text{G,bend}}}\right]^{1/2} = 1.249.$$

With $R = 0.3$ m and $D = 0.037$ m, Eq. (8.98) gives $C_2 = 1.29$. Equation (8.82) leads to $C = 12.04$. The flow quality is

$$x = \frac{G_G}{G_G + G_L} = 0.0797.$$

Equation (8.98) then gives $C_2 = 1.29$. Using this value, we can then solve Eq. (8.97), leading to $C = 12.04$. Equation (8.96) can now be applied to get $\Phi_{L0} = 11.2$. The total pressure drop in the bend will then be

$$\Delta P_{\text{bend}} = \Phi_{\text{L0}} \Delta P_{\text{L0,bend}} = \Phi_{\text{L0}} K_0 \frac{1}{2} \frac{(G_{\text{L}} + G_{\text{G}})^2}{\rho_{\text{L}}} = 11\,196\ \text{N/m}^2.$$

We now need to calculate the pressure drop in the straight segment of the pipe. Let us use the method of Chisholm *et al.*, Eq. (8.28),

$$\text{Re}_{\text{G}} = \frac{G_{\text{G}} D}{\mu_{\text{G}}} = 2.6 \times 10^5,$$

$$\text{Re}_{\text{L}} = \frac{G_{\text{L}} D}{\mu_{\text{L}}} = 6.2 \times 10^4.$$

Clearly, both phases are turbulent; therefore $C = 20$ should be used in Eq. (8.28). The Martinelli parameter can be found from Eq. (8.27), leading to $X_{\text{tt}} = 1.44$. Application of Eq. (8.28) then gives $\Phi_{\text{L}}^2 = 15.35$. The pressure drop in the straight segment can be found by writing

$$f'_{\text{L}} = 0.316\,\text{Re}_{\text{L}}^{-0.25} = 0.020$$

and so

$$\Delta P_{\text{straight}} = \Phi_{\text{L}}^2 \Delta P_{\text{L}} = \Phi_{\text{L}}^2 f'_{\text{L}} \frac{L}{D} \frac{G_{\text{L}}^2}{2\rho_{\text{L}}} = 2.81 \times 10^4\ \text{N/m}^2.$$

The total pressure drop will thus be

$$\Delta P_{\text{tot}} = \Delta P_{\text{bend}} + \Delta P_{\text{straight}} = 3.93 \times 10^4\ \text{N/m}^2.$$

EXAMPLE 8.3. Ammonia with a mass flow rate of 35 g/s, a quality of 2% and a temperature of –25 °C flows in a horizontal tube with 6 mm inner diameter. The tube has a surface roughness of 1.5 μm. Find the frictional pressure gradient. Also, a gate valve is installed on the tube. The valve has a flow diameter that is equal to the inner diameter of the tube, and is half open. Calculate the pressure loss caused by the gate valve, using the HEM assumption.

SOLUTION. We deal with a saturated mixture of liquid and vapor ammonia at −25 °C. The saturation pressure of ammonia at this temperature is 1.504 bars. Other relevant properties are

$$\begin{aligned}
\rho_{\mathrm{f}} &= 671.7\,\mathrm{kg/m^2\cdot s}\\
\rho_{\mathrm{g}} &= 1.287\,\mathrm{kg/m^2\cdot s}\\
\mu_{\mathrm{f}} &= 2.289 \times 10^{-4}\,\mathrm{kg/m\cdot s}\\
\mu_{\mathrm{g}} &= 8.295 \times 10^{-6}\,\mathrm{kg/m\cdot s}\\
h_{\mathrm{fg}} &= 1.345 \times 10^{6}\,\mathrm{J/kg}.
\end{aligned}$$

We now calculate the mass flux and the homogeneous void fraction.

$$\frac{x}{1.0 - x} = \frac{\rho_{\mathrm{g}}\alpha_{\mathrm{h}}}{\rho_{\mathrm{f}}\,(1 - \alpha_{\mathrm{h}})} \Rightarrow \alpha_{\mathrm{h}} = 0.914$$

$$A = \pi D^2/4 = \pi\,(0.006\,\mathrm{m})^2/4 = 2.827 \times 10^{-5}\,\mathrm{m}^2$$

$$G = \dot{m}/A = (35 \times 10^{-3}\,\mathrm{kg})/(2.827 \times 10^{-5}\,\mathrm{m}^2) = 1238\,\mathrm{kg/m^2\cdot s}.$$

To find the frictional pressure gradient, let us use the method of Beattie and Whalley (1982)

$$\rho_{\mathrm{h}} = \frac{1}{x/\rho_{\mathrm{g}} + \frac{1.-x}{\rho_{\mathrm{f}}}} = 58.84\,\mathrm{kg/m^2 \cdot s}$$

$$\mu_{\mathrm{TP}} = \alpha_{\mathrm{h}}\mu_{\mathrm{g}} + \mu_{\mathrm{f}}(1-\alpha_{\mathrm{h}})(1+2.5\alpha_{\mathrm{h}}) = 7.215\times10^{-5}\,\mathrm{kg/m\cdot s}$$

$$\mathrm{Re}_{\mathrm{TP}} = \frac{GD_{\mathrm{H}}}{\mu_{\mathrm{TP}}} = 1.029\times10^{5}.$$

Following the recommendation of Beattie and Whalley (1982) we will apply the Colebrook equation (Eq. (8.23)), noting that

$$\varepsilon_{\mathrm{D}}/D = (1.5\times10^{-6}\,\mathrm{m})/(0.006\,\mathrm{m}) = 2.5\times10^{-4}.$$

Iterative solution of Eq. (8.23) then gives

$$f' = 0.0159.$$

The frictional pressure gradient can now be found

$$\left(-\frac{\partial P}{\partial z}\right)_{\mathrm{fr}} = f'\frac{1}{D}\frac{1}{2}\frac{G^2}{\rho_{\mathrm{TP}}} = 0.0191\left(\frac{1}{0.006\,\mathrm{m}}\right)\frac{1}{2}\frac{\left(1238\,\mathrm{kg/m^2\cdot s}\right)^2}{58.84\,\mathrm{kg/m^2\cdot s}}$$
$$= 4.147\times10^{4}\,\mathrm{N/m^3}.$$

We now calculate the pressure drop across the valve. The all-liquid velocity through the valve, when it is fully open, will be

$$U_{\mathrm{f0}} = G/\rho_{\mathrm{f}} = \frac{1238\,\mathrm{kg/m^2\cdot s}}{671.7\,\mathrm{kg/m}} = 1.843\,\mathrm{m/s}.$$

This is the reference velocity that should be used for calculating the pressure drop across the valve. From Table 8.1 the loss coefficient for the half-open gate valve is 2.1. Assuming homogeneous flow, the pressure drop across the valve (which is the same as the pressure loss if we assume that phase density variations across the valve remain unchanged) can be found as

$$\Delta P_{f0} = K\frac{1}{2}\rho_f U_{f0}^2 = 2.1\left(\frac{1}{2}\right)(671.7\,\text{kg/m})\,(1.843\,\text{m/s})^2 = 2395\,\text{N/m}^2$$

$$\Phi_{f0} = 1.0 + x\left(\rho_f/\rho_g - 1.0\right) = 1.0 + (0.02)\,[(671.7/1.287) - 1] = 11.42$$

$$\Delta P_{\text{valve}} = \Phi_{f0}\Delta P_{f0} = 2.73 \times 10^3\,\text{N/m}^2.$$