11.2 Heterogeneous Bubble Nucleation and Ebullition

- Waiting period
  - ✓ Bubble influenced area: four times the cross section of the departing bubble
  - ✓ Hsu and Graham (1961)
    - One-dimensional transient conduction in a slab with a know thickness, δ
  - ✓ Han and Griffith(1965)

$$t_{wt} = \frac{9}{4\pi \alpha_L}\left[\frac{(T_w - T_\infty)R_C}{T_w - T_{sat}\left(1 - \frac{2\sigma}{R_C \rho_g h_{fg}}\right)}\right]^2$$

11.2 Heterogeneous Bubble Nucleation and Ebullition

- Waiting period
  - ✓ Han and Griffith(1965)

b. Transient Thermal Layer

Since the convection intensity near a solid wall is damped down due to the no slip boundary condition for a solid surface, the use of the pure conduction equation is justified in determining the temperature distribution in this thin layer of fluid near the heating surface. For this particular problem, a simplified physical model is shown in Fig. 2.

Initial condition is

$$\left.\begin{array}{ll} T = T_w & at \quad x = 0 \\ T = T_\infty & at \quad x > 0 \end{array}\right\} t = 0 \qquad (1)$$

Boundary condition is

$$\left.\begin{array}{ll} T = T_w & at \quad x = 0 \\ T = T_\infty & at \quad x = \infty \end{array}\right\} t > 0 \qquad (2)$$

The solution to this problem is found from Ref. (1) as

$$T - T_\infty = (T_w - T_\infty)\, erfc \frac{x}{2\sqrt{kt}} \qquad (3)$$

$$\frac{\partial T}{\partial x} = -\frac{T_w - T_\infty}{\sqrt{\pi k t}} e^{-\frac{x^2}{4kt}} \qquad (4)$$

$$\frac{T(x,t) - T_s}{T_i - T_s} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \qquad q_s''(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

$$\frac{T - T_i}{T_s - T_i} = 1 - erf\left[\frac{x}{2\sqrt{\alpha t}}\right]$$

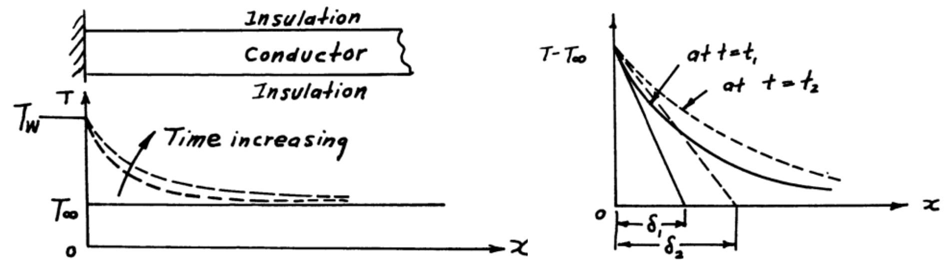


Fig. 2 Temperature Distribution in a Semi-Infinite Conductor

at $x = 0$

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = -\frac{T_w - T_\infty}{\sqrt{\pi k t}} \qquad (5)$$

If the actual temperature distribution near the wall is assumed to be a straight line distribution, the slope of this straight line is determined by equation (5). This assumption has been justified through measurements made in Reference (6). With this fact, one can introduce the notion of thickness of transient thermal layer by drawing a tangent line from x = 0 on the $T - T_\infty \sim X$ curve defined by (3), the interception of this straight line on X-axis gives the transient thermal layer thickness as shown in Fig. (3).

$$\delta = \sqrt{\pi k t} \qquad (6)$$

This means that the temperature distribution at any instant varies linearly from the wall to $x = \delta$, beyond δ, the fluid does not know whether the wall is hot or cold. The layer thickness increases with the square root of waiting time.

11.2 Heterogeneous Bubble Nucleation and Ebullition

❖ Waiting period

✓ Han and Griffith(1965)

$$\Delta P = \frac{2\sigma}{R_c} \qquad \Delta P = \frac{\Delta T}{T_{sat}} \frac{L}{\frac{1}{\rho_v} - \frac{1}{\rho}} \approx \frac{\Delta T \rho_v L}{T_{sat}}$$

where $\Delta T = T_b - T_{sat}$

$\Delta P = P_b - P_{sat}$

T_b, P_b are temperature and pressure of the vapor in the bubble at the initial stage of bubble growth.

From potential flow theory and the fluid flow analogy, the potential line in fluid flow is just equivalent to the isothermal line in heat conduction, the distance of an isothermal line passing through the top point of a waiting bubble is $\frac{3}{2}R_c$ distant from heating surface when measured on the straight part of this isothermal line.

Fluid temperature at $X = \frac{3}{2}R_c$ is

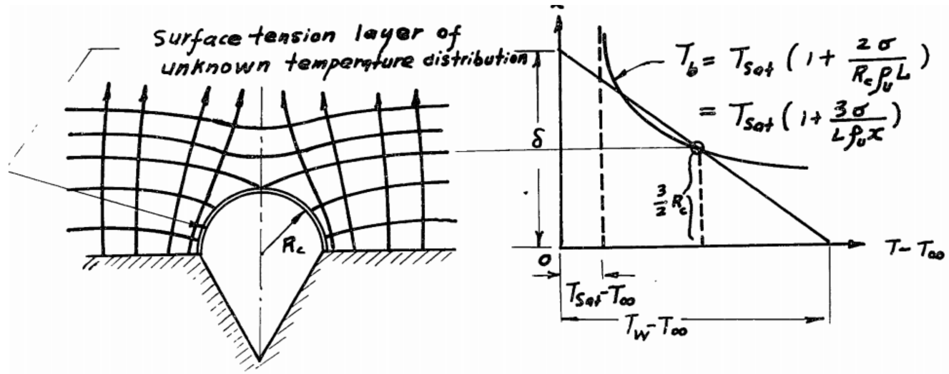
$$T_f = (T_w - T_\infty)\left(1 - \frac{\frac{3}{2}R_c}{\delta}\right) + T_\infty = T_w - (T_w - T_\infty)\frac{3R_c}{2\delta} \qquad (10)$$

Equating this temperature to the bubble temperature yields the criterion of initiation of a bubble growth from a nucleate site of cavity radius R_c as

$$T_{sat} + \frac{2\sigma T_{sat}}{R_c \rho_v L} = T_w - (T_w - T_\infty)\frac{3R_c}{2\delta}$$

or

$$\delta = \frac{3}{2}\frac{(T_w - T_\infty)R_c}{T_w - T_{sat}\left(1 - \frac{2\sigma}{R_c \rho_v L}\right)} \qquad \delta = \sqrt{\pi k t} \qquad (11)$$

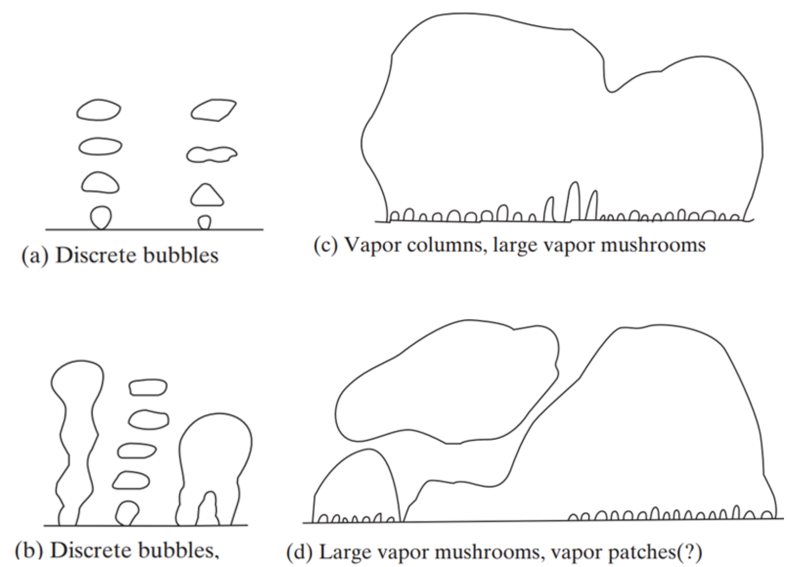


$$t_w = \frac{\delta^2}{\pi k} = \frac{9}{4\pi k}\left[\frac{(T_w - T_\infty)R_c}{T_w - T_{sat}\left(1 - \frac{2\sigma}{R_c \rho_v L}\right)}\right]^2$$

$$t_{wt} = \frac{9}{4\pi \alpha_L}\left[\frac{(T_w - T_\infty)R_C}{T_w - T_{sat}\left(1 - \frac{2\sigma}{R_C \rho_g h_{fg}}\right)}\right]^2$$

11.2 Heterogeneous Bubble Nucleation and Ebullition

- Heat Transfer Mechanisms in Nucleate Boiling
  - ✓ The phenomenology described in the previous section
    - For the isolated bubble zone of the partial boiling regime
  - ✓ Recent DNS and mechanistic model
    - Mechanistic bubble ebullition model based on microlayer evaporation predicts well the experimental data obtained with a polished surface and well-characterized artificial cavity
    - However, heated surfaces have unknown cavity characteristics
      - Limited practical and design application
  - ✓ Partial boiling regime
    - Nucleate boiling + natural convection
  - ✓ With increasing heat flux
    - The contribution of convection diminishes
      + bubble frequency & nucleation increase
    - Bubble interaction in the lateral direction
      → formation of vapor mushrooms

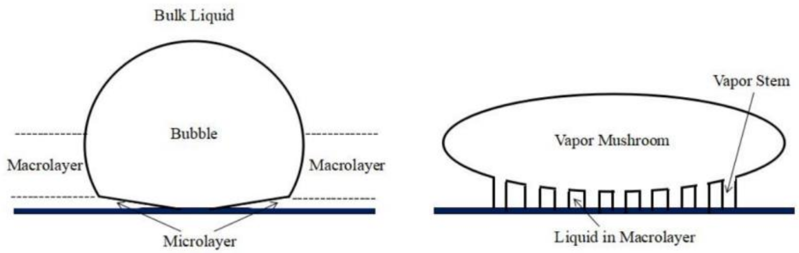


11.2 Heterogeneous Bubble Nucleation and Ebullition

- Heat Transfer Mechanisms in Nucleate Boiling
  - ✓ Transition from isolated bubbles to columns and mushrooms
    - Transition from partial to fully developed nucleate boiling
    - Evaporation at the periphery of the vapor stems in the liquid macrolayer
  - ✓ Correlation for the transition (Moissis and Berenson, 1963)

$$q''_{\mathrm{w}} = 0.11\sqrt{\theta}\,\rho_{\mathrm{g}} h_{\mathrm{fg}} \left(\frac{\sigma g}{\Delta\rho}\right)^{1/4}$$

θ (degree)



  - ✓ Main difficulty
    - Nucleation sites
    - Non-linear and conjugate nature of a multitude of subprocesses
    - Constant wall temperature or heat flux ? → invalid
    - Modeling of bubble behavior based on static force balance → invalid

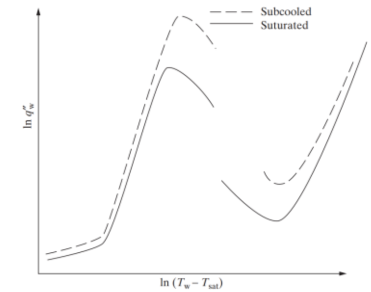
11.3 Nucleate Boiling Correlation

- Rohsenow correlation (1952)
  - ✓ The oldest and most widely used nucleate boiling correlation

$$\mathrm{Nu} = \frac{h\lambda_L}{k_f} = \frac{1}{C_{sf}}\mathrm{Re}^{1-n}\,Pr^{-m}$$

$$St^* = \frac{1}{St} = \frac{\rho C_p}{h} = \frac{RePr}{Nu} = C_{sf}Re^a\ Pr^b$$

- Its typical error in calculating q when Tw is known is about 100%
- In calculating Tw−Tsat when qw is known the error is about 25% (Lienhard and Lienhard, 2005)



  - ✓ Assumptions
    - Small effect of surface
    - Small effect of liquid pool temp.
  - ✓ Laplace length scale

$$\lambda_L = \sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}} \qquad P_\sigma = \frac{2\sigma}{\lambda_L} \qquad P_h = \rho g(2\lambda_L)$$

  - ✓ Velocity & HTC

$$U = q''_w/\rho_f h_{fg} \qquad H = \frac{q''_w}{T_w - T_{sat}}$$

  - ✓ Then,

$$C_{Pf}\frac{T_w - T_{sat}}{h_{fg}} = C_{sf}\left[\frac{q''_w}{\mu_f h_{fg}}\sqrt{\frac{\sigma}{g\Delta\rho}}\right]^n\left(\frac{\mu C_p}{k}\right)_f^{m+1}$$

Surface combination	C_{sf}	$m+1$
Water–nickel	0.006	1.0
Water–platinum	0.013	1.0
Water–emery polished copper	0.0128	1.0
Water–brass	0.006	1.0
Water–ground and polished stainless steel	0.008	1.0
Water–Teflon pitted stainless steel	0.0058	1.0
Water–chemically etched stainless steel	0.0133	1.0
Water–mechanically polished stainless steel	0.0132	1.0
Water–emery polished, paraffin treated copper	0.0147	1.0
CCl_4–emery polished copper	0.007	1.7
Benzene–chromium	0.01	1.7
n-Pentane–chromium	0.015	1.7
n-Pentane–emery polished copper	0.0154	1.7
n-Pentane–emery polished nickel	0.0127	1.7
Ethyl alcohol–chromium	0.0027	1.7
Isopropyl alcohol–copper	0.0025	1.7
35% K_2CO_3–copper	0.0054	1.7
50% K_2CO_3–copper	0.0027	1.7
n-Butyl alcohol–copper	0.0030	1.7

$n = 0.33$, $m = 0$ for water and $m = 0.7$ for other fluids.

11.3 Nucleate Boiling Correlation

❖ Rohsenow correlation (1952)

✓ The oldest and most widely used nucleate boiling correlation

$$\mathrm{Nu} = \frac{h\lambda_L}{k_f} = \frac{1}{C_{sf}}\mathrm{Re}^{1-n}\,Pr^{-m} \qquad \lambda_L = \sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}} \qquad U = q''_w/\rho_f h_{fg} \qquad H = \frac{q''_w}{T_w - T_{sat}}$$

$$\frac{q''_w}{T_w - T_{sat}}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}C_{sf} = k_f\left(\frac{\rho_f}{\mu_f}\frac{q''_w}{\rho_f h_{fg}}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}\right)^{1-n}\left(\frac{\mu_f C_{pf}}{k_f}\right)^{-m}$$

$$T_w - T_{sat} = C_{sf}\frac{q''_w}{k_f}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}\left(\frac{1}{\mu_f}\frac{q''_w}{h_{fg}}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}\right)^{n-1}\left(\frac{\mu_f C_{pf}}{k_f}\right)^{m}$$

$$C_{pf}\frac{T_w - T_{sat}}{h_{fg}} = C_{sf}\frac{C_{pf}}{h_{fg}}\frac{q''_w}{k_f}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}\left(\frac{1}{\mu_f}\frac{q''_w}{h_{fg}}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}\right)^{n-1}\left(\frac{\mu_f C_{pf}}{k_f}\right)^{m}$$

$$C_{pf}\frac{T_w - T_{sat}}{h_{fg}} = C_{sf}\frac{\mu_f C_{pf}}{k_f}\left(\frac{q''_w}{\mu_f h_{fg}}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}\right)\left(\frac{1}{\mu_f}\frac{q''_w}{h_{fg}}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}\right)^{n-1}\left(\frac{\mu_f C_{pf}}{k_f}\right)^{m}$$

$$= C_{sf}\frac{\mu_f C_{pf}}{k_f}\left(\frac{q''_w}{\mu_f h_{fg}}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}\right)\left(\frac{1}{\mu_f}\frac{q''_w}{h_{fg}}\sqrt{\frac{\sigma}{g(\rho_f-\rho_g)}}\right)^{n}\left(\frac{\mu_f C_{pf}}{k_f}\right)^{m+1}$$

11.3 Nucleate Boiling Correlation

- Rohsenow correlation (1952)

$$C_{Pf}\frac{T_w - T_{sat}}{h_{fg}} = C_{sf}\left[\frac{q''_w}{\mu_f h_{fg}}\sqrt{\frac{\sigma}{g\Delta\rho}}\right]^n \left(\frac{\mu C_p}{k}\right)_f^{m+1}$$

A METHOD OF CORRELATING HEAT TRANSFER DATA

FOR SURFACE BOILING OF LIQUIDS

by

Warren M. Rohsenow*

SUMMARY

A method based on a logical explanation of the mechanism of heat transfer associated with the boiling process is presented for correlating heat transfer data for nucleate boiling of liquids for the case of pool boiling. The suggested relation is

$$\frac{C_\ell T_x}{h_{fg}} = C_{sf}\left(\frac{q/A}{\mu_\ell h_{fg}}\sqrt{\frac{g_0\sigma}{g(\rho_\ell-\rho_v)}}\right)^{0.33}\left(\frac{C_\ell \mu_\ell}{k_\ell}\right)^{1.7}$$

where the various fluid properties are evaluated at the saturation temperature corresponding to the local pressure and C_{sf} is a function of the particular heating surface-fluid combination.

Heat transfer data for forced convection flow without boiling is correlated by the normal Nusselt number, Reynolds number based on pipe diameter and Prandtl number. For pool boiling with essentially saturated liquids, Jakob (1) shows that the heat transfer from the surface is for the most part transferred directly to the liquid, the increased heat transfer rate associated with boiling being accounted for by the resulting agitation of the fluid by motion of the liquid flowing behind the wake of the bubble departing from the surface. Rohsenow and Clark (2) showed a similar result in studying motion pictures of McAdams (3) for subcooled liquids flowing in forced convection with surface boiling but no net generation of vapor. Gunther and Kreith (4) and Gunther(5)

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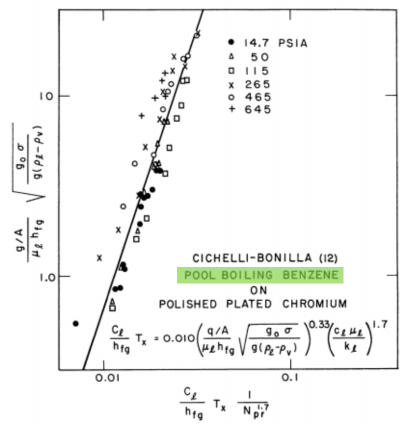
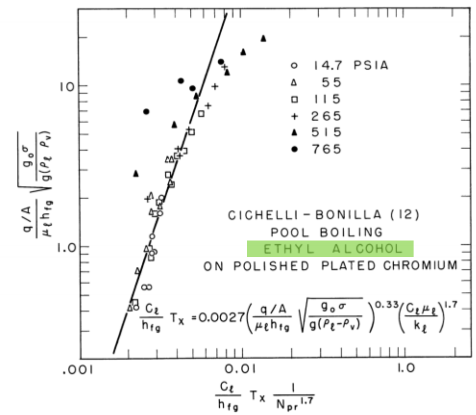
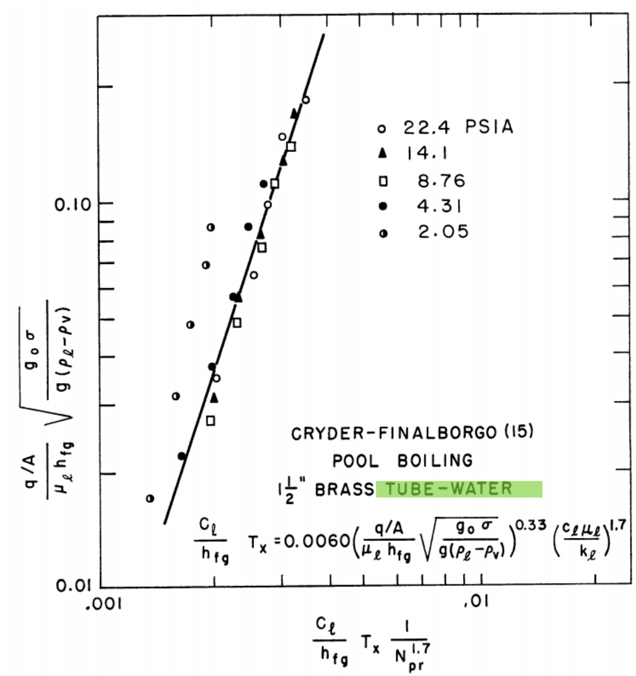






11.3 Nucleate Boiling Correlation

❖ Rohsenow correlation (1952)

$$C_{\mathrm{Pf}}\frac{T_{\mathrm{w}}-T_{\mathrm{sat}}}{h_{\mathrm{fg}}}=C_{\mathrm{sf}}\left[\frac{q''_{\mathrm{w}}}{\mu_{\mathrm{f}}h_{\mathrm{fg}}}\sqrt{\frac{\sigma}{g\Delta\rho}}\right]^{n}\left(\frac{\mu C_p}{k}\right)_{\mathrm{f}}^{m+1}$$

$n = 0.33$, $m = 0$ for water and $m = 0.7$ for other fluids.

$$q''_s=\mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}\Delta T_e}{C_{s,f}h_{fg}Pr_l^n}\right)^3$$

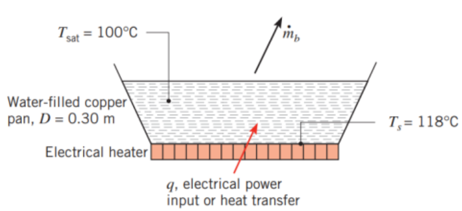
EXAMPLE 10.1

The bottom of a copper pan, 0.3 m in diameter, is maintained at 118°C by an electric heater. Estimate the power required to boil water in this pan. What is the evaporation rate? Estimate

Find:

1. Power required by electric heater to cause boiling.
2. Rate of water evaporation due to boiling.
3. Critical heat flux corresponding to the burnout point.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Water exposed to standard atmospheric pressure, 1.01 bar.
3. Water at uniform temperature T_{sat} = 100°C.
4. Large pan bottom surface of polished copper.
5. Negligible losses from heater to surroundings.

Properties: Table A.6, saturated water, liquid (100°C): $\rho_l = 1/v_f = 957.9\ \mathrm{kg/m^3}$, $c_{p,l} = c_{p,f} = 4.217\ \mathrm{kJ/kg\cdot K}$, $\mu_l = \mu_f = 279 \times 10^{-6}\ \mathrm{N\cdot s/m^2}$, $Pr_l = Pr_f = 1.76$, $h_{fg} = 2257\ \mathrm{kJ/kg}$, $\sigma = 58.9 \times 10^{-3}\ \mathrm{N/m}$. Table A.6, saturated water, vapor (100°C): $\rho_v = 1/v_g = 0.5956\ \mathrm{kg/m^3}$.

Analysis:

1. From knowledge of the saturation temperature T_{sat} of water boiling at 1 atm and the temperature of the heated copper surface T_s, the excess temperature ΔT_e is

$$\Delta T_e \equiv T_s - T_{sat} = 118°\mathrm{C} - 100°\mathrm{C} = 18°\mathrm{C}$$

According to the boiling curve of Figure 10.4, nucleate pool boiling will occur and the recommended correlation for estimating the heat transfer rate per unit area of plate surface is given by Equation 10.5.

$$q''_s=\mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}\Delta T_e}{C_{s,f}h_{fg}Pr_l^n}\right)^3$$

The values of $C_{s,f}$ and n corresponding to the polished copper surface–water combination are determined from the experimental results of Table 10.1, where $C_{s,f} = 0.0128$ and $n = 1.0$. Substituting numerical values, the boiling heat flux is

$$q''_s = 279\times10^{-6}\ \mathrm{N\cdot s/m^2}\times 2257\times10^3\ \mathrm{J/kg}$$
$$\times\left[\frac{9.8\ \mathrm{m/s^2}\,(957.9-0.5956)\ \mathrm{kg/m^3}}{58.9\times10^{-3}\ \mathrm{N/m}}\right]^{1/2}$$
$$\times\left(\frac{4.217\times10^3\ \mathrm{J/kg\cdot K}\times18°\mathrm{C}}{0.0128\times2257\times10^3\ \mathrm{J/kg}\times1.76}\right)^3 = 836\ \mathrm{kW/m^2}$$

Hence the boiling heat transfer rate is

$$q_s = q''_s \times A = q''_s \times \frac{\pi D^2}{4}$$

$$q_s = 8.36\times10^5\ \mathrm{W/m^2}\times\frac{\pi(0.30\ \mathrm{m})^2}{4} = 59.1\ \mathrm{kW} \quad \triangleleft$$

2. Under steady-state conditions all heat addition to the pan will result in water evaporation from the pan. Hence

$$q_s = m_b h_{fg}$$

where m_b is the rate at which water evaporates from the free surface to the room. It follows that

$$m_b = \frac{q_s}{h_{fg}} = \frac{5.91\times10^4\ \mathrm{W}}{2257\times10^3\ \mathrm{J/kg}} = 0.0262\ \mathrm{kg/s} = 94\ \mathrm{kg/h} \quad \triangleleft$$

11.3 Nucleate Boiling Correlation

- Forster and Zuber (1954)

$$\mathrm{Nu} = \frac{h\lambda_L}{k_f} = \frac{1}{C_{sf}} \mathrm{Re}^{1-n}\, \mathrm{Pr}^{-m}$$

- ✓ The length and velocity scales
  - Based on the growth process of microbubbles suspended in a superheated liquid

$$\dot{R} = \sqrt{\frac{\pi}{2}} \frac{k_f}{\rho_g h_{fg}} \frac{\Delta T_{sat}}{\sqrt{\alpha_f t}} \qquad R = 2\sqrt{\frac{\pi}{2}} \frac{k_f}{\rho_g h_{fg}} \frac{\Delta T_{sat}}{\sqrt{\alpha_f}} \sqrt{t} \qquad R = C\frac{2k_L(T_\infty - T_{sat})}{\sqrt{\alpha_L}\rho_v h_{fg}}\sqrt{t}$$

- ✓ Generic heat transfer correlation & length scale

$$\mathrm{Nu} = 0.0015 \mathrm{Re}^{0.62}\, \mathrm{Pr}_f^{0.33} \qquad \mathrm{Re} \sim \rho_f R \dot{R}/\mu_f \qquad \mathrm{Nu} = \frac{q''_w}{\Delta T_{sat}} \frac{l}{k_f}$$

$$l = \frac{\Delta T_{sat} \rho_f C_{Pf} \sqrt{\pi \alpha_f}}{\rho_g h_{fg}} \sqrt{\frac{2\sigma}{\Delta P}} \left[\frac{\rho_f}{\Delta P}\right]^{1/4} \qquad R^* = \frac{2\sigma}{P_{sat}(T_w) - P}$$

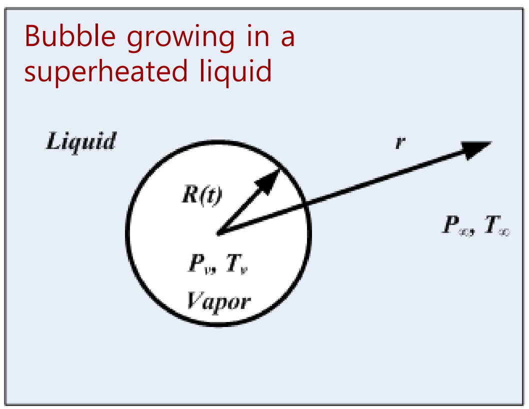
- ✓ Final form

$$\frac{q''_w}{\rho_g h_{fg}} \left(\frac{\pi}{\alpha_f}\right)^{1/2} \left[\frac{\rho_f R^{*3}}{2\sigma}\right]^{1/4} = 0.0015 \left\{ \frac{\rho_f}{\mu_f} \left[\frac{(T_w - T_{sat})k_f}{\rho_g h_{fg}}\right]^2 \frac{\pi}{\alpha_f} \right\}^{5/8} (\mu C_P/k)_f^{1/3}$$

Used in Chen correlation
No surface effect

11.3 Nucleate Boiling Correlation

- Growth of a vapor bubble in superheated liquid
  - ✓ Three phases of growth
  - ✓ First phase: Rayleigh solution, hydro-dynamically controlled
    - At low pressures, the bubble grows approximately at a constant rate ($\dot{R} \approx const$)
    - The time duration is very short. (~μs)
  - ✓ The second phase
    - Transition from hydrodynamically controlled growth to a thermally controlled growth
  - ✓ The third phase: thermally controlled
    - Inertia and surface tension effects are insignificant.



$$P_\infty \le P_v \le P_{sat}(T_\infty)$$

$$T_{sat}(P_\infty) \le T_v \le T_\infty$$

11.3 Nucleate Boiling Correlation

- Growth of a vapor bubble in superheated liquid
  - ✓ Hydro-dynamically controlled growth
    - Assumption: inviscid liquid behavior, stagnant and infinitely large liquid field
    - Potential flow theory

$$\nabla^2\phi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = 0 \qquad \frac{d\phi}{dr} = U_L = \dot{R} \quad \text{at} \quad r = R \qquad \frac{d\phi}{dr} = 0 \quad \text{for} \quad r \to \infty$$

Continuity equation

- Solution $\phi = -\frac{R^2\dot{R}}{r}$
- Momentum conservation

$$\frac{\partial u}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2u^2) = \frac{\partial u}{\partial t} + \frac{1}{r^2}\left[u\frac{\partial}{\partial r}(r^2u) + r^2u\frac{\partial}{\partial r}(u)\right] = \frac{\partial u}{\partial t} + \frac{\partial}{\partial r}\left(\frac{u^2}{2}\right) = -\frac{\partial P}{\partial r}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial\phi}{\partial t}\right) + \frac{\partial}{\partial r}\left(\frac{u^2}{2}\right) = -\frac{1}{\rho}\frac{\partial P}{\partial r}$$

$$\left[\frac{\partial\phi}{\partial t} + \left(\frac{u^2}{2}\right)\right]_R^\infty = -\frac{P_\infty - P_R}{\rho} \qquad \frac{\partial\phi}{\partial t} + \left(\frac{u^2}{2}\right) = \frac{P_\infty - P_R}{\rho} \qquad \frac{\partial\phi}{\partial t} + \frac{1}{2}U_L^2 + \frac{1}{\rho_L}P_L = \frac{1}{\rho_L}P_\infty$$

11.3 Nucleate Boiling Correlation

- Growth of a vapor bubble in superheated liquid
  - ✓ Momentum equation at the interface

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}U_L^2 + \frac{1}{\rho_L}P_L = \frac{1}{\rho_L}P_\infty \qquad U_L = \partial\phi/\partial r = R^2\dot{R}/r^2 \qquad \phi = -\frac{R^2\dot{R}}{r}$$

$$\frac{\partial}{\partial t}\left(-\frac{R^2\dot{R}}{r}\right) + \frac{1}{2}\left(\frac{R^4\dot{R}^2}{r^4}\right) = \frac{P_\infty - P_L}{\rho}$$

$$-\frac{1}{r}\left[R^2\frac{\partial \dot{R}}{\partial t} + \dot{R}\frac{\partial R^2}{\partial t}\right] + \frac{1}{2}\left(\frac{R^4\dot{R}^2}{r^4}\right) = -\frac{1}{r}\left[R^2\ddot{R} + 2R\dot{R}^2\right] + \frac{1}{2}\left(\frac{R^4\dot{R}^2}{r^4}\right) = \frac{P_\infty - P_L}{\rho}$$

$$at\ r = R$$

$$-R\ddot{R} - \frac{3}{2}\dot{R}^2 = \frac{P_\infty - P_L}{\rho}$$

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{P_L - P_\infty}{\rho}$$

Rayleigh equation

11.3 Nucleate Boiling Correlation

❖ Growth of a vapor bubble in superheated liquid

✓ Rayleigh's equation for the special case of $P_L - P_\infty$=const.

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{P_L - P_\infty}{\rho_L} \qquad \times R^2\dot{R}$$

$$\frac{d}{dt}\left(R^3\dot{R}^2\right) = \frac{2(P_L - P_\infty)}{\rho_L}R^2\dot{R} \qquad + \qquad R(0) = R_0 \qquad \dot{R}(t) = \left\{\frac{2(P_L - P_\infty)}{3\rho_L}\left[1 - \left(\frac{R_0}{R}\right)^3\right]\right\}^{1/2}$$

✓ For $R \gg R_0$, $\quad R(t) \approx \left\{\frac{2(P_L - P_\infty)}{3\rho_L}\right\}^{1/2} t$

- Hydrodynamic and liquid-inertia controlled bubble growth

✓ Coupling with the gas phase

$$P_L - P_\infty = (P_L - P_v) + (P_v - P_\infty) \qquad P_L - P_v = -\frac{2\sigma}{R} \qquad P_v - P_\infty = \frac{h_{fg}(T_B - T_{sat})}{T_{sat}(v_v - v_L)}$$

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{P_L - P_\infty}{\rho_L} \quad \Rightarrow \quad R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{2\sigma}{\rho_L R} - \frac{h_{fg}(T_B - T_{sat})}{\rho_L T_{sat}(v_v - v_L)} = 0$$

Extended Rayleigh equation

11.3 Nucleate Boiling Correlation

- Growth of a vapor bubble in superheated liquid
  - ✓ Extended Rayleigh's equation + Energy conservation eq. for liquid

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{2\sigma}{\rho_L R} - \frac{h_{fg}(T_B - T_{sat})}{\rho_L T_{sat}(v_v - v_L)} = 0$$

Two unknowns of T_B and R

$$\rho c\left[\frac{\partial T}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 UT)\right] = \rho c\left[\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial r} + \frac{T}{r^2}\frac{\partial}{\partial r}(r^2 U)\right] = \rho c\left[\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial t}\right] = \frac{k}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)$$

$$U_L = R^2\dot{R}/r^2$$

$$\frac{\partial T_L}{\partial t} + \frac{R^2}{r^2}\dot{R}\frac{\partial T_L}{\partial r} = \frac{\alpha_L}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_L}{\partial r}\right)$$

+ IC & BC

$$T_L = \begin{cases} T_\infty & \text{at } t = 0, \\ T_B & \text{at } r = R, t > 0, \\ T_\infty & \text{for } r \to \infty. \end{cases}$$

  - ✓ Neglecting viscous effect and surface tension + $\rho_L \gg \rho_v$

$$\frac{d}{dt}(R^3\dot{R}^2) = \frac{2\rho_v h_{fg}(T_B - T_{sat})}{\rho_L T_{sat}} R^2\dot{R}$$

$$R(t) \approx \left\{\frac{2(P_L - P_\infty)}{3\rho_L}\right\}^{1/2} t$$

$$R(t) = \left[\frac{2\rho_v h_{fg}(T_B - T_{sat})}{3\rho_L T_{sat}}\right]^{1/2} t$$

$$R(t) \approx \left[\frac{2\rho_v h_{fg}(T_\infty - T_B)}{3\rho_L T_B}\right]^{1/2} t$$

11.3 Nucleate Boiling Correlation

❖ Growth of a vapor bubble in superheated liquid

✓ Example

- For inertia controlled bubble growth, estimate the interface velocity of a 0.2mm diameter bubble growing in water at atmospheric pressure and 120 ℃.

$$T_{sat} = 100^{o}C, \qquad h_{lv} = 2257kJ/kg,$$

$$\rho_l = 958kg/m^3, \quad \rho_v = 0.598kg/m^3$$

$$R(t) = \left\{\left(\frac{2}{3}\left[\frac{\left(T_v - T_{sat}(P_\infty)\right)}{T_{sat}(P_\infty)}\right]\frac{h_{lv}\rho_v}{\rho_l}\right)\right\}^{1/2} t \;\Rightarrow\; \frac{dR(t)}{dt} = \left\{\left(\frac{2}{3}\left[\frac{\left(T_v - T_{sat}(P_\infty)\right)}{T_{sat}(P_\infty)}\right]\frac{h_{lv}\rho_v}{\rho_l}\right)\right\}^{1/2}$$

$$= 7.10m/s$$

11.3 Nucleate Boiling Correlation

- Growth of a vapor bubble in superheated liquid
  - ✓ For the thermally controlled growth phase

$$q'' = \rho_v h_{fg} \dot{R}$$

  - $T_B \approx T_{sat}$: since the bubble is relatively large during its thermally controlled growth
- ✓ 1D transient heat conduction equation into a semi-infinite medium

$$q'' = \frac{k_L (T_\infty - T_{sat})}{\sqrt{\pi \alpha_L t}} \quad + \text{IC} \quad R = 0 \text{ at } t = 0$$

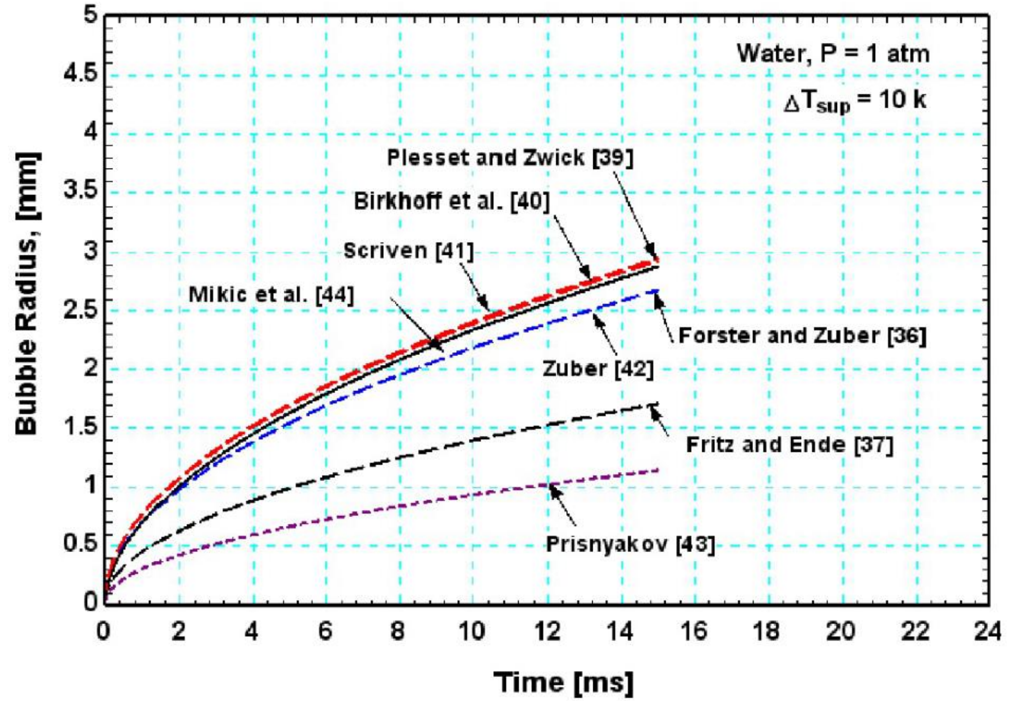
$$R = C \frac{2k_L (T_\infty - T_{sat})}{\sqrt{\alpha_L} \rho_v h_{fg}} \sqrt{t} \qquad C = 1/\sqrt{\pi}$$

Plesset and Zwick (1954) $\quad C = \sqrt{3/\pi}$

Forster and Zuber (1954) $\quad C = \sqrt{\pi/2}$

$$Ja = \frac{(T_\infty - T_{sat}) \rho_l C_{pl}}{\rho_l h_{fg}}$$

$$R = C\, Ja \sqrt{\alpha t}$$



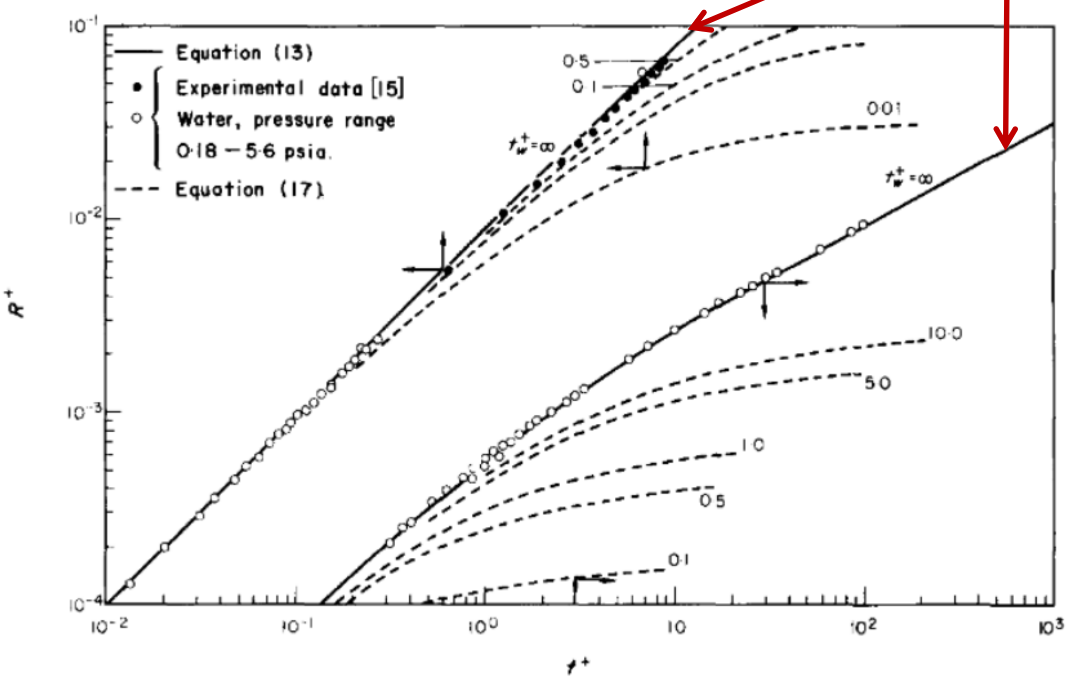
11.3 Nucleate Boiling Correlation

- Growth of a vapor bubble in superheated liquid
  - ✓ Complete bubble growth process (MIKIC, ROHSENOW and GRIFFITH, 1970)

$$R^+ = \frac{2}{3}\left[\left(t^+ + 1\right)^{3/2} - \left(t^+\right)^{3/2} - 1\right] \qquad , where \;\; R^+ = \frac{RA}{B^2} \qquad t^+ = \frac{tA^2}{B^2}$$

$$A = \left\{\frac{2\left[T_\infty - T_{sat}\left(P_\infty\right)\right]h_{lv}\rho_v}{\rho_l T_{sat}\left(P_\infty\right)}\right\}^{1/2} \qquad B = \left(\frac{12\alpha_l}{\pi}\right)^{1/2} Ja \qquad Ja = \frac{\left(T_\infty - T_{sat}\right)\rho_l c_{pl}}{\rho_v h_{lv}}$$

  - ✓ For $t^+ \ll 1, R^+ = t^+$
    (Rayleigh solution)
  - ✓ For $t^+ \gg 1, R^+ = \sqrt{t^+}$
    (Plesset and Zwick solution)



11.3 Nucleate Boiling Correlation

❖ Stephan and Abdelsalam (1980)

✓ Found to have good accuracy

$$\mathrm{Nu} = H d_{\mathrm{Bd}} / k_{\mathrm{f}}$$

- d_{Bd}: bubble departure diameter, Fritz model

✓ For water in the range $10^{-4} \le P/P_{cr} \le 0.886$

$$\mathrm{Nu} = (0.246 \times 10^{7}) \left(\frac{q''_{\mathrm{w}} d_{\mathrm{Bd}}}{k_{\mathrm{f}} T_{\mathrm{sat}}} \right)^{0.673} \left(\frac{h_{\mathrm{fg}} d_{\mathrm{Bd}}^{2}}{\alpha_{\mathrm{f}}^{2}} \right)^{-1.58} \left(C_{\mathrm{Pf}} T_{\mathrm{sat}} d_{\mathrm{Bd}}^{2} / \alpha_{\mathrm{f}}^{2} \right)^{1.26} \left(\frac{\Delta \rho}{\rho_{\mathrm{f}}} \right)^{5.22} \qquad \theta_0 = 45^{\circ}$$

✓ For refrigerants (propane, n-butane, carbon dioxide, and several refrigerants including R-12, R-113, R-114, and RC-318), in the range $3 \times 10^{-3} \le P/P_{cr} \le 0.78$

$$\mathrm{Nu} = 207 \left(\frac{q''_{\mathrm{w}} d_{\mathrm{Bd}}}{k_{\mathrm{f}} T_{\mathrm{sat}}} \right)^{0.745} \left(\frac{\rho_{\mathrm{g}}}{\rho_{\mathrm{f}}} \right)^{0.581} \mathrm{Pr}_{\mathrm{f}}^{0.533}, \quad \theta_0 = 35^{\circ}$$

11.3 Nucleate Boiling Correlation

- Gorenflo (1993)
  - ✓ Widely respected correlation

Reference surface roughness parameter

$$\frac{H}{H_0} = F_{PR}(q''_w/q''_0)^n (R_P/R_{P0})^{0.133} \quad \text{where } q''_0 = 20\,000 \text{ W/m}^2 \quad R_{P0} = 0.4\ \mu\text{m}$$

  - ✓ For water

Pressure correction factor

$$F_{PR} = 1.73P_r^{0.27} + \left(6.1 + \frac{0.68}{1 - P_r}\right) P_r^2,$$

$$n = 0.9 - 0.3P_r^{0.15}.$$

  - ✓ For other fluids

$$F_{PR} = 1.2P_r^{0.27} + \left(2.5 + \frac{1}{1 - P_r}\right) \mathrm{P_r},$$

$$n = 0.9 - 0.3P_r^{0.3}.$$

Work very well in predicting experimental data with newer refrigerants

Table 11.2. *Reference parameters for the correlation of Gorenflo (1993) for selected fluids.*

Fluid	P_{cr} (bar)	H_0 (W/m^2·K)	Fluid	P_{cr} (bar)	H_0 (W/m^2·K)
Water	220.6	5600	R-11	44.0	2800
Ammonia	113.0	7000	R-12	41.6	4000
Sulfur hexafluoride	37.6	3700	R-13	38.6	3900
Methane	46.0	7000	R-22	49.9	3900
Ethane	48.8	4500	R-23	48.7	4400
Propane	42.4	4000	R-113	34.1	2650
Benzene	48.9	2750	R-123	36.7	2600
n-Pentane	33.7	3400	R-134a	40.6	5040
i-Pentane	33.3	2500	R-152a	45.2	4000
Nitrogen (on Pt)	34.0	7000	RC-318	28.0	4200
Nitrogen (on Cu)	34.0	10 000	R-32	57.82	6550
Propane	42.48	5210	R-152a	45.17	5570
i-Butane	36.4	4320	R-143a	37.76	5410
Ethanol	63.8	4400	R-125	36.29	4940
Acetone	47.0	3950	R-227ea	29.80	4860

11.3 Nucleate Boiling Correlation

EXAMPLE 11.2. Using the correlations of Rohsenow (1952), Cooper (1984), and Gorenflo (1993), calculate the boiling heat transfer coefficient for a mechanically polished stainless-steel surface submerged in saturated water at a pressure of 17.9 bars. The wall is at $T_w = 490$ K. Assume a mean surface roughness of 2 μm.

SOLUTION. The relevant properties are $C_{Pf} = 4{,}524$ J/kg·K, $k_f = 0.647$ W/m·K, $\mu_f = 1.30 \times 10^{-4}$ kg/m·s, $\rho_f = 856.7$ kg/m^3, $\rho_g = 9.0$ kg/m^3, $T_{sat} = 480$ K, $h_{fg} = 1.913 \times 10^6$ J/kg, and $\sigma = 0.036$ N/m.

First, consider Rohsenow's correlation. From Table 11.1 we get $C_f = 0.0132$. We also have $m = 0$ and $n = 0.33$. Equation (11.29) can now be solved for q_w, resulting in

$$q''_{w,\text{Rohsenow}} = 9.147 \times 10^5 \text{ W/m}^2.$$

We now consider Cooper's correlation. We have

$$\text{P}_r = P/P_{cr} = 17.9 \text{ bars}/220.6 \text{ bars} = 0.0811,$$
$$M = 18,$$

and

$$R_P = 2,$$

and so

$$n = 0.12 - 0.21 \log_{10}(R_p) = 0.0568.$$

We can now solve Eq. (11.40) to get

$$q''_{W,\text{Cooper}} = 1.247 \times 10^6 \text{ W/m}^2.$$

Lastly, we consider the method of Gorenflo. From Table 11.2, we have $H_0 = 5600$ W/m^2. Furthermore, $q''_0 = 20{,}000$ W/m^2 and

$$n = 0.9 - 0.3 P_r^{0.15} = 0.694,$$

$$F_{PR} = 1.73\,\text{Pr}^{0.27} + \left[6.1 + \frac{0.68}{1 - \text{Pr}}\right] \text{Pr}^2 = 0.923.$$

We can now calculate the boiling heat transfer coefficient from Eq. (11.41), noting that

$$\frac{R_P}{R_{P0}} = \frac{2 \text{ μm}}{0.4 \text{ μm}} = 5.$$

Equation (11.41) must be solved simultaneously with the following equation:

$$q''_{w,\text{Gorenflo}} = H_{\text{Gorenflo}}(T_w - T_{sat}),$$

with $q''_{w,\text{Gorenflo}}$ and H_{Gorenflo} as the two unknowns. The result will be

$$q''_{w,\text{Gorenflo}} = 8.98 \times 10^5 \text{ W/m}^2.$$