- Hydrodynamic models
 - ✓ Work well with minor adjustments
- Film boiling on a horizontal, flat surface
 - ✓ Berenson (1961)
 - Standing Taylor waves with square λ_d
 - Vapor generated in a square unit cell with λ_d^2 assumed to flow toward each vapor dome
 - Circular unit cell for simplicity: $r_2 = \lambda_d / \sqrt{\pi}$



Momentum equation

 $\frac{dP}{dr} = C \frac{\mu_v \overline{U}_v}{\delta^2} \qquad \begin{array}{c} C = 12 \text{ if vapor velocity is zero at the vapor-liquid interphase} \\ C = 3 \text{ if shear stress is zero at the interphase} \end{array}$

Other equations

corrected for the effect of vapor superheating

$$\dot{m}_{\rm v} = 2\pi r \rho_{\rm v} \delta \overline{U}_{\rm v} \qquad \dot{m}_{\rm v} h_{\rm fg}' = \pi \left(r_2^2 - r^2 \right) k_{\rm v} \frac{\Delta T}{\delta} \qquad h_{\rm fg}' = h_{\rm fg} \left[1 + \frac{1}{2} C_{\rm Pg} \frac{T_{\rm w} - T_{\rm sat}}{h_{\rm fg}} \right]$$



Berenson model

$$\dot{m}_{\rm v} = 2\pi r \rho_{\rm v} \delta \overline{U}_{\rm v}$$
 $\dot{m}_{\rm v} h_{\rm fg}' = \pi \left(r_2^2 - r^2 \right) k_{\rm v} \frac{\Delta T}{\delta}$ $r_2 = \lambda_{\rm d} / \sqrt{\pi}$

 $\overline{U}_{v} = \frac{k_{v}\Delta T}{\rho_{v}h_{fg}^{\prime}\delta^{2}} \left[\frac{(\lambda_{d}^{2}/2) - \pi r^{2}}{2\pi r}\right] \text{ average vapor velocity in the vapor film}$ $\underbrace{\frac{dP}{dr}}_{r} = C\frac{\mu_{v}\overline{U}_{v}}{\delta^{2}}$



$$P_2 - P_1 = \frac{8C}{\pi} \frac{\mu_{\rm v} k_{\rm v} \Delta T}{\rho_{\rm v} h'_{\rm fg} \delta^4} \frac{\sigma}{g \Delta \rho} \qquad ???$$

 $P_2 - P_1 = g \Delta \rho \ell - \frac{2\sigma}{R_{\rm B}}$

This pressure difference is supplied by the hydrostatic and surface tension forces.

 $\checkmark\,$ The vapor film thickness and HTC

$$\delta = \left[1.09C \frac{\mu_{\rm v} k_{\rm v} \Delta T}{\rho_{\rm v} g \Delta \rho h'_{\rm fg}} \sqrt{\sigma/(g \Delta \rho)} \right]^{0.25} \qquad H = k_{\rm v}/\delta \qquad H = 0.425 \left[\frac{k_{\rm v}^3 \rho_{\rm v} \Delta \rho g h'_{\rm fg}}{\mu_{\rm v} (T_{\rm w} - T_{\rm sat}) \sqrt{\sigma/(g \Delta \rho)}} \right]^{0.25}$$

Berenson model



FIG 36 PHYSICAL MODEL OF FILM BOILING FROM A HORIZONTAL SURFACE



FIG 3 TRANSITION BOILING: (¶/A)max=170,000 BTU hr.ft.2(25)



FIG 6 TRANSITION BOILING: $q'A = 13,000 \frac{BTU}{hr, ft.^2}$ (25)



FIG 7 TRANSITION BOILING: (4/A)min=5,500 BTU hr.ft.2(25)

FIG 5 TRANSITION BOILING: 9/A= 27,000 BTU (25)

Film boiling on a vertical, flat surface

- ✓ Analysis for a coherent, laminar flow
 - Vapor momentum conservation equation

$$-\mu_{\rm v}\frac{d^2U}{dy^2} - g(\rho_{\rm L}-\rho_{\rm v}) = 0$$

BCs

U = 0 at y = 0, U = 0 at $y = \delta$ for the stagnant interphase (i.e., no-slip), or $\frac{dU}{dy} = 0$ at $y = \delta$ for the dynamic interphase (i.e., zero shear stress).

Velocity profile and vapor mass flow rate per unit width

 $U(y) = \frac{g\Delta\rho}{2\mu_{\rm v}} \begin{bmatrix} C_1 \delta y - y^2 \end{bmatrix} \qquad \begin{array}{c} C_1 = 1 \text{ for stagnant interphase} \\ C_2 = 2 \text{ for dynamic interphase} \end{array}$

Energy balance (linear temperature profile)

$$h_{\rm fg}\frac{d\Gamma_{\rm v}}{dz} = k_{\rm v}\frac{T_{\rm w} - T_{\rm sat}}{\delta}$$

• Vapor film thickness $\delta = \left[\frac{8}{3(C_1/2 - 1/3)} \frac{k_v (T_w - T_{sat}) \mu_v z}{\rho_v h_{fg} g \Delta \rho}\right]^{1/4}$



 $\Gamma_{\rm v} = \rho_{\rm v} \int\limits_{-\infty}^{0} U(y) dy$

Film boiling on a vertical, flat surface

- ✓ Analysis for a coherent, laminar flow
 - Local film boiling heat transfer coefficient

 $\delta = \left[\frac{8}{3(C_1/2 - 1/3)} \frac{k_v (T_w - T_{sat})\mu_v z}{\rho_v h_{fg} g \Delta \rho}\right]^{1/4} \qquad H = k_v / \delta$

Bromley, 1950 • Average HTC $\overline{H}_L = \frac{1}{L} \int_0^L H dz$ $\overline{H}_{FB,L} = C \left[\frac{\rho_v h_{fg} g \Delta \rho k_v^3}{(T_w - T_{sat}) \mu_v L} \right]^{1/4}$ C = 0.663 for stagnant interphase C = 0.943 for dynamic interphase

- The stagnant interphase is evidently more appropriate for film boiling in a quiescent liquid pool.

Correction for vapor superheating

$$- h_{fg} \rightarrow h'_{fg}$$
$$h'_{fg} = H_{fg} \left[1 + 0.34 \frac{C_{Pv}(T_w - T_{sat})}{h_{fg}} \right]$$

Film boiling on a vertical, flat surface

$$\overline{H}_{\text{FB},L} = C \left[\frac{\rho_{\text{v}} h_{\text{fg}} g \Delta \rho k_{\text{v}}^3}{(T_{\text{w}} - T_{\text{sat}}) \mu_{\text{v}} L} \right]^{1/4}$$

Improvements to the simple theory

- Bromley model: under-predicts experimental data when the length > 1/2 inch
 - Laminar film assumption
 - Turbulent film when $\delta^+ = \delta \sqrt{\frac{\tau_w}{\rho_v}} / \nu_v > 10$ (Hsu and Westwater, 1960)
 - Intermittency of the vapor film
- Bailey (1970)
 - The vapor film supports a spatially intermittent structure.
 - At the bottom of each spatial interval, the vapor film is initiated and grows, until it becomes unstable and eventually is dispersed by the time it reaches the top of the interval.
 - The vapor film remains laminar in the aforementioned intervals.

$$Spprox\lambda_{
m cr}=2\pi\sqrt{rac{\sigma}{g\Delta
ho}}$$



Film boiling on a vertical, flat surface

Whether the system is stable or not will depend on the sign of the square-root term. For neutral conditions, therefore

$$k_{\rm cr} = \sqrt{g\Delta\rho/\sigma}.$$
 (2.174)

The quantity $\lambda_{\rm L} = 1/k_{\rm cr} = \sqrt{\sigma/g\Delta\rho}$ is called the *Laplace length scale*. The neutral wavelength is therefore

$$\lambda_{\rm cr} = \frac{2\pi}{k_{\rm cr}} = 2\pi\lambda_{\rm L}.\tag{2.175}$$

Waves with shorter wavelength are called ripples, and these do not cause the disruption of the interphase. Waves with wavelengths longer than λ_{cr} lead to the disruption of the interphase. The fastest growing wavelength (also sometimes referred to as the most dangerous wavelength!) can be found by applying $d\omega/dk = 0$ to Eq. (2.173), which results in

$$k_{\rm d} = \sqrt{(g\Delta\rho)/(3\sigma)} = \frac{1}{\sqrt{3}\lambda_{\rm L}}.$$
(2.176)

This is equivalent to

$$\lambda_{\rm d} = 2\pi\sqrt{3}\lambda_{\rm L},\tag{2.177}$$

Film boiling on a vertical, flat surface

$$\overline{H}_{\mathrm{FB},L} = C \left[\frac{\rho_{\mathrm{v}} h_{\mathrm{fg}} g \Delta \rho k_{\mathrm{v}}^{3}}{(T_{\mathrm{w}} - T_{\mathrm{sat}}) \mu_{\mathrm{v}} L} \right]^{1/4}$$

 $\checkmark\,$ Improvements to the simple theory

- Leonard et al. (1978)
 - For vertical surfaces, L in the Bromley model should be replaced with λ_{cr} .
 - Modified Bromley correlation

	Mode of heat transfer							
User geometry default value underlined	Laminar	Natural	Turbulent	Condens ation	Nucleate boiling	Transition boiling	Film boiling	CHF
1, 100, <u>101</u> , 103-109, 114	Sellars Nu=4.36	C-Chu or McAdams	Dittus- Boelter	Nusselt/ Chato- Shah- Coburn- Hougen	Chen	Chen	Bromley	Table
<u>102</u>	ORNL ANS Nu=7.63	Elenbaas	Petukhov	"	"	"	"	Table Gambi Il- Weath erhead
121-133, <u>130</u>	Sellars	McAdams	"	"	"	"	"	Table
<u>110</u> , 112	"	C-Chu or McAdams	DB- Inayatov	"	Chen- Inayatov	"	"	"
<u>111</u> , 113	"	"	DB- Inayatov- Shah	"	"	"	"	"
<u>134</u> -137	"	Churchill- Chu	DB-ESDU	"	Polley	"	"	Folkin

- Film boiling on horizontal tubes
 - ✓ Breen and Westwater (1962)

$$H = (0.59 + 0.069C) \left[\frac{g\Delta\rho\rho_{\rm v}k_{\rm v}^3 h_{\rm fg}'}{\lambda_{\rm cr}\mu_{\rm v}(T_{\rm w} - T_{\rm sat})} \right]^{1/4} \qquad h_{\rm fg}' = h_{\rm fg} \left[1 + 0.34 \frac{C_{\rm Pv}(T_{\rm w} - T_{\rm sat})}{h_{\rm fg}} \right] \qquad C = \min(1, \lambda_{\rm cr}/D)$$

✓ Effect of thermal radiation

$$H = H_{\rm FB} + \frac{3}{4}H_{\rm rad}$$
 $H_{\rm rad} \approx \frac{\sigma \varepsilon_{\rm w} \left(T_{\rm w}^4 - T_{\rm sat}^4\right)}{T_{\rm w} - T_{\rm sat}}$

- ³/₄: empirical
- ε_w : heated surface emissivity

- Minimum film boiling (MFB)
 - ✓ Important threshold for nuclear safety (quenching)
 - ✓ Leidenfrost process (1756): phenomenon related to MFB
 - ✓ Models and correlations for the MFB temperature T_{MFB}
 - Not very accurate or generally applicable
 - Particularly true for transient boiling processes where MFB may represent the position of the "quench front."
 - ✓ Zuber (1959): hydrodynamic model for MFB
 - Improved by Berenson (1961)
 - the effect of heated surface properties is not considered.
 - ✓ Leidenfrost temperature correlation (Baumeister and Simon, 1973)

$$T_{\rm MFB} = \frac{27}{32} T_{\rm cr} \left\{ 1 - \exp\left[-0.52 \left(\frac{10^4 (\rho_{\rm w}/A_{\rm w})^{4/3}}{\sigma} \right)^{1/3} \right] \right\}$$

- A_w and ρ_w : the atomic number and density of the heated surface
- *T_{cr}*: the critical temperature of the fluid
- σ : the liquid– vapor surface tension (in dynes per centimeter)

- Minimum film boiling (MFB)
 - ✓ Zuber model (1959)
 - MFB is driven by the Taylor instability
 - Bubbles are formed on a 2D grid



- Bubbles grow as a result of the growth of Taylor waves
 - The growth rate of the Taylor wave nodes corresponds to the fastest growing wavelength in the Taylor instability.
- Bubble release takes place when the peak rises to a height of $\lambda_d/2$.
- The released bubble is a sphere with a radius of $\lambda_d/4$.
- MFB heat flux

$$q_{\rm MFB}'' = f \frac{2}{\lambda_{\rm d}^2} \frac{4\pi}{3} \left(\frac{\lambda_{\rm d}}{4}\right)^3 \rho_{\rm g} h_{\rm fg} = \frac{\pi}{24} \lambda_{\rm d} \rho_{\rm g} h_{\rm fg} f. \qquad q_{\rm CHF}'' = \rho_{\rm g} h_{\rm fg} U_{\rm g} \frac{\pi R_{\rm j}^2}{\lambda_{\rm d}^2} \quad q_{\rm CHF,Z}'' \approx \frac{\pi}{24} \rho_{\rm g}^{1/2} h_{\rm fg} (\sigma g \Delta \rho)^{1/4} d\rho_{\rm g} h_{\rm fg} f.$$

 $R_{\rm i} = \lambda_{\rm d}/4$

 The factor of 2 is based on the argument that in each complete cycle two bubbles are released from a unit cell.





- Minimum film boiling (MFB)
 - ✓ Zuber model (1959)

$$q_{\rm MFB}'' = f \frac{2}{\lambda_{\rm d}^2} \frac{4\pi}{3} \left(\frac{\lambda_{\rm d}}{4}\right)^3 \rho_{\rm g} h_{\rm fg} = \frac{\pi}{24} \lambda_{\rm d} \rho_{\rm g} h_{\rm fg} f_{\rm s}$$

- Bubble release frequency $f = (\overline{d\zeta/dt})/\lambda_d$
- Growth rate of the wave displacement
- Wave displacement $\zeta = \zeta_0 \exp[i(\omega t kx)]_d$

• Then,
$$(\overline{d\zeta/dt}) = 0.2\omega_{\rm d}\lambda_{\rm d}$$

$$f = 0.2\omega_{\rm d} = 0.2 \left[\frac{4(\Delta \rho)^3 g^3}{27\sigma (\rho_{\rm f} + \rho_{\rm g})^2} \right]^{0.25}$$

$$q_{\rm MFB}'' = C_1 \rho_{\rm v} h_{\rm fg} \left[\frac{\sigma g \Delta \rho}{\left(\rho_{\rm f} + \rho_{\rm g}\right)^2} \right]^{1/4}$$







- Minimum film boiling (MFB)
 - ✓ Berenson (1961)

•
$$C_1 = 0.091$$
, $h_{fg} \rightarrow h'_{fg}$ $h'_{fg} = h_{fg} \left[1 + 0.34 \frac{C_{Pv}(T_w - T_{sat})}{h_{fg}} \right]$

$$q_{\rm MFB}'' = C_1 \rho_{\rm v} h_{\rm fg} \left[\frac{\sigma g \Delta \rho}{\left(\rho_{\rm f} + \rho_{\rm g}\right)^2} \right]^{1/4} \qquad \clubsuit \qquad H = 0.425 \left[\frac{k_{\rm v}^3 \rho_{\rm v} \Delta \rho g h_{\rm fg}'}{\mu_{\rm v} (T_{\rm w} - T_{\rm sat}) \sqrt{\sigma/(g \Delta \rho)}} \right]^{0.25}$$

Minimum film boiling temperature

 $(T_{\rm MFB} - T_{\rm sat})_{\rm Berenson} = (q_{\rm MFB}''/H_{\rm FB})_{\rm Berenson}$

- No surface effect

✓ Henry (1974)

$$\frac{T_{\rm MFB} - T_{\rm MFB}^{*}}{T_{\rm MFB}^{*} - T_{\rm L}} = 0.42 \left[\sqrt{\frac{(\rho Ck)_{\rm f}}{(\rho Ck)_{\rm w}}} \frac{h_{\rm fg}}{C_{\rm w}(T_{\rm MFB}^{*} - T_{\rm sat})} \right]^{0.6}$$

• T^*_{MFB} : MFB temperature by Berenson's correlation

Minimum film boiling (MFB)

EXAMPLE 11.1. A horizontal circular disk that is 10 cm in diameter is submerged in a shallow pool of quiescent water that is at 95 °C. Calculate the size range of active nucleation sites for $T_w = 109$ °C, assuming that the contact angle is 50°.

EXAMPLE 11.4. Calculate the minimum film boiling heat flux and temperature for the conditions of Example 11.1. Assume that the disk is made of stainless steel.

SOLUTION. The saturation properties that are needed are $\rho_f = 958.4 \text{ kg/m}^3$, $\rho_g = 0.597 \text{ kg/m}^3$, $h_{fg} = 2.337 \times 10^6 \text{ J/kg}$, $C_{Pg} = 1,987 \text{ J/kg} \cdot \text{K}$, $\sigma = 0.059 \text{ N/m}$, $T_{\text{sat}} = 373 \text{ K}$, $C_{Pf} = 4,217 \text{ J/kg} \cdot \text{K}$ and $\alpha_f = 1.646 \times 10^{-7} \text{ m}^2/\text{s}$.

We need to estimate the mean vapor film properties. Let us use $T_{\rm film} = T_{\rm sat} + 40$ K as an estimation. The following vapor film properties accordingly represent superheated vapor at one atmosphere pressure and 413 K: $k_{\rm v} = 0.0028$ W/m·K, $\mu_{\rm v} = 1.38 \times 10^{-5}$ kg/m·s, $\rho_{\rm v} = 0.537$ kg/m³, and $h'_{\rm fg} = 2.41 \times 10^{6}$ J/kg.

Equation (11.77) is now solved using $C_1 = 0.091$, resulting in

$$q_{\rm MFB}'' = 17,679 \text{ W/m}^2.$$

The film boiling heat transfer coefficient is next calculated by using Berenson's correlation, Eq. (11.62), leading to

$$H_{\text{Berenson}} = 242.7 \text{ W/m}^2 \cdot \text{K}$$

We can now write

$$T_{\text{MFB,Berenson}} - T_{\text{sat}} = q''_{\text{MFB}} / H_{\text{Berenson}} = 73 \text{ K} \Rightarrow T_{\text{MFB,Berenson}} = 446 \text{ K}$$

We now apply the correlation of Henry (1974), Eq. (11.79), noting that $T_{\text{MFB}}^* = 446$ K, and $T_{\text{L}} = T_{\text{sat}}$. For the solid properties, let us use the properties of AISI 302 stainless steel at 446 K, whereby $\rho_{\text{w}} = 7,998$ kg/m³, $C_{\text{w}} = 523$ J/kg·K, and $k_{\text{w}} = 17.9$ W/m·K. Equation (11.79) then gives $T_{\text{MFB}} = 579$ K.

11.7 Transition boiling

Surface partially in nucleate boiling and partially in film boiling

- ✓ Poorly understood, received relatively little research attention
- \checkmark Industrial systems usually are not designed to operate in this regime
- In nuclear engineering
 - ✓ Reflood phase of a loss-of-coolant accident (LOCA)
- Parametric trends
 - ✓ Surface roughness: moves the transition boiling line toward the left.
 - ✓ Improved wettability: improves the transition boiling heat transfer coefficient.
 - ✓ Transient heating > transient cooling
 - ✓ Contaminants: improves heat transfer





11.7 Transition boiling

Bjonard and Griffith (1977)

✓ Interpolation between CHF and MFB points

 $q_{\rm TB}^{\prime\prime}(T_{\rm w}) = Cq_{\rm CHF}^{\prime\prime} + (1-C)q_{\rm MFB}^{\prime\prime}$

$$C = \left(\frac{T_{\rm MFB} - T_{\rm w}}{T_{\rm MFB} - T_{\rm CHF}}\right)^2$$

Haramura (1999)

 $\frac{\ln[q_{\rm TB}^{\prime\prime}(T_{\rm w})/q_{\rm MFB}^{\prime\prime}]}{\ln(q_{\rm CHF}^{\prime\prime}/q_{\rm MFB}^{\prime\prime})} = \frac{\ln[\Delta T_{\rm MFB}/(T_{\rm w}-T_{\rm sat})]}{\ln(\Delta T_{\rm MFB}/\Delta T_{\rm CHF})}$

 $\Delta T_{\rm MFB} = T_{\rm MFB} - T_{\rm sat}$ and $\Delta T_{\rm CHF} = T_{\rm CHF} - T_{\rm sat}$