458.401 Process & Product Design

08

Engineering Economic Analysis

Time Value of Money (TVM)

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Definitions

- P - Principal or Present Value (of an investment); 투자시점 가치
- F_n - Future Value (of an investment); 미래가치
- n - Years (or other time units) between P and F
- i - Interest Rate (based on time interval of n) per annum

Basic premise: Money when invested earns money

$1 today is worth more than $1 in the future

Interest

- Simple Interest - Annual Basis
  - Interest paid in any year = Pi_s
    - Pi_s - Fraction of investment paid as interest per year
  - After n years, total interest paid = Pi_sn
  - Total investment is worth = $P + Pi_sn$
  - Total investment after 1 year ($n = 1$) = $P(1 + i_s)$
  - What is the drawback of simple interest?
- We can earn interest on earned interest

Compound Interest

- At time 0, we have P
- At the end of Year 1, we have $F_1 = P(1 + i)$
- At the end of Year 2, we have $F_2 = P(1 + i)^2$
- … … …
- At the end of Year n, we have $F_n = P(1 + i)^n$ or $P = F_n/(1 + i)^n$

Example-01

How much would I need to invest at 8% p.a. to yield $5,000 in 10 years?

Solution-01

$$i = 0.08,\ n = 10,\ F_{10} = 5,000$$

What if interest rate changes with time?

$$F_n = P\prod_{j=1}^{n}(1+i_j) = P(1+i_1)(1+i_2)\cdots(1+i_n)$$

Different Time Basis for Interest Calculations

- Relates to statement “Your loan is 6% p.a., compounded monthly”
- Define actual interest rate per compounding period as r
  - i_{nom} = Nominal annual interest rate (명목이율)
  - *m* = Number of compounding periods per year (일년에 몇 번 이자 계산을 하는가. 월별 12, 분기별 4)

Different Time Basis for Interest Calculations

- i_{eff} = effective annual interest rate

$$r = \frac{i_{nom}}{m}$$

- Look at condition after 1 year

$$F_1 = P(1 + i_{eff}) = P\left(1 + \frac{i_{nom}}{m}\right)^m$$

$$\boxed{i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m - 1}$$

EFFECT(inom%,m)

Example-02

I invest $1,000 at 10% p.a. compounded monthly. How much do I have in 1 year, 10 years?

Example 2 Cont.

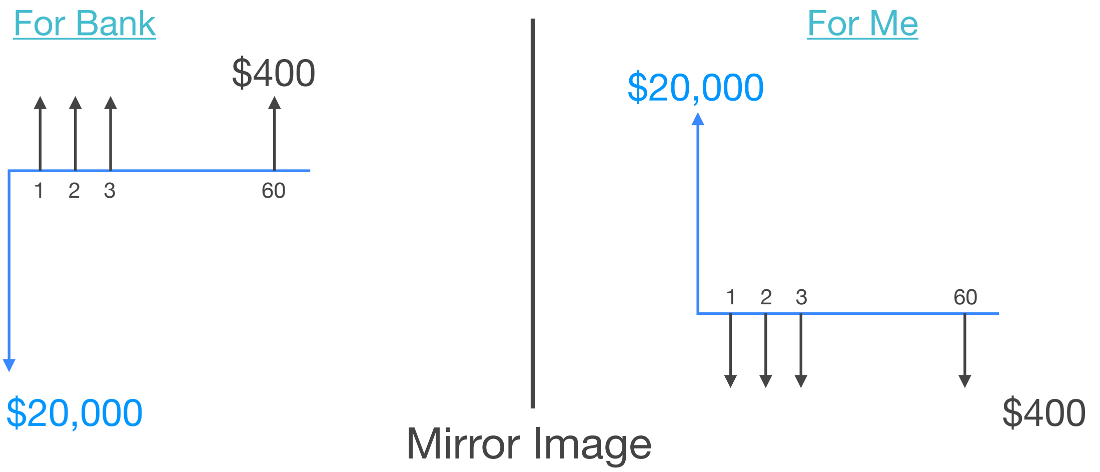
- As *m* increases i_{eff} increases
- Is there a limit as *m* goes to infinity
  - Yes. Continuously compounded interest
  - Derivation: pp. 265-266
  - i_{eff} (continuous) = $e^{i_{nom}} - 1$

Cash Flow Diagram (CFD)

- Represent timings and approximate magnitude of investment on a cad
  - x-axis is time and y-axis is magnitude
  - both positive and negative investments are possible
- In order to determine direction (sign) of cash flows, we must define what system is being considered.
- Disbursements = Outflow of money, “—”
- Receipts = Inflow of money, “+”
- Beginning of the first year: Traditionally defined as “Time 0”

Consider a Discrete Cash Flow Diagram

- Discrete refers to individual CFDs that are plotted
- Example
  - I borrow $20K for a car and repay as a $400 monthly payment for 5 years.


For Bank

$400

1 2 3 60

$20,000

For Me

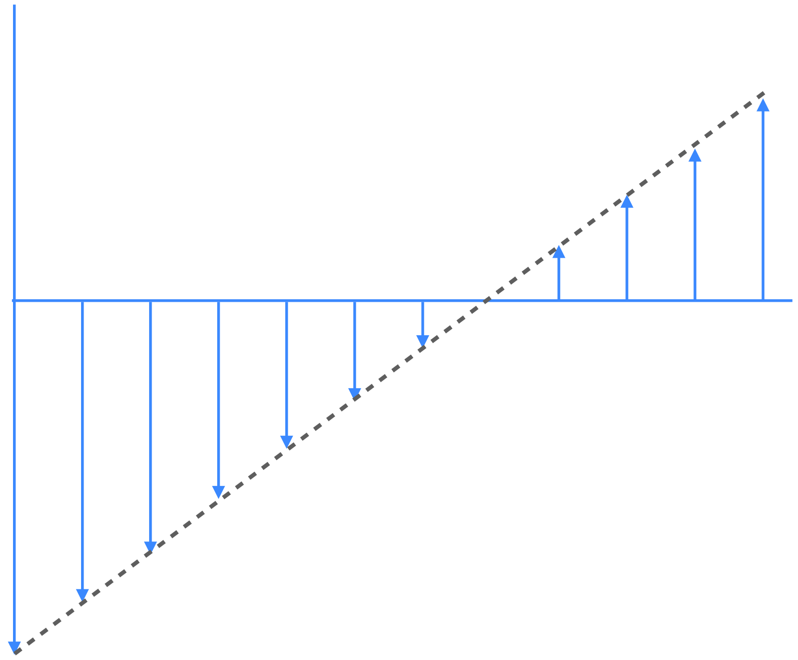
$20,000

1 2 3 60

$400

Mirror Image

Cumulative CFD



Equivalence

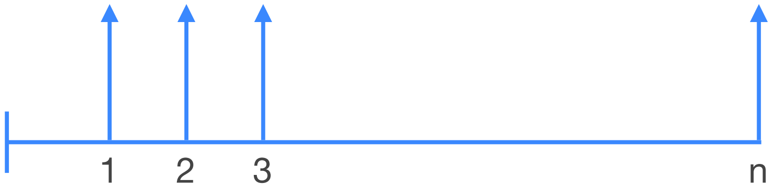
Translating cash-flows over time into common units;

1. Present values of future payments
2. Future value of present payments
3. Present value of continuous uniform payments
4. Continuous payments equivalent to present payment

Interests, Type and Interconversion

- Simple Interest vs. Compounding Interest
- Single Payment Factor (F/P and P/F)
- Standard Factor Notation (F/P, i, n)
  - 이자율이 i이고 기간이 n일 때 P를 F로 바꿈.
- Excel Functions, **FV(i%,n,,P), PV(i%,n,,F)**
- Uniform Series (Annuity): P/A, A/P

Annuities (연금)



Uniform series of equally spaced, equal value cash flows

Note: The first payment is at the beginning of year 1 not at t = 0

Annuities

What is future value F_n?

$$F_n = A(1+i)^{n-1} + A(1+i)^{n-2} + \cdots + A$$

Geometric progression

$$F_n = S_n = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Discount Factors (P/A)

Just a shorthand symbol for a formula in *i* and *n*

$$P = \frac{F}{(1+i)^n} \quad \Rightarrow \quad \left(\frac{P}{F}, i, n\right) = \frac{1}{(1+i)^n}$$

$$P = F\left(\frac{P}{F}, i, n\right) = F\left(\frac{1}{(1+i)^n}\right)$$

See Table 9.1

$$\Rightarrow \quad A \;\rightarrow\; P \;\Rightarrow\; \left(\frac{P}{A}, i, n\right) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

Discount Factors

$$P/A = F/A \times P/F$$

$$\left(\frac{F}{A}, i, n\right) = \frac{(1+i)^n - 1}{i}$$

$$\left(\frac{P}{F}, i, n\right) = \frac{1}{(1+i)^n}$$

therefore

$$\left(\frac{P}{A}, i, n\right) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

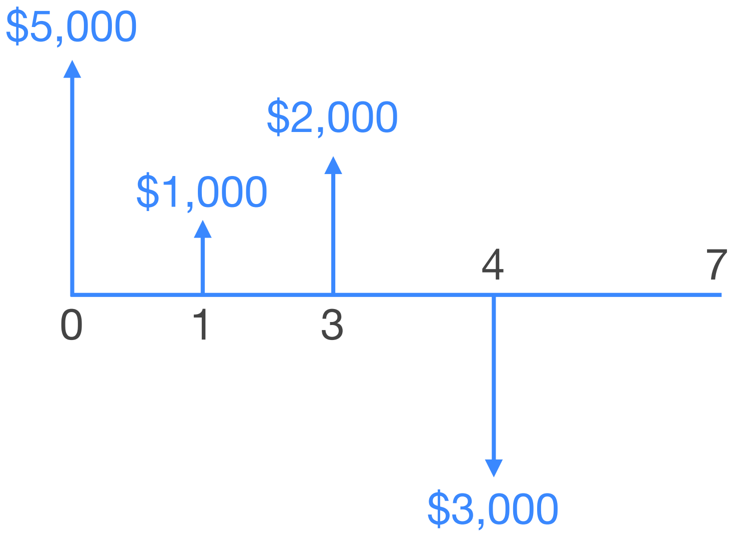
Table 9.1 has six versions of these

Three are reciprocals of the other three

Commonly Used Factors for Cash Flow Diagram Calculations

Conversion	Symbol	Common Name	Formula
P to F	$(F/P, i, n)$	Single Payment Compound Amount Factor	$(1+i)^n$
F to P	$(P/F, i, n)$	Single Payment Present Worth Factor	$\frac{1}{(1+i)^n}$
A to F	$(F/A, i, n)$	Uniform Series Compound Amount Factor, Future Worth of Annuity	$\frac{(1+i)^n - 1}{i}$
F to A	$(A/F, i, n)$	Sinking Fund Factor	$\frac{i}{(1+i)^n - 1}$
P to A	$(A/P, i, n)$	Capital Recovery Factor (CRF)	$\frac{i(1+i)^n}{(1+i)^n - 1}$
A to P	$(P/A, i, n)$	Uniform Series Present Worth Factor, Present Worth of Annuity	$\frac{(1+i)^n - 1}{i(1+i)^n}$

Calculations with Cash Flow Diagrams



- Invest 5K, 1K, 2K at end of Years 0, 1, 3 and take 3K at end of Year 4
- Note that annuity payments are all at the end of the year

Example-03

How much in account at end of Year 7 if *i* = 8% p.a.?

Solution-03

$$F_7 = \$9,097.84$$

Example-04

What would investment be at Year 0 to get this amount at Year 7?

Solution-04

Example-05

What should my annual monthly car payment be if interest rate is 8% p.a. compounded monthly? Compare at n = 60

Solution-05

PMT(i%,m,P)

$A = \$405.53$ Interest paid = $4,331.80

Example-06

You buy a house where you finance $200K at 6% p.a. interest, compounded monthly. What is your monthly payment, and how much interest do you pay over the lifetime of the loan for a 15-year and a 30-year mortgage in current dollars?

Solution-06

For a 15-year mortgage

$1687.71/month, total of $303,788 paid, $103,788 interest

For a 30-year mortgage

$1199.10/month, total of $431,676 paid, $231,676 interest

Example-07

You invest $5,000/year (the maximum, for now) in a Roth IRA, starting at age 25 for 40 years. Assuming a return of 8% p.a., how much will you have at age 65 in future dollars?

Example-08

Repeat the previous calculation, assuming that you do not start investing until age 35 or age 45.

Depreciation

- Total Capital Investment = Fixed Capital + Working Capital
  - Fixed Capital: All costs associated with new construction, but land cannot be depreciated
  - Working Capital: Float of material to start operations cannot depreciate
- Definitions
  - Salvage Value, *S*
    - Value of FCI_L at end of project
    - Often = 0
  - Life of Equipment
    - n: set by IRS
    - Not related to actual equipment life
  - Total Capital for Depreciation
    - $FCI_L - S$

S는 공장의 수명이 다한 후 (예, 30년)의 잔존가치가 아닌 회계 처리 규정 (미국 IRS)에 의한 장부상의 잔존가치(book value)이다.

FCI_L = TIC - WC - Land

4 Basic Methods for Depreciation

- Straight Line
- Sum of Years Digits (SOYD)
- Double Declining Balance (DDB)
- Modified Accelerated Cost Recovery System (MACRS)

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① Straight Line

$$d_k^{SL} = \frac{FCI_L - S}{n}$$

n = # of years over which depreciation is taken

② Sum of Years Digits (SOYD)

$$d_k^{SOYD} = \frac{[(n+1-k)(FCI_L - S)]}{\boxed{\frac{1}{2}n(n+1)}}$$

SOYD

③ Double Declining Balance (DDB)

$$d_k^{DDB} = \frac{\textcircled{2}}{n} \underbrace{\left[FCI_L - \sum_{j=0}^{k-1} d_j \right]}_{\text{Book Value at k-1}}$$

④ MACRS

Current IRS-approved method

Year	Depreciation Percentage
1	20.00
2	32.00
3	19.20
4	11.52
5	11.52
6	5.76

See Chapter 9

Based on combination of DDB and SL

Example-09

The fixed capital investment (excluding the cost of land) of a new project is estimated to be \$150.0 million, and the salvage value of the plant is \$10.0 million. Assuming a seven-year equipment life, estimate the yearly depreciation allowances.

Solution-05

$FCI_L = \$150 \times 10^6$ $\quad S = \$10.0 \times 10^6$ $\quad n = 7$

year 2

$$d_2^{SL} = (\$150 \times 10^6 - \$10 \times 10^6)/7 = \$20 \times 10^6$$

$$d_2^{SOYD} = (7 + 1 - 2)(\$150 \times 10^6 - \$10 \times 10^6)/28 = \$30 \times 10^6$$

$$d_2^{DDB} = (2/7)(\$150 \times 10^6 - \$42.86 \times 10^6) = \$30.6 \times 10^6$$

Table E9.21

Year (k)	d_k^{SL}	d_k^{SOYD}	d_k^{DDB}	Book Value $FCI_L - Sd_k^{DDB}$
0				(15 - 0) = 15
1	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-1)(15-1)}{28} = 3.5$	$\frac{(2)(15)}{7} = 4.29$	(15 - 4.29) = 10.71
2	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-2)(15-1)}{28} = 3.0$	$\frac{(2)(10.71)}{7} = 3.06$	(10.71 - 3.06) = 7.65
3	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-3)(15-1)}{28} = 2.5$	$\frac{(2)(7.65)}{7} = 2.19$	(7.65 - 2.19) = 5.46
4	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-4)(15-1)}{28} = 2.0$	$\frac{(2)(5.46)}{7} = 1.56$	(5.46 - 1.56) = 3.90
5	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-5)(15-1)}{28} = 1.5$	$\frac{(2)(3.90)}{7} = 1.11$	(3.90 - 1.11) = 2.79
6	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-6)(15-1)}{28} = 1.0$	$\frac{(2)(2.79)}{7} = 0.80$	(2.79 - 0.80) = 1.99
7	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-7)(15-1)}{28} = 0.5$	1.99 - 1.0 = 0.99	(1.99 - 0.99) = 1.00
Total	14.0	14.0	14.0	1.0 = S

The depreciation allowance in the final year of the DDB method is adjusted to give a final book value equal to the salvage value

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MACRS

- uses a double declining balance method and
- switches to a straight-line method when the straight line method yields a greater depreciation allowance for that year, then using a half-a year convention
- In the first year, the depreciation is only half of that for full year. Likewise, in the last year of depreciation, the depreciation is again for one-half-year convention

MACRS: Basic Methods

time remaining

k	dk (DDB)	dk (SL)
1	0.4(100)(0.5) = 20	
2	0.4(100-20) = 32	(100-20)/4.5 = 17.78
3	0.4(100-52) = 19.2	(100-52)/3.5 = 13.71
4	0.4(100-71.2) = 11.52	(100-71.2)/2.5 = 11.52
5	0.4(100-82.72) = 6.91	(100-82.72)/1.5 = 11.52
6		(0.5)(100-94.24)/0.5 = 5.76

Taxation, Cash Flow, and Profit

- Tables 9.3 - 9.4
- Sales Revenue = R
- Expenses = $COM_d + d_k$
- Income Tax = $(R - COM_d - d_k)t$
- After Tax (net) Profit = $(R - COM_d - d_k)(1 - t)$; 세후수익 (감가상각은 세금 대상이 아님)
- After Tax Cash Flow = After-Tax Profit + Depreciation = $(R - COM_d - d_k)(1 - t) + d_k$ (회계상 감가상각은 세금 낸 후 돌아옴)
- Cash Flow at the end of project life = Above + Recovery of WC + Salvage Value (Working Capital은 운영하기 위한 초기 비용이므로 회수)

Inflation

$ Net Worth Now vs. $ Next Year

$$\boxed{\mathrm{CEPCI}(j+n) = (1+f)^n \mathrm{CEPCI}(j)}$$

f = average inflation rate between years j and n

Example

CEPCI (1993) = 359 CEPCI (2003) = 402

$$(1+f)^{10} = \frac{402}{359} \qquad f = \left(\frac{402}{359}\right)^{0.1} - 1 = 0.0114 \text{ or } 1.14\%$$

Inflation

- Effect of inflation on interest rate
  f affects the purchasing power of the $
- Look at the purchasing power of future worth, then

$$F' = \frac{F}{(1+f)^n}$$

- If this future worth was obtained by investing at a rate i, then the inflation adjusted interest rate, i’ is given by

$$F' = \frac{F}{(1+f)^n} = P\frac{(1+i)^n}{(1+f)^n} = P\left(\frac{1+i}{1+f}\right)^n = P(1+i')^n$$

$$\boxed{i' = \frac{1+i}{1+f} - 1 \approx i - f}$$