SEOUL NATIONAL UNIVERSITY

Nuclear Thermal Hydraulic Engineering Lab.

원자로 열유체 실험

Uncertainty Analysis for Measurement Parameters

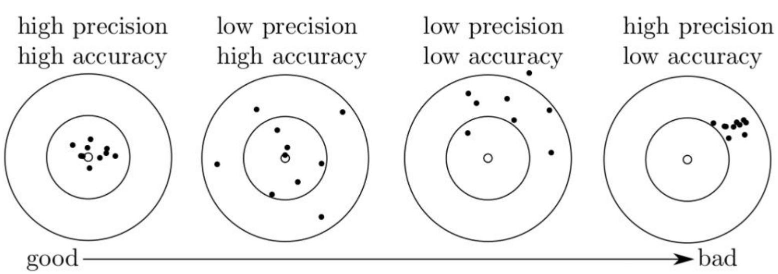
Department of Nuclear Engineering, Seoul National Univ.
Hyoung Kyu Cho

Basic Definitions

- **Accuracy : closeness of agreement between measured and true value**
- **Error : difference between measured and true value**
  - ✓ **Systematic error : bias of data to the true value**
  - ✓ **Random error : precision error**
    - Precision : a measure of the agreement replicate measurements
- **Uncertainty (U) : estimate of errors in measurements of individual variables $X_i(U_{xi})$ of results (U_r)**
  - ✓ **Estimate of U at a certain (eg. 95%) confidence level, on large data samples (at least 10 measurements)**

	Results A	Results B	Results C
Accuracy	Low	High	Low
Precision	High	High	Low
Systematic error	High	None	No
Random Error	No	None	High

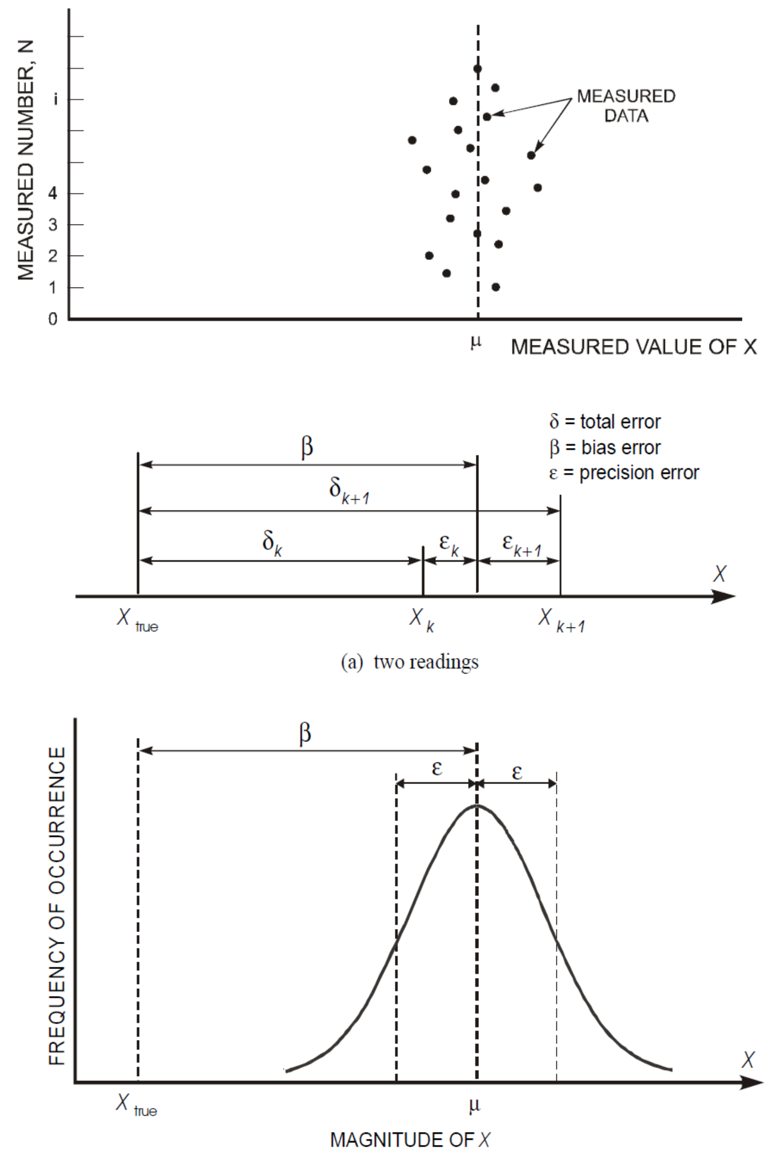
Image source: Bishop, M. *Precision vs Accuracy (Fig 1.12)*. From *Preparatory Chemistry* (ebook) via http://preparatorychemistry.com/Bishop_Book_1_eBook.pdf



Basic Definitions

- **Bias Error :** β
  - Fixed, systematic, constant component of total error
- **Precision Error :** ε
  - Randomness, repeatability
- **Total Error :** δ

$$\delta = \beta + \varepsilon$$



(a) two readings

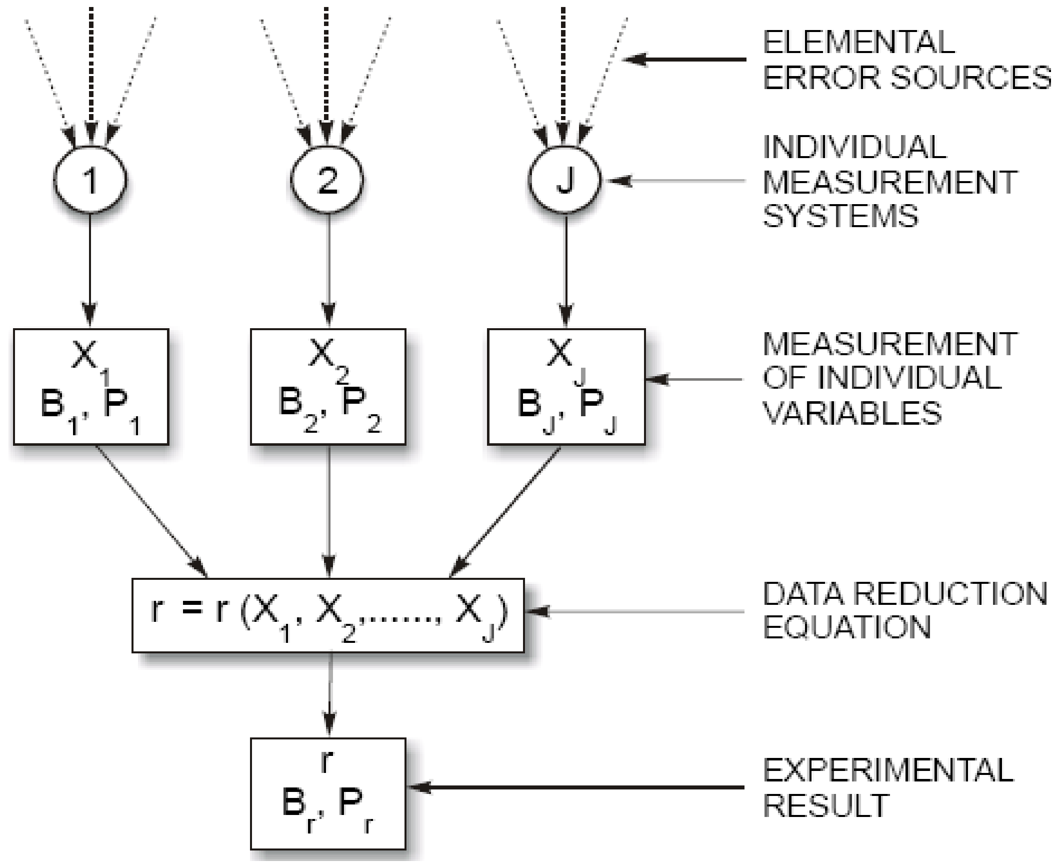
(b) infinite number of readings

Theory of Uncertainty Analysis

❖ Measurement system for individual variable X_i [1]

- ✓ **Instrumentation system, DAS, reduction procedures and so on.**
- ✓ **Data reduction equations**
  - $r = r(X_1, X_2 \ldots\ldots X_{j,})$
  - Example : Pitot tube

$$V = \sqrt{2\Delta p / \rho}$$



Theory of Uncertainty Analysis

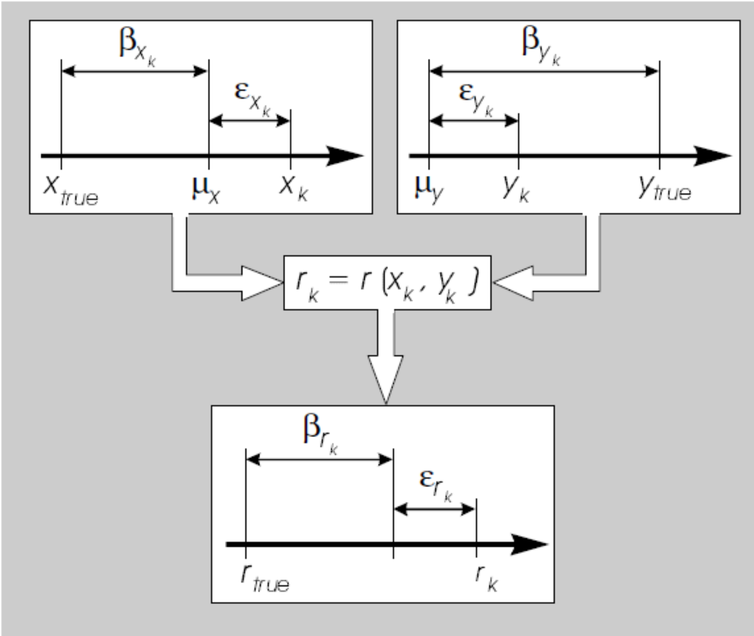
❖ Uncertainty propagation of two measured variables

✓ Two measured variables

$$r = r(x, y)$$

$$x_k = x_{true} + \beta_{x_k} + \varepsilon_{x_k}$$

$$y_k = y_{true} + \beta_{y_k} + \varepsilon_{y_k}$$



✓ Taylor expansion of r

$$r_k - r_{true} = \frac{\partial r}{\partial x}(x_k - x_{true}) + \frac{\partial r}{\partial y}(y_k - y_{true}) + R_2$$

✓ Total error of variable of r

$$\delta_{r_k} = r_k - r_{true} = \theta_x(\beta_{x_k} + \varepsilon_{x_k}) + \theta_y(\beta_{y_k} + \varepsilon_{y_k})$$

$$\theta_x = \partial r / \partial x \qquad \theta_y = \partial r / \partial y$$

Theory of Uncertainty Analysis

❍ N 개 데이터 취득 시 분산

$$\sigma^2_{\delta_r} = \lim_{N\to\infty}\left[\frac{1}{N}\sum_{k=1}^{N}\left(\delta_{r_k}\right)^2\right] \qquad (2)$$

$$\delta_{r_k} = r_k - r_{true} = \theta_x\left(\beta_{x_k} + \varepsilon_{x_k}\right) + \theta_y\left(\beta_{y_k} + \varepsilon_{y_k}\right) \qquad (1)$$

❍ eq.(1) → eq.(2)

- N: 무한대, x, y 변수의 precision error, bias error가 상호 독립적

$$\sigma^2_{\delta_r} = \lim_{N\to\infty}\left[\frac{1}{N}\sum_{k=1}^{N}\left[\theta_x\left(\beta_{x_k} + \varepsilon_{x_k}\right) + \theta_y\left(\beta_{y_k} + \varepsilon_{y_k}\right)\right]^2\right]$$

$$= \lim_{N\to\infty}\left[\frac{1}{N}\sum_{k=1}^{N}\left[\theta_x^2\left(\beta_{x_k} + \varepsilon_{x_k}\right)^2 + \theta_y^2\left(\beta_{y_k} + \varepsilon_{y_k}\right)^2 + 2\theta_x\theta_y\left(\beta_{x_k} + \varepsilon_{x_k}\right)\left(\beta_{y_k} + \varepsilon_{y_k}\right)\right]\right]$$

$$= \lim_{N\to\infty}\left[\frac{1}{N}\sum_{k=1}^{N}\left[\theta_x^2\left(\beta_{x_k}^2 + \cancelto{0}{2\beta_{x_k}\varepsilon_{x_k}} + \varepsilon_{x_k}^2\right) + \theta_y^2\left(\beta_{y_k}^2 + \cancelto{0}{2\beta_{y_k}\varepsilon_{y_k}} + \varepsilon_{y_k}^2\right)\right.\right.$$
$$\left.\left. + 2\theta_x\theta_y\left(\beta_{x_k}\beta_{y_k} + \cancelto{0}{\varepsilon_{x_k}\beta_{y_k}} + \cancelto{0}{\varepsilon_{y_k}\beta_{x_k}} + \varepsilon_{x_k}\varepsilon_{y_k}\right)\right]\right]$$

$$\sigma^2_{\delta_r} = \theta_x^2\sigma^2_{\beta_x} + \theta_y^2\sigma^2_{\beta_y} + 2\theta_x\theta_y\sigma_{\beta_x\beta_y} + \theta_x^2\sigma^2_{\varepsilon_x} + \theta_y^2\sigma^2_{\varepsilon_x} + 2\theta_x\theta_y\sigma_{\varepsilon_x\varepsilon_y} \qquad (3)$$

Theory of Uncertainty Analysis

$$\sigma_{\delta_r}^2 = \theta_x^2\sigma_{\beta_x}^2 + \theta_y^2\sigma_{\beta_y}^2 + 2\theta_x\theta_y\sigma_{\beta_x\beta_y} + \theta_x^2\sigma_{\varepsilon_x}^2 + \theta_y^2\sigma_{\varepsilon_x}^2 + 2\theta_x\theta_y\sigma_{\varepsilon_x\varepsilon_y} \qquad (3)$$

$$\theta_x = \partial r/\partial x \qquad \theta_y = \partial r/\partial y$$

- σ : 정답을 모르기에 알 수 없는 값. 추정 필요.
- u^2_c : $\sigma^2_{\delta r}$ 추정치
- b^2_x , b^2_y , b_{xy} : Bias error 추정치
  - S^2_x , S^2_y , S_{xy} : Precision error 추정치

$$u_c^2 = \theta_x^2 b_x^2 + \theta_y^2 b_y^2 + 2\theta_x\theta_y b_{xy} + \theta_x^2 S_x^2 + \theta_y^2 S_y^2 + 2\theta_x\theta_y S_{xy}$$

- 95% 신뢰도의 불확실도 U_r

$U_r = Ku_c$ K: 포함인자, coverage factor

  - K : 오차 분포에 따라 다른 값을 가짐
  - 충분한 데이터 측정 시 (N>10), 95% 신뢰도일 때 $K = 2$

Theory of Uncertainty Analysis

$$u_c^2 = \theta_x^2 b_x^2 + \theta_y^2 b_y^2 + 2\theta_x \theta_y b_{xy} + \theta_x^2 S_x^2 + \theta_y^2 S_y^2 + 2\theta_x \theta_y S_{xy}$$

$$U_r = K u_c$$

❐ 관심 변수가 J 개의 측정 변수에 영향을 받을 경우

$$U_r^2 = \sum_{i=1}^{J} \theta_i^2 B_i^2 + 2\sum_{i=1}^{J-1} \sum_{k=i+1}^{J} \theta_i \theta_k B_{ik} + \sum_{i=1}^{J} \theta_i^2 P_i^2 + 2\sum_{i=1}^{J-1} \sum_{k=i+1}^{J} \theta_i \theta_k P_{ik}$$

$\theta_i = \dfrac{\partial r}{\partial X_i}$, $B_i = tb_i$, $B_{ik} = t^2 b_{ik}$, $P_i = tS_i$, $P_{ik} = t^2 S_{ik}$ and $t = 2$ for $N \geq 10$.

$$U_r^2 = B_r^2 + P_r^2$$

$$B_r^2 = \sum_{i=1}^{J} \theta_i^2 B_i^2 + 2\sum_{i=1}^{J-1} \sum_{k=i+1}^{J} \theta_i \theta_k B_{ik}$$

❐ 두 변수가 독립적인 경우

$$U_r^2 = \sum_{i=1}^{J} \theta_i^2 B_i^2 + \sum_{i=1}^{J} \theta_i^2 P_i^2$$

Theory of Uncertainty Analysis

$$U_r^2 = B_r^2 + P_r^2$$

- Precision error for single and multiple tests
  - Single test
    - 한번만 실험이 진행되는 경우. 실험 시 여러 개의 샘플 측정
    - 예: 긴 시간 동안 많은 수의 측정 데이터를 평균하여 사용
    - Precision limit for single tests

      $$P_r = tS_r$$

      – t: 포함인자, S_r: N 개 데이터의 표준편차
    - r 변수가 J 개의 변수로 도출 될 경우

      $$P_r = \sum_{i=1}^{J} \left(\theta_i P_i\right)^2 \text{ with } P_i = t_i S_i$$

  - Multiple testes
    - Repeat of single tests

$$P_{\bar{r}} = \frac{tS_{\bar{r}}}{\sqrt{M}} \quad \text{with} \quad S_{\bar{r}} = \left[\sum_{k=1}^{M} \frac{(r_k - \bar{r})^2}{M-1}\right]^{1/2}, \quad \bar{r} = \frac{1}{M}\sum_{k=1}^{M} r_k$$

$$P_{\bar{r}} = \sum_{i=1}^{J} \left(\theta_i P_{\bar{i}}\right)^2 \quad \text{with} \quad P_{\bar{i}} = \frac{tS_{\bar{i}}}{\sqrt{M}}, \quad S_{\bar{i}} = \left[\sum_{k=1}^{M} \frac{\left(X_k - \bar{X}_{\bar{i}}\right)^2}{M-1}\right]^{1/2}$$

Example: Heat exchanger

Heat Exchanger

Problem

- Two plastic-encased thermistor probes are to be used to measure the inlet and outlet temperatures of water that flow through a heat exchanger. What is the bias error in the temperature difference ?

$$\Delta T = T_2 - T_1$$

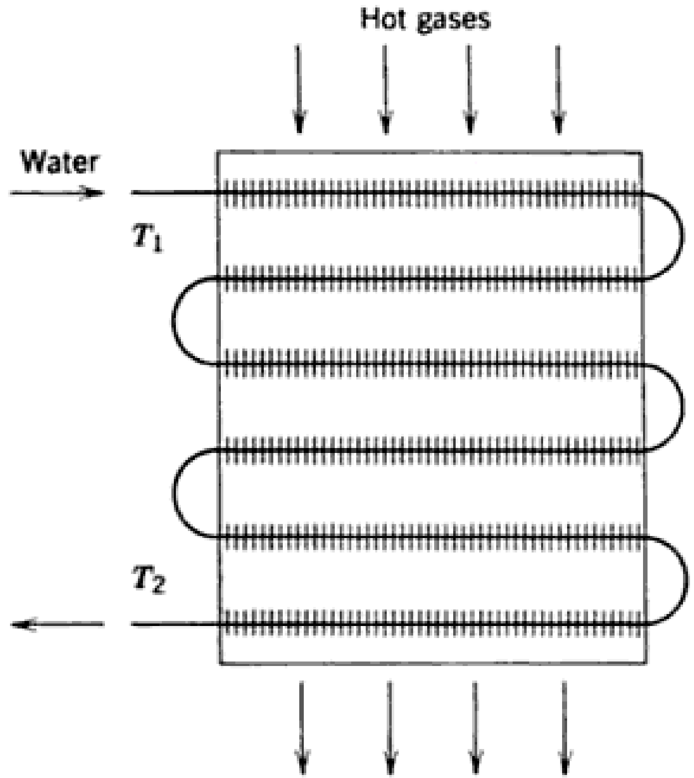


Table : Source for Bias Error

Elemental sources	Bias T$_1$ (^{0}C)	Bias T$_2$ (^{0}C)	*(note)*
As-delivered probe specification	1.0	1.0	*Provide by manufacturer*
Reference thermometer - calibration	0.3	0.3	*Bias of the calibrated thermometer*
Bath non-uniformity - calibration	0.1	0.1	*Non-uniformity of constant temperature bath*
Spatial variation – data acquisition	0.1	0.2	*Measuring position*

Example: Heat exchanger

❍ Uncertainty analysis

✓ **Data reduction and uncertainty analysis equations**

$$\Delta T = T_2 - T_1$$

$$B_{\Delta T}^2 = \theta_{T_1}^2 B_{T_1}^2 + \theta_{T_2}^2 B_{T_2}^2 + 2\theta_{T_1}\theta_{T_2} B_{T_1}' B_{T_2}'$$

$$\theta_{T_1} = \frac{\partial \Delta T}{\partial T_1} = -1 \qquad \theta_{T_2} = \frac{\partial \Delta T}{\partial T_2} = 1$$

✓ **Uncertainty of measured temperature without any calibration for the sensors : source 1&4**

$$B_{T1} = [(1.0)^2 + (0.1)^2]^{1/2} \sim 1.0^o C$$

$$B_{T2} = [(1.0)^2 + (0.2)^2]^{1/2} \sim 1.0^o C$$

✓ **Uncertainty of measured temperature with calibration of the two sensors : source 2,3&4**

$$B_{T1} = [(0.3)^2 + (0.1)^2 + (0.1)^2]^{1/2} \sim 0.33^o C$$

$$B_{T2} = [(0.3)^2 + (0.1)^2 + (0.2)^2]^{1/2} \sim 0.37^o C$$

Example: Heat exchanger

Uncertainty analysis

✓ Total uncertainty

- No calibration

$$B_{\Delta T}^2 = \theta_{T_1}^2 B_{T_1}^2 + \theta_{T_2}^2 B_{T_2}^2 + 2\theta_{T_1}\theta_{T_2} B'_{T_1} B'_{T_2}$$

$$B'_{T_1} = B'_{T_2} = 0$$

$$B_{T_1}^2 = (1.0)^2 + (0.1)^2 = 1.01 \qquad B_{T_1}^2 = (1.0)^2 + (0.2)^2 = 1.04$$

$$B_{\Delta T} = \left[(-1)^2 \times 1.01 + (1)^2 \times 1.04\right]^{1/2} = 1.4\ (^{o}\mathrm{C})$$

- With calibration

$$B_{\Delta T}^2 = (-1)^2 (0.33)^2 + (1)^2 (0.37)^2 + (2)(-1)(1) B'_{T1} B'_{T2}$$

Type	Calibration1	Calibration2	Calibration3
Calibration	-Different time -Different 2 reference Temps	-The same time -The same reference Temps -Different location in bath	-The same time -The same reference Temps -Adjacent location in bath
B'_{T1}, B'_{T2}	$B'_{T1} = B'_{T2} = 0$	$B'_{T1} = B'_{T2} = 0.3^{o}C$ -No bath non-uniformity	$B'_{T1} = B'_{T2}$ $= \left[(0.3)^2 + (0.1)^2\right]^{1/2} = 0.32^{o}C$
$B_{\Delta T}$	$0.5^{o}C$	$0.26^{o}C$	$0.2^{o}C$

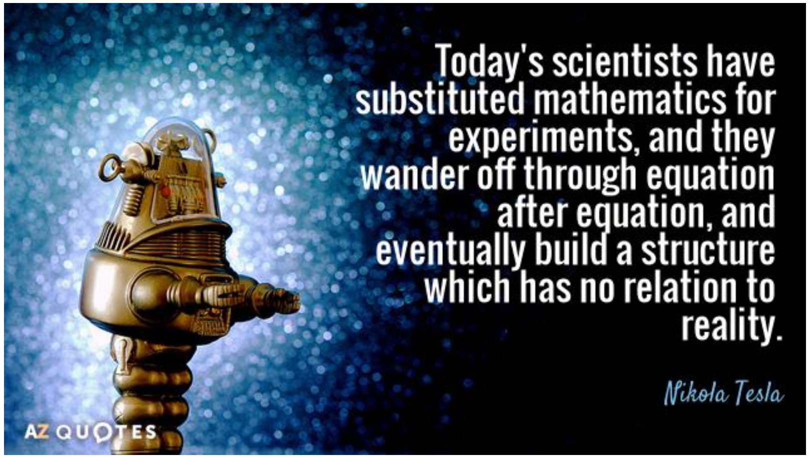
Final exam.

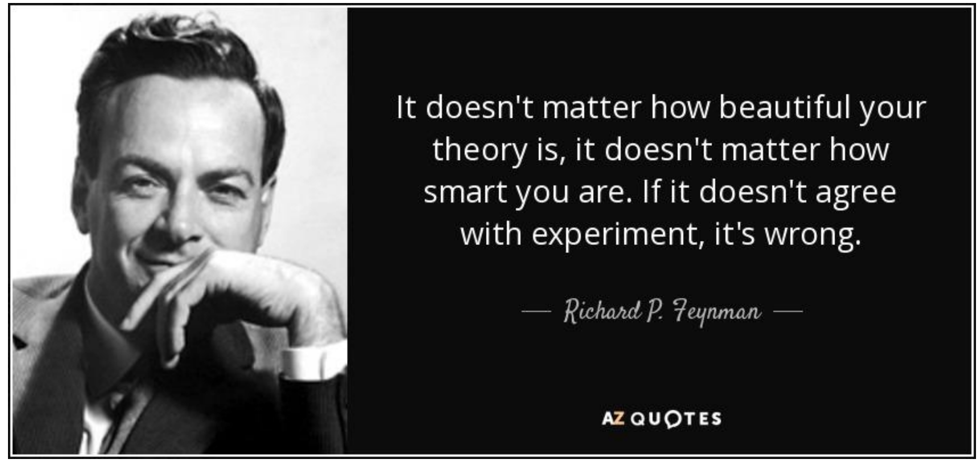
- 6월 18일 (금) 오전 9시~12시
- 장소: 32동 109호

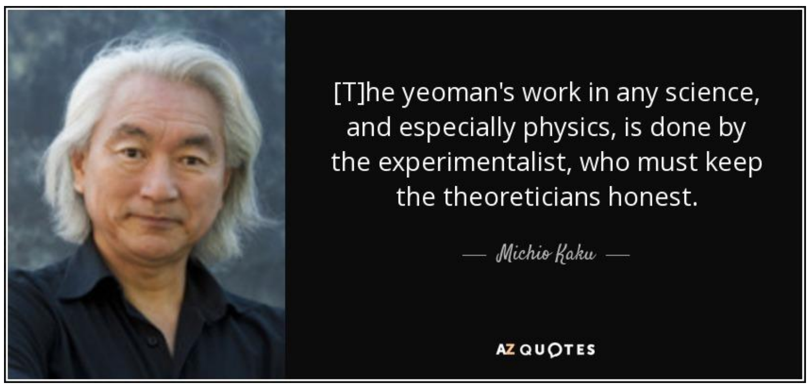
Summary

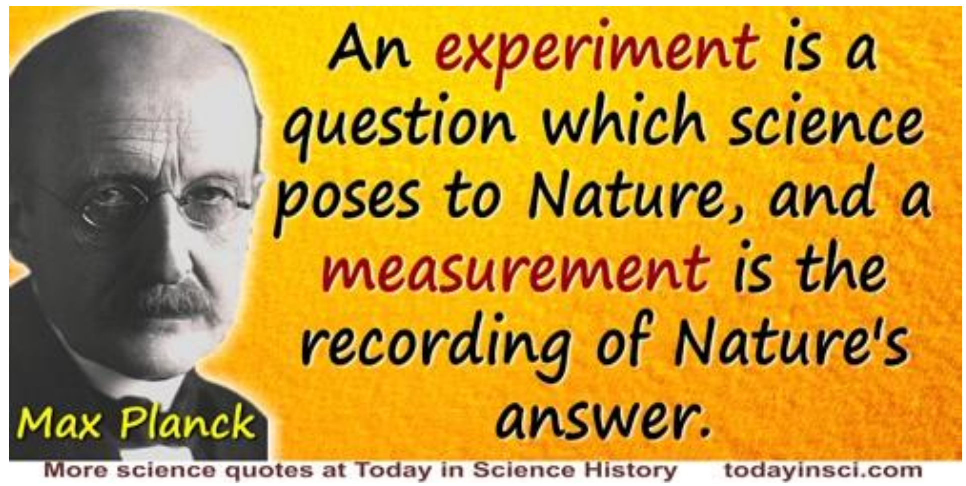
- 원자로 열유체 실험 개요
- Navier-Stokes eq.
- 계측 시스템 구성, 온도 측정: TC, RTD
- 유량계 (차압식, Vortex, 터빈, 전자기, 코리올리, 초음파, 열식)
- 국소 속도 측정: Pitot 관, hot wire/film, LDV, PIV
- 압력 측정, 수위 측정
- SCR, SSR, 인버터, PID 제어, on/off 제어
- 영상처리, 카메라 이론
- 유체기계, 압력강하 계산
- 열수력 기초, 열유체 변수 (열전도도, 건도, 유동양식, 기포율, 계면면적밀도)
- IR 카메라 이론, CHF 이론, 기포 종단속도, ECC bypass
- 불확실도
- 실험전문가 세미나
  - ✓ 종합효과실험, 개별효과실험, 고온실험, 격납건물 열수력실험, SMR 종합효과실험

Summary

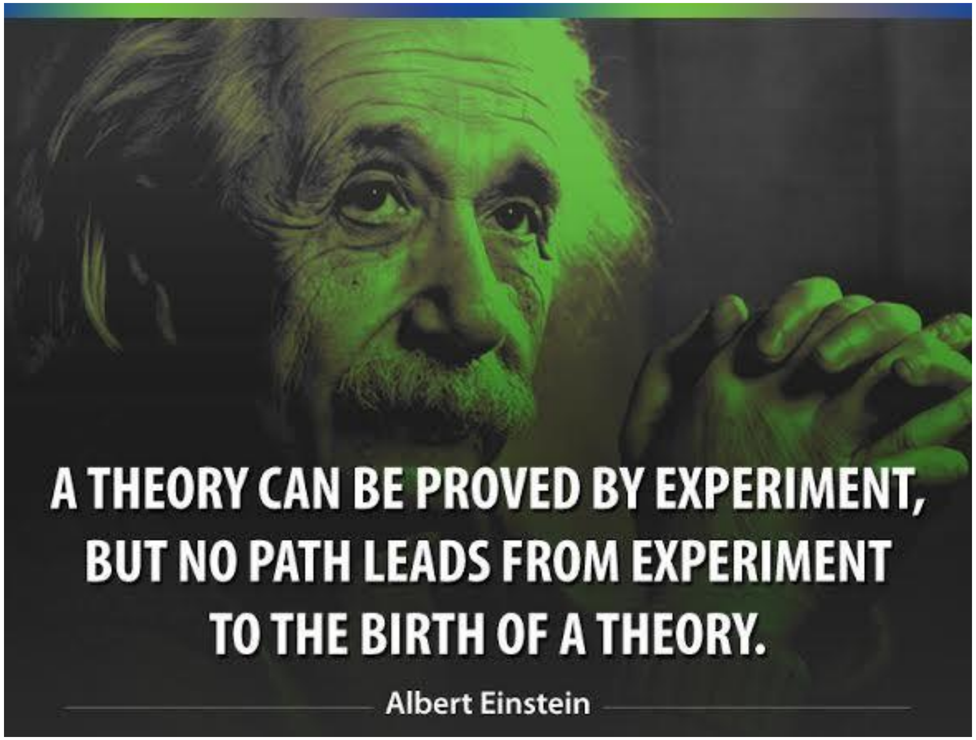

Today's scientists have substituted mathematics for experiments, and they wander off through equation after equation, and eventually build a structure which has no relation to reality.
Nikola Tesla
AZ QUOTES

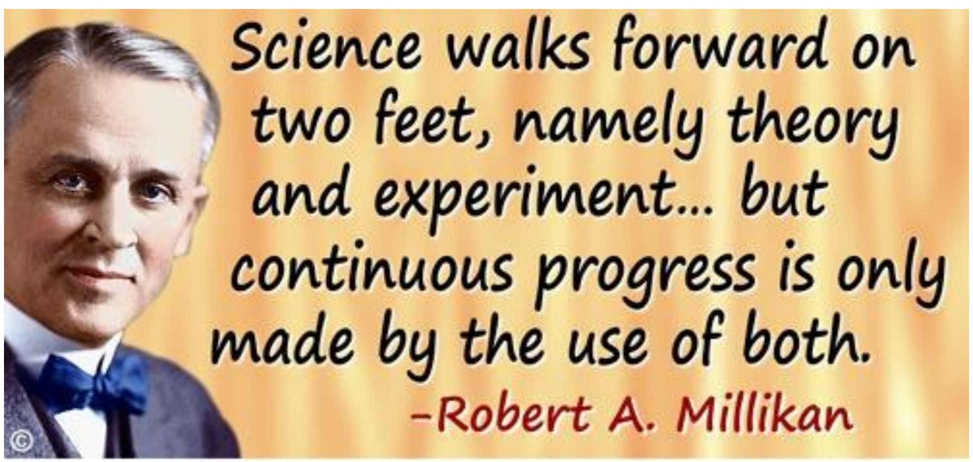

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.
— Richard P. Feynman —
AZ QUOTES

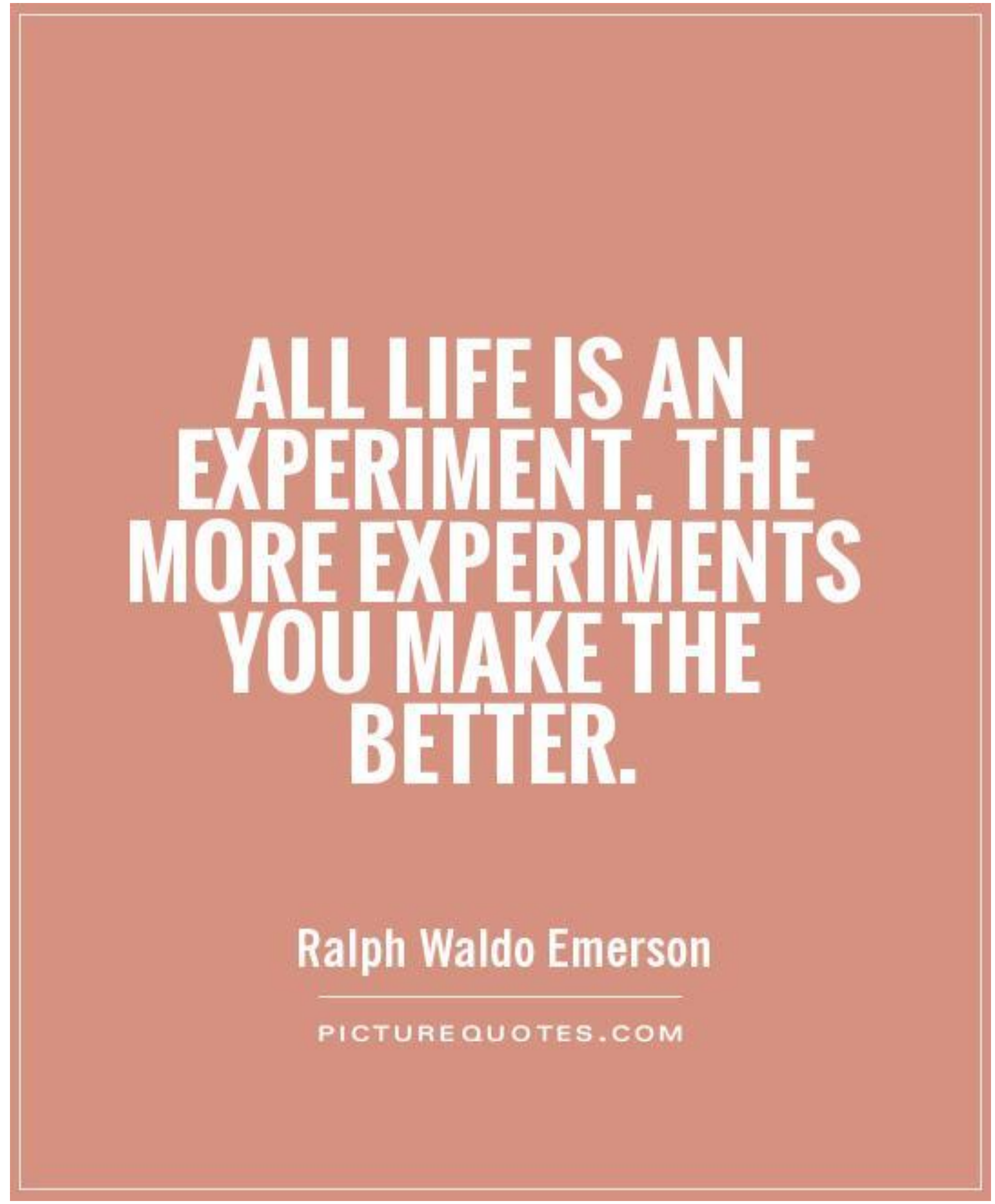

[T]he yeoman's work in any science, and especially physics, is done by the experimentalist, who must keep the theoreticians honest.
— Michio Kaku —
AZ QUOTES


An experiment is a question which science poses to Nature, and a measurement is the recording of Nature's answer.
Max Planck
More science quotes at Today in Science History todayinsci.com

Summary


A THEORY CAN BE PROVED BY EXPERIMENT,
BUT NO PATH LEADS FROM EXPERIMENT
TO THE BIRTH OF A THEORY.
Albert Einstein


Science walks forward on two feet, namely theory and experiment... but continuous progress is only made by the use of both.
-Robert A. Millikan


ALL LIFE IS AN EXPERIMENT. THE MORE EXPERIMENTS YOU MAKE THE BETTER.
Ralph Waldo Emerson
PICTUREQUOTES.COM