Electric and Magnetic Field Lenses

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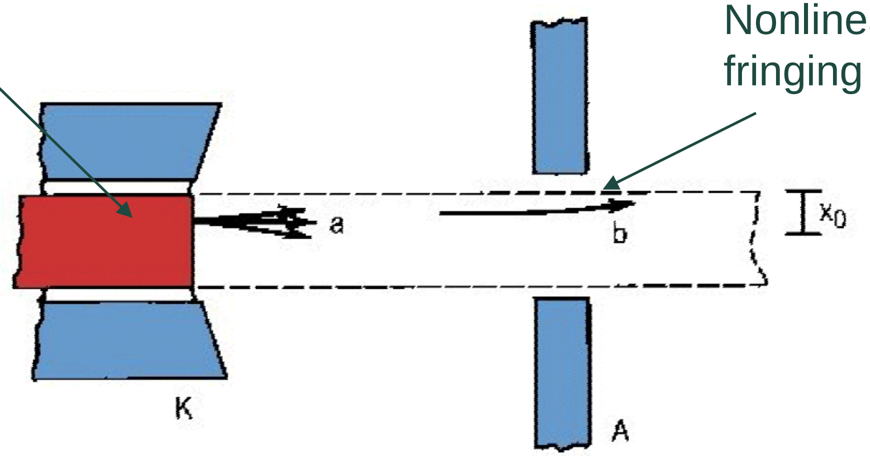
Introduction

- Particles in beams always have components of velocity perpendicular to the main direction of motion. The concern in charged particle optics is the control of the transverse motion of particles by shaped electric and magnetic fields.

Thermionic emitter
operating at T

Nonlinear deflection by
fringing electric field

$$\bar{v}_x = \sqrt{\frac{2kT}{\pi m}}$$



- In many practical cases, beam particles have small velocity perpendicular to the main direction of motion. Also, it is often permissible to use special forms for the electric and magnetic fields near the beam axis.
- With these approximations, the transverse forces acting on particles are linear; they increase proportional to distance from the axis. Such treatment is called linear or Gaussian charged particle optics.

Functions of charged particle lenses

- Charged particle lenses perform three types of operations: 1) confine a beam or maintain a constant or slowly-varying radius. 2) focus a beam or compress it to the smallest possible radius. 3) form an image.

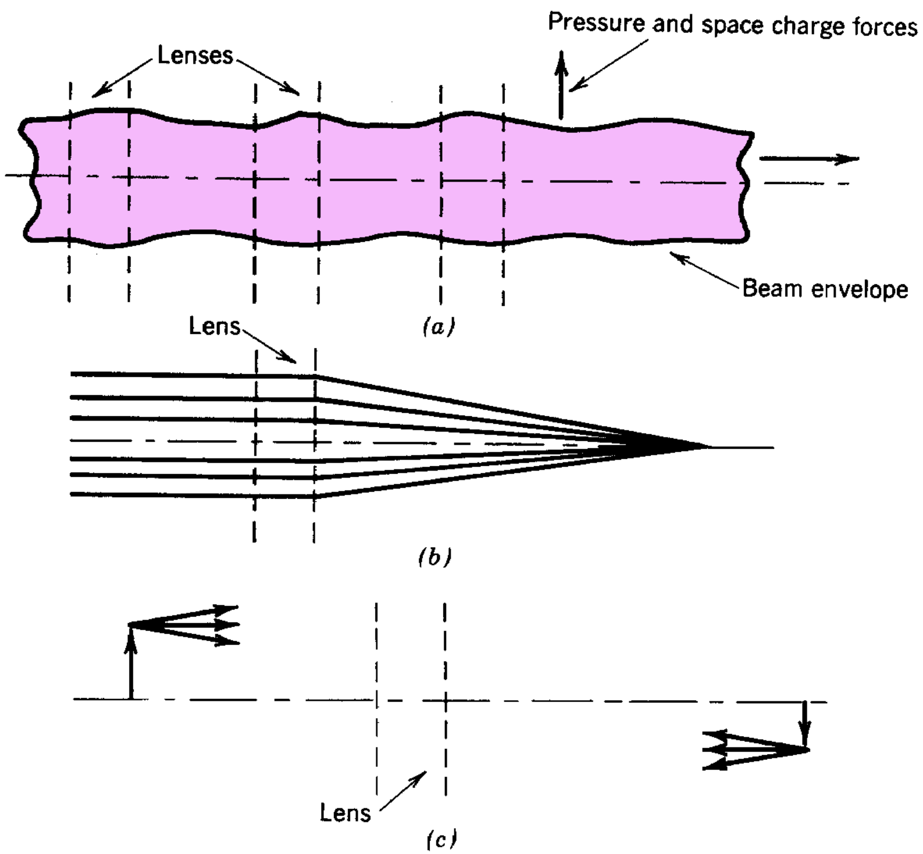
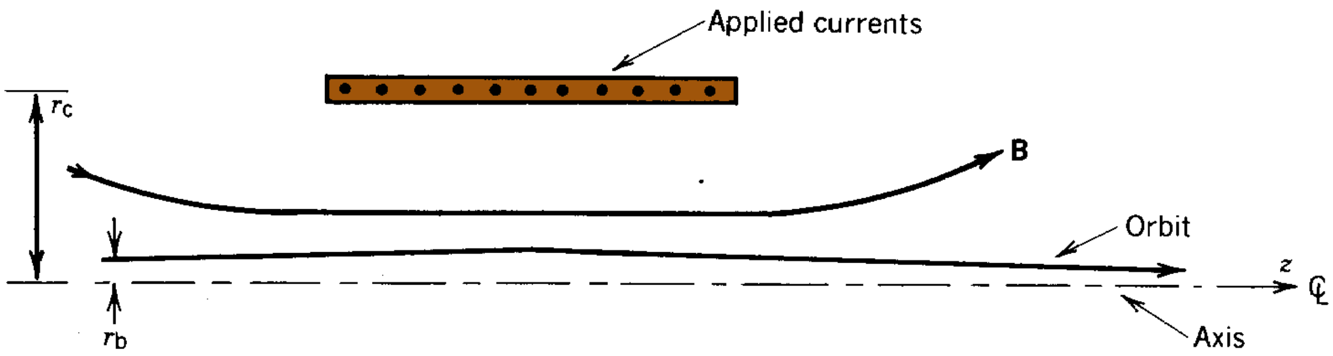


Figure 6.1 Functions of charged particle lenses. (a) Beam confinement. (b) Focusing to a spot. (c) Imaging.

Paraxial approximation

- Many particle beam applications require cylindrical beams. The electric and magnetic fields of lenses for cylindrical beams are azimuthally symmetric.
- The term paraxial comes from the Greek para meaning "alongside of."



$$\nabla \cdot \boldsymbol{B} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0$$

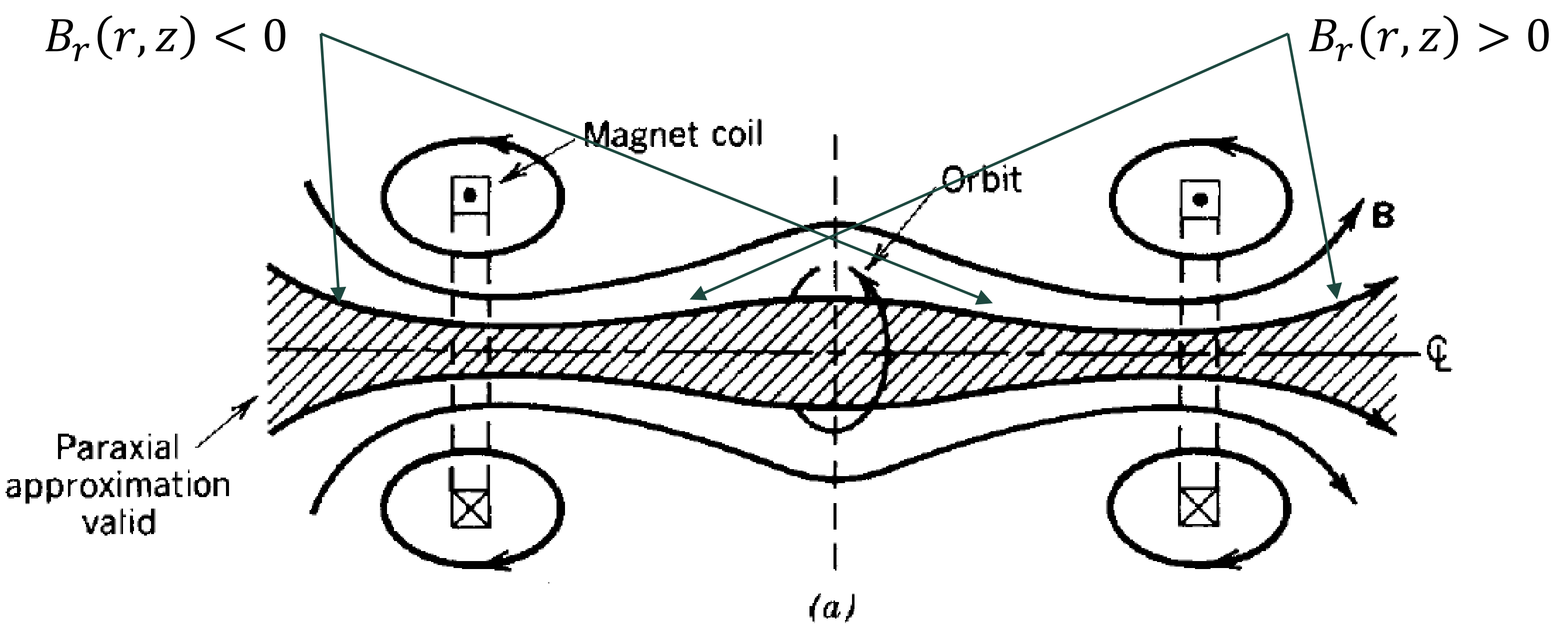
Figure 6.2 Validity conditions for the paraxial field approximation in a magnetic field lens. Paraxial approximation: $r_b \ll r_c$; $v_r \ll v_z$; $B_r \ll B_z$.

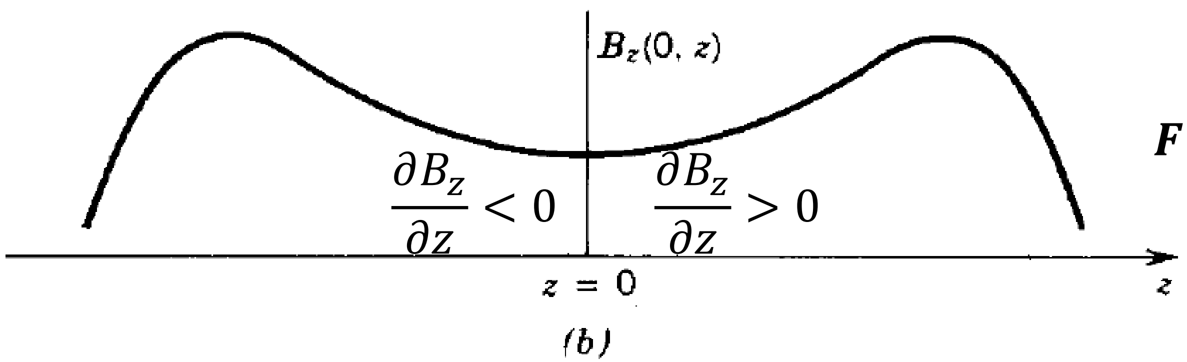
- In the paraxial limit ($E_r \ll E_z$ and $B_r \ll B_z$), we can relate the radial components of applied fields to the axial field by:

$$E_r(r,z) \approx -\frac{r}{2}\left(\frac{\partial E_z}{\partial z}\right)_{r=0} \qquad B_r(r,z) \approx -\frac{r}{2}\left(\frac{\partial B_z}{\partial z}\right)_{r=0}$$

- Note that all transverse forces are linear in the paraxial approximation.

Magnetic mirror field

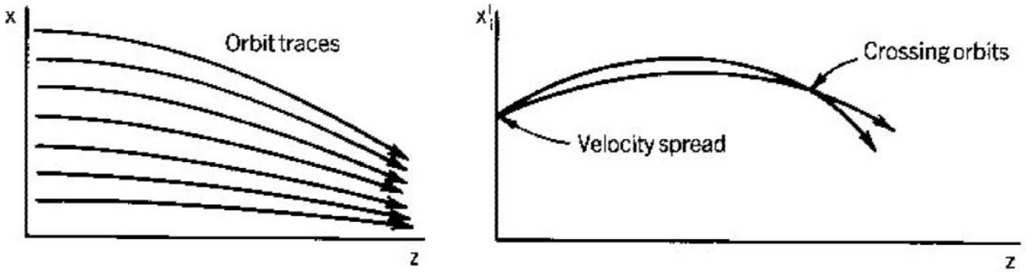




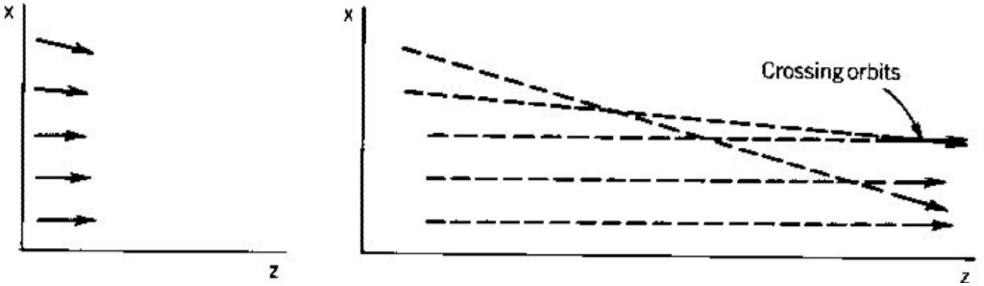
$$\boldsymbol{F}_{\parallel} = -\mu \boldsymbol{\nabla}_{\parallel} \boldsymbol{B}$$

Laminar and non-laminar beams

- Beams with good parallelism are easier to transport than beams with large random transverse velocity components. Ordered beams can focus to a small spot size.
- The ideal charged particle beam has laminar particle orbits. Orbits in a laminar beam flow in layers that never intersect. A laminar beam satisfies two conditions:
  - All particles at a position have identical transverse velocities. If this is not true, the orbits of two particles that start at the same position could separate and later cross each other.



  - The magnitude of the transverse particle velocity is linearly proportional to the displacement from the axis of beam symmetry.



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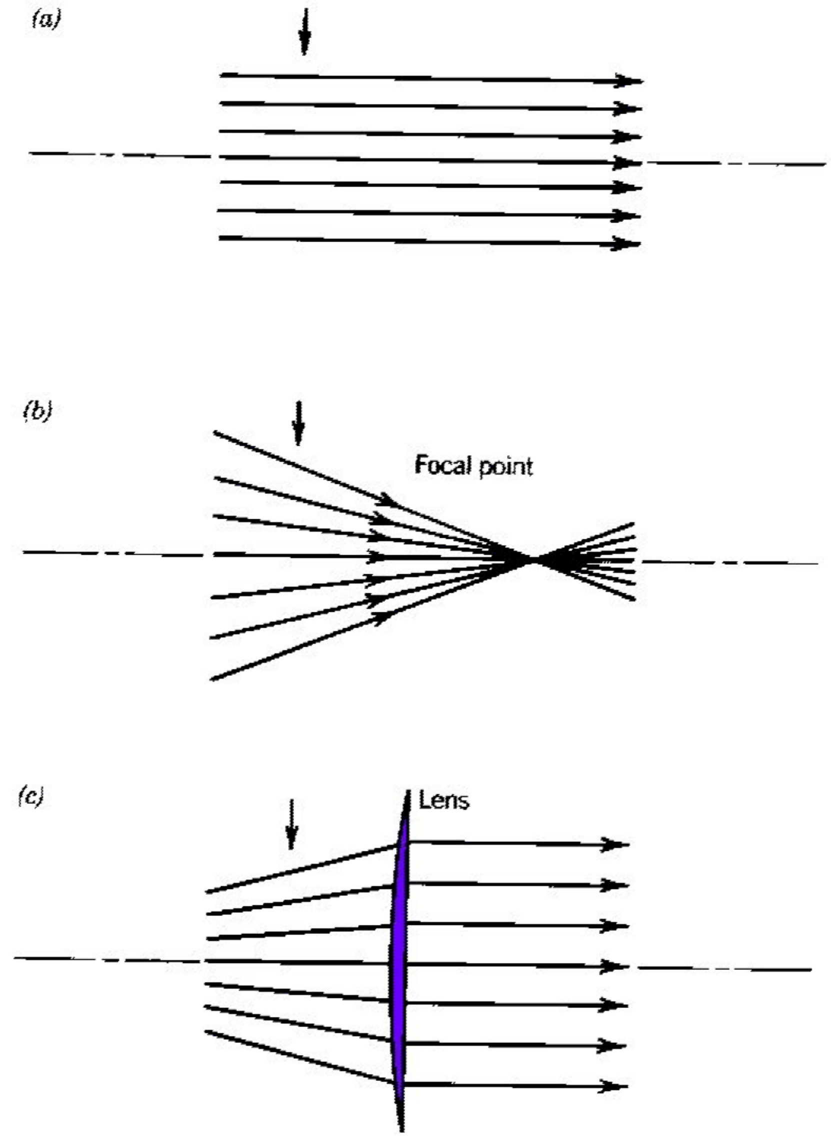
Some examples of orbits in laminar beams

a. Ideal parallel beam (all particles have zero transverse velocity)

b. Converging laminar beam, where orbits pass through a common focus

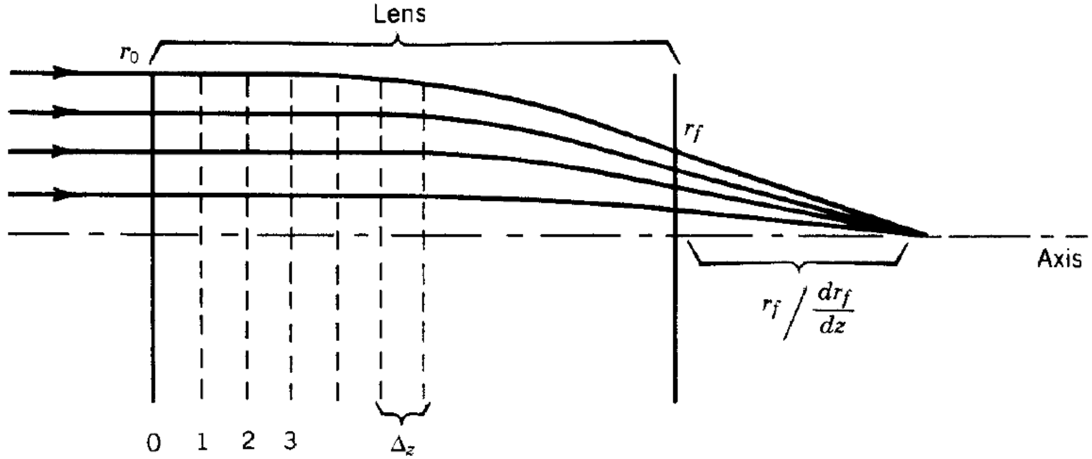
c. Diverging laminar beam converted to a parallel beam by a linear lens



Focusing properties of linear fields

- Consider one-dimensional transverse paraxial particle motion along the z axis in the presence of a linear force, $F_r(r) = -A(z, v_r)r$.
- The nonrelativistic equation of motion in the paraxial approximation:

$$\frac{d^2r}{dt^2} = \frac{F_r(r,z,v_z)}{m_0} \xrightarrow{\frac{d}{dt} \approx v_z \frac{d}{dz}} \frac{d^2r}{dz^2} = \frac{F_r(r,z,v_z)}{m_0 v_z^2} - \underbrace{\frac{v_z' r'}{v_z}}_{\approx 0}$$



- In the thin-lens approximation, rays are deflected but have little change in radius passing through the lens. This approximation is often invalid for charged particle optics; the particle orbits undergo a significant radius change in the field region. Lenses in which this occurs are called thick lenses.

Lens properties

- If the exit orbits are projected backward in a straight line, they intersect the forward projection of entrance orbits in a plane perpendicular to the axis. This is called the principal plane (H_1). The distance from H_1 to the point where orbits intersect is called the focal length, f_1.
- There is also a principal plane (H_2) and focal length (f_2) for particles with negative v_z. The focal lengths need not be equal, e.g. the aperture lens and the immersion lens.
- A thin lens is defined as one where the axial length is small compared to the focal length. A thin lens has only one principal plane ($H_1 = H_2$). Particles emerge at the same radius they entered but with a change in direction.

$$f \cong -\frac{r_0}{r_f'} \cong -\frac{r_0}{v_{rf}/v_{zf}}$$

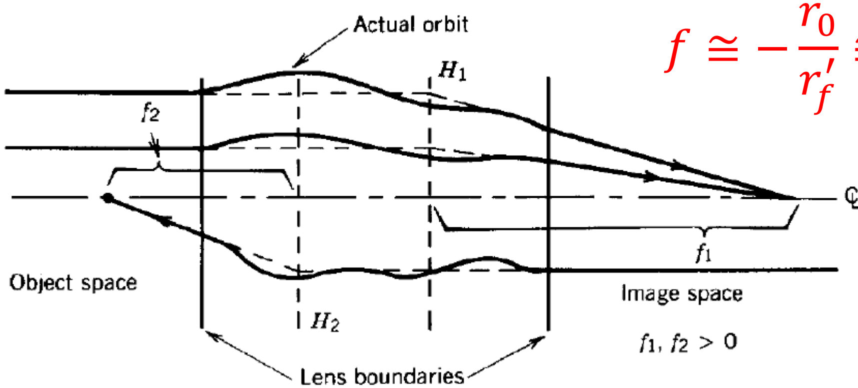


Figure 6.5 Charged particle lens. Definition of quantities used in linear approximation (Gaussian optics): principal planes (H_1, H_2) and focal lengths (f_1, f_2).

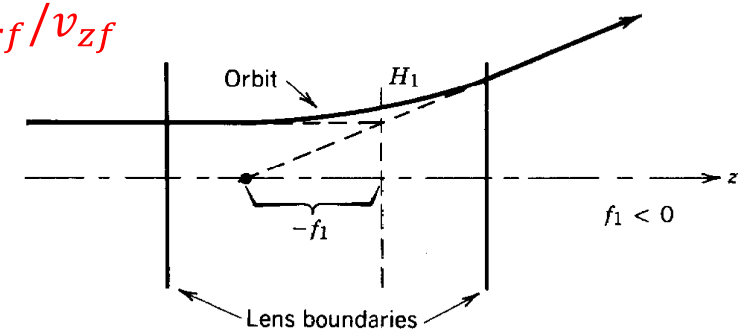


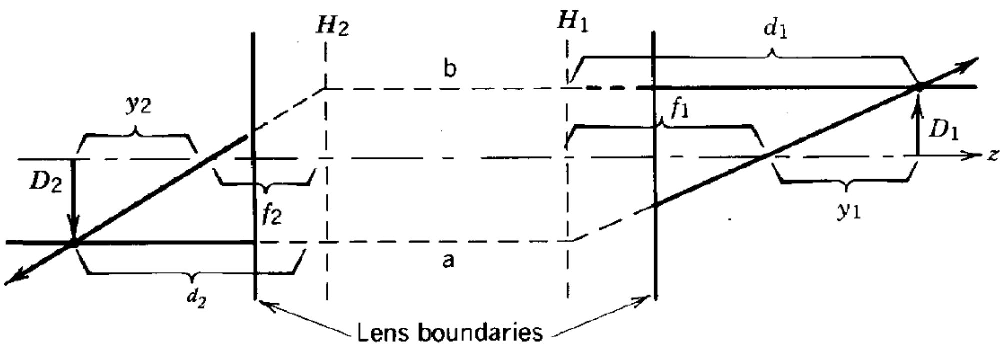
Figure 6.6 Construction of a particle orbit in a lens with a negative focal length.

Lens properties

- The strength of a lens is determined by how much it bends orbits. Shorter focal lengths mean stronger lenses. The lens power P is the inverse of the focal length, $P = 1/f$. If the focal length is measured in meters, the power is given in m^{-1} or diopters.
- The F-number is the ratio of focal length to the lens diameter: $F = f/D$. The F-number is important for describing focusing of non-laminar beams. It characterizes different optical systems in terms of the minimum focal spot size and maximum achievable particle flux.

$$M_{21} = \frac{D_1}{D_2}$$

$$\frac{D_2}{y_2} = \frac{D_1}{f_2}$$



$$M_{12} = \frac{D_2}{D_1}$$

$$\frac{D_1}{y_1} = \frac{D_2}{f_1}$$

$$f_1 f_2 = y_1 y_2 = (d_1 - f_1)(d_2 - f_2) \quad \Rightarrow \quad \frac{f_1}{d_1} + \frac{f_2}{d_2} = 1$$

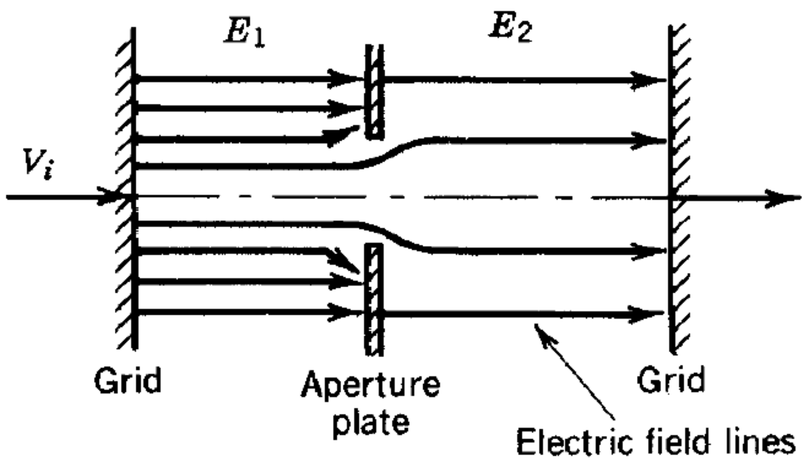
- For magnetic lenses or unipotential electrostatic lenses where the particle energy does not change in the lens,

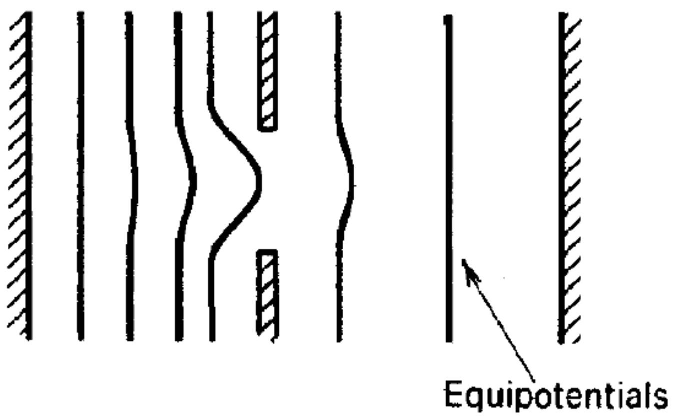
$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

Electrostatic aperture lens

- The electrostatic aperture lens is an axi-centered hole in an electrode separating two regions of axial electric field. The fields may be produced by grids with applied voltage relative to the aperture plate. If the upstream and downstream electric fields differ, there will be radial components of electric field near the hole which focus or defocus particles.
- Under the assumptions that (1) the relative change in radius passing through the aperture is small (thin lens approximation) and (2) the relative change in axial velocity is small in the vicinity of the aperture,

$$\frac{dv_r}{dz} = \frac{qE_r}{m_0 v_z} \cong -\frac{q}{2m_0 v_z} r \frac{dE_z(0,z)}{dz} \quad \Rightarrow \quad \frac{v_{rf}}{v_{zf}} = r_f' \cong -\frac{qr(E_{z2} - E_{z1})}{2m_0 v_{zf}^2}$$



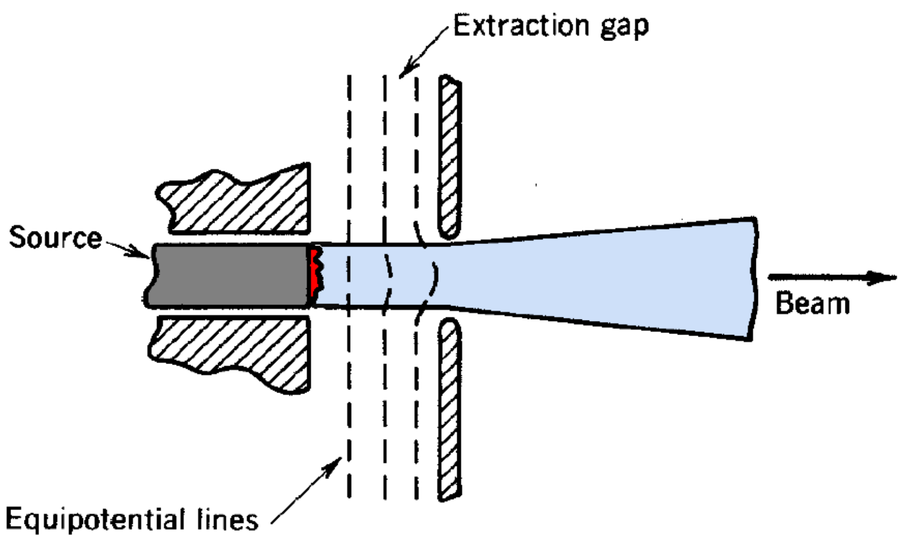


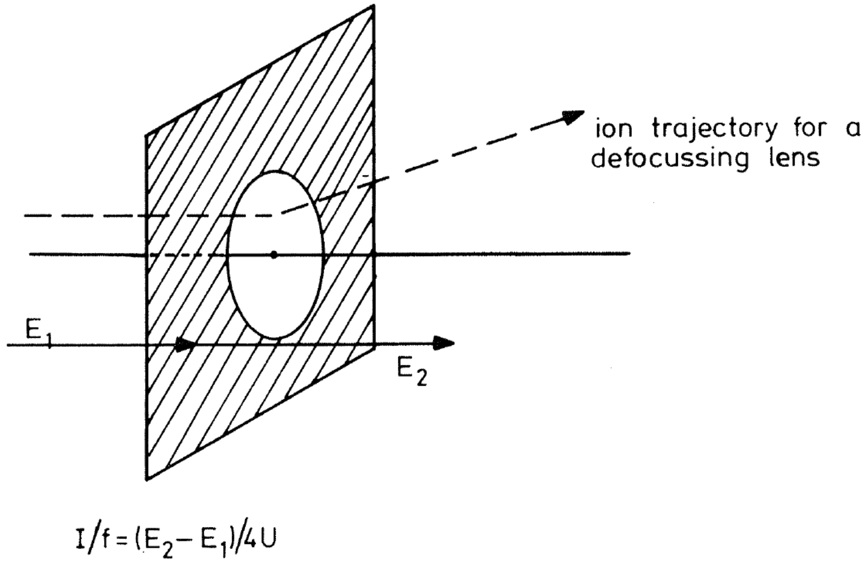
Electrostatic aperture lens

- The focal length is related to the final radial position (r_f) and the ratio of the radial velocity to the final axial velocity:

$$f = -\frac{r_0}{r_f'} \cong \frac{2m_0 v_{zf}^2}{q(E_{z2} - E_{z1})} = \frac{4T}{q(E_{z2} - E_{z1})} = \frac{4V}{V_2' - V_1'}$$

- The charged particle extractor is a frequently encountered application of aperture lens. The hole acts as an aperture lens, with $E_1 > 0$ and $E_2 = 0$. The focal length is negative; the beam emerging will diverge. This is called the negative lens effect in extractor design.





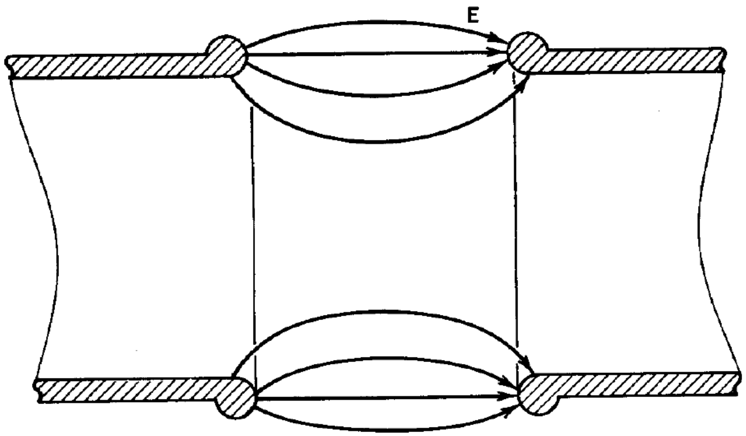
Electrostatic immersion lens

- The electrostatic immersion lens consists of two tubes at different potential separated by a gap. Acceleration gaps between drift tubes of a standing-wave linear accelerator have this geometry. The one-dimensional version of this lens occurs in the gap between the Dees of a cyclotron.

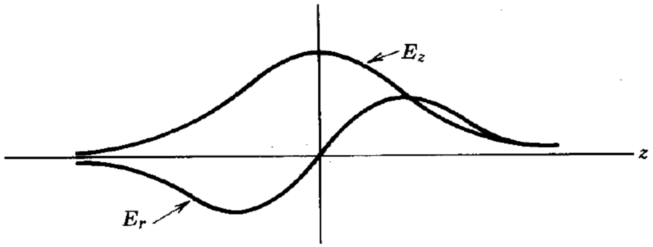
- Following the treatment used for the aperture lens, the change in radial velocity of a particle passing through the gap would be zero because the radial electric field is symmetric if the particle radius and axial velocity are constant.

$$\Delta v_r = v_{rf} = \int \frac{qE_r(r,z)}{m_0 v_z} dz$$

- In contrast to the aperture lens, the focusing action of the immersion lens arises from changes in r and v_z in the gap region.



(a)



(b)

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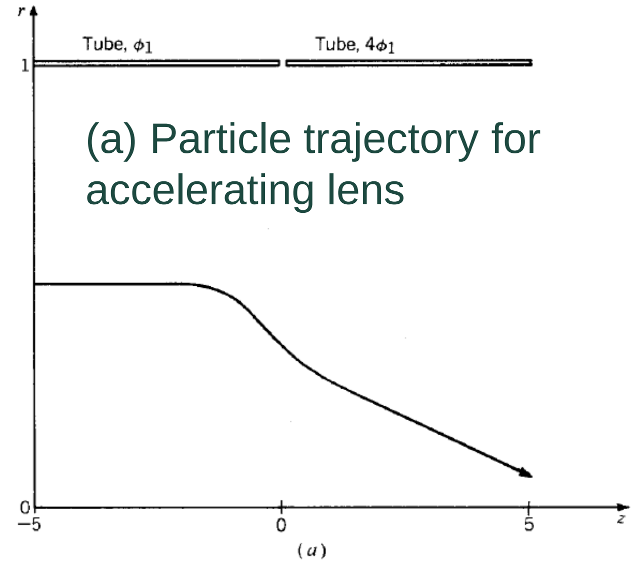
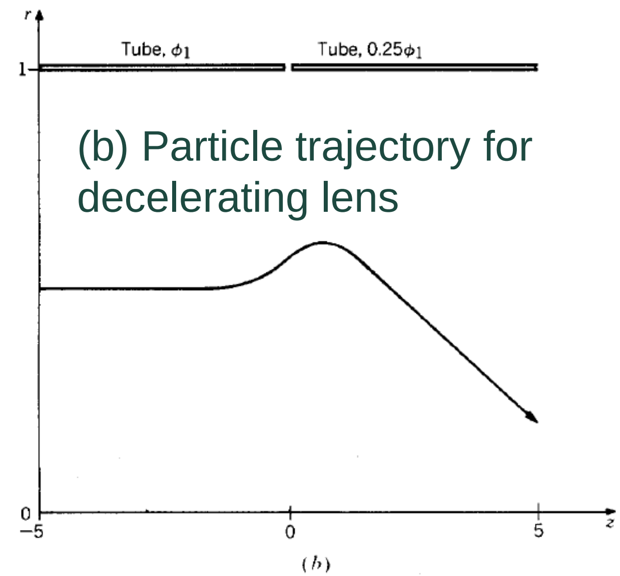
Electrostatic immersion lens

- When the longitudinal gap field accelerates particles, they are deflected inward on the entrance side of the lens and outward on the exit side. The outward impulse is smaller because (1) the particles are closer to the axis and (2) they move faster on the exit side.
- The converse holds for a decelerating gap. Particles are deflected to larger radii on the entrance side and are therefore more strongly influenced by the radial fields on the exit side. Furthermore, v_z is lower at the exit side enhancing focusing.
- The focal length for either polarity or charge sign is positive.



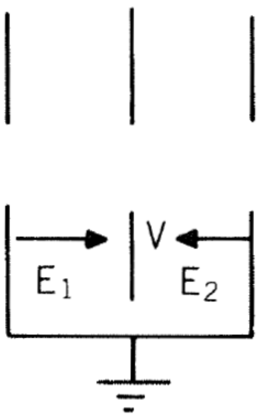
(a) Particle trajectory for accelerating lens



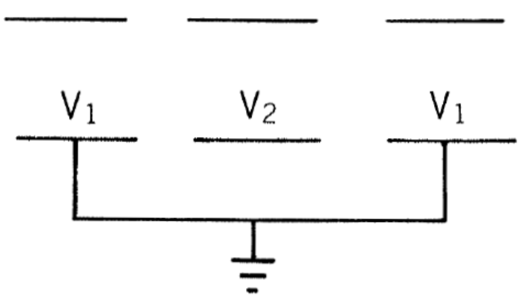
(b) Particle trajectory for decelerating lens

Einzel lens

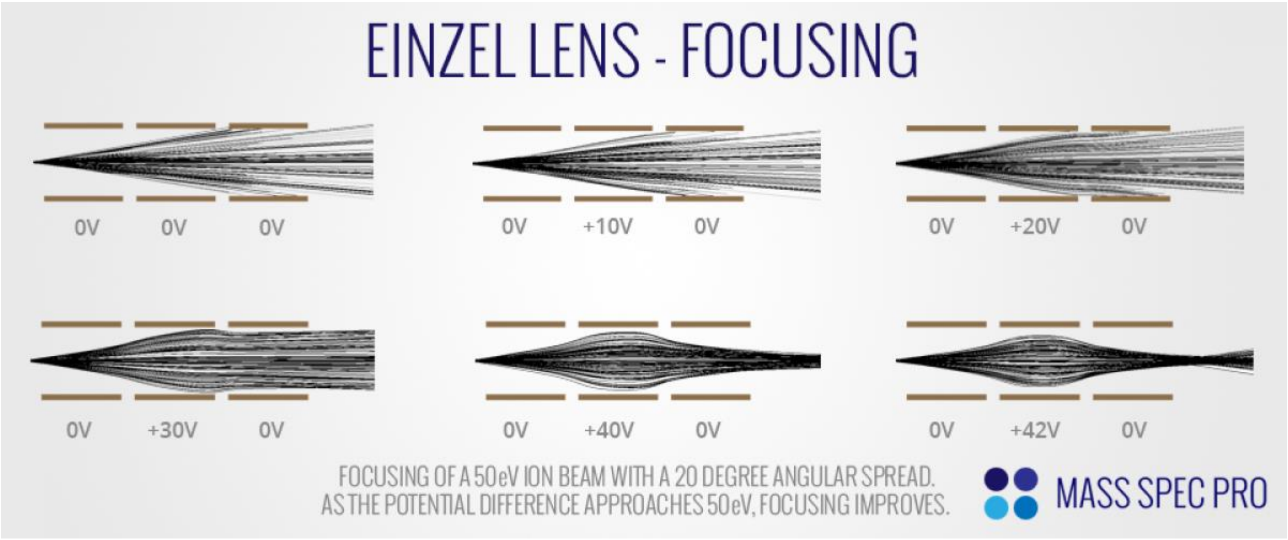
- The einzel lens is a variant of the immersion lens often encountered in low-energy electron guns. It consists of three colinear tubes, with the middle tube elevated to high potential. The einzel lens consists of two immersion lenses in series; it is a unipotential lens.



Einzel lens
(always focusing)

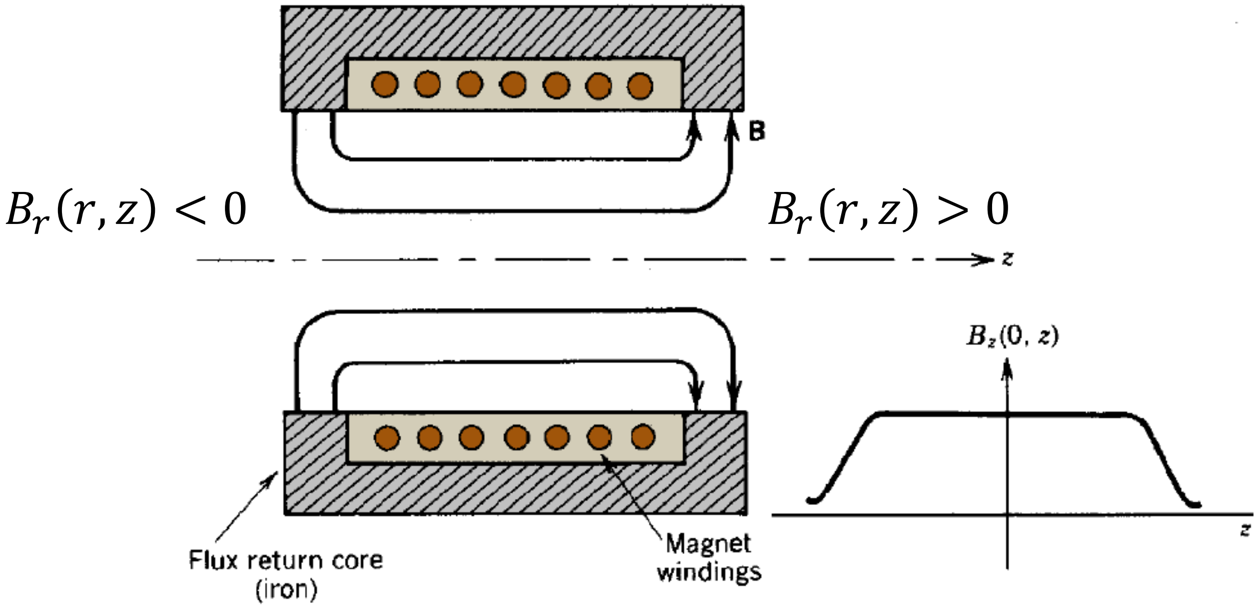


Cylindrical lens
(always focusing)



Solenoidal magnetic lens

- The solenoidal magnetic lens consists of a region of cylindrically symmetric radial and axial magnetic fields produced by axi-centered coils carrying azimuthal current. This lens is the only possible magnetic lens geometry consistent with cylindrical paraxial beams.
- It is best suited to electron focusing. It is used extensively in cathode ray tubes, image intensifiers, electron microscopes, and electron accelerators.
- Since the magnetic field is static, there is no change of particle energy passing through the lens.



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Solenoidal magnetic lens

- Particles enter the lens through a region of radial magnetic fields. The Lorentz force ($qv_z \times B_r$) is azimuthal. The resulting v_θ leads to a radial force when crossed into the B_z fields inside the lens. The net effect is a deflection toward the axis, independent of charge state or transit direction.
- The equations of motion (assuming constant γ) are

$$\gamma m_0 \frac{dv_\theta}{dt} = qv_z B_r \quad \Rightarrow \quad \gamma m_0 v_z \frac{dv_\theta}{dz} = -qv_z \frac{r}{2}\frac{dB_z(0,z)}{dz}$$

$$\Rightarrow \quad v_\theta = -\frac{qrB_z}{2\gamma m_0}$$

$$\gamma m_0 \frac{dv_r}{dt} = qv_\theta B_z + \gamma m_0 \frac{v_\theta^2}{r} \quad \Rightarrow \quad \gamma m_0 v_z \frac{dv_r}{dz} = -\frac{(qB_z)^2 r}{4\gamma m_0}$$

$$r_f' = \frac{v_{rf}}{v_z} \cong -\frac{1}{4}\int \left(\frac{qB_z(0,z)}{\gamma m_0 v_z}\right)^2 r\,dz$$

- The focal length is:

$$\boxed{f = -\frac{r_0}{r_f'} \cong \frac{4}{\int \left(\frac{qB_z(0,z)}{\gamma m_0 v_z}\right)^2 dz} \cong \frac{(2\gamma m_0 v_z)^2}{q^2 B_z^2 L}}$$

Magnetic sector lens

- A sector magnet consists of a gap with uniform magnetic field extending over a bounded region. Focusing about an axis results from the location and shape of the field boundaries rather than variations of the field properties.
- Sector field magnets are used to bend beams in transport lines and circular accelerators and to separate particles according to momentum in charged particle spectrometers.

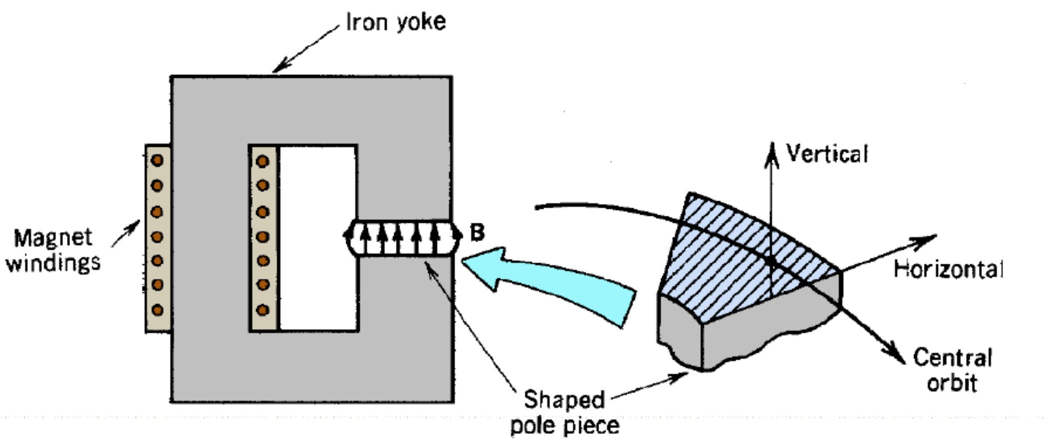


Figure 6.15 Sector field magnet for deflecting a beam showing definition of horizontal and vertical directions with respect to the main beam orbit.

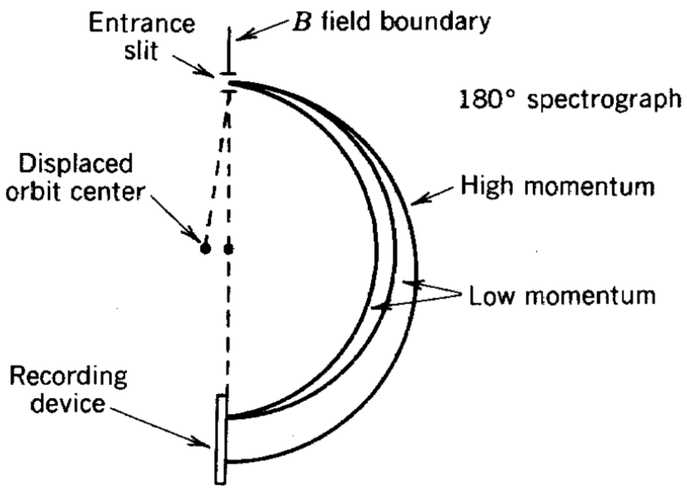


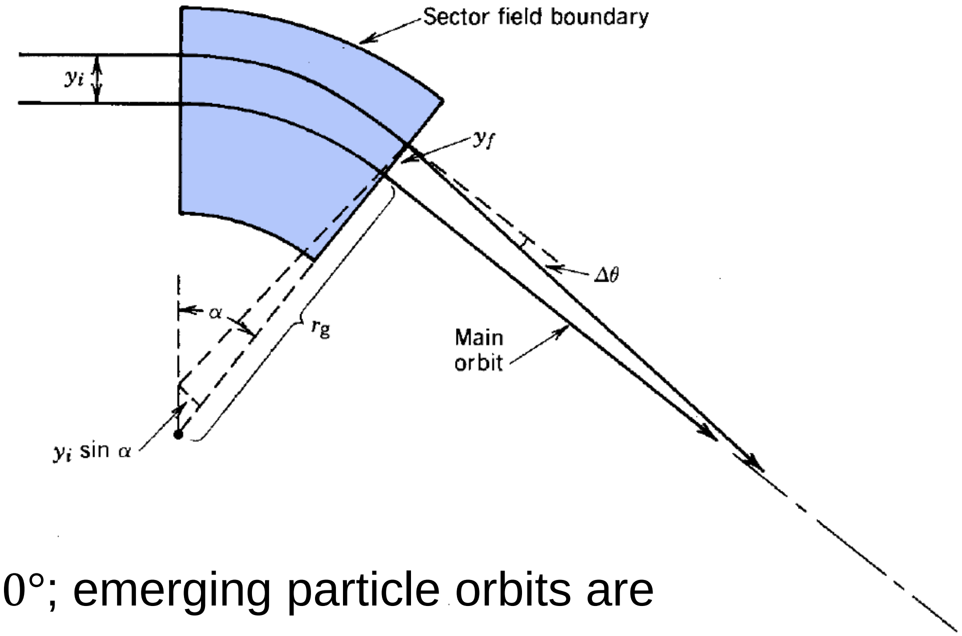
Figure 6.16 Particle focusing by a 180° sector magnet.

Magnetic sector lens

- A sector field with angular extent less than 180° can act as a thick lens to produce a horizontal convergence of particle orbits after exiting the field. Focusing occurs because off-axis particles travel different distances in the field and are bent a different amount. If the field boundaries are perpendicular to the central orbit, the difference in bending is linearly proportional to the distance from the axis.
- The exit position and angle are

$$y_f = y_i \cos\alpha$$

$$\Delta\theta = -y_i \sin\alpha / r_g$$



- The distance from the field boundary to the focal point is

$$\boxed{f \cong -\frac{y_f}{\Delta\theta} = \frac{r_g}{\tan\alpha}}$$

- The focal distance is positive for $\alpha < 90°$; emerging particle orbits are convergent.
- $f = 0$ at $\alpha = 90°$; initially parallel particles are focused to a point at the exit of a 90° sector.

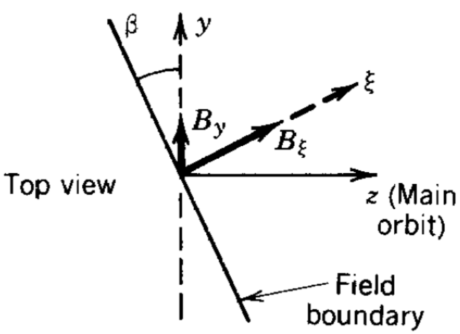
Edge focusing

- The term edge focusing refers to the vertical forces exerted on charged particles at a sector magnet boundary that is inclined with respect to the main orbit.
- When the inclination angle β is nonzero, there is a component of **B** in the y direction which produces a vertical force at the edge when crossed into the particle v_z.
- The momentum change in the vertical direction

$$\Delta P_x = -\int q v_z B_y \, dt = -\int q B_\xi \sin\beta \; dz = -\int q B_\xi \sin\beta \; d\xi / \cos\beta$$

$$\Delta v_x = -\frac{q}{\gamma m_0}\int B_\xi \tan\beta \; d\xi \approx -\frac{q B_0 x \tan\beta}{\gamma m_0} \Rightarrow f_x = -\frac{x}{x'} \approx -\frac{v_z}{\Delta v_x} x = \frac{\gamma m_0 v_z}{q B_0 \tan\beta}$$

$$= \frac{r_g}{\tan\beta}$$



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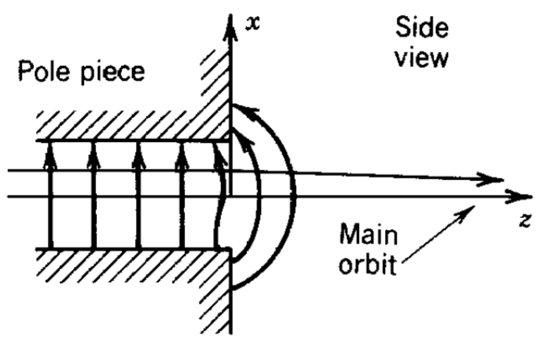

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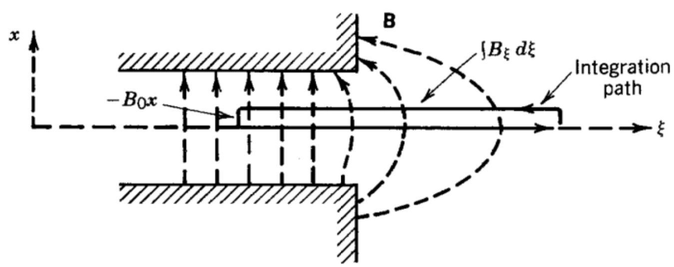


Principle of dual-focusing magnetic spectrometer

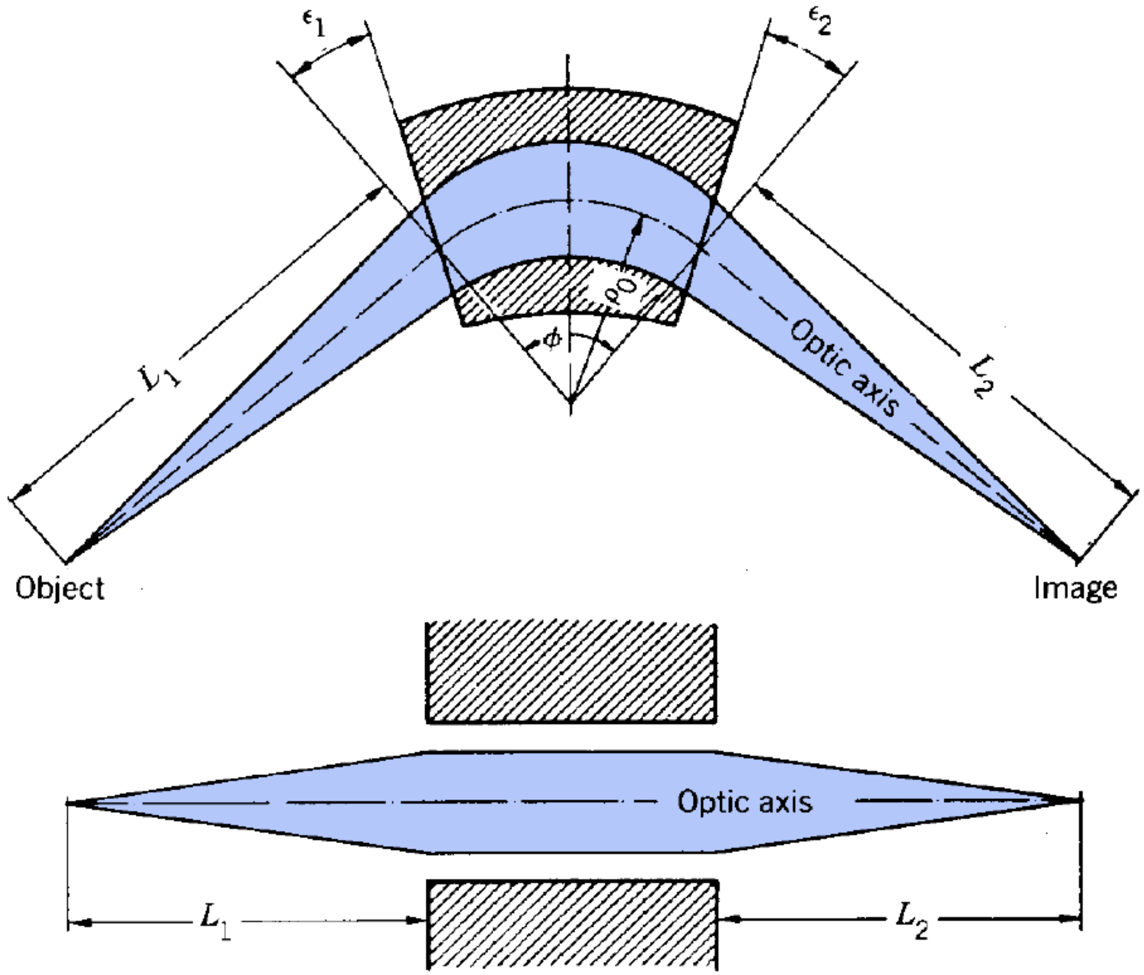


Figure 6.22 A homogeneous sector field in which the particle beam crosses the boundary obliquely. Proper choice of the angles ε_1 and ε_2 gives stigmatic focusing in both the radial and vertical directions. (Courtesy, H. Wollnik.)

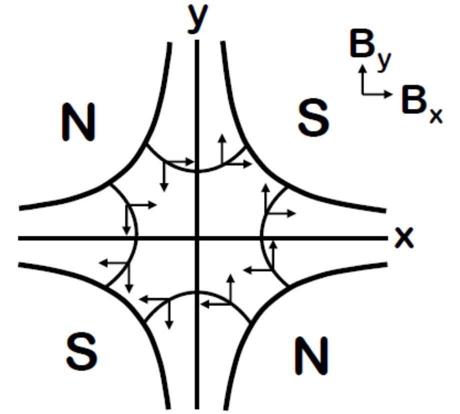
Magnetic quadrupole lens

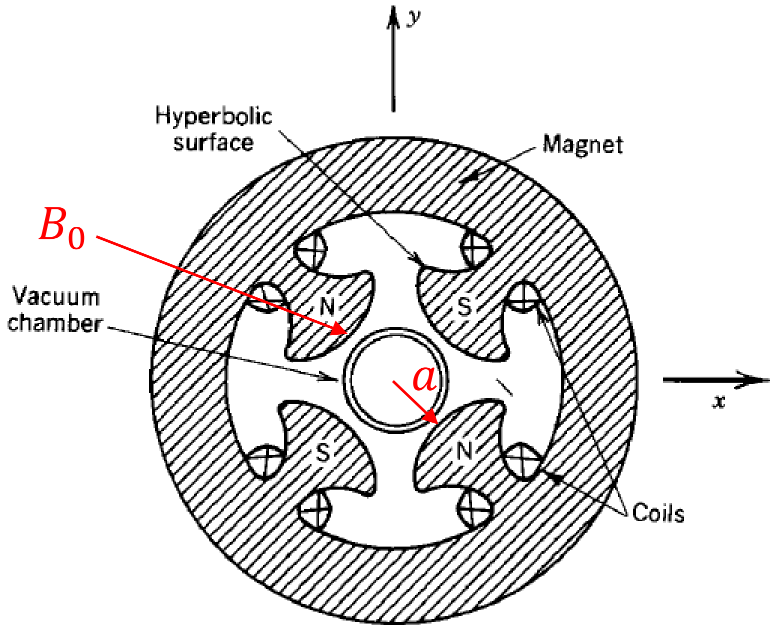
- The fields depend linearly on the distance from the center of the magnet.

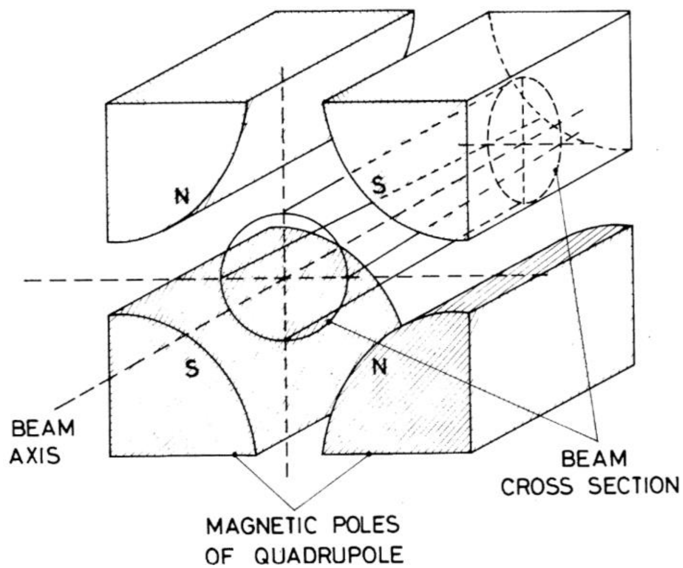
$$B_x = B_0 y/a \qquad B_y = B_0 x/a \qquad \Rightarrow \qquad F_x \sim x \qquad F_y \sim y$$

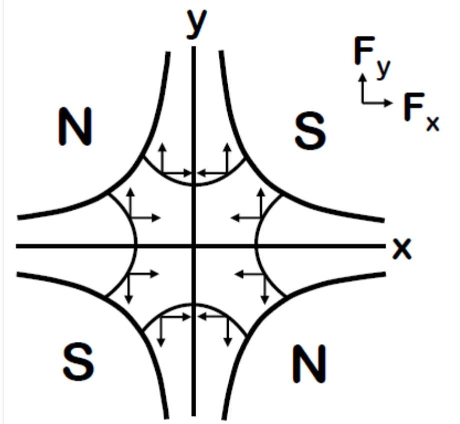
- Motions in the transverse directions are independent, and the forces are linear.
- The orbit equations are

$$\frac{d^2y}{dz^2} = \frac{qB_0}{\gamma m_0 a v_z} y \qquad \frac{d^2x}{dz^2} = -\frac{qB_0}{\gamma m_0 a v_z} x$$









Magnetic quadrupole lens

- The solutions are

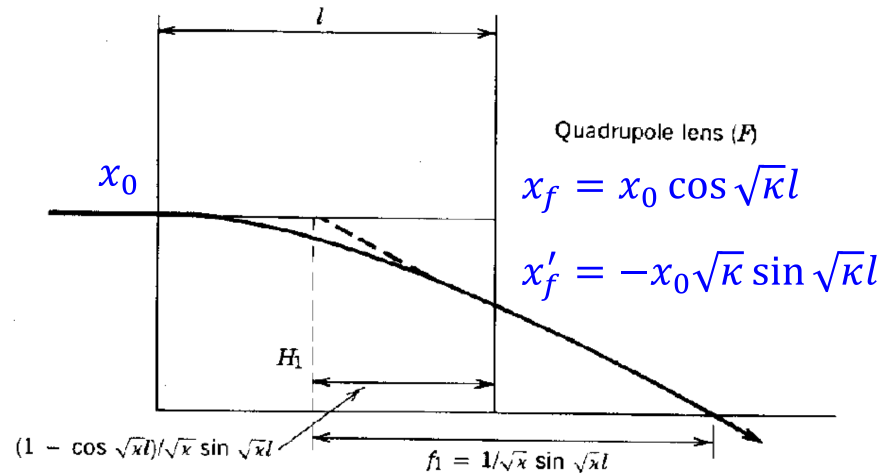
$$x(z) = x_0 \cos\sqrt{\kappa}z + x_0' \sin\sqrt{\kappa}z \,/\sqrt{\kappa}$$

$$x'(z) = -x_0\sqrt{\kappa} \sin\sqrt{\kappa}z + x_0' \cos\sqrt{\kappa}z$$

$$y(z) = y_0 \cosh\sqrt{\kappa}z + y_0' \sinh\sqrt{\kappa}z \,/\sqrt{\kappa}$$

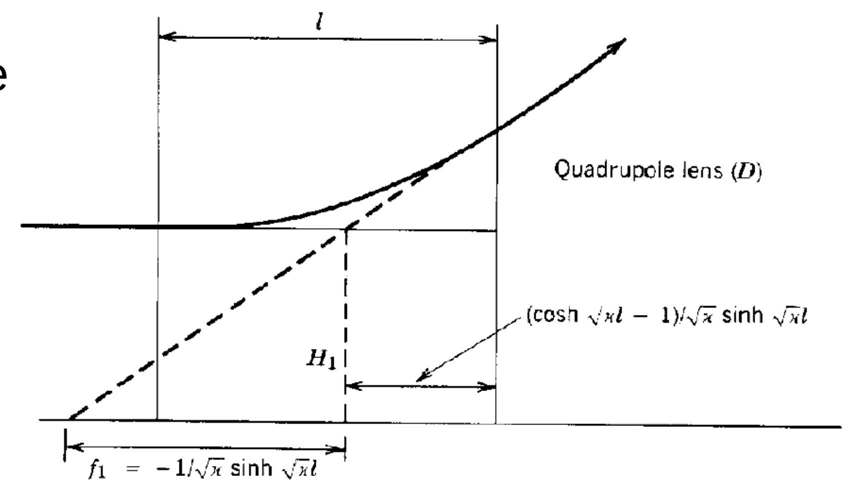
$$y'(z) = y_0\sqrt{\kappa} \sinh\sqrt{\kappa}z + y_0' \cosh\sqrt{\kappa}z$$

$$\kappa = \frac{qB_0}{\gamma m_0 a v_z} \qquad f \cong -\frac{x_0}{x_f'} \cong \frac{1}{\sqrt{\kappa}\sin\sqrt{\kappa}l}$$



- The quadrupole lens focuses in one plane and defocuses in the other, hence breaking rotational symmetry. However, the focal length is much shorter than a solenoid with equal field

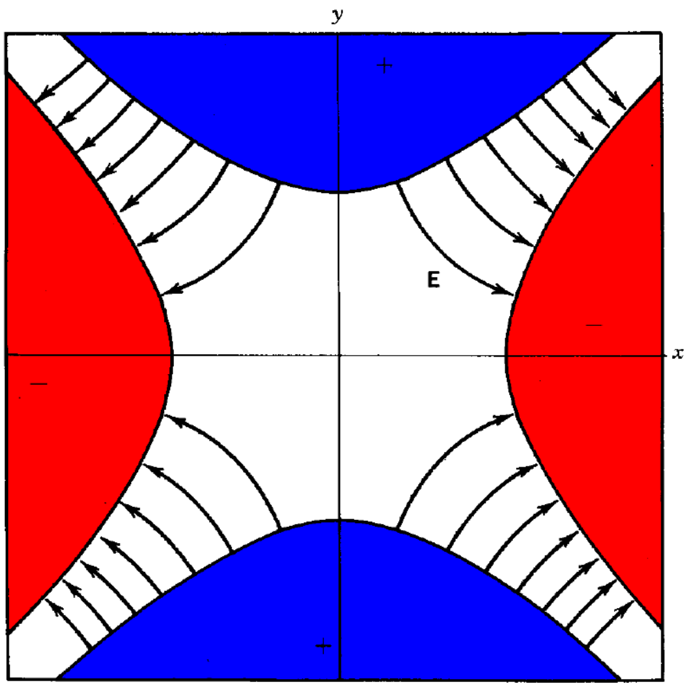
$$\boxed{\left|\frac{1}{f}\right| = \sqrt{\kappa}\sin\sqrt{\kappa}l}$$



Homework

- The two-dimensional electric field distribution in a quadrupole electrostatic lens:

$$E_x = -\frac{\partial \varphi}{\partial x} = +kx = E_0 \frac{x}{a} \qquad E_y = -\frac{\partial \varphi}{\partial y} = -ky = -E_0 \frac{y}{a}$$



- Find the solution for $x(z), x'(z), y(z), y'(z)$, and the focal length.