SYSTEMS OF PARTICLES

->Single Particle ? ~ Satellite around the Earth ?

* Control ? Structural design ?

All matter is in constant interaction with other matter !!

Newton's law : Originally formulated for a single particle !

~> ? Multiple, infinite number of particles

System of N particles each of mass m_j (j= 1,…,N)

Definition: mass center of a system of particles

Global axes X-Y-Z-O

Total mass M ~ for each m_j with displacement vector r_j (Fig.21)

Rc = $\frac{1}{M} m_j \vec{r}_j \ldots (j = 1,,, N)$ (1.41) where $M = \sum_{j=1}^{N} m_j$ (1.42)

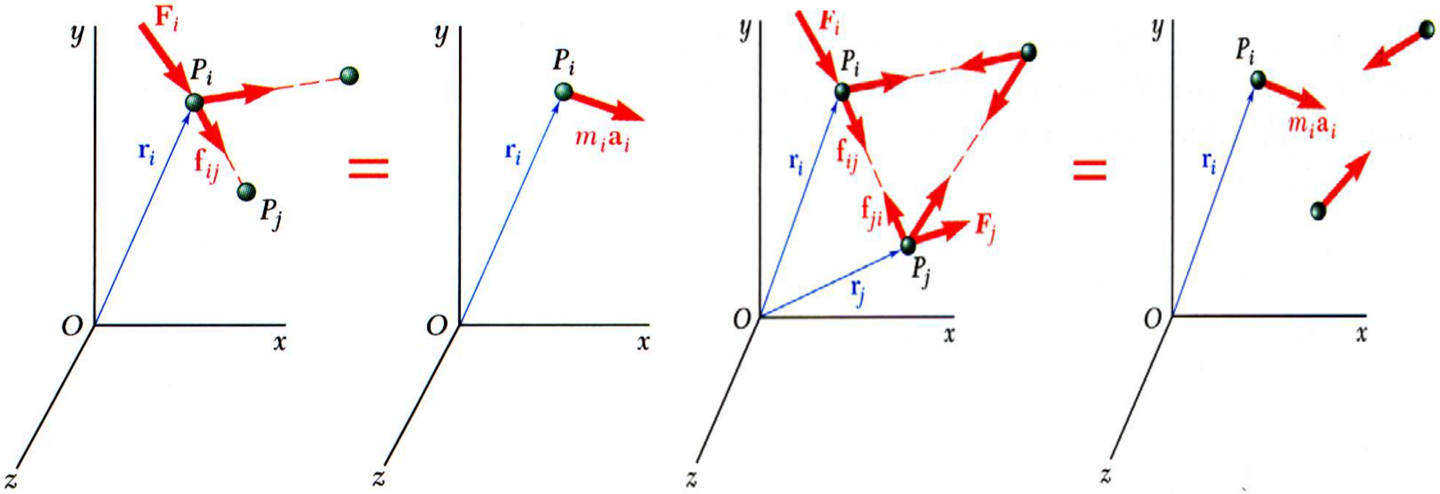
(Fig.1.22)

Center of mass of the System : Rc ~ Weighted average of locations r_j

.

(1.44)~(1.45) :

Newton's 2nd law



Newton's second law for each particle P_i in a system of n particles,

$$\vec{F}_i + \sum_{j=1}^{n} \vec{f}_{ij} = m_i \vec{a}_i ; \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{n} \left(\vec{r}_i \times \vec{f}_{ij} \right) = \vec{r}_i \times m_i \vec{a}_i$$

$\vec{F}_i =$ external force $\quad \vec{f}_{ij} =$ internal forces $\quad m_i \vec{a}_i =$ effective force

If $i=j$? (Fig.1.23)

- **Summing over all the elements,**

$$\sum_{i=1}^{n}\vec{F}_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\vec{f}_{ij} = \sum_{i=1}^{n} m_i\vec{a}_i ; \sum_{i=1}^{n}\left(\vec{r}_i \times \vec{F}_i\right) + \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\vec{r}_i \times \vec{f}_{ij}\right) = \sum_{i=1}^{n}\left(\vec{r}_i \times m_i\vec{a}_i\right)$$

- **Since the <u>internal forces occur in equal and opposite collinear pairs,</u> the resultant force and couple due to the internal forces are zero,**

$$\sum \vec{F}_i = \sum m_i\vec{a}_i : \sum\left(\vec{r}_i \times \vec{F}_i\right) = \sum\left(\vec{r}_i \times m_i\vec{a}_i\right)$$

= > (1.50) !

<u>This is very important result : .....</u>

<u>Principle of Impulse and Momentum</u> : $\displaystyle \sum_{j=1}^{N}\int_{t_1}^{t_2}\vec{F}_j\,dt = ..$

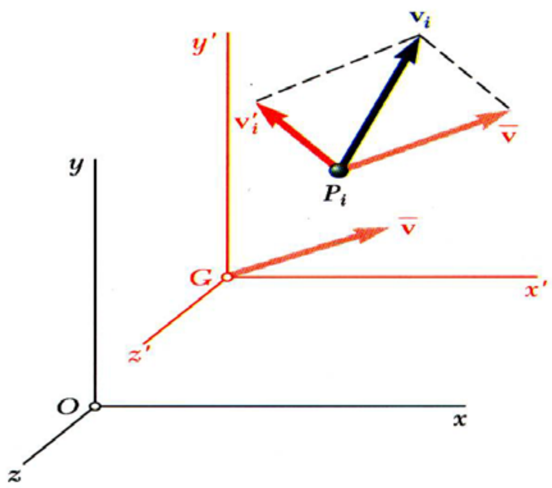
: **Absolute coordinate system (X,Y)**

Moving reference frame attached at Rc

$$\mathbf{R}_j = \mathbf{R}_C + \mathbf{r}_j \qquad (1.54)$$

<u>Angular momentum</u>

Kinetic Energy



- **Kinetic energy of a system of particles,**

$$T = \frac{1}{2}\sum_{i=1}^{n} m_i\left(\vec{v}_i \bullet \vec{v}_i\right) = \frac{1}{2}\sum_{i=1}^{n} m_i v_i^2$$

Expressing the velocity in terms of the centroidal reference frame,

$$T = \frac{1}{2}\sum_{i=1}^{n}\left[m_i\left(\vec{v}_G + \vec{v}_i'\right)\bullet\left(\vec{v}_G + \vec{v}_i'\right)\right] = \frac{1}{2}\left(\sum_{i=1}^{n} m_i\right)v_G^2 + \vec{v}_G \bullet \sum_{i=1}^{n} m_i \vec{v}_i' + \frac{1}{2}\sum_{i=1}^{n} m_i v_i'^2 = \frac{1}{2} m \vec{v}_G^2 + \frac{1}{2}\sum_{i=1}^{n} m_i v_i'^2$$

- **Kinetic energy is equal to kinetic energy of mass center plus kinetic energy relative to the centroidal frame.**
- **Although** $\vec{f}_{ij}$ and $\vec{f}_{ji}$ **are equal and opposite, <u>the work of these forces will not, in general, cancel out.</u>**

- <u>Total Kinetic Energy</u> =

  Energy associated with the motion as a single entity

(orbital kinetic energy)

+ Energy of motion of the individual particle about the mass center (spin kinetic energy)

Moving constraint

MOTION IN NONINERTIAL REFERENC FRAMES

Newton's law is based on the inertial reference frame:

~ Allow moving frame with constant (uniform) velocity

A noninertial reference frame:

With linear acceleration wrt some I.R.F.

or

With some angular velocity wrt an I.R.F.

~ Both will occur in various cases.
Any reference frame attached to the Earth :

Inertial reference frame ?

Acceptable assumption ? : negligible errors

Gun ?   I.C.B.M. ?

: Most practical problems involve reference frames ~

~ Non-inertial frame !

Objective of this section :
How dynamics can be formulated and analyzed using moving reference frame ?
Reference frame :

o Inertial reference : OXYZ with unit vector I,J,K.

o A moving (ω) reference frame :
oxyz with unit vector i,j,k

Refer to Fig.1.28,

$$r_p(t) = r_B(t) + r_{rel}(t)$$

Then

$$v_p = v_B + (\varpi \times r_{rel}) + v_{rel}$$

~ Motion of a Particle P in a Box !

= ?

Acceleration:

$$\frac{d}{dt}\mathrm{v}_{p} = \frac{d}{dt}\mathrm{v}_{B} + \frac{d}{dt}(\varpi \times \mathrm{r}_{rel}) + \frac{d}{dt}\mathrm{v}_{rel}$$

$$\mathrm{a}_{p} = \mathrm{a}_{B} + (\dot{\varpi} \times \mathrm{r}_{rel}) + \varpi \times \frac{d}{dt}\mathrm{r}_{rel} + \frac{d}{dt}\mathrm{v}_{rel}$$

Remember

$$\frac{d}{dt}\mathrm{r}_{rel} = (\varpi \times \mathrm{r}_{rel}) + \mathrm{v}_{rel}$$

$$\frac{d}{dt}\mathrm{v}_{rel} = (\varpi \times \mathrm{v}_{rel}) + \mathrm{a}_{rel}$$

Then,

$$\mathrm{a}_{p} = \mathrm{a}_{B} + (\dot{\varpi} \times \mathrm{r}_{rel}) + \varpi \times \frac{d}{dt}\mathrm{r}_{rel} + \frac{d}{dt}\mathrm{v}_{rel}$$

Finally,

$$\mathrm{a}_{p} = \mathrm{a}_{B} + (\dot{\varpi} \times \mathrm{r}_{rel}) + \varpi \times (\varpi \times \mathrm{r}_{rel}) + 2(\varpi \times \mathrm{v}_{rel}) + \mathrm{a}_{rel}$$

This is based only on kinematics !

Absolute frame (Inertial frame : OXYZ)

~ Moving frame(Non-inertal frame : oxyz)

Keep in mind : Both observing the same particle (Remenber Fig.1.28)

Coriolis force :

(PLANA MOTION OF RIGID BODY)

VIRTUAL WORK

Method of Virtual Work :
Actual ?

What is virtual (δ **) ?   What is actual (d) ?**

What is statics ?   What is dynamics ?

Time is always involved ! (implicit or explicit)

Mechanics:

- **o Vector Mechanics : Free body diagram for isolated body**

  ~ Reaction should be involved ! Force ! ~ Vector !

- **o Analytical Mechanics**

  ~ System as a whole !