Nucleus and Nuclear Radiation

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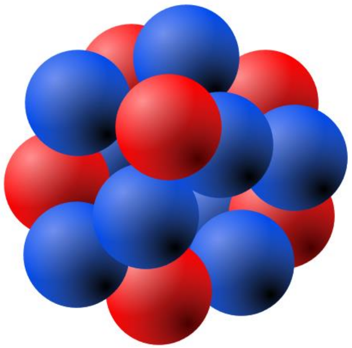
Atomic nucleus

- The atomic nucleus is the small, dense region consisting of protons and neutrons at the center of an atom, discovered in 1911 by Ernest Rutherford based on the 1909 Geiger–Marsden gold foil experiment.
- The nucleus of an atom of atomic number Z and mass number A consists of Z protons and N = A – Z neutrons.
- The atomic masses of all individual atoms are nearly integers, and A gives the total number of nucleons (i.e., protons and neutrons) in the nucleus.
- A species of atom, characterized by its nuclear constitution—its values of Z and A (or N)—is called a nuclide.
- Nuclides of an element that have different A (or N) are called isotopes; nuclides having the same number of neutrons are called isotones.

${}^{1}_{1}H$, ${}^{2}_{1}H$, and ${}^{3}_{1}H$,

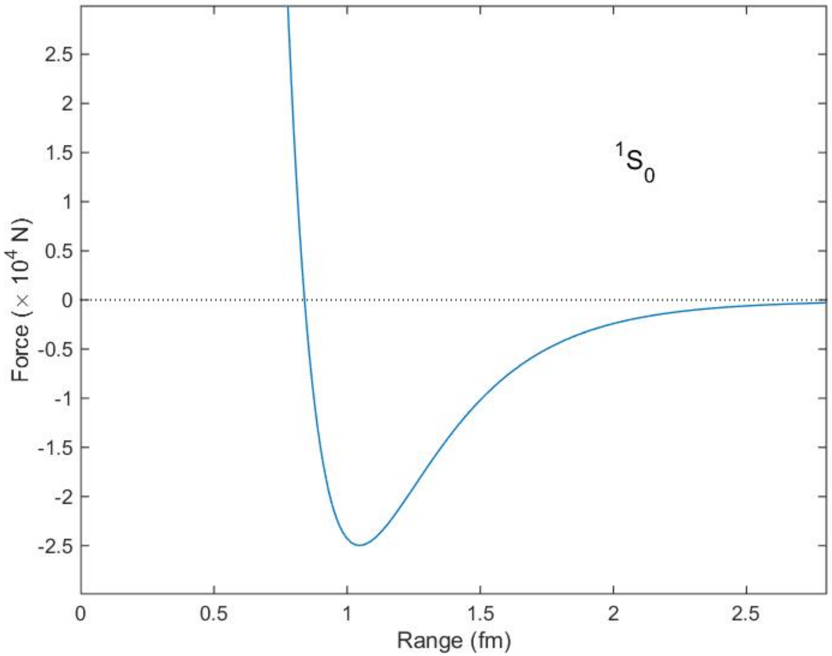
${}^{206}_{82}Pb$ and ${}^{204}_{80}Hg$



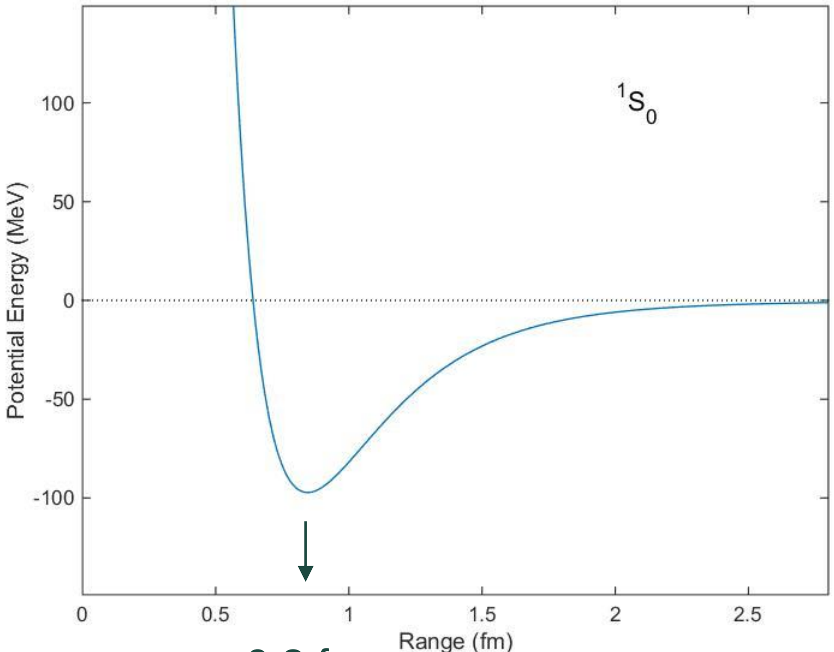
Nuclear potential

- Nucleons are bound together in a nucleus by the action of the strong, or nuclear force. The range of this force is only of the order of nuclear dimensions, ~10^{-15} m, and it is powerful enough to overcome the Coulomb repulsion of the protons in the nucleus.

Force between two nucleons (p-n) as a function of distance



Corresponding potential energy of two nucleons as a function of distance



0.8 fm

Nuclear dimensions

- The volume of a nucleus is directly proportional to the number of nucleons it contains, which is its mass number A. This suggests that the density of nucleons is very nearly the same in the interiors of all nuclei.
- An effective spherical nuclear radius is

$$R = R_0 A^{1/3} \quad (R_0 \approx 1.2 \times 10^{-15}\ \mathrm{m})$$

- The mass density of a nucleus is nearly constant due to short range nature of nuclear force

$$\rho_{nucleus} = \frac{m_{nucleus}}{V_{nucleus}} = \frac{A/N_A}{(4/3)\pi R^3}$$

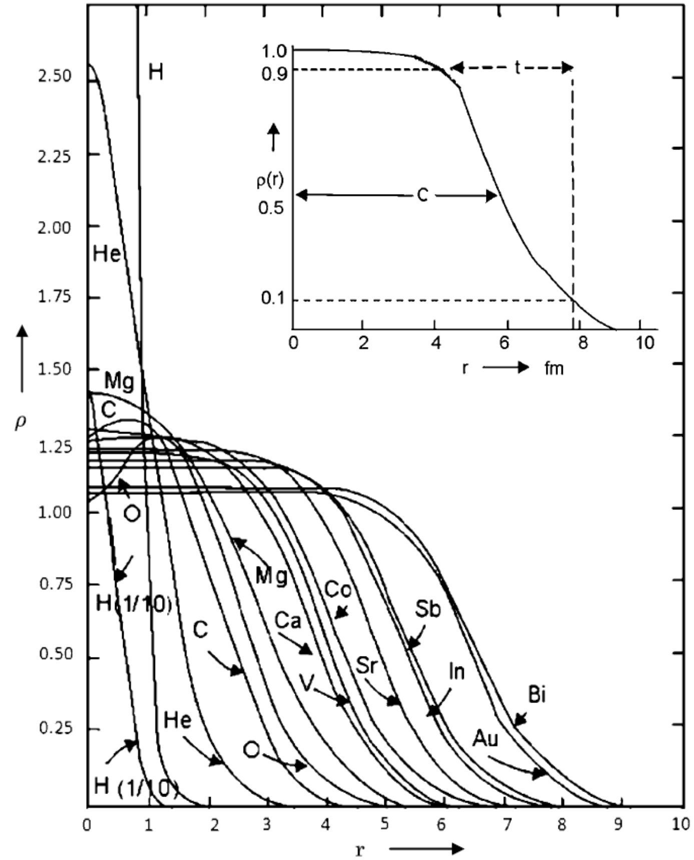
$$\approx 2.3 \times 10^{14}\,\frac{\mathrm{g}}{\mathrm{cm}^3} \approx \text{constant}$$

- Fermi model

$$\rho(r) = \frac{\rho(0)}{1 + e^{\frac{r-c}{b}}}$$

$$c = 1.07 \times A^{1/3} \times 10^{-13}\ \mathrm{cm}$$

$$t = 4.4b$$



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Atomic mass

- Atomic masses refer to the masses of neutral atoms, not of bare nuclei. Thus, an atomic mass always includes the masses of its Z electrons.
- Atomic masses are expressed in mass units (u or amu), which are so defined that the mass of a ${}^{12}_{6}C$ atom, the most abundant isotope of carbon, is exactly 12 amu.

$$1\ \text{AMU} = 1/(6.02 \times 10^{23}) = 1.66 \times 10^{-24}\ \text{g} = 1.66 \times 10^{-27}\ \text{kg}.$$

Using the Einstein relation and $c = 3 \times 10^{8}\ \text{m s}^{-1}$, we obtain

$$\begin{aligned} 1\ \text{AMU} &= (1.66 \times 10^{-27})(3 \times 10^{8})^{2} \\ &= 1.49 \times 10^{-10}\ \text{J} \\ &= \frac{1.49 \times 10^{-10}\ \text{J}}{1.6 \times 10^{-13}\ \text{J MeV}^{-1}} = 931\ \text{MeV}. \end{aligned}$$

More precisely, $\underline{1\ \text{AMU} = 931.49\ \text{MeV.}}$

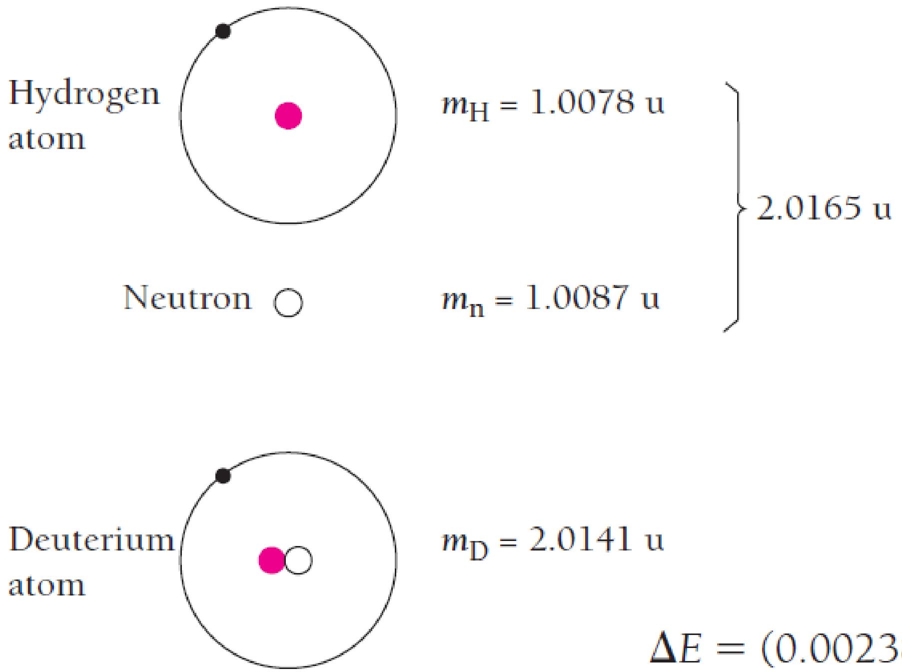
Nuclear mass

- An atom of ${}^{A}_{Z}X$ is formed by combining Z electrons with its nucleus to form the neutral atom. As these electrons bind to the nucleus, energy is emitted equal to the binding energy of the electrons.
- To remove all the electrons from the atom would thus require an ionization energy of BE_{Ze}. This electron binding energy comes from a decrease in the mass of the atom compared to the sum of nuclear and electron masses, by an amount BE_{Ze}/c^2. Thus, atomic and nuclear masses are related by

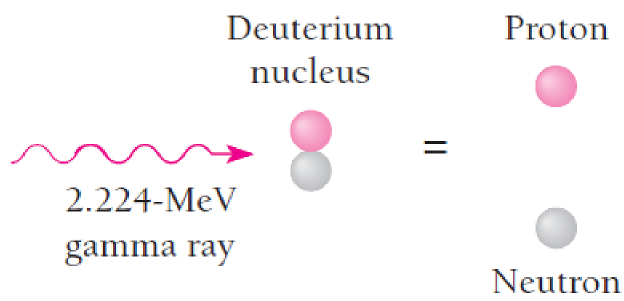
$$\underbrace{m\left({}^{A}_{Z}X\right)}_{\text{Atomic mass}} = \underbrace{m\left({}^{A}_{Z}X \text{ nucleus}\right)}_{\text{Nuclear mass}} + Zm_e - \frac{\mathrm{BE}_{ze}}{c^2}$$

- The binding energy term in this relation is often neglected since it is always very small compared to the other terms. For example, it requires 13.6 eV to ionize the hydrogen atom. This electron binding energy represents a mass change of BE_{1e}/c^2 = 13.6 (eV)/931.49 (MeV/u) = 1.4×10^{-8} u. This mass change is negligible compared to the mass of the hydrogen atom and nucleus (each about 1 u) and even the electron (about 5.5×10^{-4} u).

Missing mass



$$^1_0\mathrm{n} + {}^1_1\mathrm{H} \rightarrow {}^2_1\mathrm{H}$$



$$\Delta E = (0.002388 \text{ u})(931.49 \text{ MeV/u}) = 2.224 \text{ MeV}$$

- The "missing" mass (0.002388 u) correspond to energy given off when a $^2_1\mathrm{H}$ nucleus is formed from a free proton and neutron.

Particle	Mass (kg)	Mass (u)	Mass (MeV/c^2)
Proton	1.6726×10^{-27}	1.007276	938.28
Neutron	1.6750×10^{-27}	1.008665	939.57
Electron	9.1095×10^{-31}	5.486×10^{-4}	0.511
$^1_1$H atom	1.6736×10^{-27}	1.007825	938.79

Binding energy

- To examine the energies involved with nuclear forces in the nucleus, consider the binding energy of a nucleus composed of Z protons and N = A − Z neutrons. The formation of such a nucleus from its constituents is described by the reaction

$$Z \text{ protons } + (A-Z) \text{ neutrons} \longrightarrow \text{nucleus}({}^{A}_{Z}\mathrm{X}) + \mathrm{BE}$$

- The binding energy is determined from the change of mass between the left- and right-hand sides of the reaction, i.e.,

$$\text{Mass defect} = \frac{\mathrm{BE}}{c^2} = Zm_p + Nm_n - m\left({}^{A}_{Z}X \text{ nucleus}\right)$$

- In terms of atomic masses, we obtain the binding energy as follows.

$$\text{Mass defect} = \frac{\mathrm{BE}}{c^2} = Zm_p + Nm_n - m\left({}^{A}_{Z}X\right) + Zm_e - \frac{\mathrm{BE}_{ze}}{c^2}$$

$$\mathrm{BE}\left({}^{A}_{Z}X\right) = \left[Zm({}^{1}_{1}\mathrm{H}) + Nm_n - m\left({}^{A}_{Z}X\right)\right]c^2$$

Binding energy

- The binding energy E_b in MeV of the nucleus ${}^{A}_{Z}X$, which has $N = A - Z$ neutrons, is given by

$$E_b = [Zm({}^{1}_{1}\mathrm{H}) + Nm(n) - m({}^{A}_{Z}X)](931.49\ \mathrm{MeV/u})$$

where $m({}^{1}_{1}\mathrm{H})$ is the atomic mass of ${}^{1}_{1}\mathrm{H}$, $m(n)$ is the neutron mass, and $m({}^{A}_{Z}X)$ is the atomic mass of ${}^{A}_{Z}X$, all in mass units. As mentioned before, atomic masses, not nuclear masses, are used in such calculations; the electron masses subtract out.

Example 11.4

The binding energy of the neon isotope ${}^{20}_{10}\mathrm{Ne}$ is 160.647 MeV. Find its atomic mass.

Solution

Here $Z = 10$ and $N = 10$. From Eq. (11.7),

$$m({}^{A}_{Z}X) = [Zm({}^{1}_{1}\mathrm{H}) + Nm(n)] - \frac{E_b}{931.49\ \mathrm{MeV/u}}$$

$$m({}^{20}_{10}\mathrm{Ne}) = [10(1.007825\ \mathrm{u}) + 10(1.008665)] - \frac{160.647\ \mathrm{MeV}}{931.49\ \mathrm{MeV/u}} = 19.992\ \mathrm{u}$$

Binding energy

- Mass excess $\Delta = m({}^{A}_{Z}X) - A$

$$ {}^{1}_{0}\mathrm{n} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + {}^{0}_{0}\gamma $$

$$ Q = 8.0714 + 7.2890 - 13.1359 = 2.2245 \text{ MeV} $$

Example

Find the binding energy of the nuclide ${}^{24}_{11}\mathrm{Na}$.

Solution

One can work in terms of either AMU or MeV. The atom consists of 11 protons, 13 neutrons, and 11 electrons. The total mass in AMU of these separate constituents is, with the help of the data in Appendix A,

$$ 11(1.0073) + 13(1.0087) + 11(0.00055) = 24.199 \text{ AMU.} \tag{3.9} $$

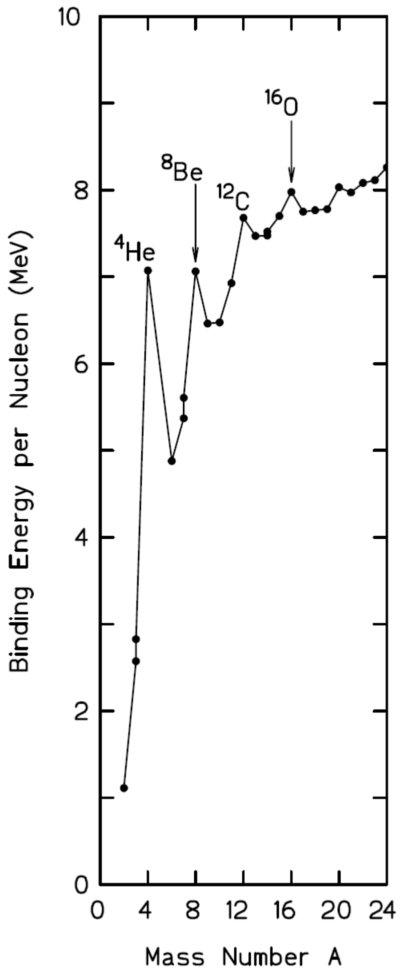
From Appendix D, $\Delta = -8.418$ MeV gives the difference $M - A$. Thus, the mass of the ${}^{24}_{11}\mathrm{Na}$ nuclide is less than 24 by the amount 8.418 MeV/(931.49 MeV AMU^{-1}) = 0.0090371 AMU. Therefore, the nuclide mass is $M = 23.991$ AMU. Comparison with (3.9) gives for the binding energy

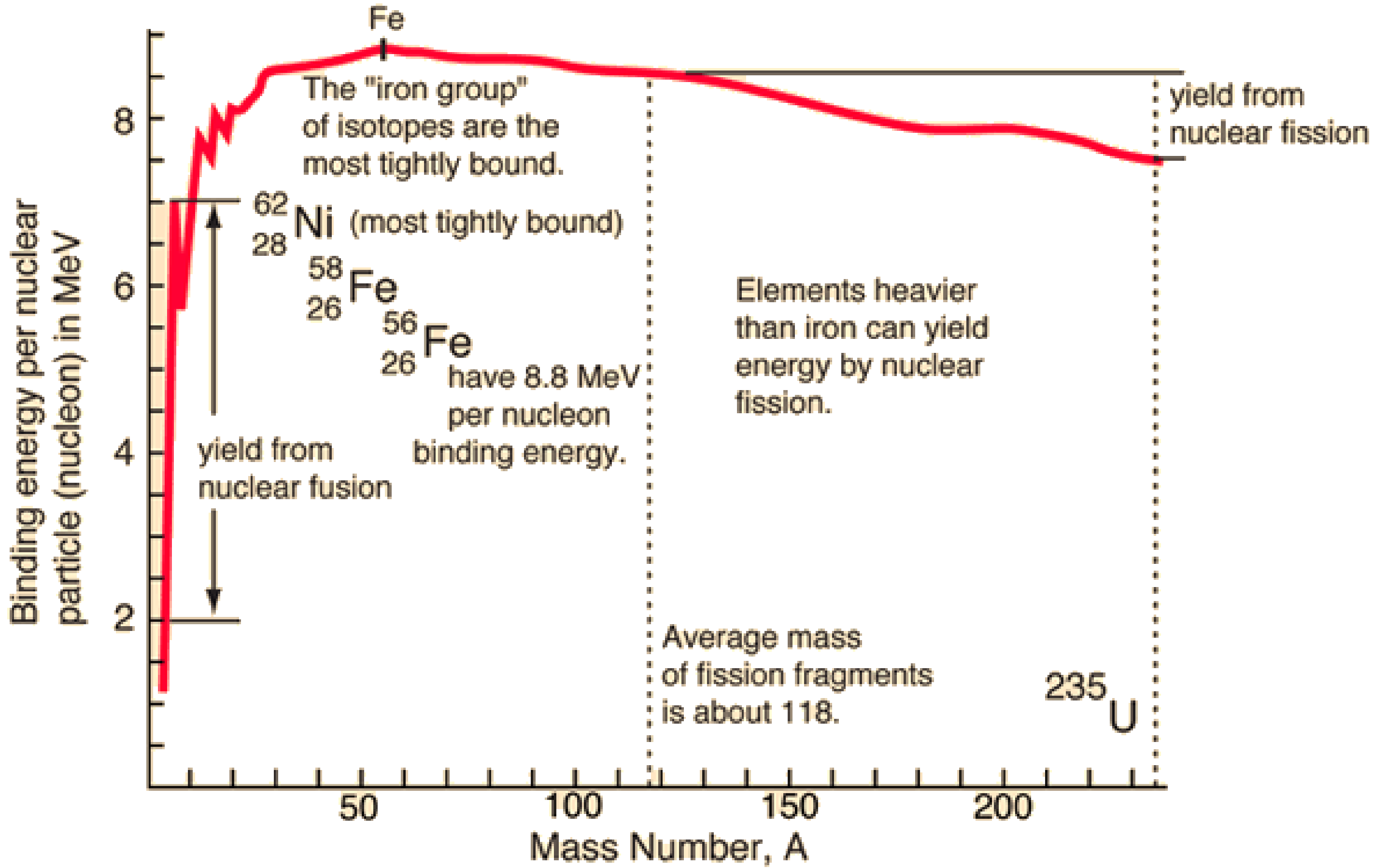
$$ \mathrm{BE} = 24.199 - 23.991 = 0.208 \text{ AMU} = 194 \text{ MeV.} \tag{3.10} $$

Appendix D

Nuclide	Natural Abundance (%)	Mass Difference $\Delta = M - A$ (MeV) (at. mass – at. mass No.)
${}^{1}_{0}\mathrm{n}$	—	8.0714
${}^{1}_{1}\mathrm{H}$	99.985	7.2890
${}^{2}_{1}\mathrm{H}$	0.015	13.1359
${}^{3}_{1}\mathrm{H}$	—	14.9500
${}^{3}_{2}\mathrm{He}$	0.00013	14.9313
${}^{4}_{2}\mathrm{He}$	99.99+	2.4248
${}^{6}_{3}\mathrm{Li}$	7.42	14.088
${}^{7}_{3}\mathrm{Li}$	92.58	14.907
${}^{7}_{4}\mathrm{Be}$	—	15.769
${}^{10}_{5}\mathrm{B}$	19.7	12.052
${}^{11}_{6}\mathrm{C}$	—	10.648
${}^{12}_{6}\mathrm{C}$	98.892	0
${}^{14}_{6}\mathrm{C}$	—	3.0198
${}^{14}_{7}\mathrm{N}$	99.635	2.8637
${}^{15}_{7}\mathrm{N}$	0.356	0.100
${}^{16}_{8}\mathrm{O}$	99.759	−4.7366
${}^{17}_{8}\mathrm{O}$	0.037	−0.808
${}^{22}_{10}\mathrm{Ne}$	8.82	−8.025

Binding energy per nucleon (E_b/A)





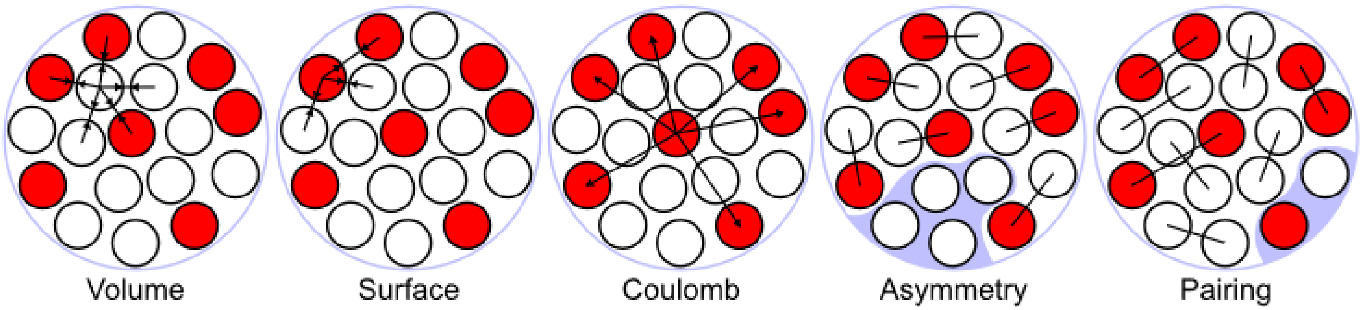
Liquid-drop model

- The liquid drop model, was first proposed by George Gamow and further developed by Niels Bohr and John Archibald Wheeler.
- It treats the nucleus as a drop of incompressible fluid of very high density, held together by the nuclear force (a residual effect of the strong force), there is a similarity to the structure of a spherical liquid drop.
- While a crude model, the liquid drop model accounts for the spherical shape of most nuclei, and makes a rough prediction of binding energy.
- Semi-empirical binding energy formula

$$E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} (\pm, 0) \frac{a_5}{A^{3/4}}$$

$a_1 = 14.1$ MeV $\quad a_2 = 13.0$ MeV $\quad a_3 = 0.595$ MeV

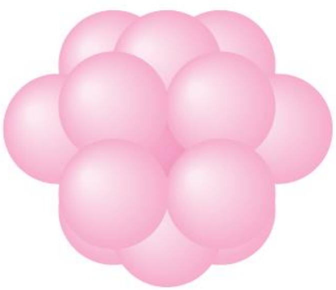
$a_4 = 19.0$ MeV $\quad a_5 = 33.5$ MeV

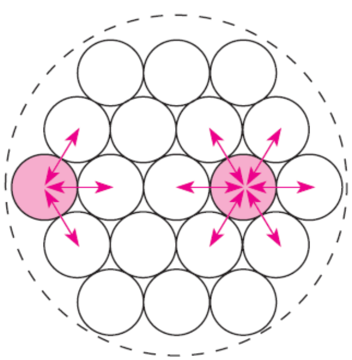


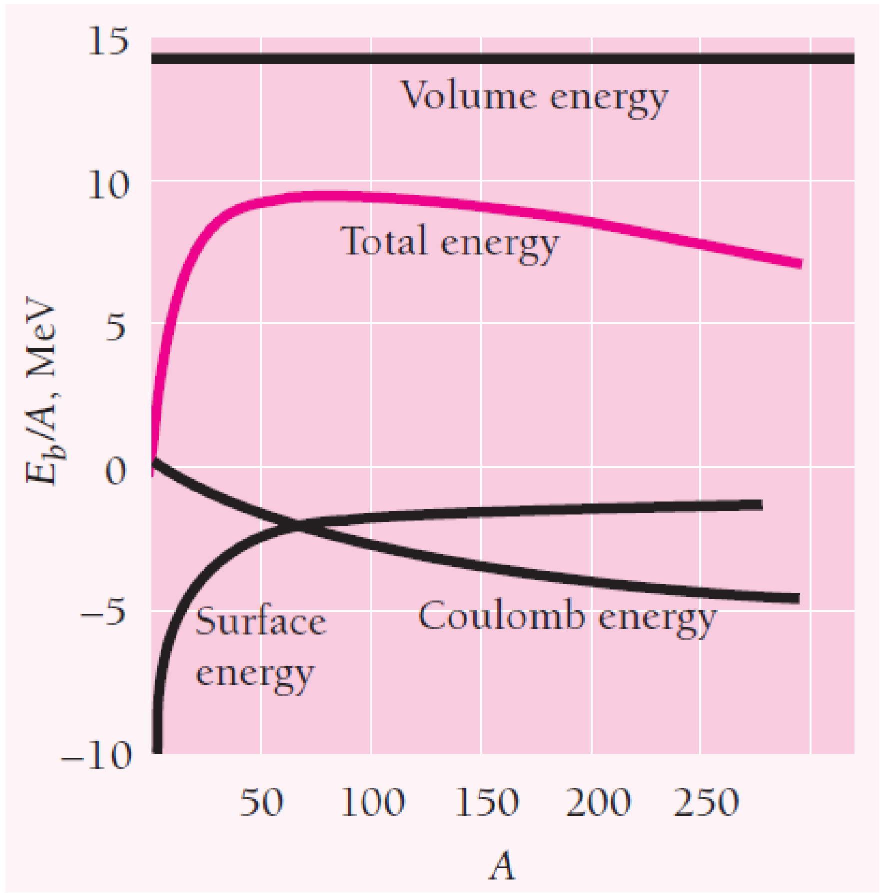
Liquid-drop model

- The semi-empirical binding energy per nucleon curve (first 3 terms only)

$$\frac{E_b}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3\frac{Z(Z-1)}{A^{4/3}}$$







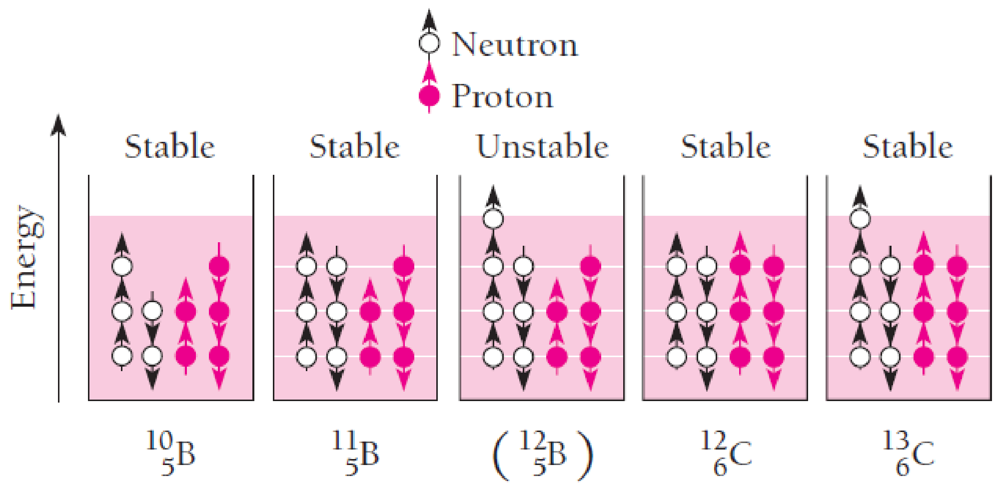
$$E_v = a_1 A$$

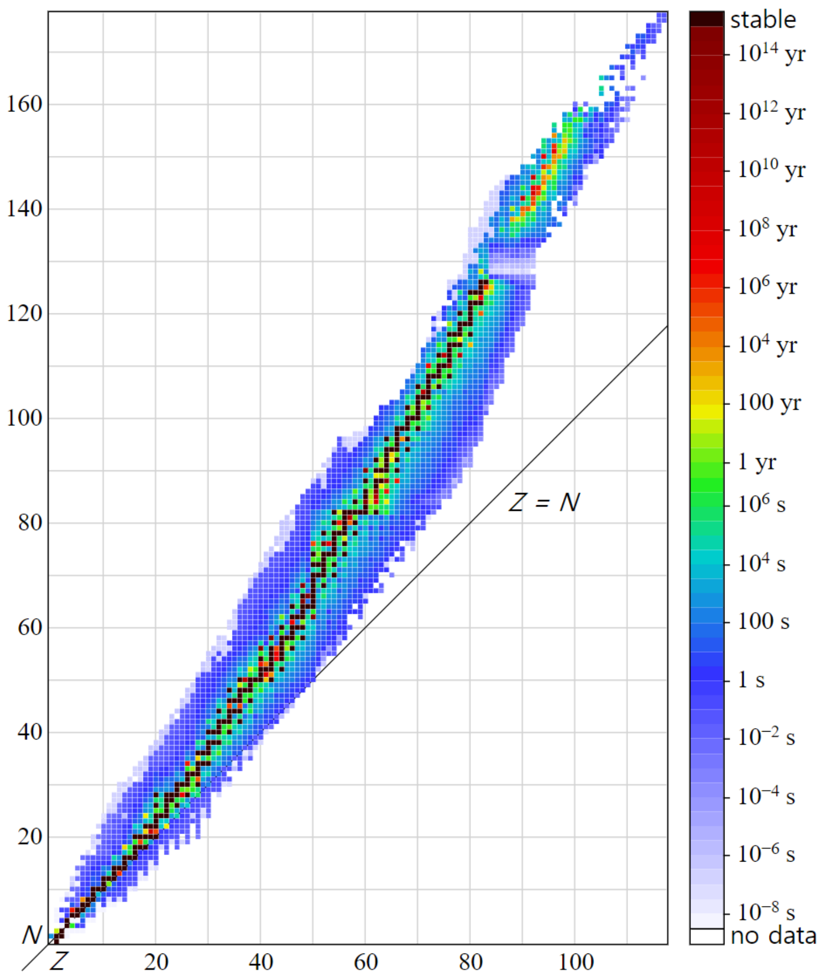
$$E_s = -a_2 A^{2/3}$$

$$E_c = -a_3\frac{Z(Z-1)}{A^{1/3}}$$

Chart of nuclides

- The tendency for N to equal Z follows from the existence of nuclear energy levels.
- Energy levels in nuclei are filled in sequence, just as energy levels in atoms are, to achieve configurations of minimum energy and therefore maximum stability.
- Sixty percent of stable nuclides have both even Z and even N; these are called “even-even” nuclides.





http://atom.kaeri.re.kr/nuchart/

Nuclear reactions

- Nuclear reactions play a very important role in nuclear science and engineering, since it is through such reactions that various types of radiation are produced or detected, or information about the internal structure of a nucleus is gained. There are two main categories of nuclear reactions.

- Radioactive decay: the initial reactant X is a single atom or nucleus that spontaneously changes by emitting one or more particles, i.e.,

$$X \longrightarrow Y_1 + Y_2 + \cdots$$

- Binary reactions: two nuclear particles (nucleons, nuclei or photons) interact to form different nuclear particles. The most common types of such nuclear reactions are those in which some nucleon or nucleus x moves with some kinetic energy and strikes and interacts with a nucleus X to form a pair of product nuclei y and Y , i.e.,

$$x + X \longrightarrow Y + y$$

$$X(x, y)Y$$

Examples of binary nuclear reactions

(α, n) reaction: In 1932, Chadwick discovered the neutron by bombarding beryllium with alpha particles to produce neutrons from the reaction

$$ {}^{4}_{2}\mathrm{He} + {}^{9}_{4}\mathrm{Be} \longrightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n} \qquad \text{or} \qquad {}^{9}_{4}\mathrm{Be}(\alpha, \mathrm{n}){}^{12}_{6}\mathrm{C}. $$

(γ, n) reaction: Energetic photons (gamma rays) can also interact with a nucleus. For example, neutrons can be produced by irradiating deuterium with sufficiently energetic photons according to the reaction

$$ \gamma + {}^{2}_{1}\mathrm{H} \longrightarrow {}^{1}_{1}\mathrm{H} + {}^{1}_{0}\mathrm{n} \qquad \text{or} \qquad {}^{2}_{1}\mathrm{H}(\gamma, n){}^{1}_{1}\mathrm{H} \qquad \text{or} \qquad {}^{2}_{1}\mathrm{H}(\gamma, \mathrm{p}){}^{1}_{0}\mathrm{n}. $$

(p, γ) reaction: Protons can cause nuclear reactions such as the radiative capture of a proton by ${}^{7}\mathrm{Li}$, namely,

$$ {}^{1}_{1}\mathrm{H} + {}^{7}_{3}\mathrm{Li} \longrightarrow {}^{8}_{4}\mathrm{Be} + \gamma \qquad \text{or} \qquad {}^{7}_{3}\mathrm{Li}(\mathrm{p}, \gamma){}^{8}_{4}\mathrm{Be}. $$

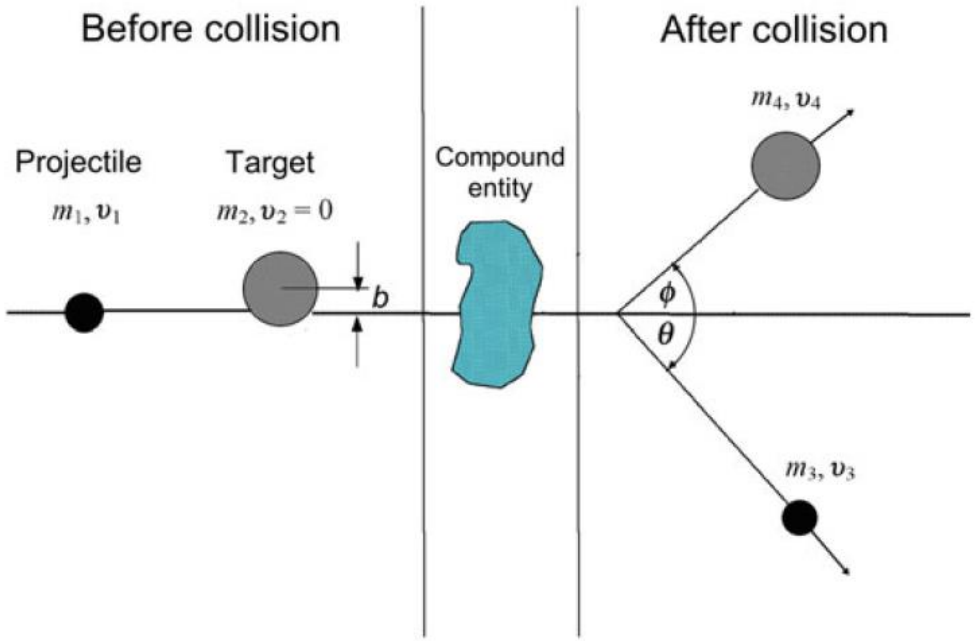
The product nucleus ${}^{8}_{4}\mathrm{Be}$ is not bound and breaks up (radioactively decays) almost immediately into two alpha particles, i.e.,

$$ {}^{8}_{4}\mathrm{Be} \longrightarrow {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He}. $$

$(\gamma, \alpha\mathrm{n})$ reaction: As an example of a reaction in which more than two products are produced, a high-energy photon can cause ${}^{17}\mathrm{O}$ to split into ${}^{12}\mathrm{C}$, an α particle, and a neutron through the reaction

$$ \gamma + {}^{17}_{8}\mathrm{O} \longrightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} \qquad \text{or} \qquad {}^{17}_{8}\mathrm{O}(\gamma, \mathrm{n}\alpha){}^{12}_{6}\mathrm{C}. $$

Kinematics of nuclear reactions



- In any nuclear reaction a number of physical quantities must be conserved, most notably: *charge*, *linear momentum* and *mass-energy*.
- In addition, the sum of atomic numbers Z and the sum of atomic mass numbers A for before and after the collision must also be conserved, i.e.,

$$\sum Z \text{ (before collision)} = \sum Z \text{ (after collision)}$$

and

$$\sum A \text{ (before collision)} = \sum A \text{ (after collision)}.$$

Kinematics of nuclear reactions

- Conservation of momentum

The conservation of momentum in a two-particle nuclear collision is expressed through the vector relationship

$$m_1 \boldsymbol{v}_1 = m_3 \boldsymbol{v}_3 + m_4 \boldsymbol{v}_4, \tag{5.1}$$

that can be resolved into a component along the incident direction and a component perpendicular to the incident direction to obtain

$$m_1 v_1 = m_3 v_3 \cos\theta + m_4 v_4 \cos\phi \tag{5.2}$$

and

$$0 = m_3 v_3 \sin\theta - m_4 v_4 \sin\phi, \tag{5.3}$$

where the angles θ and ϕ are defined in Fig. 5.1 and v_1, v_3, and v_4 are magnitudes of velocity vectors $\boldsymbol{v}_1$, $\boldsymbol{v}_3$, and $\boldsymbol{v}_4$, respectively.

Kinematics of nuclear reactions

- Conservation of energy

The total energy of the projectile m_1 and target m_2 before the interaction (collision) must equal to the total energy of reaction products m_3 and m_4 after the collision

$$\left\{m_{10}c^2 + (E_\mathrm{K})_\mathrm{i}\right\} + \left(m_{20}c^2 + 0\right) = \left\{m_{30}c^2 + (E_\mathrm{K})_3\right\} + \left\{m_{40}c^2 + (E_\mathrm{K})_4\right\}, \quad (5.4)$$

where

$m_{10}c^2$	is the rest energy of the projectile.
$m_{20}c^2$	is the rest energy of the target.
$m_{30}c^2$	is the rest energy of the reaction product m_3.
$m_{40}c^2$	is the rest energy of the reaction product m_4.
$(E_\mathrm{K})_\mathrm{i}$	is the kinetic energy of the projectile (incident particle).
$(E_\mathrm{K})_3$	is the kinetic energy of the reaction product m_3.
$(E_\mathrm{K})_4$	is the kinetic energy of the reaction product m_4.

Kinematics of nuclear reactions

- Conservation of energy

Inserting into (5.4) the so-called Q value for the collision in the form

$$Q = \left(m_{10}c^2 + m_{20}c^2\right) - \left(m_{30}c^2 + m_{40}c^2\right), \tag{5.5}$$

we get the following relationship for the conservation of energy

$$(E_\mathrm{K})_\mathrm{i} + Q = (E_\mathrm{K})_3 + (E_\mathrm{K})_4 . \tag{5.6}$$

Each two-particle collision possesses a characteristic Q value that can be either positive, zero, or negative.

- For $Q > 0$, the collision is *exothermic* (also called exoergic) and results in release of energy.
- For $Q = 0$, the collision is termed *elastic*.
- For $Q < 0$, the collision is termed *endothermic* (also called endoergic) and, to take place, it requires an energy transfer from the projectile to the target.

$$Q = \sum_{i,\text{before}} M_i c^2 - \sum_{i,\text{after}} M_i c^2. \qquad Q = \sum_{i,\text{after}} E_\mathrm{B}(i) - \sum_{i,\text{before}} E_\mathrm{B}(i).$$

Nuclear decay

Alpha decay

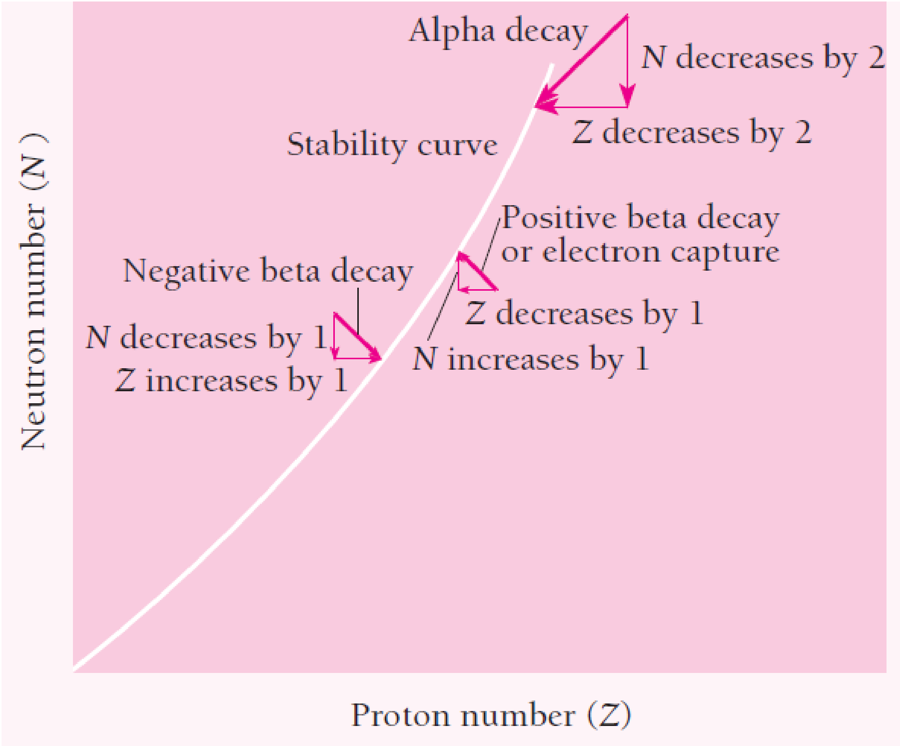
$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He $$

Parent nucleus → Daughter nucleus + Alpha particle

Beta decay $\quad n^0 \rightarrow p^+ + e^-$

Positron emission $\quad p^+ \rightarrow n^0 + e^+$

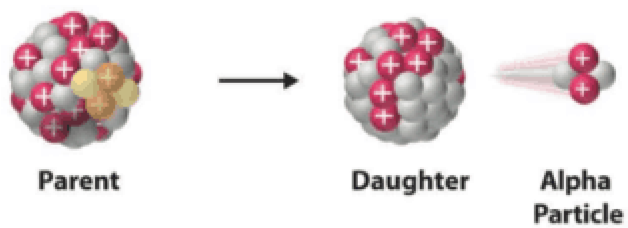
Electron capture $\quad p^+ + e^- \rightarrow n^0$



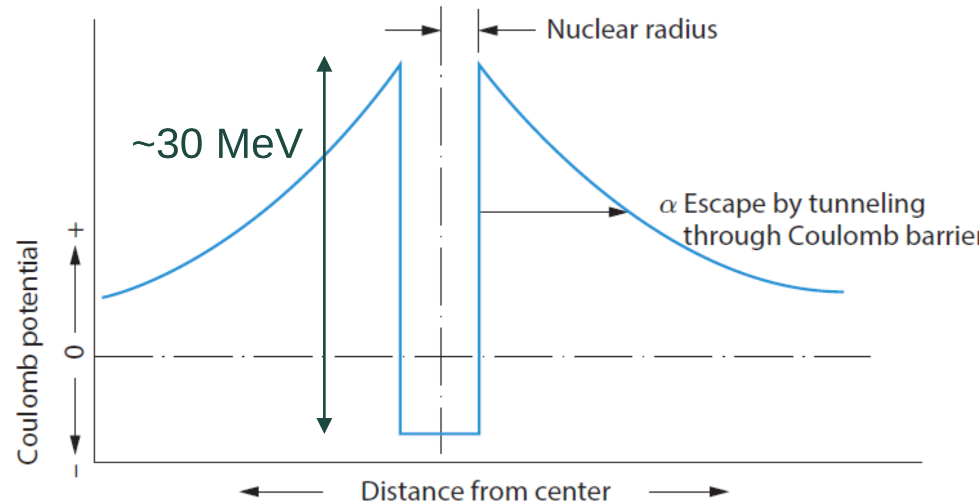
Alpha decay

- In alpha decay the number of protons and neutrons is conserved by producing a ^{4}He nucleus (α-particle) and lowering the parent's A and Z by 4 and 2, respectively, i.e.,

$$ {}^{A}_{Z}\mathrm{P} \rightarrow {}^{A-4}_{Z-2}\mathrm{D} + {}^{4}_{2}\mathrm{He} = {}^{A-4}_{Z-2}\mathrm{D} + \alpha $$

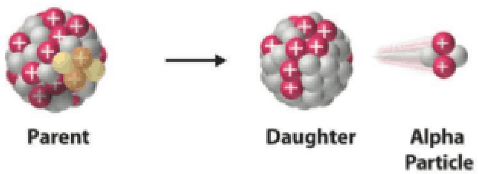


$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He $$



- The energetic alpha particle slows down in moving through the absorber medium and captures two electrons from its surroundings to become a neutral ^{4}He atom.
- Typical kinetic energies of alpha particles released by naturally occurring radionuclides are between 4 MeV and 9 MeV, corresponding to a range in air of about 1 cm to 10 cm, respectively, and in tissue of about 10^{-3} cm and 10^{-2} cm, respectively.

Kinematics of alpha decay



$$^{A}_{Z}X \rightarrow \, ^{A-4}_{Z-2}Y + \, ^{4}_{2}He$$

$$^{226}_{88}\mathrm{Ra} \rightarrow \, ^{222}_{86}\mathrm{Rn} + \, ^{4}_{2}\mathrm{He}.$$

- The energy Q released in the decay arises from a net loss in the masses M_{Ra}, M_{Rn}, and M_{He} of the radium, radon, and helium nuclei:

$$Q = M_{\mathrm{Ra,N}} - M_{\mathrm{Rn,N}} - M_{\mathrm{He,N}}.$$

- In terms of atomic mass difference:

$$Q_\alpha = \Delta_{\mathrm{P}} - \Delta_{\mathrm{D}} - \Delta_{\mathrm{He}}. \quad \Rightarrow \quad Q = 23.69 - 16.39 - 2.42 = 4.88 \text{ MeV}.$$

- The Q value is shared by the alpha particle and the recoil radon nucleus, and we can calculate the portion that each acquires

$$mv = MV.$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}MV^2 = Q.$$

$$\Rightarrow$$

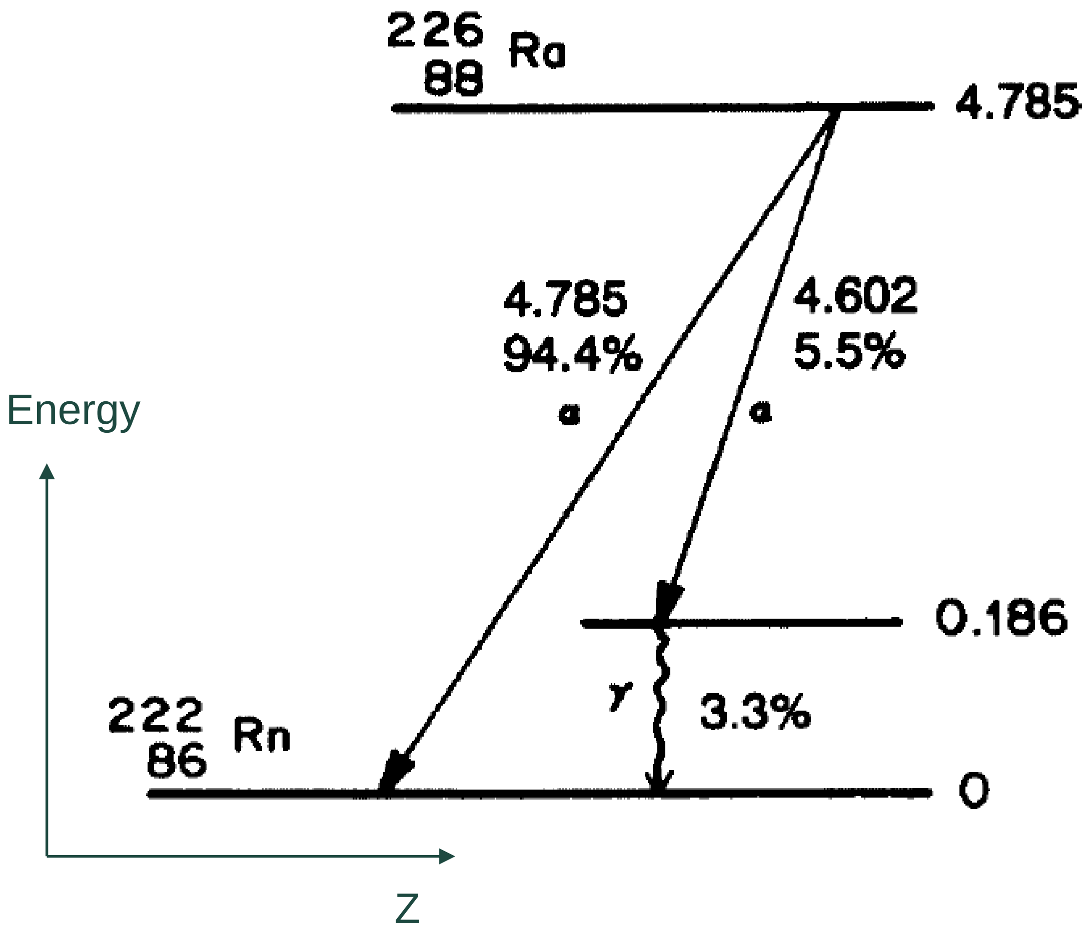
$$E_\alpha = \tfrac{1}{2}mv^2 = \frac{MQ}{m+M}.$$

$$E_\alpha = \frac{222 \times 4.88}{4 + 222} = 4.79 \text{ MeV}.$$

$$E_{\mathrm{N}} = \tfrac{1}{2}MV^2 = \frac{mQ}{m+M}. \quad = 0.09 \text{ MeV}$$

Decay-scheme diagram

$$ {}^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}. $$



Internal conversion = 2.2%
(Not shown in diagram)

Beta decay

- In beta decay, a nucleus simultaneously emits an electron, or negative beta particle, ${}_{-1}^{0}\beta$, and an antineutrino, ${}_{0}^{0}\bar{\nu}$.

$$ {}_{27}^{60}\mathrm{Co} \rightarrow {}_{28}^{60}\mathrm{Ni} + {}_{-1}^{0}\beta + {}_{0}^{0}\bar{\nu}. $$

- The energy Q released in the decay arises from a net loss in the masses M_{Co}, M_{Ni}, and one electron (m):

$$ Q = M_{\mathrm{Co,N}} - (M_{\mathrm{Ni,N}} + m). $$

- In terms of atomic mass difference:

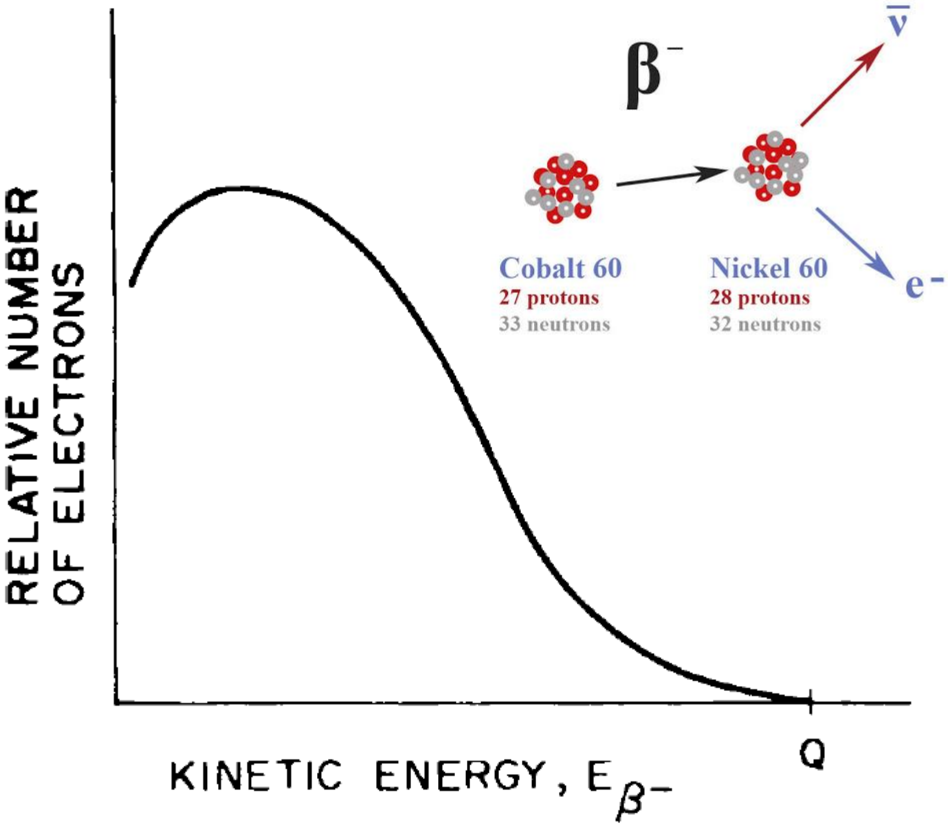
$$ Q_{\beta^-} = \Delta_{\mathrm{P}} - \Delta_{\mathrm{D}}. \quad \Rightarrow \quad Q = -61.651 - (-64.471) = 2.820 \text{ MeV}. $$

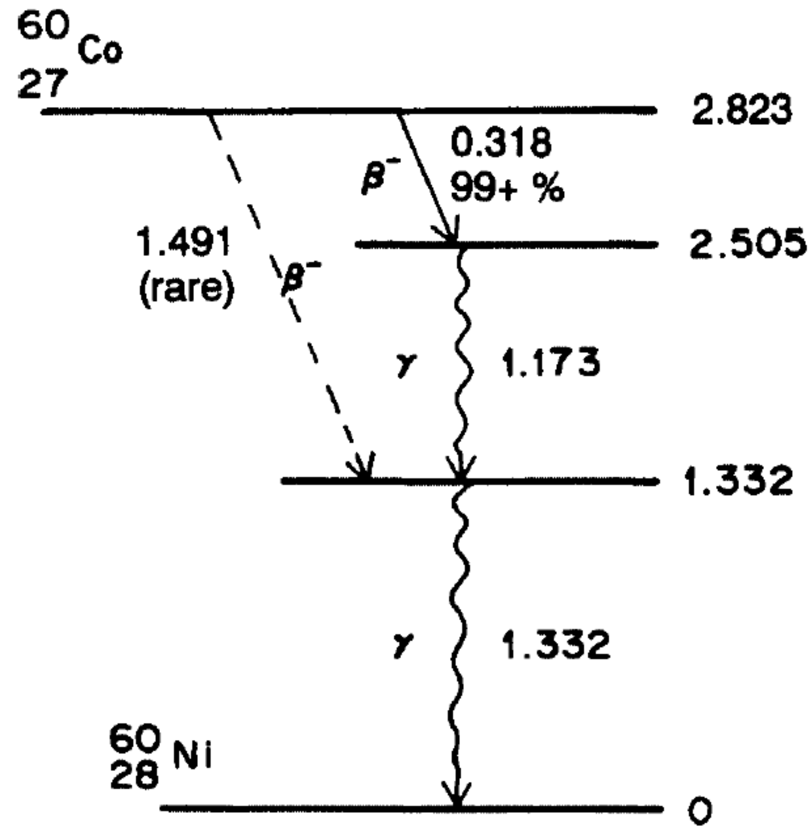
- The Q value is shared by the beta particle, antineutrino, and recoil ^{60}Ni nucleus.

$$ E_{\beta^-} + E_{\bar{\nu}} = Q, $$

Beta decay

- Depending on the relative directions of the momenta of the three decay products (β^-, $\bar{\nu}$, and recoil nucleus), E_{β^-} and $E_{\bar{\nu}}$ can each have any value between zero and Q. Thus the spectrum of beta-particle energies E_{β^-} is continuous, with $0 \le E_{\beta^-} \le Q$, in contrast to the discrete spectra of alpha particles.





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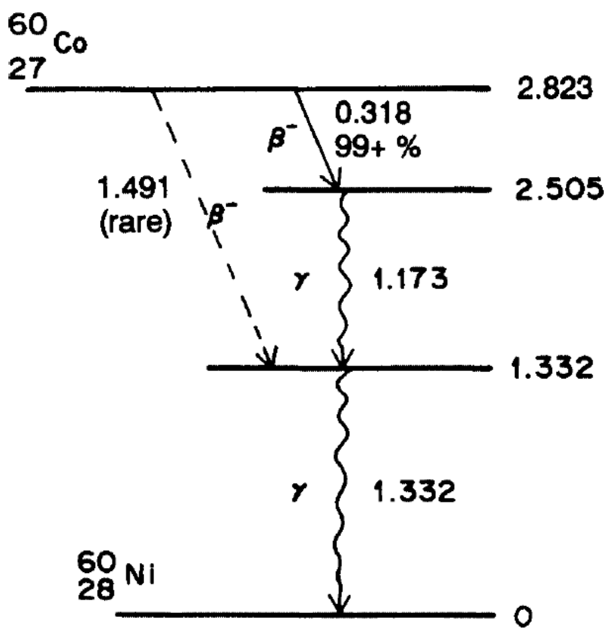
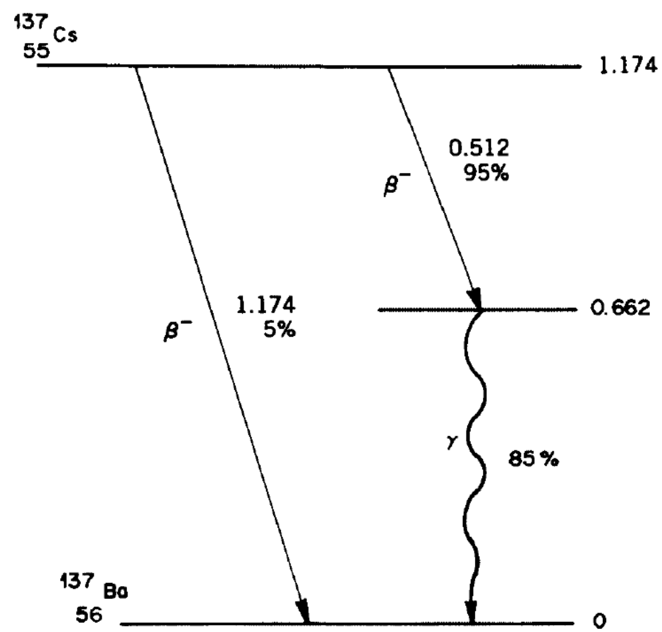
Gamma decay

- A nucleus can exist in states whose energies are higher than that of its ground state, just as an atom can. An excited nucleus is denoted by an asterisk after its usual symbol, for instance ${}^{87}_{38}Sr^*$.
- Excited nuclei return to their ground states (lifetime ~ 10^{-10} s) by emitting photons whose energies correspond to the energy differences between the various initial and final states in the transitions involved. The photons emitted by nuclei range in energy up to several MeV are traditionally called gamma rays.
- ^{60}Co gamma-rays: actually emitted by the daughter ^{60}Ni nucleus





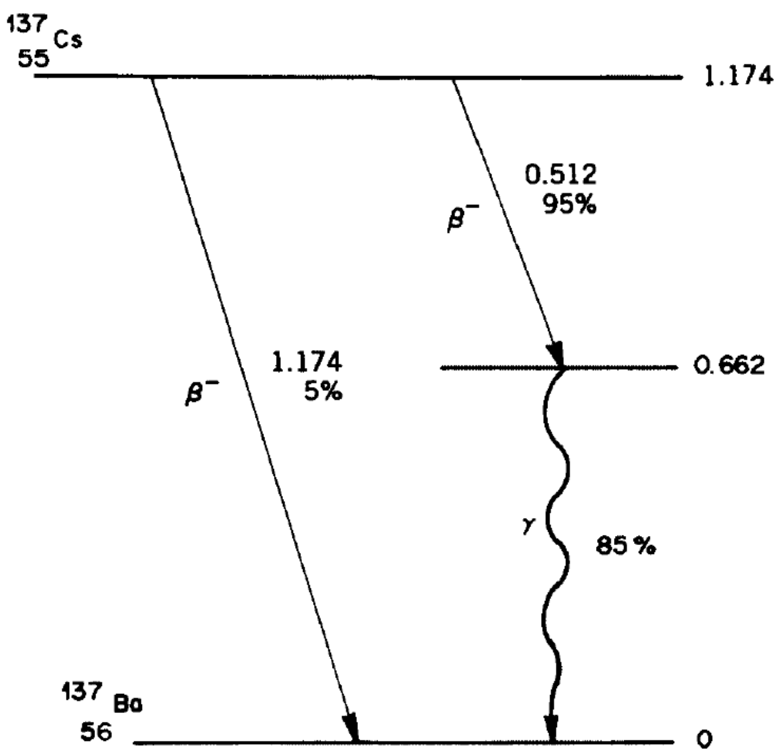
“metastable”

${}^{137m}_{56}\mathrm{Ba}$

($T_{1/2}$ ~ 2.55 min)

Gamma decay

$$^{137}_{55}\mathrm{Cs} \rightarrow {}^{137}_{56}\mathrm{Ba} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}.$$



Internal conversion = 10%
(Not shown in diagram)

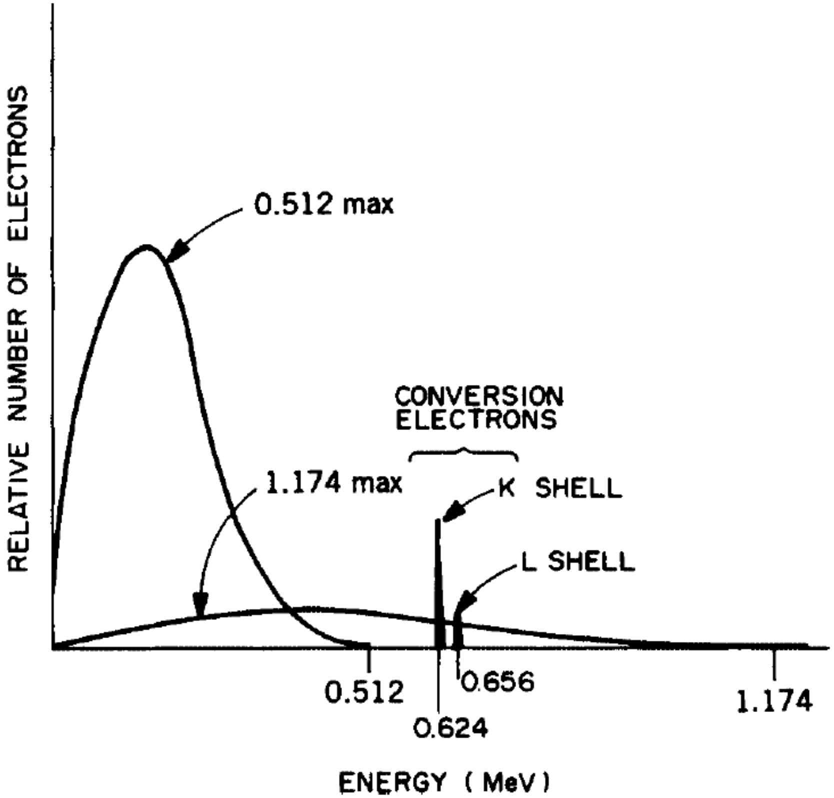


Fig. 3.9 Sources of electrons from $^{137}_{55}\mathrm{Cs}$ and their energy spectra. There are two modes of β^- decay, with maximum energies of 0.512 MeV (95%) and 1.174 MeV (5%). Internal-conversion electrons also occur, with discrete energies of 0.624 MeV (from the K shell) and 0.656 MeV (L shell) with a total frequency of 10%. See decay scheme in Fig. 3.8. The total spectrum of emitted electrons is the sum of the curves shown here.

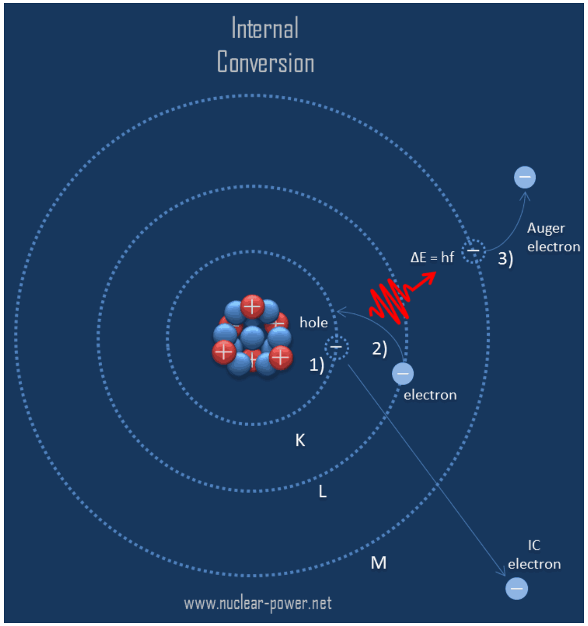
Internal conversion

- Internal conversion is the process in which the energy of an excited nuclear state is transferred to an atomic electron, most likely a K- or L-shell electron, ejecting it from the atom. It is an alternative to emission of a gamma photon from the nucleus. Do not confuse with photoelectric effect (external conversion).
- The kinetic energy of the ejected atomic electron is very nearly equal to the excitation energy E^* of the nucleus minus the binding energy E_B of the electron in its atomic shell:

$$E_e = E^* - E_B$$

- The internal conversion coefficient α for a nuclear transition is defined as the ratio of the number of conversion electrons N_e and the number of competing gamma photons N_γ for that transition:

$$\alpha = \frac{N_e}{N_\gamma} \sim \frac{Z^3}{E^*}$$



- Since internal conversion necessarily leaves an inner-atomic-shell vacancy, characteristic X-rays and/or Auger electrons of the daughter are always emitted.

Electron capture

- Some nuclei undergo a radioactive transformation by capturing an atomic electron, usually from the K shell, and emitting a neutrino.

$$^{103}_{46}\mathrm{Pd} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{103\mathrm{m}}_{45}\mathrm{Rh} + {}^{0}_{0}\nu.$$

- The captured electron releases its total mass, $m - E_B$, to the nucleus when it is absorbed there, E_B being the mass equivalent of the binding energy of the electron in the atomic shell. Therefore, the energy released is given by

$$Q = M_{\mathrm{Pd,N}} + m - E_{\mathrm{B}} - M_{\mathrm{m_{Rh}},\mathrm{N}}.$$

Q is acquired by the neutrino

- Since the palladium atom has one more electron than the rhodium atom, it follows that (neglecting the small difference in the electron binding energies) Q is equal to the difference in the two atomic masses, less the energy E_B.

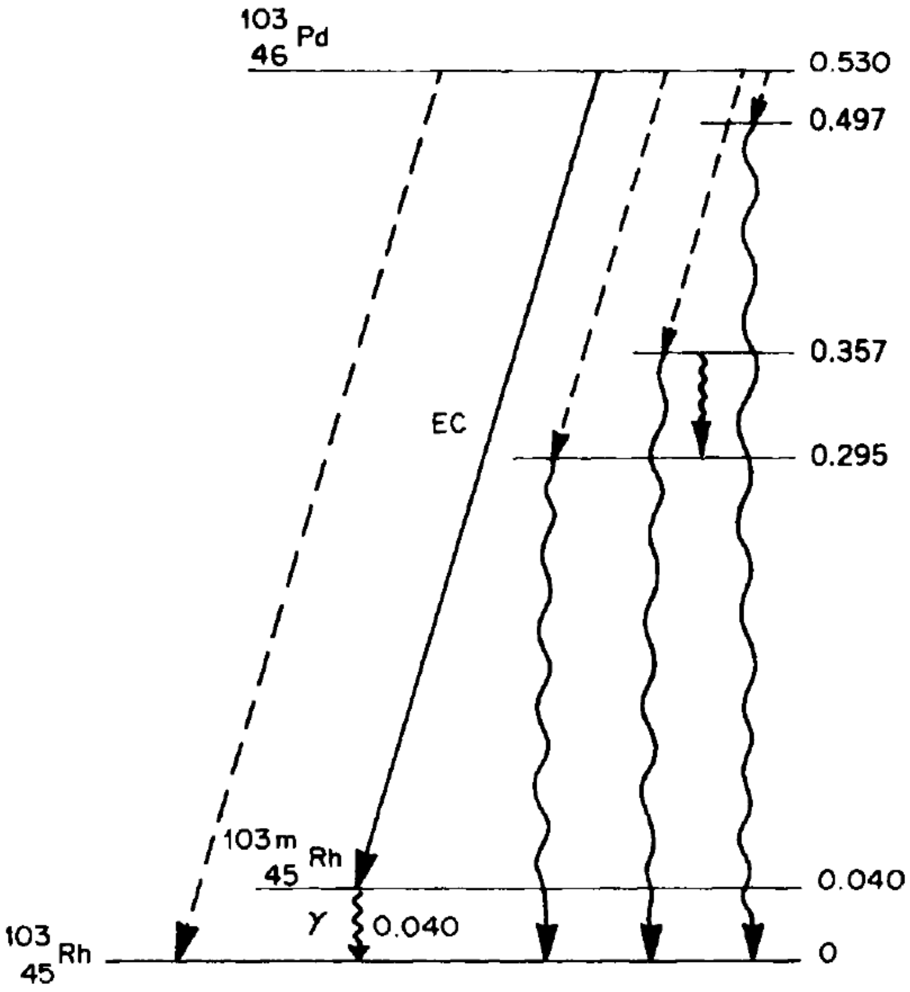
$$Q_{\mathrm{EC}} = \Delta_{\mathrm{P}} - \Delta_{\mathrm{D}} - E_{\mathrm{B}}. \quad \Rightarrow \quad Q = -87.46 - (-87.974) - 0.024 = 0.490\ \mathrm{MeV}.$$

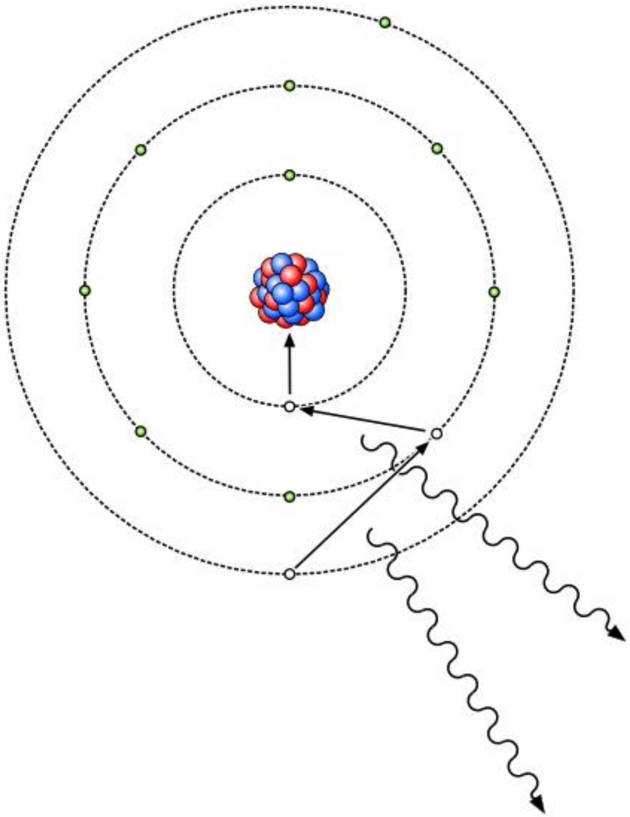
Orbital electron capture thus cannot take place unless $\Delta_{\mathrm{P}} - \Delta_{\mathrm{D}} > E_{\mathrm{B}}$.

- Since electron capture necessarily leaves an inner-atomic-shell vacancy, characteristic X-rays and/or Auger electrons of the daughter are always emitted.

Electron capture

$$^{103}_{46}\mathrm{Pd} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{103\mathrm{m}}_{45}\mathrm{Rh} + {}^{0}_{0}\nu.$$





Positron decay

- Some nuclei, such as ${}^{22}_{11}\mathrm{Na}$, disintegrate by emitting a positively charged electron (positron, β^+) and a neutrino:

$$ {}^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + {}^{0}_{1}\beta + {}^{0}_{0}\nu. $$

- Positron decay has the same net effect as electron capture, reducing Z by one unit and leaving A unchanged. The energy released is given in terms of the masses of the sodium and neon nuclei by

$$ Q = M_{\mathrm{Na,N}} - M_{\mathrm{Ne,N}} - m. $$

- In terms of atomic mass:

$$ Q = M_{\mathrm{Na,N}} + 11m - (M_{\mathrm{Ne,N}} + 10m) - 2m $$

$$ = M_{\mathrm{Na,A}} - M_{\mathrm{Ne,A}} - 2m, $$

- In terms of atomic mass difference:

$$ Q_{\beta^+} = \Delta_{\mathrm{P}} - \Delta_{\mathrm{D}} - 2mc^2. \quad \Rightarrow \quad Q_{\beta^+} = -5.182 - (-8.025) - 1.022 = 1.821\ \mathrm{MeV}. $$

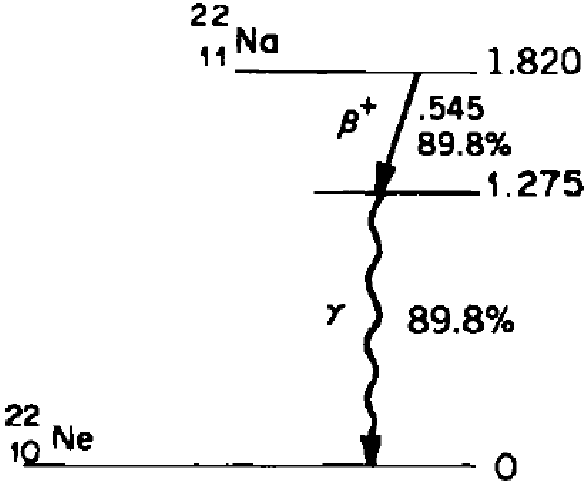
Positron decay occurs only when $\Delta_P > \Delta_D + 1.022\ \mathrm{MeV}$

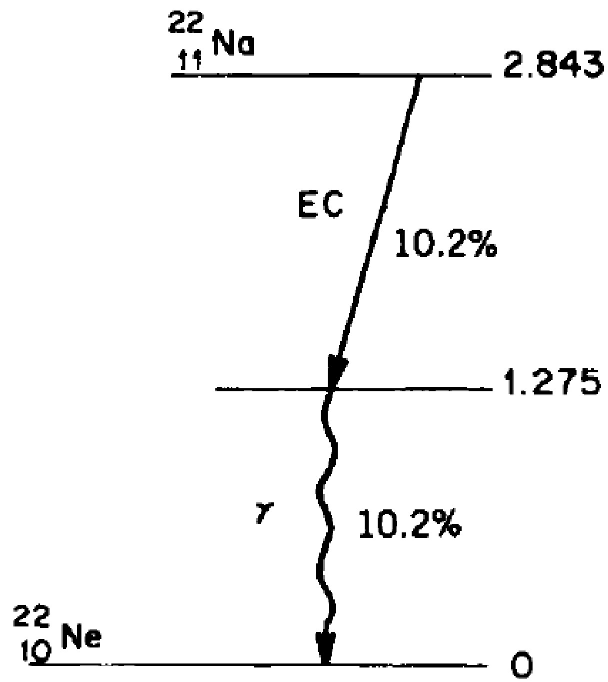
Positron decay

- A positron slows down in matter and then annihilates with an atomic electron, giving rise to two photons, each having energy 0.511 MeV and traveling in opposite directions.
- Electron capture and positron decay are competitive processes:

$$^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + {}^{0}_{1}\beta + {}^{0}_{0}\nu.$$

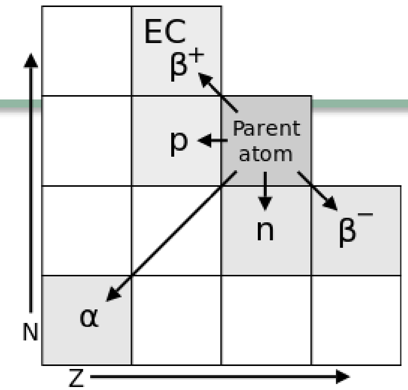
$$^{0}_{-1}\mathrm{e} + {}^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + {}^{0}_{0}\nu.$$



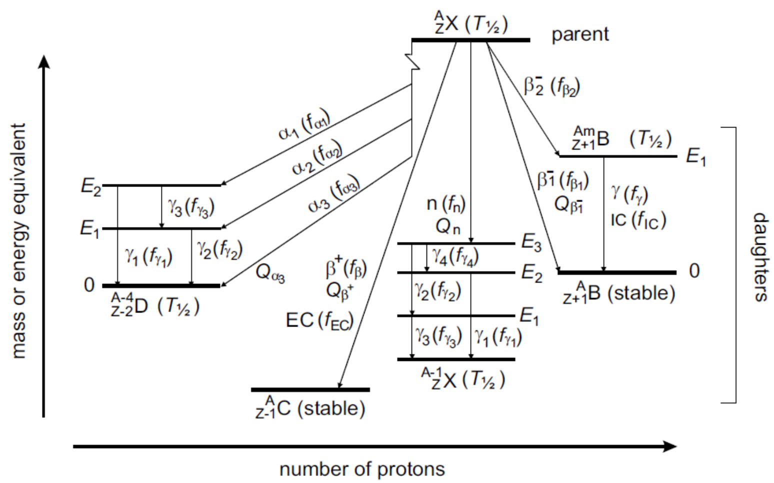


Summary of nuclear radiation

TYPE OF EMISSION	CLASSIFICATION	INSTABILITY	ENERGY SPECTRUM	EMISSION	COMPETING PROCESSES
Alpha, α	Nuclear	$Z > 82$ (Except Sm-147), low n to p$^+$ ratio	Monoenergetic	${}^4_2He^{+2}$	Fission (β^+)
Beta, β^-	Isobaric	High n to p$^+$ ratio	Polyenergetic to a maximum due to anti-neutrino emission	$n \to p^+ + e^- + \bar{\nu}$	None
Positron, β^+	Isobaric	Low n to p$^+$ ratio (α not energetically possible)	Polyenergetic to a maximum due to neutrino emission	$p^+ \to n + e^+ + \bar{\nu}$	Electron capture
Electron capture, EC, ε	Isobaric	Low n to p$^+$ ratio	Characteristic X-rays and/or Auger e^-	$p^+ + e^- \to n + \bar{\nu}$	β^+
Gamma ray, γ	Isomeric	Following particle emission or nuclear excitation	Monoenergetic	Photon (long metastable states possible)	IC
Internal Conversion, IC	Isomeric	Following particle emission or nuclear excitation. Heavy nuclei, low nuclear excitation energies	e^- monoenergetic followed by characteristic X-rays and/or Auger electrons	e^- and X-rays and/ or Auger electrons	γ
Characteristic X-rays	Atomic	Following IC, EC, or any removal of orbital electrons. Optical transitions only	Monoenergetic, X-rays specific to daughter	Photon, energy may be ionizing or nonionizing, depending on element	Auger electrons
Auger electrons	Atomic	Following IC, EC, or any removal of orbital electrons. Can be non-optical transitions	Monoenergetic, X-rays to daughter	Low energy e^-	Characteristic X-rays



Type of decay	Formula
α	$Q_\alpha = \Delta_P - \Delta_D - \Delta_{He}$
β^-	$Q_{\beta^-} = \Delta_P - \Delta_D$
γ	$Q_{IT} = \Delta_P - \Delta_D$
EC	$Q_{EC} = \Delta_P - \Delta_D - E_B$
β^+	$Q_{\beta^+} = \Delta_P - \Delta_D - 2mc^2$



Homework

- J. Turner, Atoms, Radiation, and Radiation Protection, Wiley (2007), chapter 3

  Problems: 5, 15, 16, 30