Radioactive Decay

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Activity

- Radiation: transportation of mass and energy through space
- Radioactivity: the process through which nuclei spontaneously emit subatomic particles
  - Radioactivity was discovered by Henri Becquerel in 1896.
  - The term radioactivity was suggested by Marie Curie about four years later.

- Activity: the number of radioactive decays in a particular time
  - SI unit: becquerel (Bq), 1 decay per second
  - Old unit: curie (Ci), the activity of 1 g of radium-226
  - The becquerel is a measure only of quantity of radioactive material

$$\mathcal{A}(t) = \lambda N(t)$$

- Unit conversion 1 Ci = 3.7×10^{10} Bq

- Specific activity: the activity per unit mass

$$a = \frac{\mathcal{A}}{M} = \frac{\lambda N}{M} = \frac{\lambda N_A}{A}$$

$$1\ \frac{\text{Ci}}{\text{g}} = \frac{3.7\times 10^{10}}{10^{-3}}\ \frac{\text{Bq}}{\text{kg}} = 3.7\times 10^{13}\ \frac{\text{Bq}}{\text{kg}} = 37\ \frac{\text{TBq}}{\text{kg}}$$

Exponential decay law (Rutherford and Soddy, 1902)

- Radioactive decay is a random process and has been observed to follow Poisson distribution. What this essentially means is that the rate of decay of radioactive nuclei in a large sample depends only on the number of decaying nuclei in the sample:

$$-\frac{dN}{dt} \propto N \quad \Rightarrow \quad \frac{dN}{dt} = -\lambda_d N \quad \Rightarrow \quad N(t) = N_0 e^{-\lambda_d t}$$

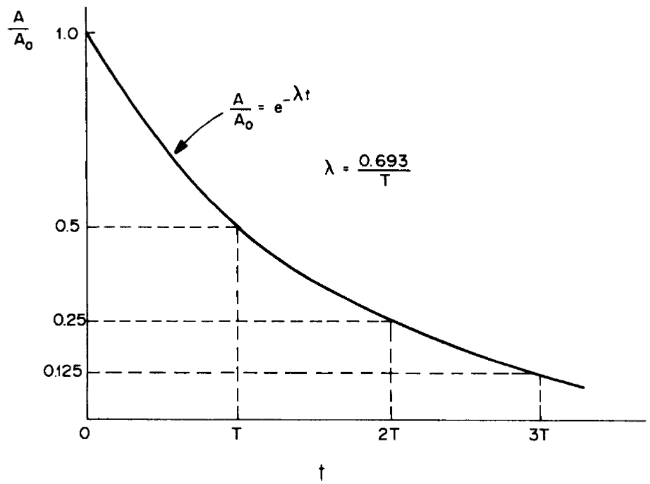
λ_d : decay constant

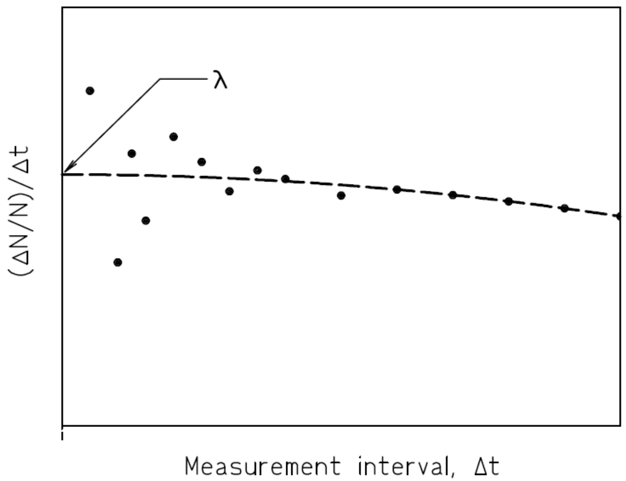
- Activity $A = -\frac{dN}{dt} = \lambda_d N \quad \Rightarrow \quad A = A_0 e^{-\lambda_d t} = A_0 e^{-t/\tau}$

τ : decay time

- Half-life $T_{1/2} = \frac{\ln 2}{\lambda_d} = 0.693\tau$

- Decay constant $\lambda \equiv \lim_{\Delta t \to 0} \frac{(\Delta N/N)}{\Delta t}$





Mean lifetime

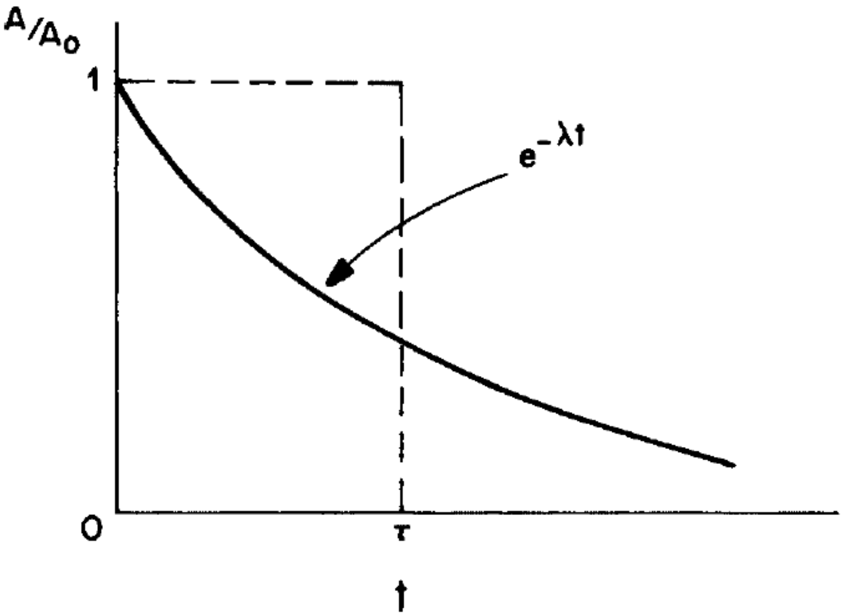
- Given an assembly of elements, the number of which decreases ultimately to zero, the mean lifetime, τ (also called simply the lifetime) is the expected value of the amount of time before an object is removed from the assembly. The mean lifetime is the arithmetic mean of the individual lifetimes.

$$\tau = \langle T \rangle = \int_0^\infty t \, \underline{\exp(-\lambda_d t)} \, \lambda_d dt = \frac{1}{\lambda_d}$$

Decay probability between t and $t + dt$

Expectation

Probability of surviving until time t



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Exponential decay

Example

Calculate the activity of a 30-MBq source of $^{24}_{11}Na$ after 2.5 d. What is the decay constant of this radionuclide?

Solution

The problem can be worked in several ways. We first find λ from Eq. (4.11) and then the activity from Eq. (4.8). The half-life $T = 15.0$ h of the nuclide is given in Appendix D. From (4.11),

$$\lambda = \frac{0.693}{T} = \frac{0.693}{15.0\ \mathrm{h}} = 0.0462\ \mathrm{h}^{-1}. \tag{4.15}$$

With $A_0 = 30$ MBq and $t = 2.5\ \mathrm{d} \times 24\ \mathrm{h\,d}^{-1} = 60.0\ \mathrm{h}$,

$$A = 30\ \mathrm{e}^{-(0.0462\ h^{-1} \times 60\ h)} = 1.88\ \mathrm{MBq}. \tag{4.16}$$

Exponential decay

Example

A solution contains 0.10 μCi of ^{198}Au and 0.04 μCi of ^{131}I at time $t = 0$. What is the total beta activity in the solution at $t = 21$ d? At what time will the total activity decay to one-half its original value?

Solution

Both isotopes decay to stable daughters, and so the total beta activity is due to these isotopes alone. (A small fraction of ^{131}I decays into ^{131m}Xe, which does not contribute to the beta activity.) From Appendix D, the half-lives of ^{198}Au and ^{131}I are, respectively, 2.70 days and 8.05 days. At the end of 21 days, the activities A_{Au} and A_I of the nuclides are, from Eq. (4.12),

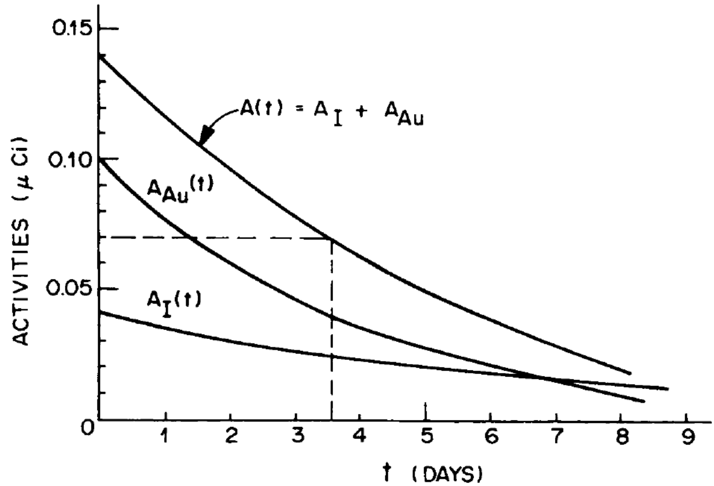
$$A_{Au} = 0.10e^{-0.693\times 21/2.70} = 4.56 \times 10^{-4}\ \mu\text{Ci} \tag{4.17}$$

and

$$A_I = 0.04e^{-0.693\times 21/8.05} = 6.56 \times 10^{-3}\ \mu\text{Ci}. \tag{4.18}$$

The total activity at $t = 21$ days is the sum of these two activities, 7.02×10^{-3} μCi. To find the time t in days at which the activity has decayed to one-half its original value of $0.10 + 0.04 = 0.14$ Ci, we write

$$0.07 = 0.1e^{-0.693t/2.70} + 0.04e^{-0.693t/8.05}. \tag{4.19}$$



Specific activity

- The specific activity of a sample is defined as its activity per unit mass, for example, Bq g^{-1} or Ci g^{-1}.
- Since the number of atoms per gram of the nuclide is $N = 6.02 \times 10^{23}/M$ (M: atomic weight), the specific activity is given by

$$SA = \frac{6.02 \times 10^{23}\, \lambda_d}{M} \approx \frac{4.17 \times 10^{23}}{A T_{1/2}}$$

Example

Calculate the specific activity of ^{226}Ra in Bq g^{-1}.

Solution

From Appendix D, $T = 1600$ y and $M = A = 226$. Converting T to seconds, we have

$$\mathrm{SA} = \frac{4.17 \times 10^{23}}{226 \times 1600 \times 365 \times 24 \times 3600} \tag{4.25}$$

$$= 3.66 \times 10^{10}\ \mathrm{s^{-1}\,g^{-1}} = 3.7 \times 10^{10}\ \mathrm{Bq\,g^{-1}}. \tag{4.26}$$

This, by definition, is an activity of 1 Ci.

Decay with production

- If $Q(t)$ is the rate at which the radionuclide of interest is being created, the rate of change of the number of radionuclides is

$$\frac{dN(t)}{dt} = -\lambda N(t) + Q(t) \quad \Rightarrow \quad N(t) = N_0 e^{-\lambda t} + \int_0^t Q(\tau) e^{-\lambda(t-\tau)} d\tau$$

- For constant production rate, i.e., $Q(t) = Q_0$,

$$N(t) = N_0 e^{-\lambda t} + \frac{Q_0}{\lambda}\{1 - e^{-\lambda t}\}$$

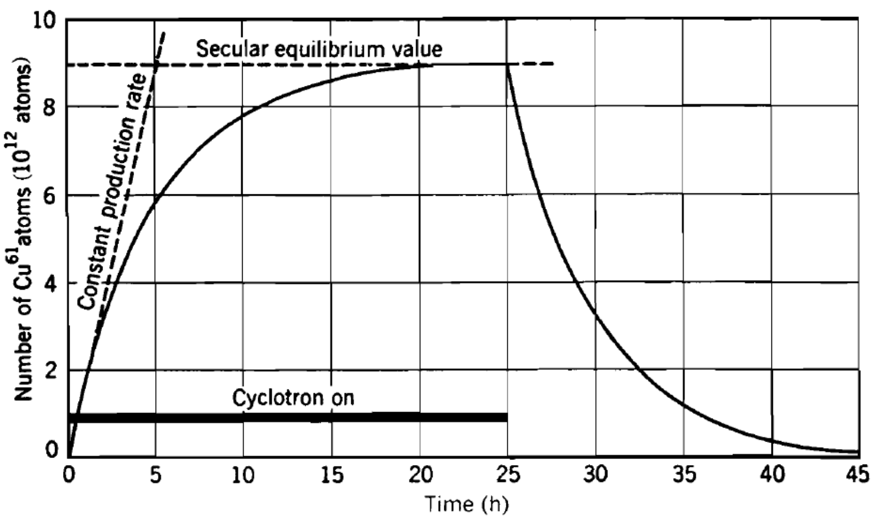


Figure 6.5 A plot of the number of radioactive ^{61}Cu atoms present in a Ni target at various times during and after bombardment with deuterons in a cyclotron.

Serial radioactive decay

- We can calculate the activity of a sample in which one radionuclide produces one or more radioactive offspring in a chain.

$$N_1 \overset{\lambda_1}{\Longrightarrow} N_2 \overset{\lambda_2}{\Longrightarrow} N_3 \overset{\lambda_3}{\Longrightarrow} \cdots$$

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \quad \Rightarrow \quad N_1(t) = N_{10} e^{-\lambda_1 t}$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 N_2(t)$$

$$\xrightarrow[N_{20} = 0]{} \quad N_2(t) = N_{10} \frac{\lambda_1}{\lambda_2 - \lambda_1} \left\{ e^{-\lambda_1 t} - e^{-\lambda_2 t} \right\}$$

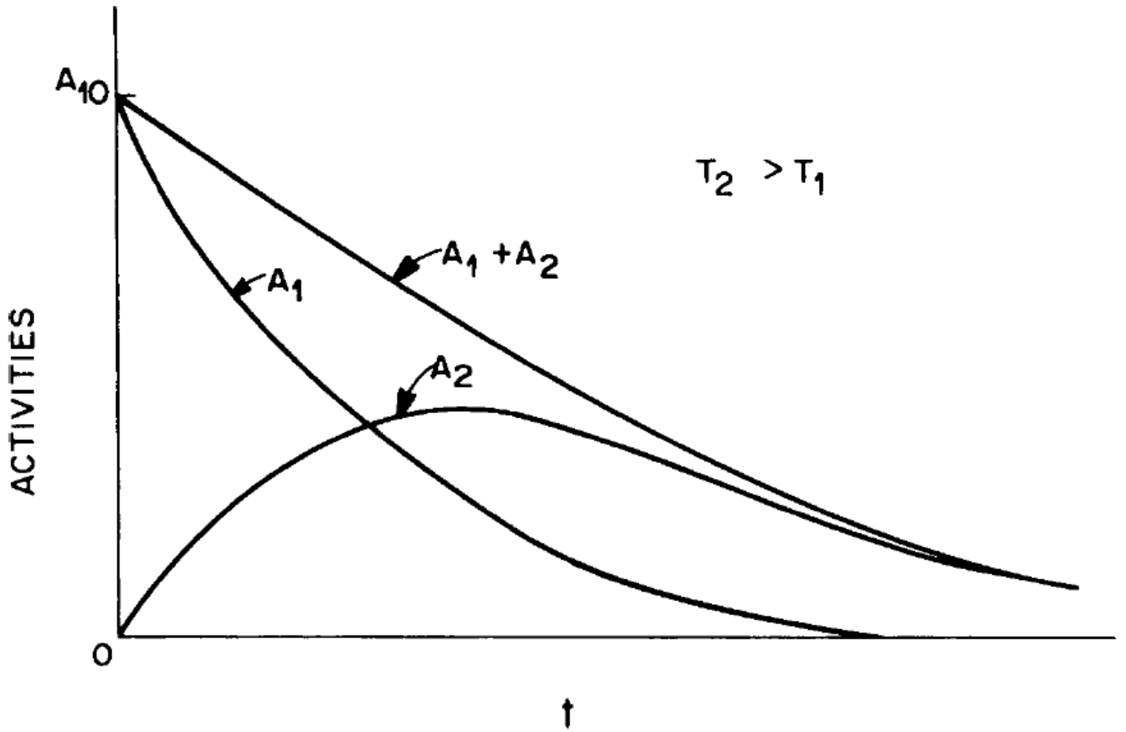
- The activity of the daughter nuclei

$$A_2(t) = \lambda_2 N_2(t) = \lambda_2 N_{10} \frac{\lambda_1}{\lambda_2 - \lambda_1} \left\{ e^{-\lambda_1 t} - e^{-\lambda_2 t} \right\}$$

$$= A_1(t) \frac{\lambda_2}{\lambda_2 - \lambda_1} \left\{ 1 - e^{-(\lambda_2 - \lambda_1) t} \right\}$$

Serial radioactive decay

- No equilibrium ($T_1 < T_2$): When the daughter, initially absent ($N_{20} = 0$), has a longer half-life than the parent, its activity builds up to a maximum and then declines. Because of its shorter half life, the parent eventually decays away and only the daughter is left. No equilibrium occurs.



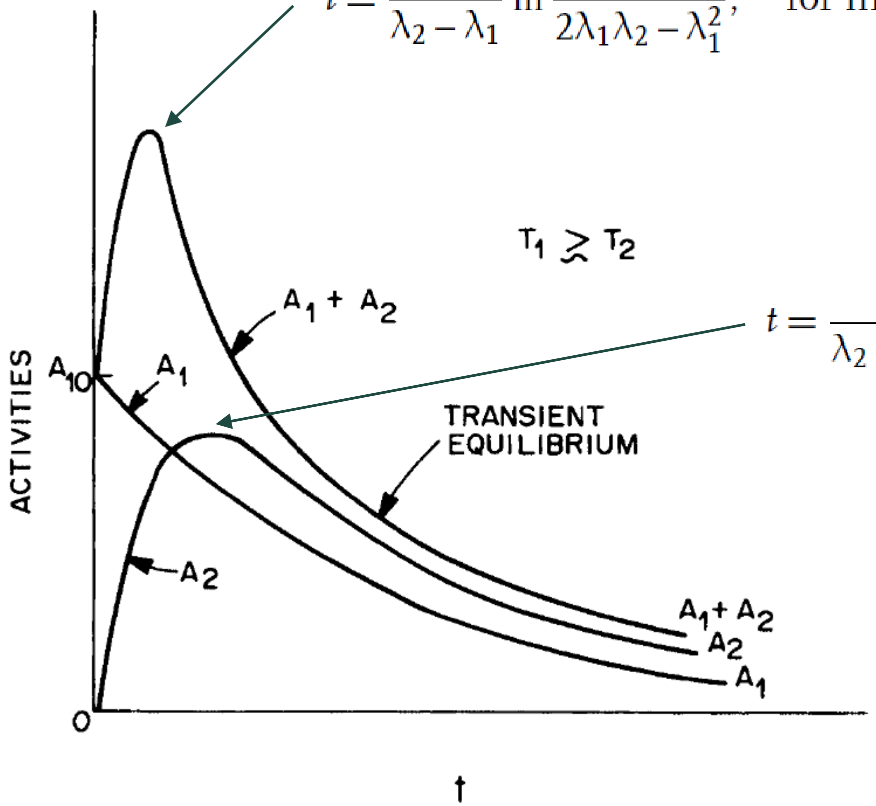
Serial radioactive decay

- Transient equilibrium ($T_1 \gtrsim T_2$)

$$A_2(t) = \lambda_2 N_2(t) = A_1(t)\frac{\lambda_2}{\lambda_2 - \lambda_1}\left\{1 - e^{-(\lambda_2-\lambda_1)t}\right\} \rightarrow \frac{\lambda_2}{\lambda_2 - \lambda_1}A_1(t)$$

$$t = \frac{1}{\lambda_2 - \lambda_1}\ln\frac{\lambda_2^2}{2\lambda_1\lambda_2 - \lambda_1^2}, \quad \text{for maximum } A_1 + A_2.$$

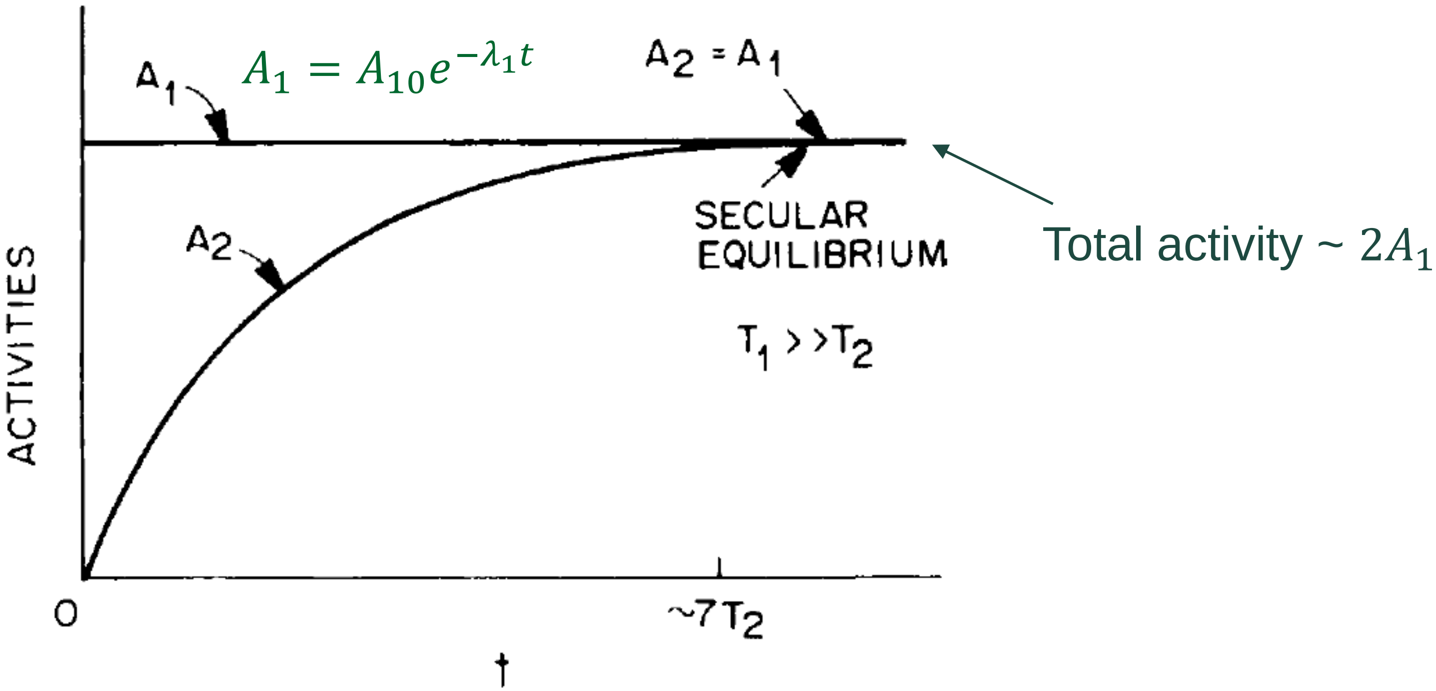
$$t = \frac{1}{\lambda_2 - \lambda_1}\ln\frac{\lambda_2}{\lambda_1}, \quad \text{for maximum } A_2.$$



Serial radioactive decay

- Secular equilibrium ($T_1 \gg T_2$ or $\lambda_2 \gg \lambda_1$)

$$A_2(t) = \lambda_2 N_2(t) = A_1(t)\frac{\lambda_2}{\lambda_2 - \lambda_1}\left\{1 - e^{-(\lambda_2-\lambda_1)t}\right\} \approx A_1(t)\left\{1 - e^{-\lambda_2 t}\right\} \to A_1(t)$$

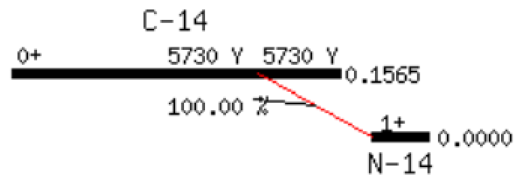


- A chain of n short-lived radionuclides can all be in secular equilibrium with a long-lived parent. Then the activity of each member of the chain is equal to that of the parent and the total activity is $n+1$ times the activity of the original parent.

Carbon dating

- Formation of radiocarbon

$$^{14}_{7}\text{N} + ^{1}_{0}n \rightarrow ^{14}_{6}\text{C} + ^{1}_{1}\text{H}$$



Example 12.5

A piece of wood from the ruins of an ancient dwelling was found to have a ^{14}C activity of 13 disintegrations per minute per gram of its carbon content. The ^{14}C activity of living wood is 16 disintegrations per minute per gram. How long ago did the tree die from which the wood sample came?

Solution

If the activity of a certain mass of carbon from a plant or animal that was recently alive is R_0 and the activity of the same mass of carbon from the sample to be dated is R, then from Eq. (12.2)

$$R = R_0 e^{-\lambda t}$$

To solve for the age t we proceed as follows:

$$e^{\lambda t} = \frac{R_0}{R} \qquad \lambda t = \ln\frac{R_0}{R} \qquad t = \frac{1}{\lambda}\ln\frac{R_0}{R}$$

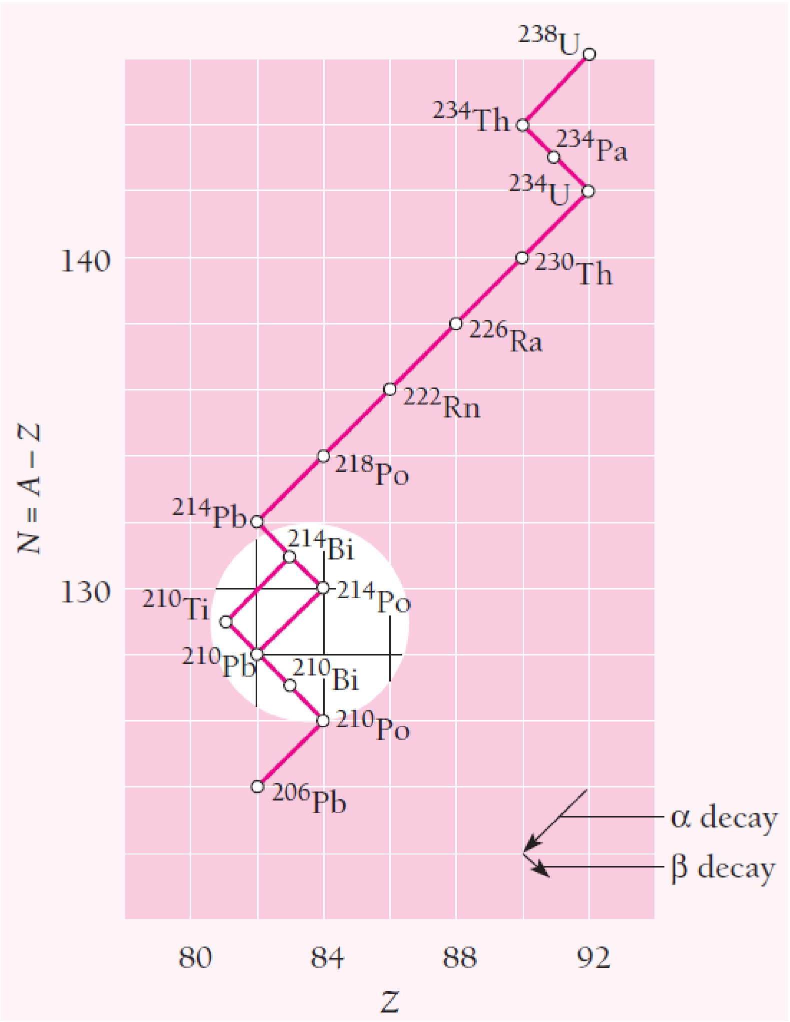
From Eq. (12.3) the decay constant λ of radiocarbon is $\lambda = 0.693/T_{1/2} = 0.693/5760$ y. Here $R_0/R = 16/13$ and so

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \frac{5760\text{ y}}{0.693}\ln\frac{16}{13} = 1.7 \times 10^3\text{ y}$$

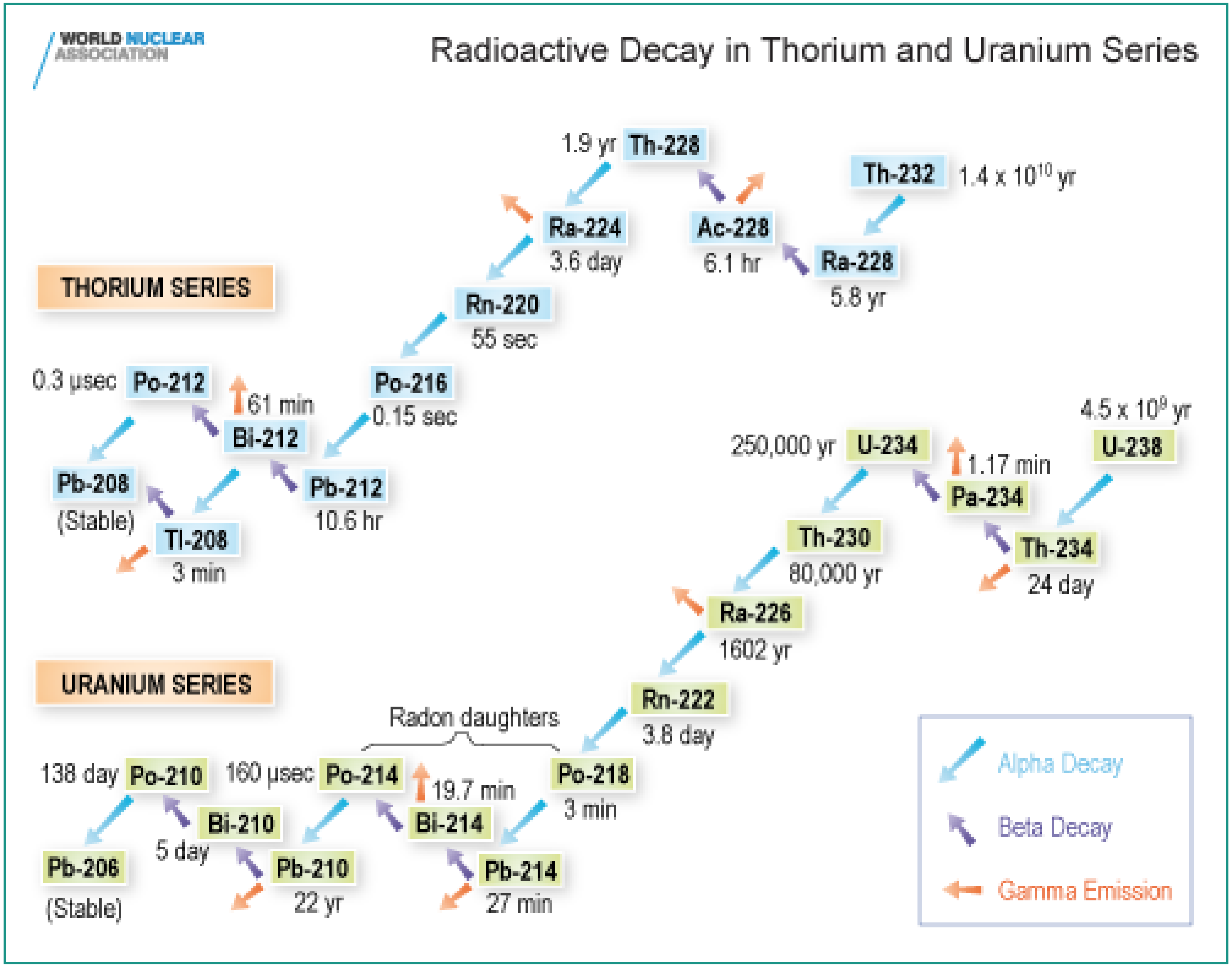
Radioactive series

- Most of the radionuclides found in nature are members of four radioactive series, with each series consisting of a succession of daughter products all ultimately derived from a single parent nuclide.
- The reason that there are exactly four series follows from the fact that alpha decay reduces the mass number of a nucleus by 4.

Mass Numbers	Series	Parent	Half-Life, Years	Stable End Product
$4n$	Thorium	$^{232}_{90}Th$	1.39×10^{10}	$^{208}_{82}Pb$
$4n + 1$	Neptunium	$^{237}_{93}Np$	2.25×10^{6}	$^{209}_{83}Bi$
$4n + 2$	Uranium	$^{238}_{92}U$	4.47×10^{9}	$^{206}_{82}Pb$
$4n + 3$	Actinium	$^{235}_{92}U$	7.07×10^{8}	$^{207}_{82}Pb$

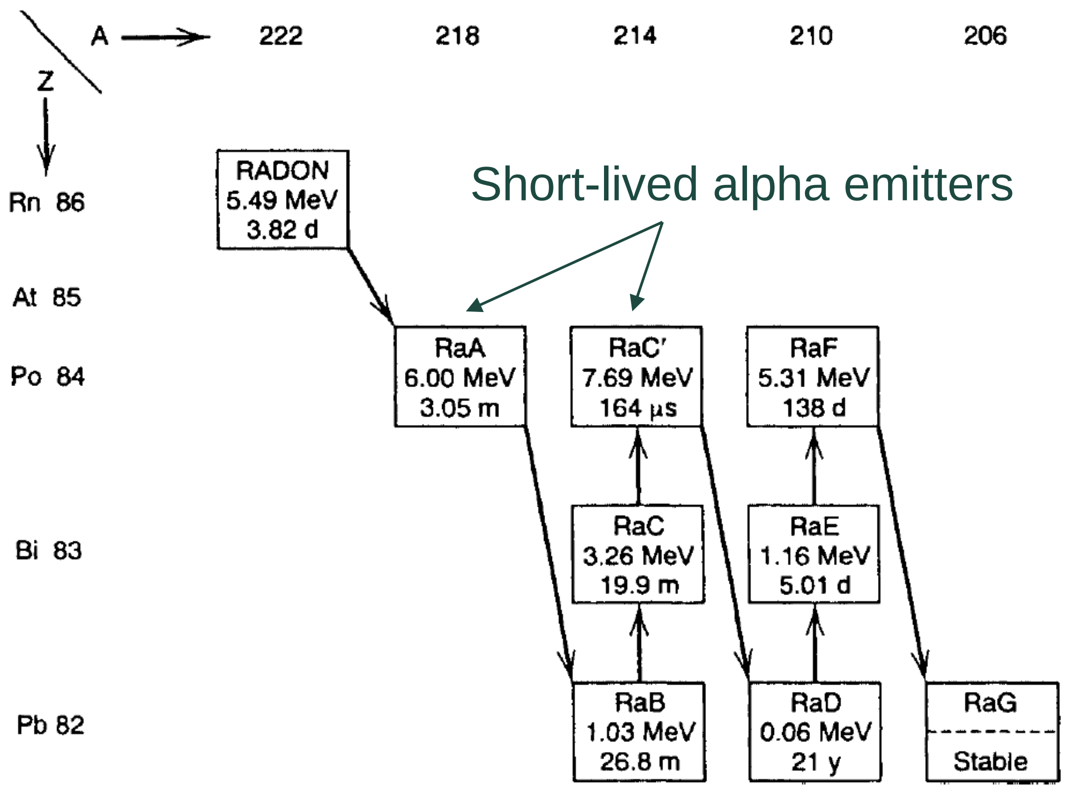


Radioactive decay of Th-232 and U-238



Radon and radon daughters

- The potential health hazard arises when radon in the air decays, producing nongaseous radioactive daughters. When inhaled, the airborne daughters can be trapped in the respiratory system, where they are likely to decay before being removed by normal lung-clearing mechanisms of the body.

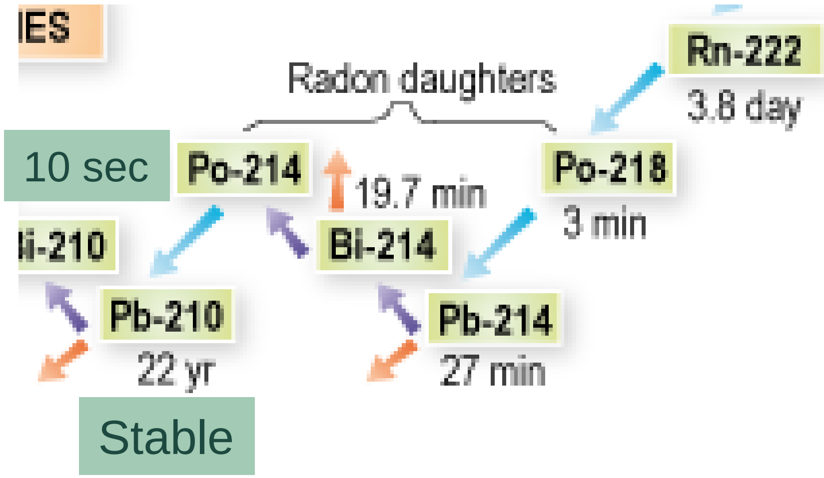


Homework

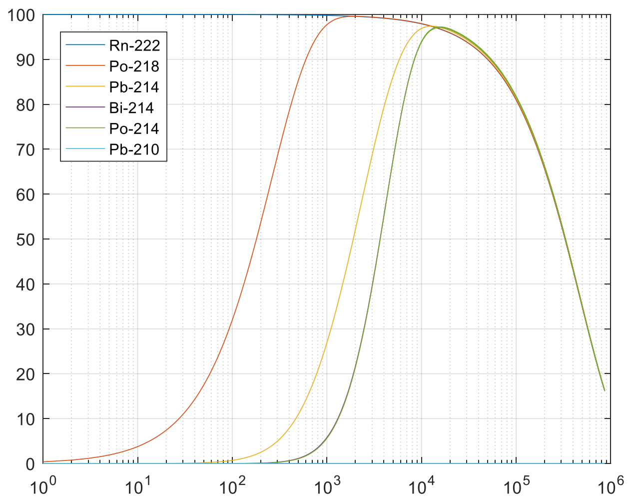
- J. Turner, Atoms, Radiation, and Radiation Protection, Wiley (2007), chapter 4

  Problems: 5, 13, 21

- Setup the decay equations and solve them (analytically or numerically). Plot the activities of each nuclide and the total activity versus time (up to 10 days). Initial activity of Rn-222 is 100 Bq and the others are zero.



$A(t)$



- Setup the differential equations for the following decay chain and find $N_1(t)$, $N_2(t)$, and $N_3(t)$. Assume initial values of $N_1(0)$, $N_2(0)$, and $N_3(0)$.

$$X_1 \xrightarrow{\lambda_1} X_2 \xrightarrow{\lambda_2} X_3(\text{ stable})$$