Interaction of Photons with Matter

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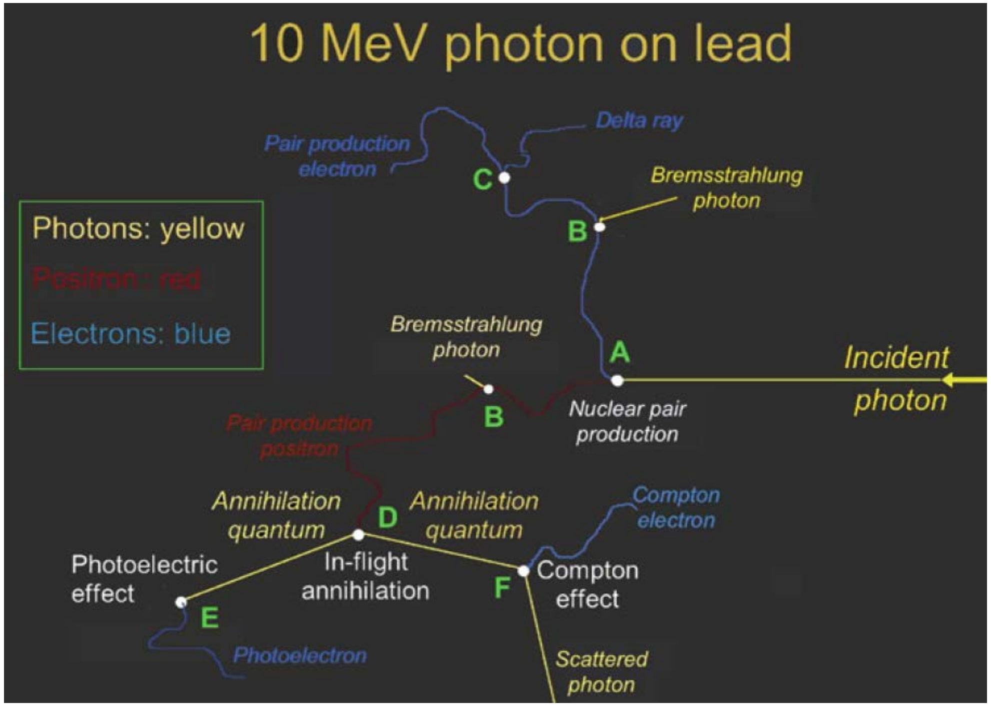
Interaction of photons with matter (absorber)

- Photons with energy exceeding the ionization energy of atoms are called indirectly ionizing radiation because they transfer energy to an absorber by an indirect process (photoelectric effect, Compton effect, and pair production).
- In each of these effects, a portion of the photon energy is transferred to energetic light charged particles (electron or positron) which travel through the absorber and lose their energy through direct Coulomb interactions with nuclei and orbital electrons of absorber atoms.



Interaction mechanisms

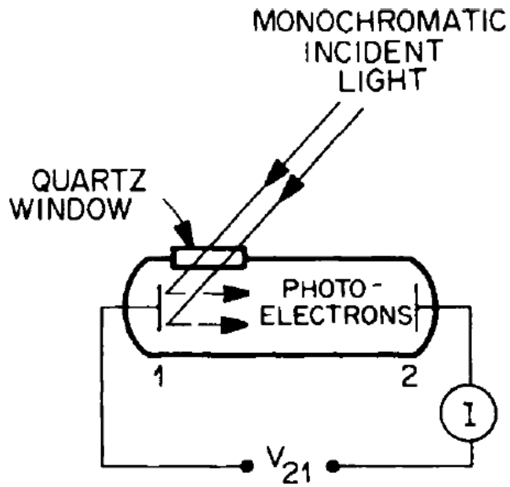
- Unlike charged particles, photons are electrically neutral and do not steadily lose energy as they penetrate matter. Instead, they can travel some distance before interacting with an atom.
- How far a given photon will penetrate is governed statistically by a probability of interaction per unit distance traveled, which depends on the specific medium traversed and on the photon energy.
- When the photon interacts, it might be absorbed and disappear or it might be scattered, changing its direction of travel, with or without loss of energy.

- Thomson (interaction of a photon with a free electron, low-energy limit of Compton scattering) and Rayleigh scattering (interaction of a photon with an atom) are two processes by which photons interact with matter without appreciable transfer of energy.

- The principal mechanisms of energy deposition by photons in matter are photoelectric absorption, Compton scattering, pair production, and photonuclear reactions.

Photoelectric effect

- The ejection of electrons from a surface as a result of light absorption is called the photoelectric effect.



$$T_{\max} = eV_0.$$

The stopping potential V_0 varies linearly with the frequency ν of the monochromatic light used. A threshold frequency ν_0 is found below which no photoelectrons are emitted, even with intense light. The value of ν_0 depends on the metal used for electrode 1.

- Einstein's explanation (1905)

$$T = h\nu - \varphi.$$

$$T_{\max} = h\nu - \varphi_0.$$

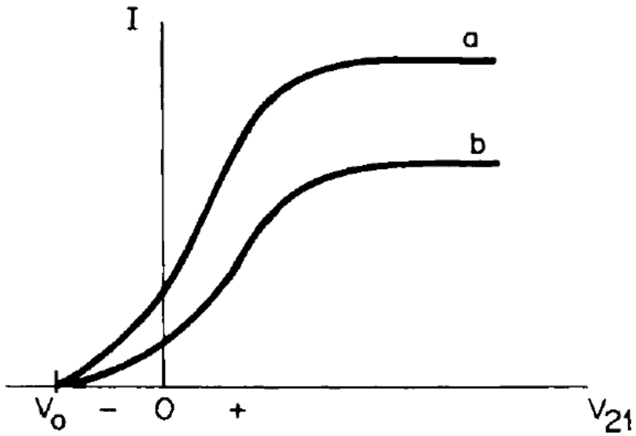


Fig. 8.1 Experiment on photoelectric effect. With electrode 1 illuminated with monochromatic light of constant intensity, the current *I* is measured as a function of the potential difference V_{21} between electrodes 2 and 1. Curves (a) and (b) represent data at two different intensities of the incident light.

Photoelectric effect

- The probability of producing a photoelectron when light strikes an atom is strongly dependent on the atomic number Z and the energy $h\nu$ of the photons.
- The probability varies as $Z^4/(h\nu)^3$.

Example

(a) What threshold energy must a photon have to produce a photoelectron from Al, which has a work function of 4.20 eV? (b) Calculate the maximum energy of a photoelectron ejected from Al by UV light with a wavelength of 1500 Å. (c) How does the maximum photoelectron energy vary with the intensity of the UV light?

Solution

(a) The work function $\varphi_0 = 4.20$ eV represents the minimum energy that a photon must have to produce a photoelectron. (b) The energy of the incident photons in eV is given in terms of the wavelength λ in angstroms by Eq. (2.26):

$$E = h\nu = \frac{12400}{\lambda} = \frac{12400}{1500} = 8.27 \text{ eV}. \tag{8.4}$$

From Eq. (8.3),

$$T_{\max} = 8.27 - 4.20 = 4.07 \text{ eV}. \tag{8.5}$$

(c) $T_{\max}$ is independent of the light intensity.

Main features of photoelectric effect

1. The extra energy and momentum carried by the photon are transferred to the absorbing atom; however, because of the relatively large nuclear mass, the atomic recoil energy is exceedingly small and may be neglected. The kinetic energy E_K of the ejected photoelectron is assumed to be equal to the incident photon energy $h\nu$ less the binding energy E_B of the orbital electron, i.e.,

$$E_K = h\nu - E_B. \tag{7.140}$$

2. When the photon energy $h\nu$ exceeds the K-shell binding energy $E_B(K)$ of the absorber, i.e., $h\nu > E_B(K)$, about 80% of all photoelectric absorptions occur with the K-shell electrons of the absorber and the remaining 20% occur with less tightly bound higher shell electrons.
3. The energy uptake by the photoelectron may be insufficient to bring about its ejection from the atom in a process referred to as atomic ionization but may be sufficient to raise the photoelectron to a higher orbit in a process referred to as atomic excitation.
4. The vacancy that results from the emission of the photoelectron from a given shell will be filled by a higher shell electron and the transition energy will be emitted either as a characteristic (fluorescence) photon or as an Auger electron, the probability for each governed by the fluorescence yield ω, as discussed in Sects. 4.1 and 7.5.7.

Cross section and attenuation coefficient for photoelectric effect

- Atomic cross section for photoelectric effect

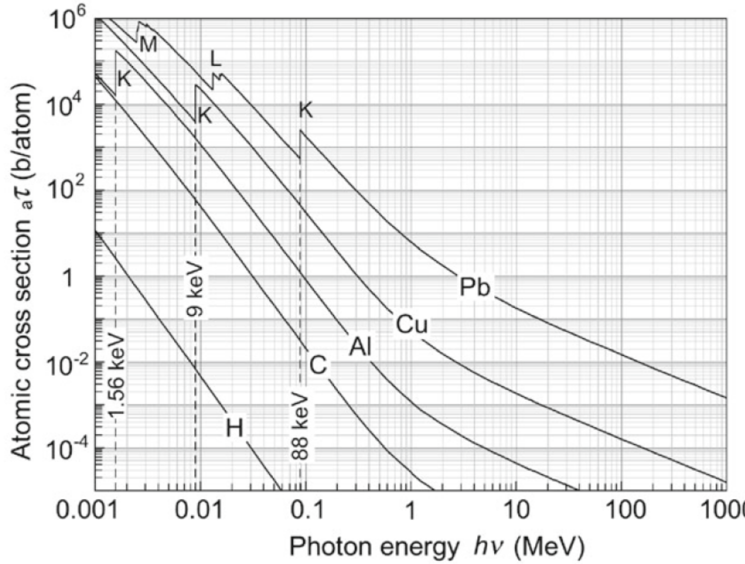
$$ {}_{\mathrm{a}}\tau_{\mathrm{K}} = \alpha^4 \left({}_{\mathrm{e}}\sigma_{\mathrm{Th}}\right) Z^n \sqrt{\frac{32}{\varepsilon^7}}, \tag{7.141} $$

where

ε	is the usual normalized photon energy, i.e., $\varepsilon = h\nu/(m_{\mathrm{e}}c^2)$.
α	is the fine structure constant (1/137).
Z	is the atomic number of the absorber.
${}_{\mathrm{e}}\sigma_{\mathrm{Th}}$	is the Thomson electronic cross section given in (7.41).
n	is the power for the Z dependence of ${}_{\mathrm{a}}\tau_{\mathrm{K}}$ ranging from $n = 4$ at relatively low photon energies to $n = 4.6$ at high photon energies.

- Mass attenuation coefficient

$$ \frac{\tau}{\rho} = \frac{N_{\mathrm{A}}}{A}\,{}_{\mathrm{a}}\tau $$



Compton effect

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THE

PHYSICAL REVIEW

A QUANTUM THEORY OF THE SCATTERING OF X-RAYS BY LIGHT ELEMENTS

By Arthur H. Compton

Abstract

A quantum theory of the scattering of X-rays and γ-rays by light elements.—The hypothesis is suggested that when an X-ray quantum is scattered it spends all of its energy and momentum upon some particular electron. This electron in turn scatters the ray in some definite direction. The change in momentum of the X-ray quantum due to the change in its direction of propagation results in a recoil of the scattering electron. The energy in the scattered quantum is thus less than the energy in the primary quantum by the kinetic energy of recoil of the scattering electron. The corresponding *increase in the wave-length of the scattered beam* is $\lambda_\theta - \lambda_0 = (2h/mc)\sin^2\frac{1}{2}\theta = 0.0484\sin^2\frac{1}{2}\theta$, where h is the Planck constant, m is the mass of the scattering electron, c is the velocity of light, and θ is the angle between the incident and the scattered ray. Hence the increase is independent of the wave-length. *The distribution of the scattered radiation* is found, by an indirect and not quite rigid method, to be concentrated in the forward direction according to a definite law (Eq. 27). The total energy removed from the primary beam comes out less than that given by the classical Thomson theory in the ratio $1/(1+2\alpha)$, where $\alpha = h/mc\lambda_0 = 0.0242/\lambda_0$. Of this energy a fraction $(1+\alpha)/(1+2\alpha)$ reappears as scattered radiation, while the remainder is truly absorbed and transformed into kinetic energy of recoil of the scattering electrons. Hence, if σ_0 is the *scattering absorption coefficient* according to the classical theory, the coefficient according to this theory is $\sigma = \sigma_0/(1+2\alpha) = \sigma_s + \sigma_a$, where σ_s is the true scattering coefficient $[(1+\alpha)\sigma/(1+2\alpha)^2]$, and σ_a is the coefficient of absorption due to scattering $[\alpha\sigma/(1+2\alpha)^2]$. Unpublished experimental results are given which show that for graphite and the Mo-K radiation the scattered radiation is longer than the primary, the observed difference ($\lambda_{\pi/2} - \lambda_0 = .022$) being close to the computed value .024. In the case of scattered γ-rays, the wave-length has been found to vary with θ in agreement with the theory, increasing from .022 A (primary) to .068 A ($\theta = 135°$). Also the velocity of secondary β-rays excited in light elements by γ-rays agrees with the suggestion that they are recoil electrons. As for the predicted variation of absorption with λ, Hewlett's results for carbon for wave-lengths below 0.5 A are in excellent agreement with this theory; also the predicted concentration in the forward direction is shown to be in agreement with the experimental results,

Compton effect

- Each plot shows peaks at two values of λ': one at the wavelength λ of the incident photons and another at a longer wavelength, $\lambda' > \lambda$. The appearance of scattered radiation at a longer wavelength is called the Compton effect.

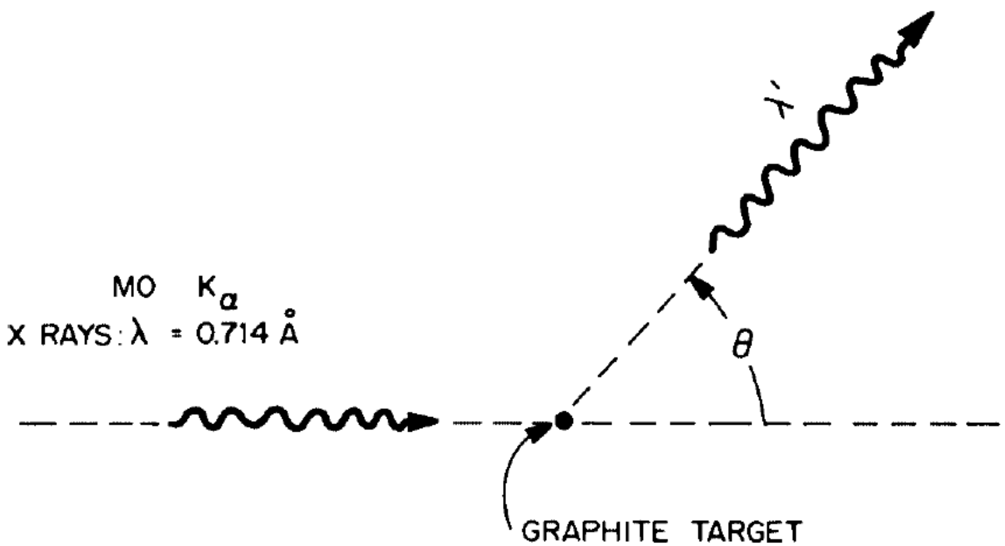


Fig. 8.2 Compton measured the intensity of scattered photons as a function of their wavelength λ' at various scattering angles θ. Incident radiation was molybdenum K_α X rays, having a wavelength $\lambda = 0.714$ Å.

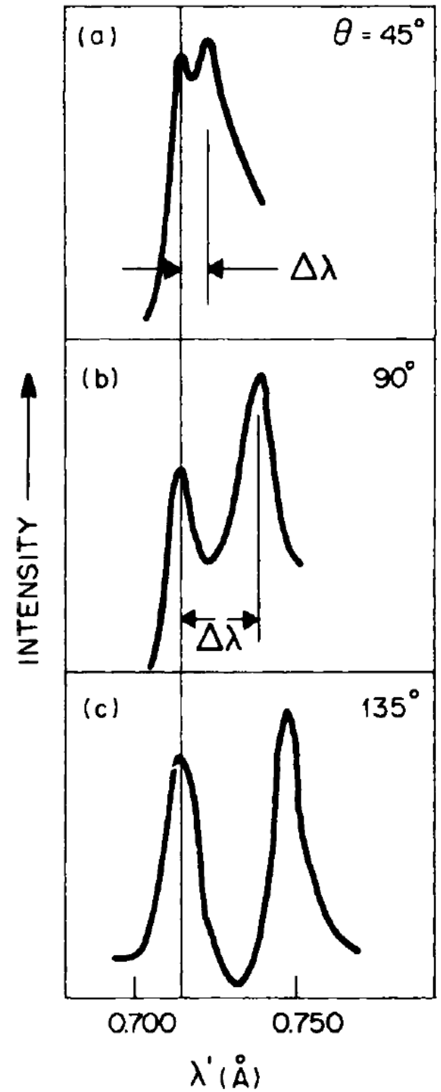


Fig. 8.3 Intensity vs. wavelength λ' of photons scattered at angles (a) $\theta = 45°$, (b) 90°, and (c) 135°.

Kinematics of Compton scattering

- Energy conservation

$$h\nu + mc^2 = h\nu' + E'.$$

- Momentum conservation

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + P'\cos\varphi \qquad \frac{h\nu'}{c}\sin\theta = P'\sin\varphi.$$

- Energy and wavelength of scattered photon

$$h\nu' = \frac{h\nu}{1 + (h\nu/mc^2)(1-\cos\theta)}.$$

$$\Delta\lambda = \lambda' - \lambda = c\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{\underline{mc}}(1-\cos\theta).$$

Compton wavelength

$$\frac{h}{mc} = \frac{6.63\times10^{-34}}{9.11\times10^{-31}\times3.00\times10^{8}} = 2.43\times10^{-12}\ \mathrm{m}$$

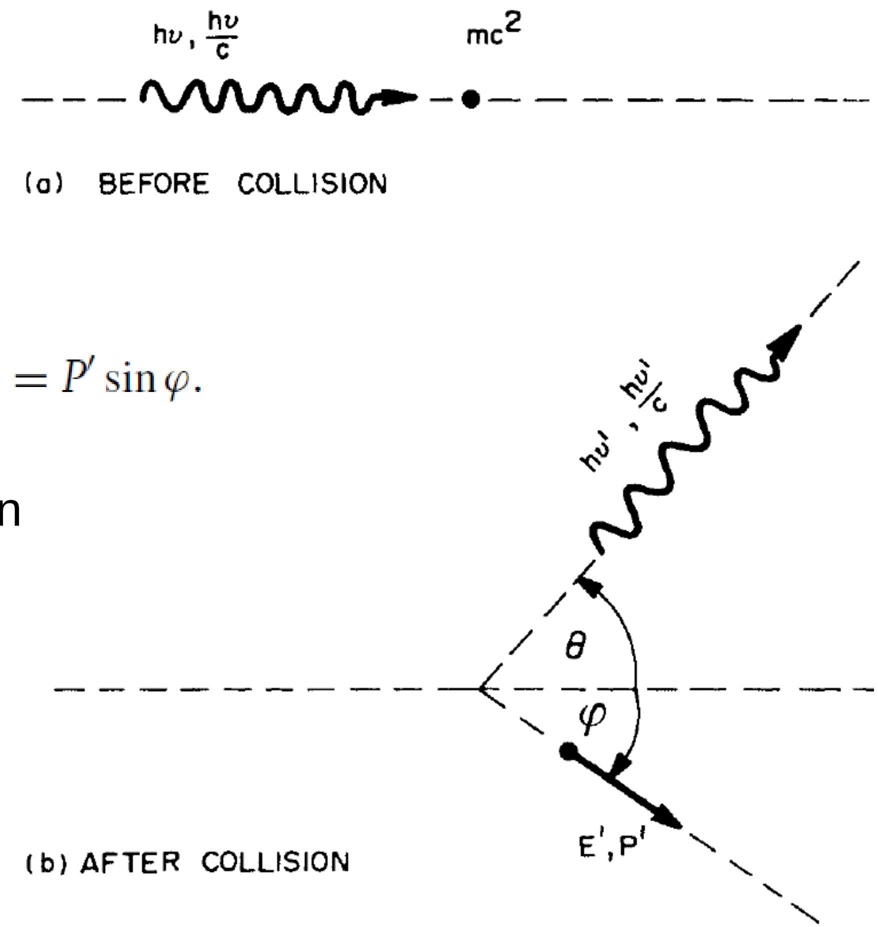


Fig. 8.4 Diagram illustrating Compton scattering of photon (energy $h\nu$, momentum $h\nu/c$) from electron, initially free and at rest with total energy mc^2. As a result of the collision, the photon is scattered at an angle θ with reduced energy $h\nu'$ and momentum $h\nu'/c$, and the electron recoils at angle φ with total energy E' and momentum P'.

Kinematics of Compton scattering

Example

A 1.332-MeV gamma photon from ^{60}Co is Compton scattered at an angle of 140°. Calculate the energy of the scattered photon and the Compton shift in wavelength. What is the momentum of the scattered photon?

Solution

The energy of the scattered photons is given by Eq. (8.12):

$$h\nu' = \frac{1.332 \text{ MeV}}{1 + (1.332/0.511)[1 - (-0.766)]} = 0.238 \text{ MeV}. \quad (8.15)$$

The Compton shift is given by Eq. (8.13) with $h/mc = 0.0243$ Å:

$$\Delta\lambda = (0.0243 \text{ Å})[1 - (-0.766)] = 0.0429 \text{ Å}. \quad (8.16)$$

The momentum of the scattered photon is

$$\frac{h\nu'}{c} = \frac{0.238 \text{ MeV} \times 1.6 \times 10^{-13} \text{ J MeV}^{-1}}{3 \times 10^{8} \text{ m s}^{-1}} = 1.27 \times 10^{-22} \text{ kg m s}^{-1}, \quad (8.17)$$

where we have used the fact that $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$ (Appendix B).

Kinematics of Compton scattering

- Energy and wavelength of scattered photon

$$h\nu' = \frac{h\nu}{1 + (h\nu/mc^2)(1 - \cos\theta)}.$$

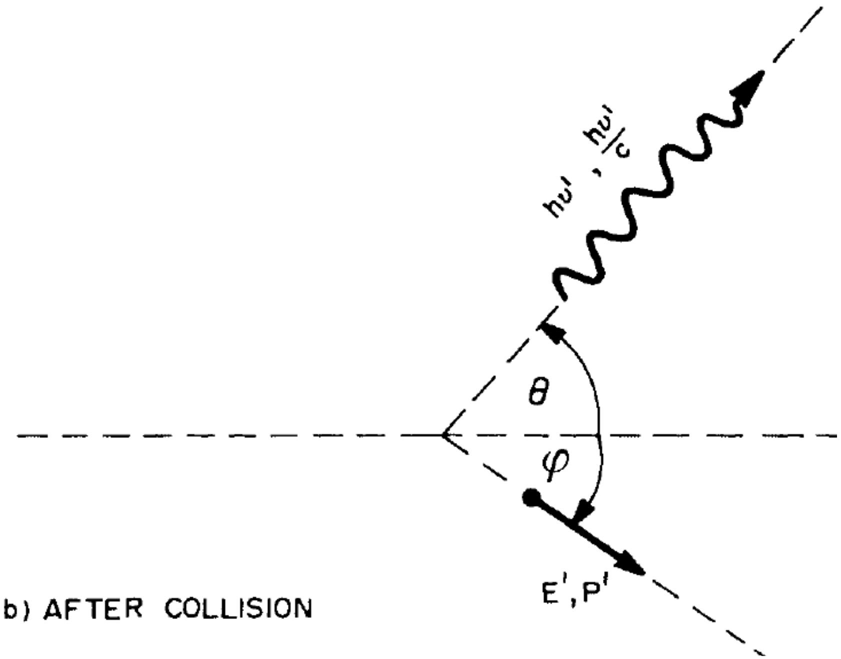
- Extreme case

$$\theta = 0 \qquad h\nu' = h\nu'_{\max} = h\nu$$

$$\theta = \pi/2 \qquad h\nu' = \frac{h\nu}{1 + \dfrac{h\nu}{mc^2}}$$

$$\theta = \pi \qquad h\nu' = h\nu'_{\min} = \frac{h\nu}{1 + 2\dfrac{h\nu}{mc^2}} \rightarrow \frac{mc^2}{2} = 255\ \text{keV}$$

$$h\nu \gg mc^2$$



(b) AFTER COLLISION

Energy transfer in Compton scattering

- The kinetic energy acquired by the secondary electron is given by

$$T = h\nu - h\nu' = h\nu \frac{1-\cos\theta}{mc^2/h\nu + 1 - \cos\theta}.$$

- The maximum kinetic energy, $T_{\max}$, that a secondary electron can acquire occurs when $\theta = 180°$.

$$T_{\max} = \frac{2h\nu}{2 + mc^2/h\nu}. \longrightarrow \quad T_{\max} = h\nu - h\nu_{\min} \rightarrow h\nu - \frac{1}{2}mc^2 \quad \text{for } h\nu \gg mc^2$$

- Recoil angle of the electron

$$\tan\varphi = \frac{h\nu' \sin\theta}{h\nu - h\nu' \cos\theta}. \longrightarrow \quad \cot\frac{\theta}{2} = \left(1 + \frac{h\nu}{mc^2}\right)\tan\varphi.$$

When θ is small, $\cot\theta/2$ is large and φ is near 90°. In this case, the photon travels in the forward direction, imparting relatively little energy to the electron, which moves off nearly at right angles to the direction of the incident photon. As θ increases from 0° to 180°, $\cot\theta/2$ decreases from ∞ to 0. Therefore, φ decreases from 90° to 0°. The electron recoil angle φ in Fig. 8.4 is thus always confined to the forward direction ($0 \le \varphi \le 90°$), whereas the photon can be scattered in any direction.

Energy transfer in Compton scattering

Example

In the previous example a 1.332-MeV photon from ^{60}Co was scattered by an electron at an angle of 140°. Calculate the energy acquired by the recoil electron. What is the recoil angle of the electron? What is the maximum fraction of its energy that this photon could lose in a single Compton scattering?

Solution

Substitution into Eq. (8.19) gives the electron recoil energy,

$$T = 1.332 \frac{1-(-0.766)}{0.511/1.332 + 1 - (-0.766)} = 1.094 \text{ MeV}. \tag{8.25}$$

Note from Eq. (8.15) that $T + h\nu' = 1.332 \text{ MeV} = h\nu$, as it should. The angle of recoil of the electron can be found from Eq. (8.24). We have

$$\tan\varphi = \frac{\cot(140°/2)}{1 + 1.332/0.511} = 0.101, \tag{8.26}$$

from which it follows that $\varphi = 5.76°$. This is a relatively hard collision in which the photon is backscattered, retaining only the fraction $0.238/1.332 = 0.179$ of its energy and knocking the electron in the forward direction. From Eq. (8.20),

$$T_{\max} = \frac{2 \times 1.332}{2 + 0.511/1.332} = 1.118 \text{ MeV}. \tag{8.27}$$

The maximum fractional energy loss is $T_{\max}/h\nu = 1.118/1.332 = 0.839$.

Compton collision cross section

- Klein–Nishina formula for Compton scattering (1928)

$$\frac{\mathrm{d}_{\mathrm{e}}\sigma}{\mathrm{d}\Omega} = \frac{k_0^2 e^4}{2m^2c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin^2\theta\right)\ \mathrm{m}^2\,\mathrm{sr}^{-1}. \tag{8.28}$$

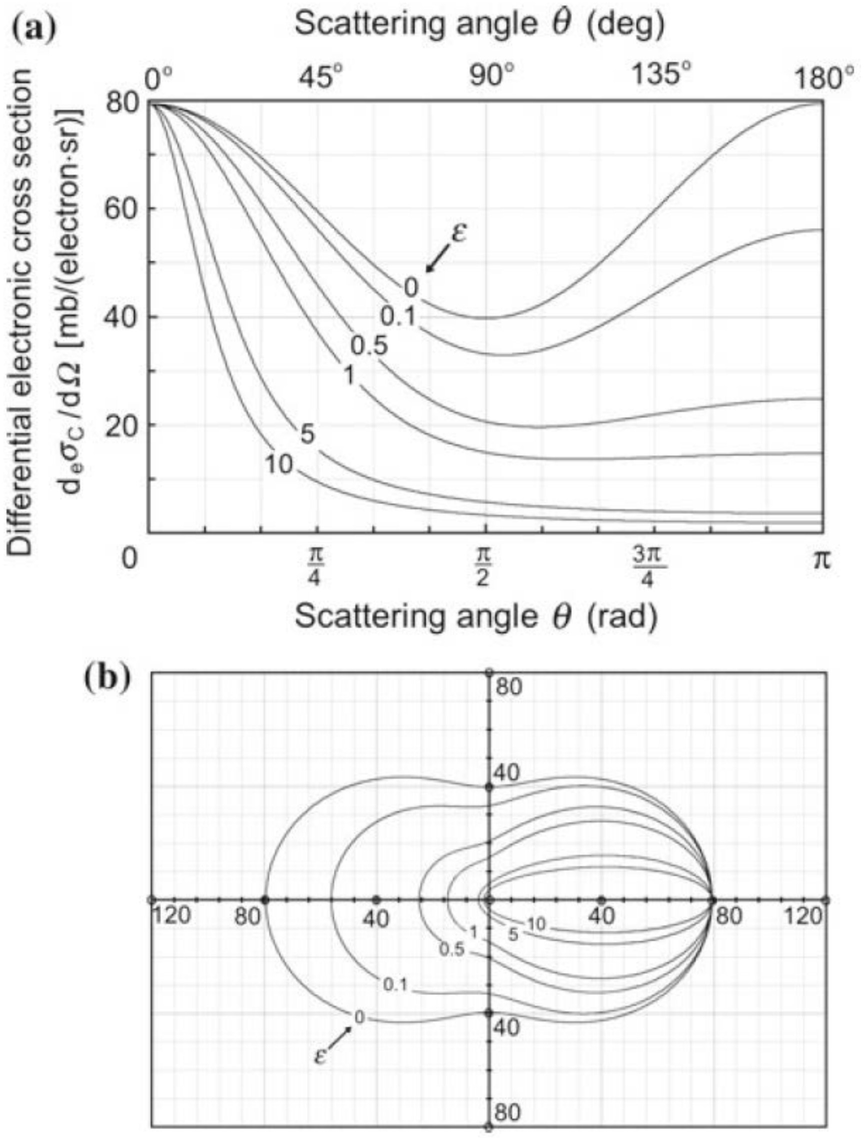
Here $\mathrm{d}_{\mathrm{e}}\sigma/\mathrm{d}\Omega$, called the *differential scattering cross section*, is the probability per unit solid angle in steradians (sr) that a photon, passing normally through a layer of material containing one electron m^{-2}, will be scattered into a solid angle $\mathrm{d}\Omega$ at angle θ. The integral of the differential cross section over all solid angles, $\mathrm{d}\Omega = 2\pi\sin\theta\,\mathrm{d}\theta$, is called the *Compton collision cross section*. It gives the probability ${}_{\mathrm{e}}\sigma$ that the photon will have a Compton interaction per electron m^{-2}:

$${}_{\mathrm{e}}\sigma = 2\pi\int\frac{\mathrm{d}_{\mathrm{e}}\sigma}{\mathrm{d}\Omega}\sin\theta\,\mathrm{d}\theta\ \mathrm{m}^2. \tag{8.29}$$

The Compton cross section, ${}_{\mathrm{e}}\sigma$, can be thought of as the cross-sectional area, like that of a target, presented to a photon for interaction by one electron m^{-2}. It is thus rigorously the stated interaction *probability*. However, the area ${}_{\mathrm{e}}\sigma$, which depends on the energy of the photon, is not the physical size of the electron.

Angular distribution of scattered photons

Fig. 7.13 Differential electronic cross section for Compton effect $d_e\sigma_C^{KN}/d\Omega$ against scattering angle θ for various values of $\varepsilon = h\nu/(m_e c^2)$, as given by (7.89). The differential electronic cross section for Compton effect $d_e\sigma_C^{KN}/d\Omega$ for $\varepsilon = 0$ is equal to the differential electronic cross section for Thomson scattering $d_e\sigma_{Th}/d\Omega$ (see Fig. 7.5). **a** Displays the data in Cartesian coordinate system; **b** in polar coordinate system



$$\varepsilon = \frac{h\nu}{m_e c^2}$$

Energy transfer cross section

- For photons of a given energy $h\nu$, one can write the differential Klein–Nishina energy-transfer cross section (per electron m^{-2})

$$\frac{\mathrm{d}_{\mathrm{e}}\sigma_{\mathrm{tr}}}{\mathrm{d}\Omega} = \frac{T}{h\nu}\frac{\mathrm{d}_{\mathrm{e}}\sigma}{\mathrm{d}\Omega} \quad \Rightarrow \quad T_{\mathrm{avg}} = h\nu \frac{{}_{\mathrm{e}}\sigma_{\mathrm{tr}}}{{}_{\mathrm{e}}\sigma}$$

- Energy transfer cross section (the average fraction of the incident photon energy that is transferred to Compton electrons per electron m^{-2} in the material traversed)

$${}_{\mathrm{e}}\sigma_{\mathrm{tr}} = 2\pi \int \frac{\mathrm{d}_{\mathrm{e}}\sigma_{\mathrm{tr}}}{\mathrm{d}\Omega} \sin\theta \, \mathrm{d}\theta$$

- Similarly, the differential cross section for energy scattering (i.e., the energy carried by the scattered photons) is defined by

$$\frac{\mathrm{d}_{\mathrm{e}}\sigma_{\mathrm{s}}}{\mathrm{d}\Omega} = \frac{\nu'}{\nu}\frac{\mathrm{d}_{\mathrm{e}}\sigma}{\mathrm{d}\Omega} \quad \Rightarrow \quad (h\nu')_{\mathrm{avg}} = h\nu \frac{{}_{\mathrm{e}}\sigma_{\mathrm{s}}}{{}_{\mathrm{e}}\sigma}$$

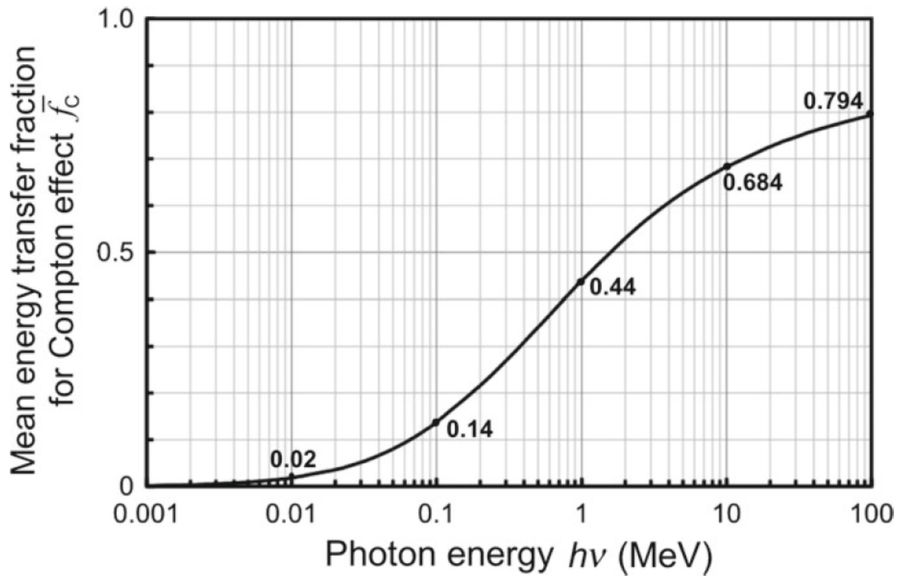
$$\frac{T_{\mathrm{avg}}}{h\nu} + \frac{(h\nu')_{\mathrm{avg}}}{h\nu} = 1 \quad \Rightarrow \quad {}_{\mathrm{e}}\sigma = {}_{\mathrm{e}}\sigma\left[\frac{T_{\mathrm{avg}}}{h\nu} + \frac{(h\nu')_{\mathrm{avg}}}{h\nu}\right] = {}_{\mathrm{e}}\sigma\left[\frac{{}_{\mathrm{e}}\sigma_{\mathrm{tr}}}{{}_{\mathrm{e}}\sigma} + \frac{{}_{\mathrm{e}}\sigma_{\mathrm{s}}}{{}_{\mathrm{e}}\sigma}\right] = {}_{\mathrm{e}}\sigma_{\mathrm{tr}} + {}_{\mathrm{e}}\sigma_{\mathrm{s}}$$

Mean energy transfer fraction

Table 8.1 Average Kinetic Energy, T_{avg}, of Compton Recoil Electrons and Fraction of Incident Photon Energy, $h\nu$

Photon Energy $h\nu$ (MeV)	Average Recoil Electron Energy T_{avg} (MeV)	Average Fraction of Incident Energy $T_{avg}/h\nu$
0.01	0.0002	0.0187
0.02	0.0007	0.0361
0.04	0.0027	0.0667
0.06	0.0056	0.0938
0.08	0.0094	0.117
0.10	0.0138	0.138
0.20	0.0432	0.216
0.40	0.124	0.310
0.60	0.221	0.368
0.80	0.327	0.409
1.00	0.440	0.440
2.00	1.06	0.531
4.00	2.43	0.607
6.00	3.86	0.644
8.00	5.34	0.667
10.0	6.84	0.684
20.0	14.5	0.727
40.0	30.4	0.760
60.0	46.6	0.776
80.0	62.9	0.787
100.0	79.4	0.794

$$T_{avg} = h\nu \frac{{}_e\sigma_{tr}}{{}_e\sigma}$$



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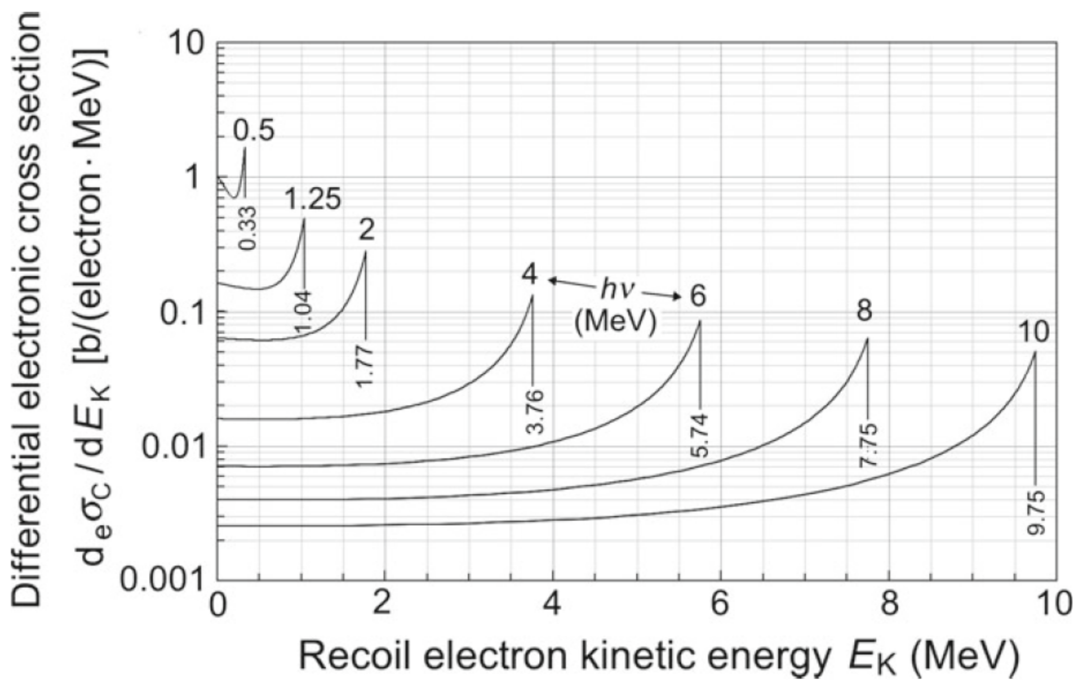
Energy distribution of recoil electrons

$$\frac{\mathrm{d}_e\sigma_C^{KN}(E_K)}{\mathrm{d}E_K} = \frac{\mathrm{d}_e\sigma_C^{KN}}{\mathrm{d}\Omega}\frac{\mathrm{d}\Omega}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}E_K}$$

$$= \frac{\pi r_e^2}{\varepsilon h\nu}\left\{2 - \frac{2E_K}{\varepsilon(h\nu - E_K)} + \frac{E_K^2}{\varepsilon^2(h\nu - E_K)^2} + \frac{E_K^2}{h\nu(h\nu - E_K)}\right\}$$

$$T_{max} = \frac{2h\nu}{2 + mc^2/h\nu}.$$



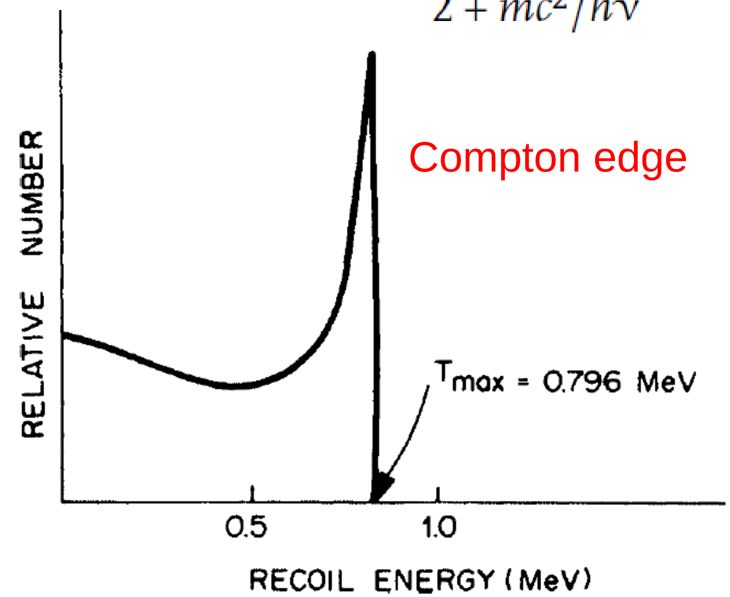


Fig. 8.5 Relative number of Compton recoil electrons as a function of their energy for 1-MeV photons.

Compton attenuation coefficient

If the material traversed consists of N atoms m^{-3} of an element of atomic number Z, then the number of electrons m^{-3} is $n = NZ$. The Compton interaction probability per unit distance of travel for the photon in the material is then

$$\sigma = NZ\,{}_e\sigma = n\,{}_e\sigma. \qquad (8.37)$$

Compton macroscopic cross section

The quantity $Z\,{}_e\sigma$ is the Compton collision cross section per atom, and σ is the Compton macroscopic cross section, or attenuation coefficient, having the dimensions of inverse length. If the material is a compound or mixture, then the cross sections of the individual elements contribute additively to σ as $NZ\,{}_e\sigma$.

The (Compton) linear attenuation coefficients for energy transfer and energy scattering can be obtained from Eqs. (8.31) and (8.34) after multiplication by the density of electrons. They are

$$\sigma_{tr} = \sigma \frac{T_{avg}}{h\nu} \qquad (8.38)$$

and

$$\sigma_s = \sigma \frac{(h\nu')_{avg}}{h\nu}. \qquad (8.39)$$

From Eq. (8.36), the (total) Compton attenuation coefficient can be written

$$\sigma = \sigma_{tr} + \sigma_s. \qquad (8.40)$$

Compton attenuation coefficient

Example

The Compton collision cross section for the interaction of an 8-MeV photon with an electron is 5.99×10^{-30} m^2. For water, find the following quantities for Compton scattering: (a) the collision cross section per molecule; (b) the linear attenuation coefficient; (c) the linear attenuation coefficient for energy transfer; and (d) the linear attenuation coefficient for energy scattering.

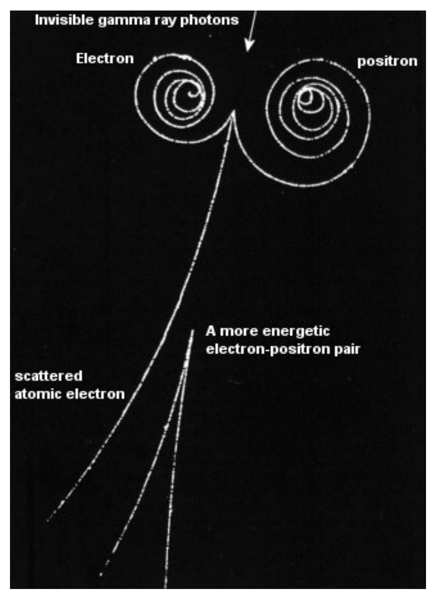
Solution

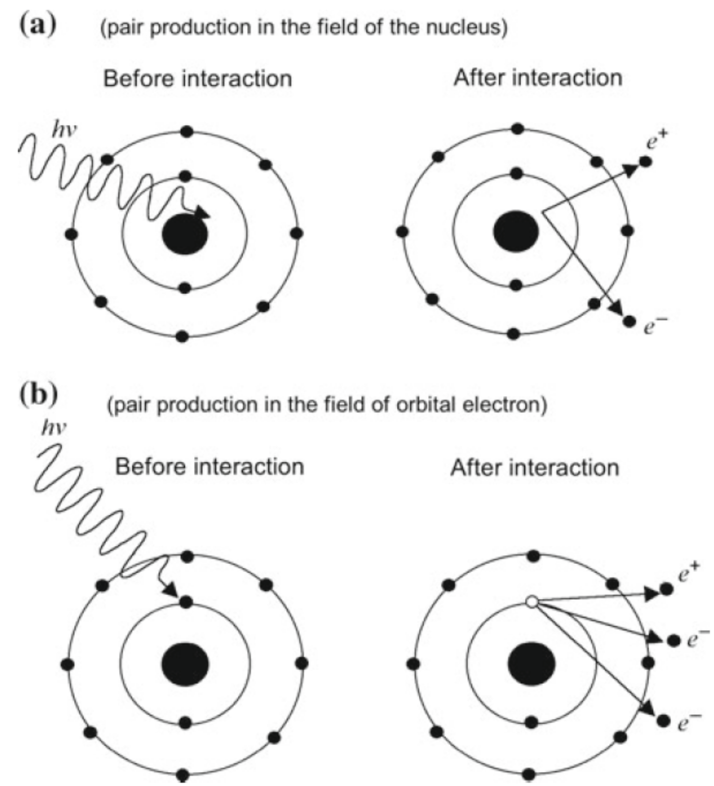
(a) The Compton collision cross section, defined by Eq. (8.29), is ${}_e\sigma = 5.99 \times 10^{-30}$ m^2. Since there are 10 electrons in a water molecule, the Compton cross section per molecule is $10_e\sigma = 5.99 \times 10^{-29}$ m^2. [See discussion in connection with Eq. (8.37).]
(b) Using Eq. (8.37) with $n = 3.34 \times 10^{29}$ m^{-3} (calculated in Section 5.9), we find for the linear attenuation coefficient, $\sigma = n_e\sigma = (3.44 \times 10^{29}\ \text{m}^{-3}) \times (5.99 \times 10^{-30}\ \text{m}^2) = 2.06\ \text{m}^{-1}$.
(c) In Table 8.1 we find that $T_{avg}/h\nu = 0.667$ for 8-MeV photons. It follows from Eq. (8.38) that the linear attenuation coefficient for energy transfer is $\sigma_{tr} = 0.667\sigma = 0.667 \times 2.06 = 1.37\ \text{m}^{-1}$.
(d) From Eq. (8.39) we find that $\sigma_s = 0.333\sigma = 0.333 \times 2.06 = 0.686\ \text{m}^{-1}$. Alternatively, one can use Eq. (8.40) and the answer from (c): $\sigma_s = \sigma - \sigma_{tr} = 2.06 - 1.37 = 0.69\ \text{m}^{-1}$.

Pair production

- A photon with an energy of at least twice the electron rest energy, $h\nu \geq 2mc^2$, can be converted into an electron–positron pair in the field of an atomic nucleus.
- Pair production can also occur in the field of an atomic electron, but the probability is considerably smaller and the threshold energy is $4mc^2$. (This process is often referred to as “triplet” production because of the presence of the recoiling atomic electron in addition to the pair.)
- Energy conservation (neglecting recoil energy)

$$h\nu = 2mc^2 + T_+ + T_-.$$



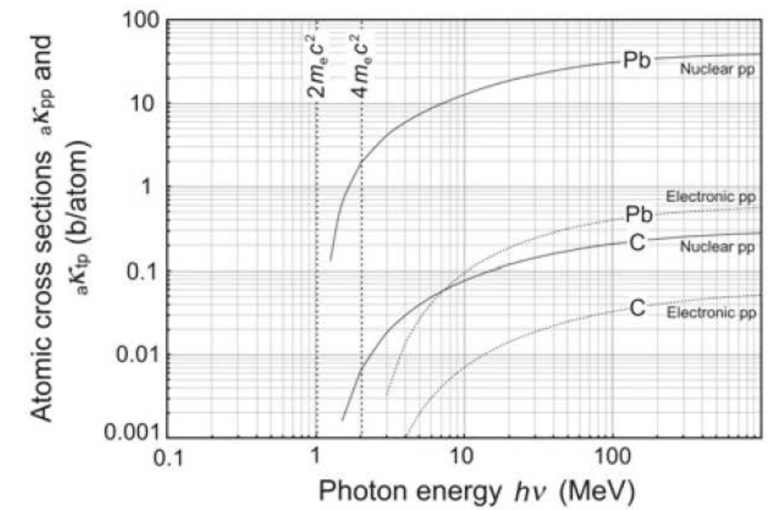


Pair production

- Atomic cross section for pair production

$$_{\mathrm{a}}\kappa = \alpha r_{\mathrm{e}}^{2} Z^{2} P(\varepsilon, Z)$$

Fig. 7.35 Atomic cross sections for nuclear pair production ${}_{\mathrm{a}}\kappa_{\mathrm{NPP}}$ (*solid curves*) and for triplet production (electronic pair production) ${}_{\mathrm{a}}\kappa_{\mathrm{TP}}$ (*dotted curves*) against incident photon energy $h\nu$ for carbon and lead. Data are from the NIST



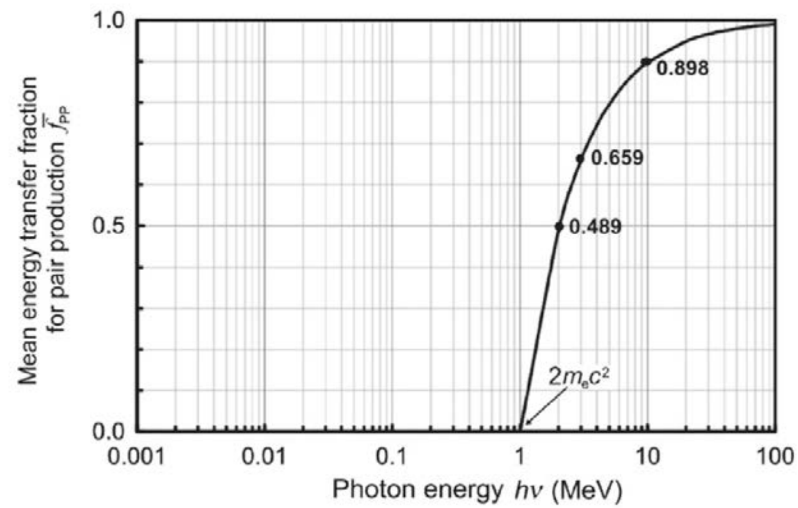
- Mass attenuation coefficient

$$\frac{\kappa}{\rho} = \frac{N_{\mathrm{A}}}{A}\, {}_{\mathrm{a}}\kappa$$

- Energy transfer to charged particles

$$\overline{E}_{\mathrm{tr}}^{\mathrm{PP}} = h\nu - 2m_{\mathrm{e}}c^{2}.$$

$$\overline{f}_{\mathrm{PP}} = \frac{\overline{E}_{\mathrm{tr}}^{\mathrm{PP}}}{h\nu} = 1 - \frac{2m_{\mathrm{e}}c^{2}}{h\nu}$$



Attenuation coefficient

- Photon penetration in matter is governed statistically by the probability per unit distance traveled that a photon interacts by one physical process or another.
- This probability, denoted by μ, is called the linear attenuation coefficient (or macroscopic cross section) and has the dimensions of inverse length. The coefficient μ depends on photon energy and on the material being traversed.
- We let $N(x)$ represent the number of photons that reach a depth x without having interacted. The number that interact within the next small distance $\mathrm{d}x$ is proportional to N and to $\mathrm{d}x$.

$$\mathrm{d}N = -\mu N \mathrm{d}x$$

$$N(x) = N_0 \underline{e^{-\mu x}}$$

Fraction of 'uncollided photon'

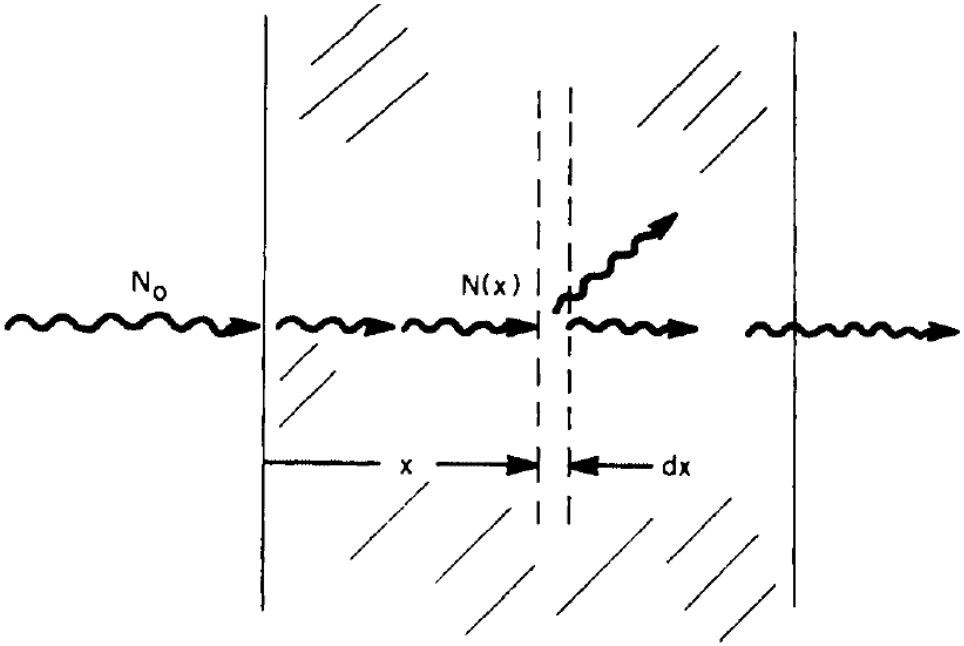


Fig. 8.6 Pencil beam of N_0 monoenergetic photons incident on slab. The number of photons that reach a depth x without having an interaction is given by $N(x) = N_0 e^{-\mu x}$, where μ is the linear attenuation coefficient.

Attenuation coefficient

- Experimental setup: A narrow beam of monoenergetic photons is directed toward an absorbing slab of thickness x. A small detector of size d is placed at a distance $R \gg d$ behind the slab directly in the beam line.
- Under these conditions, referred to as "narrow-beam" or "good" scattering geometry, only photons that traverse the slab without interacting will be detected. One can measure the relative rate at which photons reach the detector as a function of the absorber thickness to obtain the value of μ.

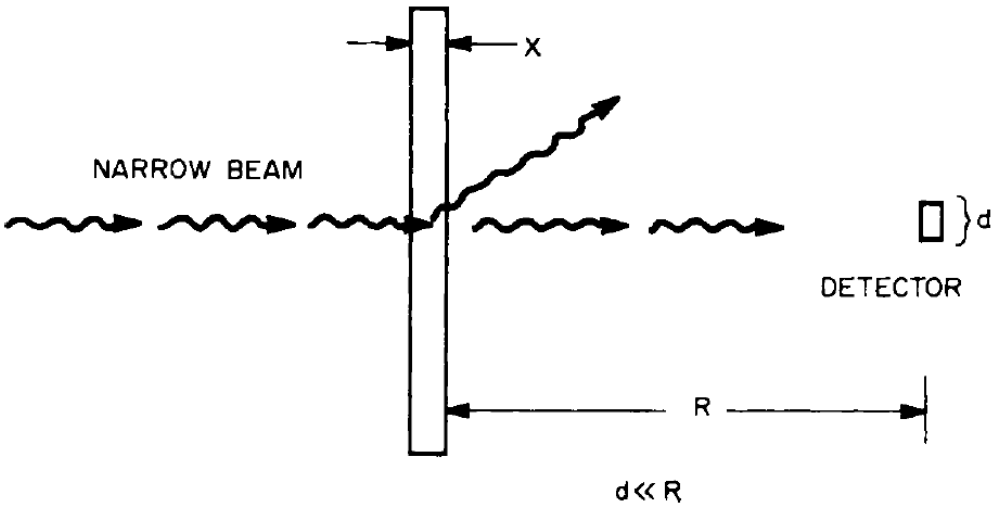


Fig. 8.7 Illustration of "good" scattering geometry for measuring linear attenuation coefficient μ. Photons from a *narrow* beam that are absorbed or scattered by the absorber do not reach a small detector placed in beam line some distance away.

Attenuation coefficient for photons

- The linear attenuation coefficient for photons of a given energy in a given material comprises the individual contributions from the various physical processes that can remove photons from the narrow beam.

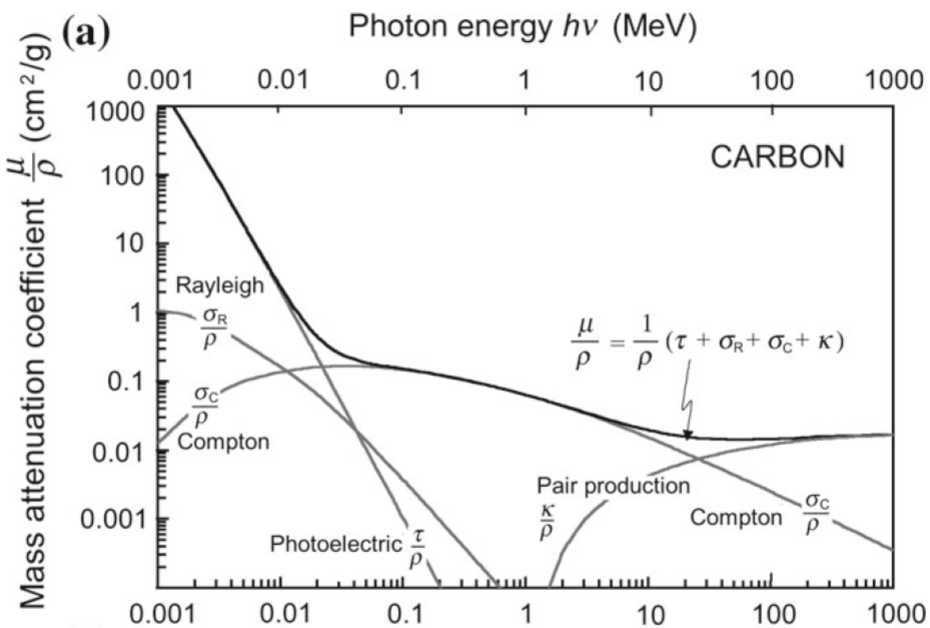
$$\mu = \tau + \sigma + \kappa$$

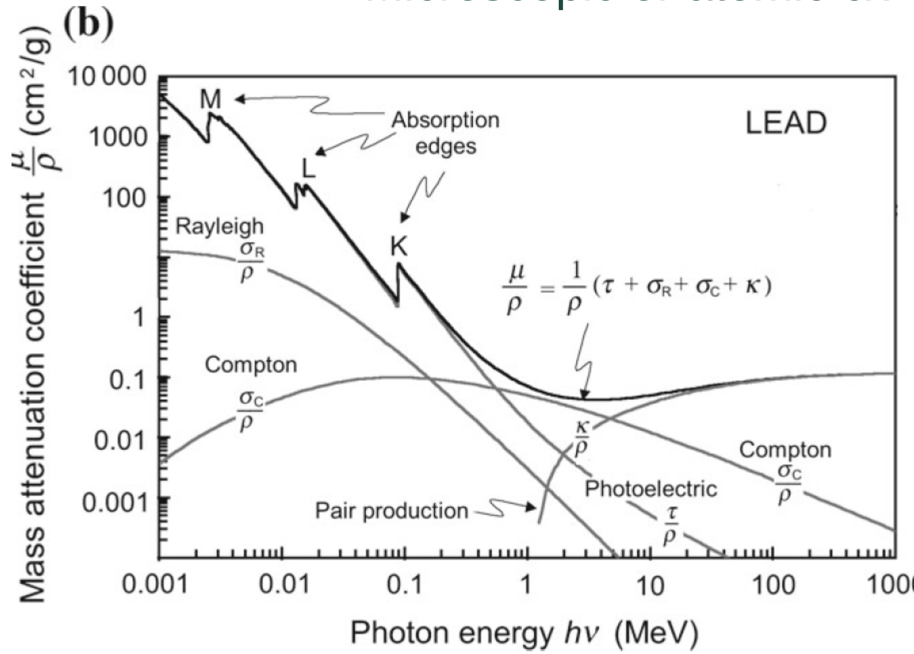
Number density

$$\mu = N\sigma = \frac{N_A}{A}\rho\sigma$$

Microscopic or atomic cx

- Mass attenuation coefficient $\dfrac{\mu}{\rho} = \dfrac{\tau}{\rho} + \dfrac{\sigma}{\rho} + \dfrac{\kappa}{\rho}$





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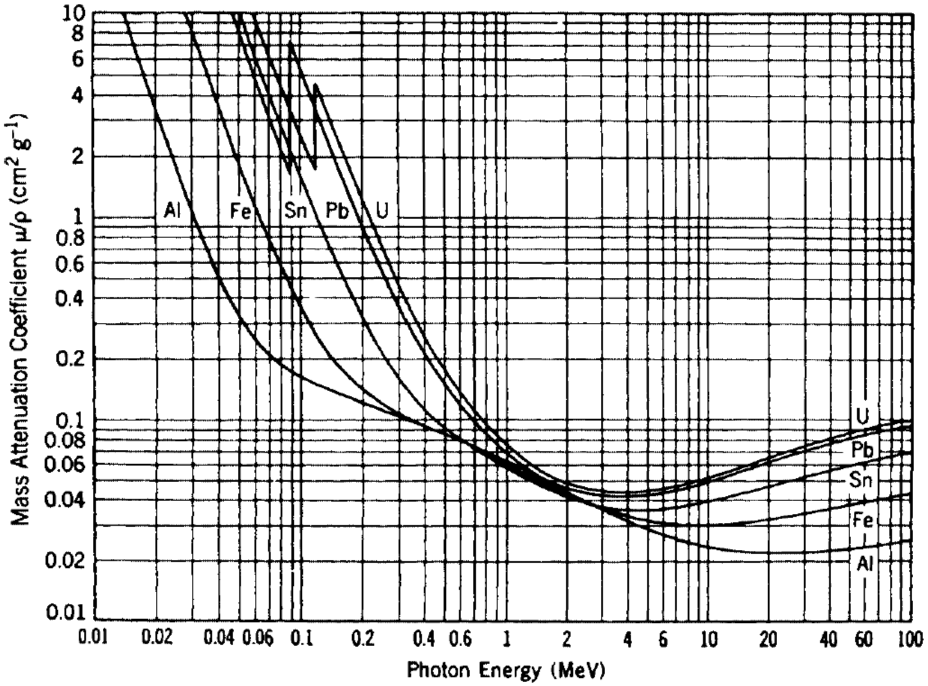
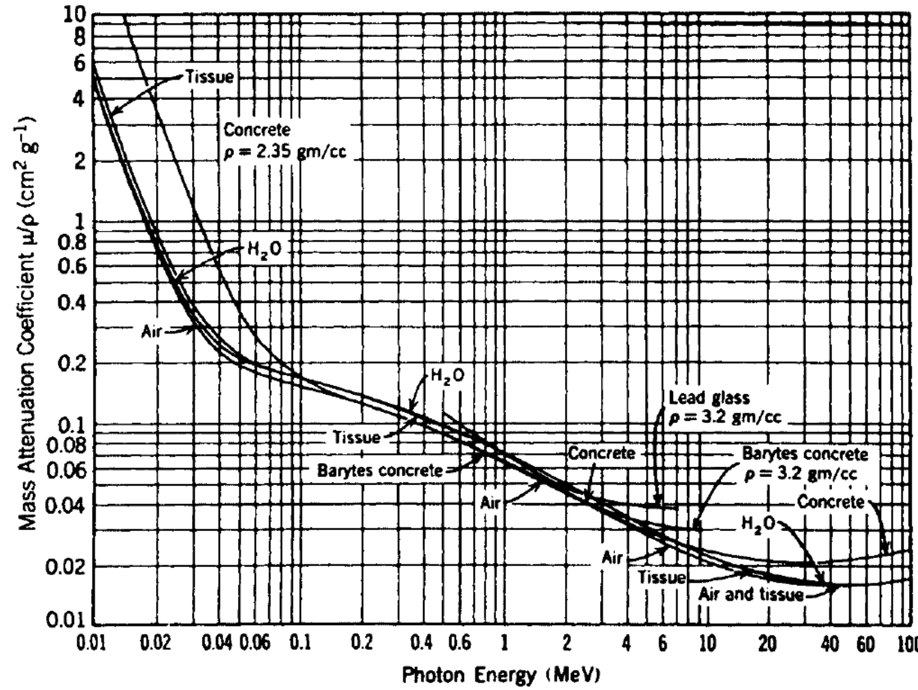
Attenuation coefficient for photons





Example

What is the atomic cross section of lead for 500-keV photons?

Solution

From Fig. 8.8, the mass attenuation coefficient is $\mu/\rho = 0.16\ \text{cm}^2\,\text{g}^{-1}$. The gram atomic weight of lead is 207 g. We find from Eq. (8.50) that

$$\sigma_A = \left(\frac{\mu}{\rho}\right)\left(\frac{A}{N_0}\right) = (0.16\ \text{cm}^2\,\text{g}^{-1})\left(\frac{207\ \text{g}}{6.02\times10^{23}}\right) = 5.50\times10^{-23}\ \text{cm}^2. \quad (8.51)$$

Alternatively, $\sigma_A = 55.0$ barn.

Energy transfer and absorption

- The incident fluence Φ_0 is the number of photons per unit area that cross a plane perpendicular to the beam.
- The number that cross per unit area per unit time at any instant is called the fluence rate, or flux density:

$$\dot{\Phi}_0 = \mathrm{d}\Phi_0/\mathrm{d}t \ (= \varphi_0) \qquad \mathrm{m^{-2}\,s^{-1}}$$

- The energy that passes per unit area is called the energy fluence Ψ_0, having the units J m^{-2}.
- The corresponding instantaneous rate of energy flow per unit area per unit time is the energy fluence rate, or energy flux density:

$$\dot{\Psi}_0 = \mathrm{d}\Psi_0/\mathrm{d}t \ (= \psi_0) \qquad \mathrm{J\,m^{-2}\,s^{-1} = W\,m^{-2}}$$

$$\Psi_0 = \Phi_0 h\nu \quad \text{and} \quad \dot{\Psi}_0 = \dot{\Phi}_0 h\nu.$$

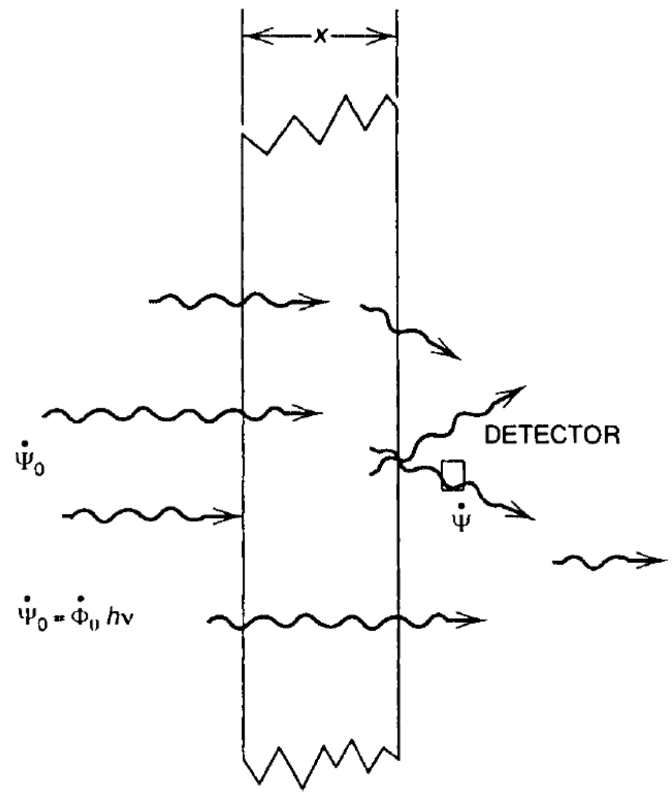


Fig. 8.10 Broad, uniform, parallel beam of monoenergetic photons normally incident on an absorber of thicknesses x. Incident energy fluence rate is $\dot{\Psi}_0$, and transmitted energy fluence rate is $\dot{\Psi}$.

Mass energy-transfer coefficient

- Photoelectric effect

$$\frac{\tau_{tr}}{\rho} = \frac{\tau}{\rho}\underline{\left(1 - \frac{\delta}{h\nu}\right)}$$

The average energy emitted as fluorescence radiation following photoelectric absorption in the material

The fraction of the incident intensity transferred to electrons (i.e., the photoelectron and the Auger electrons)

- Compton scattering

$$\frac{\sigma_{tr}}{\rho} = \frac{\sigma}{\rho}\underline{\frac{T_{avg}}{h\nu}}$$

The average fraction of the incident photon energy that is converted into the initial kinetic energy of the Compton electrons

- Pair production

$$\frac{\kappa_{tr}}{\rho} = \frac{\kappa}{\rho}\underline{\left(1 - \frac{2mc^2}{h\nu}\right)}$$

The fraction of the initial electron-positron kinetic energy

- Total mass energy-transfer coefficient

$$\frac{\mu_{tr}}{\rho} = \frac{\tau_{tr}}{\rho} + \frac{\sigma_{tr}}{\rho} + \frac{\kappa_{tr}}{\rho}$$

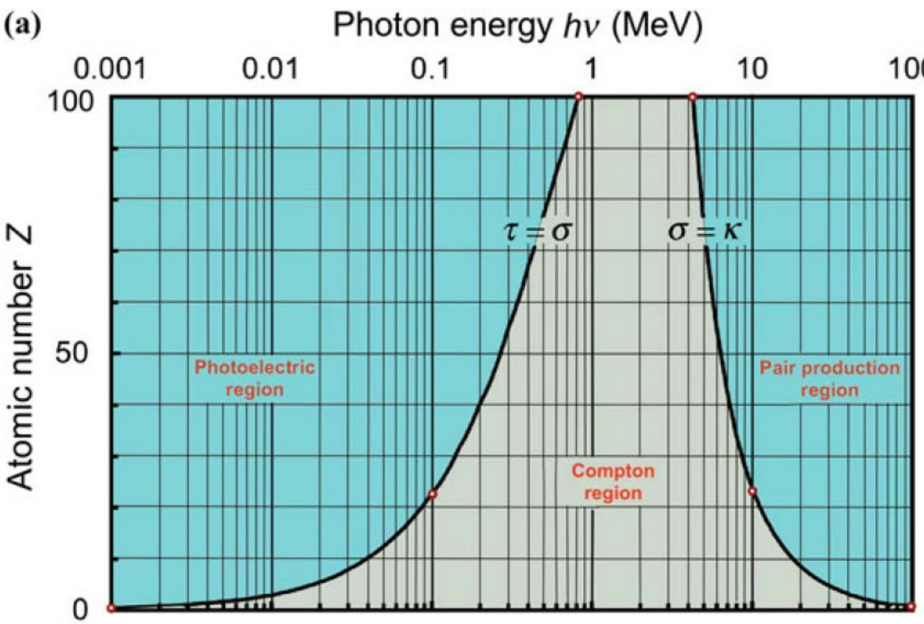
$$= \frac{\tau}{\rho}\left(1 - \frac{\delta}{h\nu}\right) + \frac{\sigma}{\rho}\left(\frac{T_{avg}}{h\nu}\right) + \frac{\kappa}{\rho}\left(1 - \frac{2mc^2}{h\nu}\right)$$

Mass energy-transfer coefficient

- Total mass energy-transfer coefficient: determines the total initial kinetic energy of all electrons produced by the photons, both directly (as in photoelectric absorption, Compton scattering, and pair production) and indirectly (as Auger electrons).
- Except for the subsequent bremsstrahlung that the electrons might emit, the energy absorbed in the immediate vicinity of the interaction site would be the same as the energy transferred there.

$$\frac{\mu_{tr}}{\rho} = \frac{\tau_{tr}}{\rho} + \frac{\sigma_{tr}}{\rho} + \frac{\kappa_{tr}}{\rho}$$

$$= \frac{\tau}{\rho}\left(1 - \frac{\delta}{h\nu}\right) + \frac{\sigma}{\rho}\left(\frac{T_{avg}}{h\nu}\right) + \frac{\kappa}{\rho}\left(1 - \frac{2mc^2}{h\nu}\right)$$

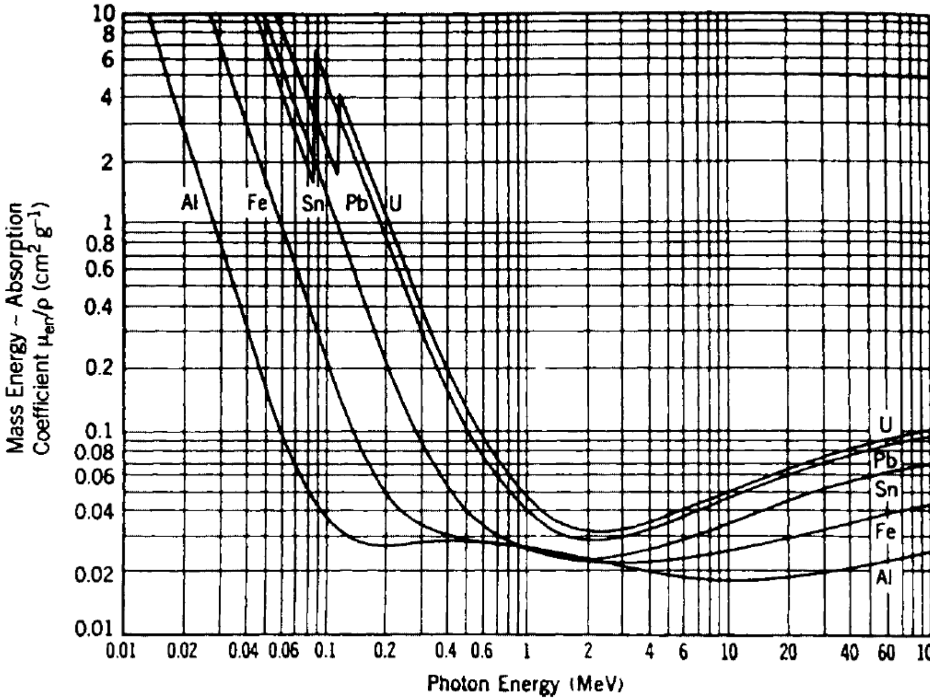


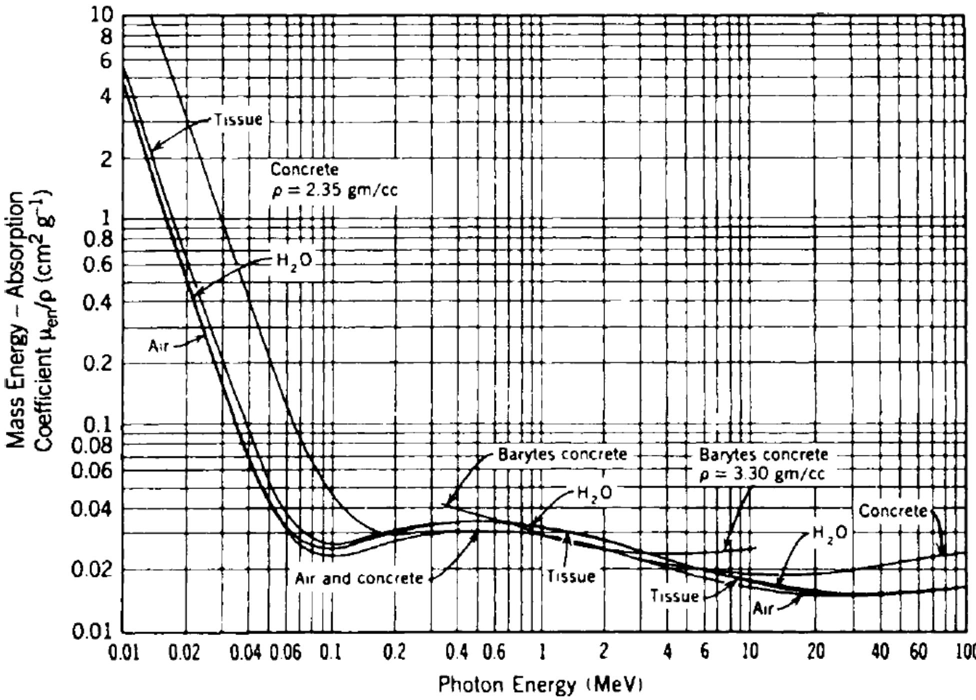
Mass energy-absorption coefficient

- Letting g represent the average fraction of the initial kinetic energy transferred to electrons that is subsequently emitted as bremsstrahlung, one defines the mass energy-absorption coefficient as

$$\frac{\mu_{en}}{\rho} = \frac{\mu_{tr}}{\rho}(1-g)$$

$$\frac{\mu_{tr}}{\rho} = \frac{\tau}{\rho}\left(1-\frac{\delta}{h\nu}\right)+\frac{\sigma}{\rho}\left(\frac{T_{avg}}{h\nu}\right)+\frac{\kappa}{\rho}\left(1-\frac{2mc^2}{h\nu}\right)$$

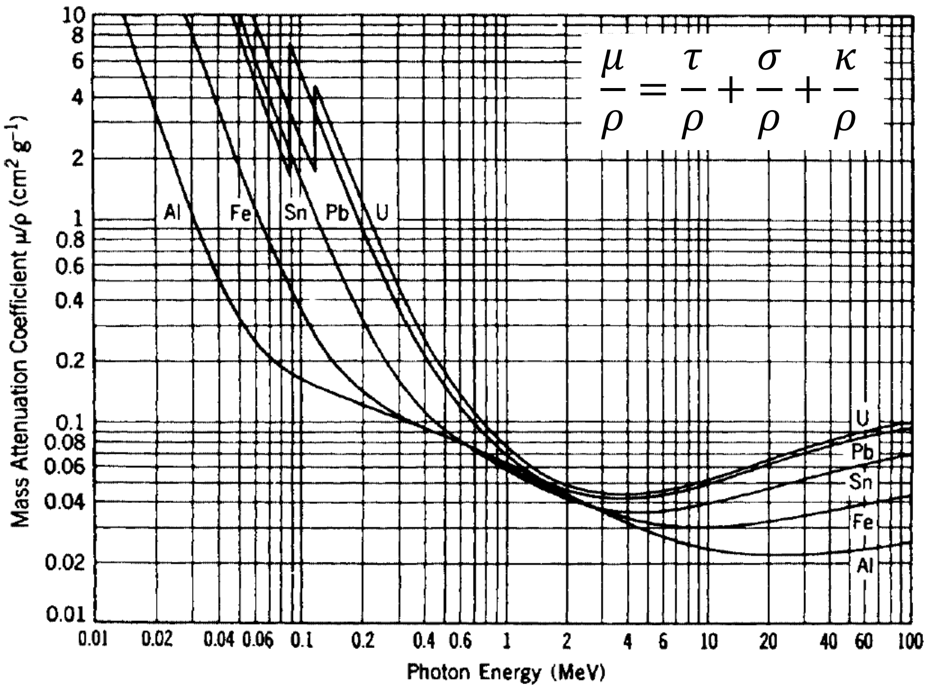


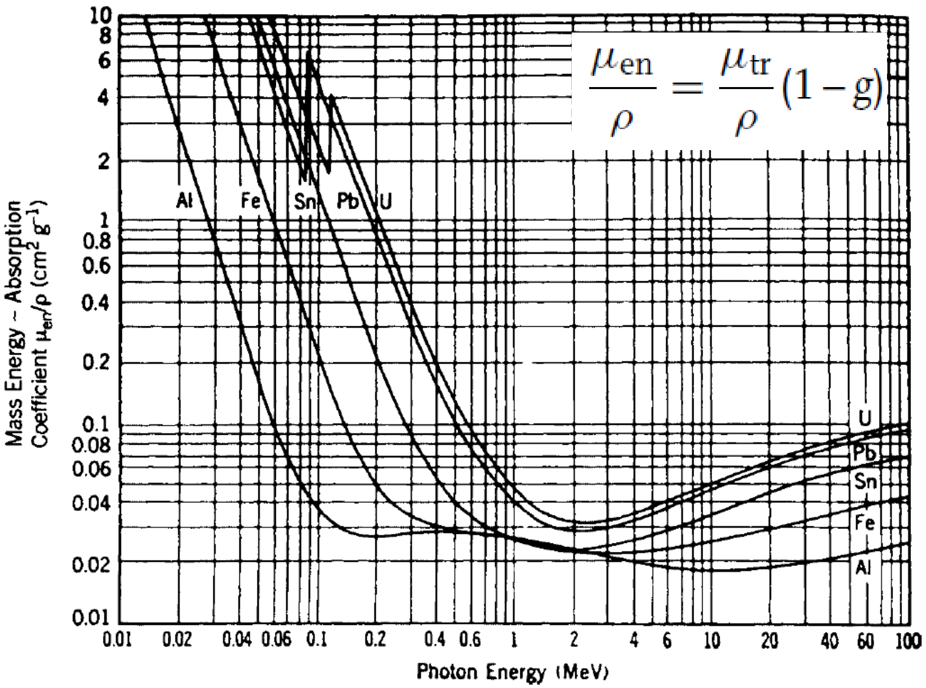


Mass attenuation vs. energy-absorption coefficient

Table 8.3 Mass Attenuation, Mass Energy-Transfer, and Mass Energy-Absorption Coefficients ($cm^2\ g^{-1}$) for Photons in Water and Lead

Photon Energy (MeV)	Water μ/ρ	Water μ_{tr}/ρ	Water μ_{en}/ρ	Lead μ/ρ	Lead μ_{tr}/ρ	Lead μ_{en}/ρ
0.01	5.33	4.95	4.95	131.	126.	126.
0.10	0.171	0.0255	0.0255	5.55	2.16	2.16
1.0	0.0708	0.0311	0.0310	0.0710	0.0389	0.0379
10.0	0.0222	0.0163	0.0157	0.0497	0.0418	0.0325
100.0	0.0173	0.0167	0.0122	0.0931	0.0918	0.0323





Individual physical processes contribute to the interaction coefficients as functions of the photon energy

- Linear attenuation coefficient

$$\mu = \tau + \sigma + \kappa$$

$$\frac{\mu_{tr}}{\rho} = \frac{\tau}{\rho}\left(1-\frac{\delta}{h\nu}\right) + \frac{\sigma}{\rho}\left(\frac{T_{avg}}{h\nu}\right) + \frac{\kappa}{\rho}\left(1-\frac{2mc^2}{h\nu}\right)$$

- Linear energy-absorption coefficient

$$\mu_{en} = \mu_{tr}(1-g)$$

It is instructive to see how the individual physical processes contribute to the interaction coefficients as functions of the photon energy. Figure 8.13 for water shows τ, σ_s, σ_{tr}, and κ as well as the coefficients μ and μ_{en}. Also shown for comparison is the attenuation coefficient σ_r for Raleigh scattering, which we have ignored. At the lowest energies (<15 keV), the photoelectric effect accounts for virtually all of the interaction. As the photon energy increases, τ drops rapidly and goes below σ_s. Between about 100 keV and 10 MeV, most of the attenuation in water is due to the Compton effect. Above about 1.5 MeV, $\sigma_{tr} > \sigma_s$. The Compton coefficients then fall off with increasing energy, and pair production becomes the dominant process at high energies.

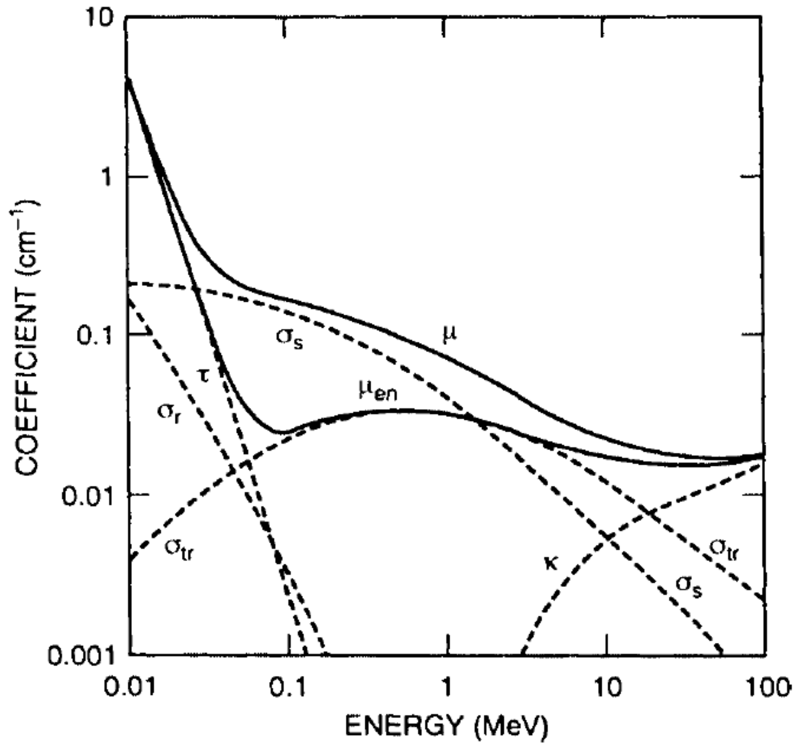


Fig. 8.13 Linear attenuation and energy-absorption coefficients as functions of energy for photons in water.

Calculation of energy absorption and energy transfer

- We begin by assuming that the slab is thin compared with the mean free paths of the incident and secondary photons, so that (1) multiple scattering of photons in the slab is negligible and (2) virtually all fluorescence and bremsstrahlung photons escape from it. On the other hand, we assume that the secondary electrons produced by the photons are stopped in the slab.

duced by the photons are stopped in the slab. Under these conditions, the transmitted intensity in Fig. 8.10 is given by

$$\dot{\Psi} = \dot{\Psi}_0 e^{-\mu_{en}x}. \tag{8.59}$$

For $\mu_{en}x \ll 1$, which is consistent with our assumptions, one can write $e^{-\mu_{en}x} \approx 1 - \mu_{en}x$. Equation (8.59) then implies that

$$\dot{\Psi}_0 - \dot{\Psi} = \dot{\Psi}_0 \mu_{en} x. \tag{8.60}$$

With reference to Fig. 8.14, the rate at which energy is absorbed in the slab over an area A is $(\dot{\Psi}_0 - \dot{\Psi})A = \dot{\Psi}_0 \mu_{en} x A$. Since the mass of the slab over this area is $\rho A x$, where ρ is the density, the rate of energy absorption per unit mass, $\dot{D}$, in the slab is

$$\dot{D} = \frac{\dot{\Psi}_0 \mu_{en} x A}{\rho A x} = \dot{\Psi}_0 \frac{\mu_{en}}{\rho}. \tag{8.61}$$

The quantity $\dot{D}$ is, by definition, the average dose rate in the slab. As discussed in Chapter 12, under the condition of electronic equilibrium Eq. (8.61) also implies that the dose rate at a point in a medium is equal to the product of the intensity, or energy fluence rate, at that point and the mass energy-absorption coefficient.

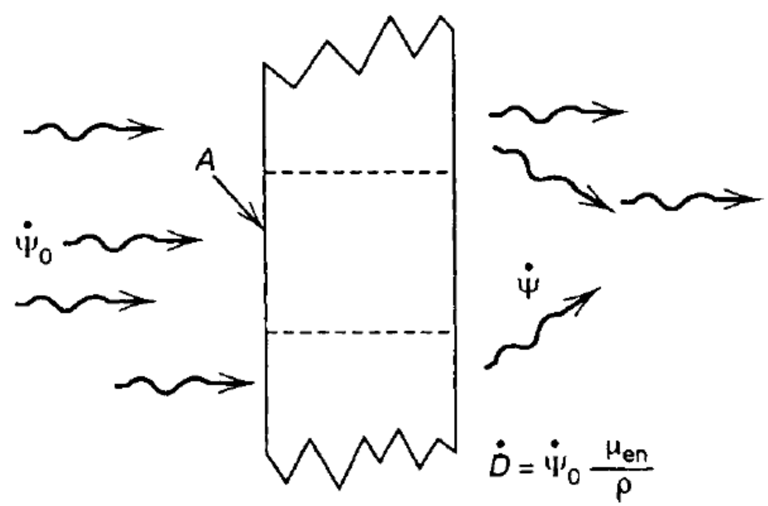


Fig. 8.14 Rate of energy absorption per unit mass in thin slab (dose rate, $\dot{D}$) is equal to the product of the incident intensity and mass energy-absorption coefficient.

Calculation of energy absorption and energy transfer

The mass energy-transfer coefficient can be employed in a similar derivation. The quantity thus obtained,

$$\dot{K} = \dot{\Psi}_0 \frac{\mu_{tr}}{\rho}, \tag{8.62}$$

is called the average *kerma rate* in the slab. Equation (8.62) also gives the kerma rate at a point in a medium in terms of the energy fluence rate at that point, irrespective of electronic equilibrium. As described in Section 12.10, kerma is defined generally as the total initial kinetic energy of all charged particles liberated by uncharged radiation (photons and/or neutrons) per unit mass of material.

The ideal geometry and other conditions represented by Fig. 8.10 and Eq. (8.59) are approached in practice only to various degrees of approximation. The nonuniformity and finite width of real beams, for example, are two factors that usually deviate significantly from the ideal. The computation of attenuation and energy absorption in thick slabs is treated with the help of buildup factors (Charter 15). Nevertheless, μ_{en} is frequently useful for estimating absorbed energy in a number of situations, as the following examples illustrate.

Homework

- J. Turner, Atoms, Radiation, and Radiation Detection, Wiley (2007), chapter 8

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