Electrostatic Accelerators and Pulsed High Voltage

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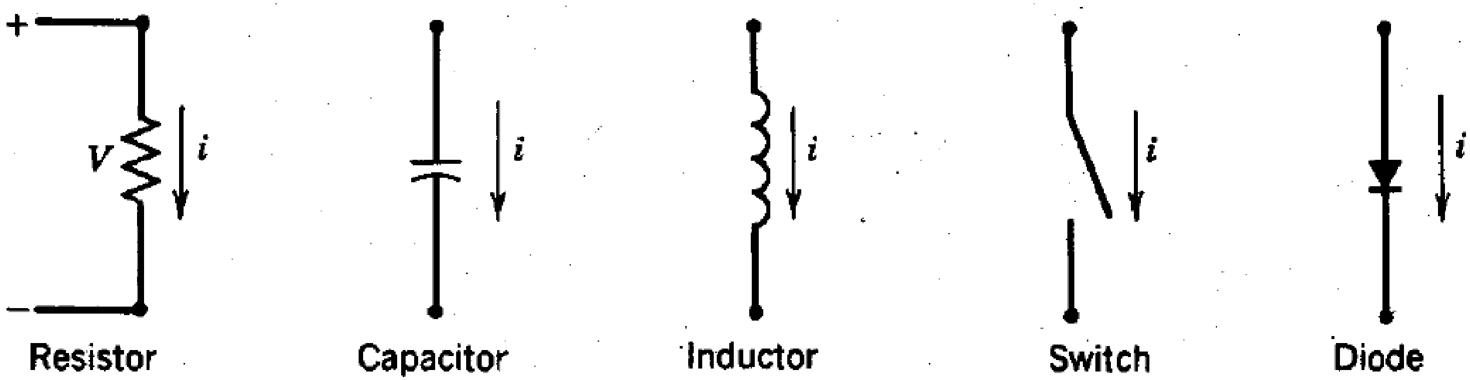
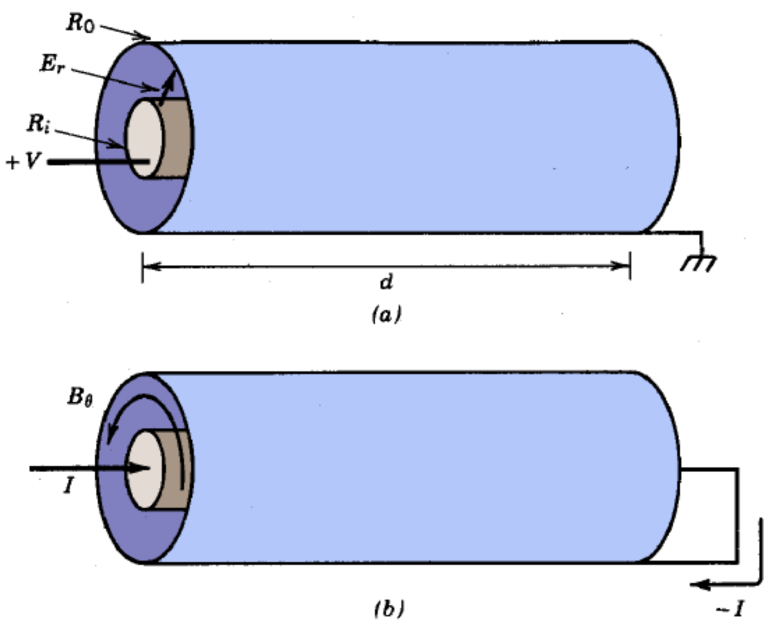
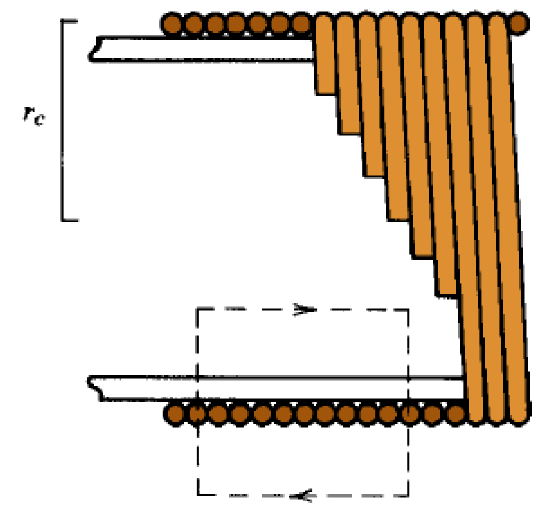
Resistors, capacitors and inductors





$$C = \frac{2\pi\epsilon d}{\ln\left(\frac{R_o}{R_i}\right)}$$

$$L = \frac{\mu d}{2\pi}\ln\left(\frac{R_o}{R_i}\right)$$



$$L = \frac{\mu N^2 A}{d}$$

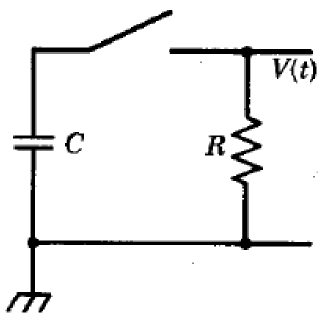
RC circuit

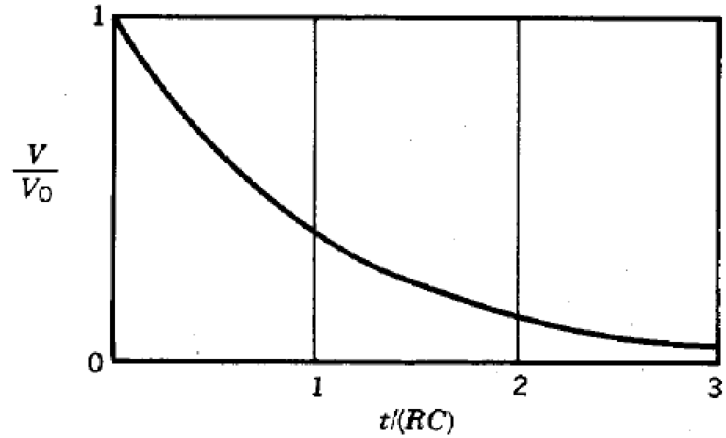
- This is the simplest model for a pulsed voltage circuit; electrical energy is stored in a capacitor and then dumped into a load resistor via a switch.

$$C\frac{dV}{dt}+\frac{V}{R}=0$$

$$\Downarrow$$

$$V(t)=V_0\exp\left(-\frac{t}{RC}\right)$$



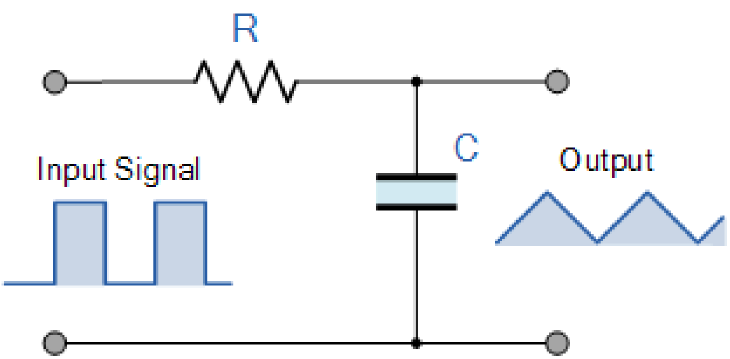


- Passive integrator (keep $V_{out} \ll V_{in}$ by keeping the product RC large)

$$I=C\frac{dV_{out}}{dt}=\frac{V_{in}-V_{out}}{R}\approx\frac{V_{in}}{R}$$

$$\Downarrow$$

$$V_{out}(t)\approx\frac{1}{RC}\int V_{in}(t)dt$$



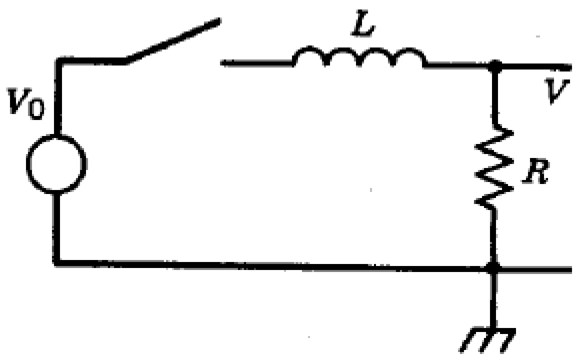
RL circuit

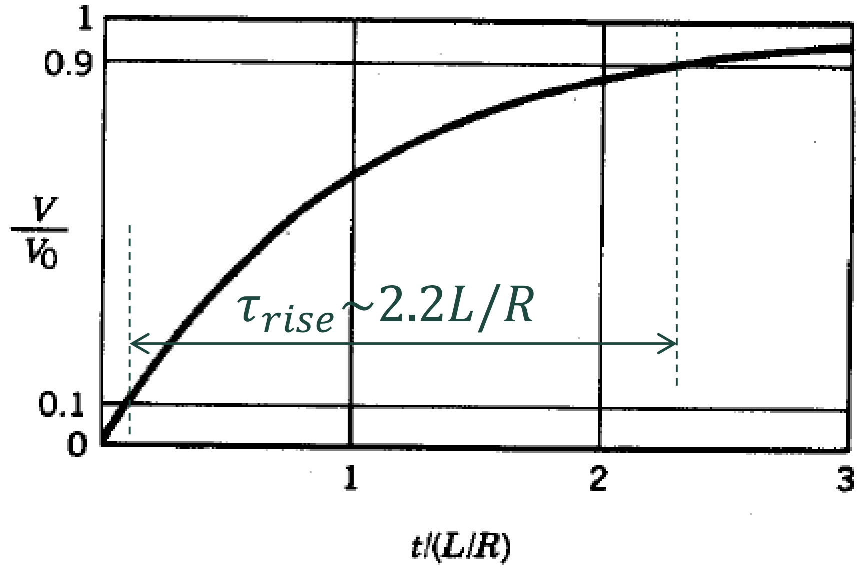
- Usually, we want a rapid rise time for power into the load. The time for initiation of current flow to the load is limited by the undesirable (or parasitic) inductance.

$$L\frac{di}{dt} + Ri = 0$$

$$\Downarrow$$

$$V(t) = V_0\left[1 - \exp\left(-\frac{t}{(L/R)}\right)\right]$$

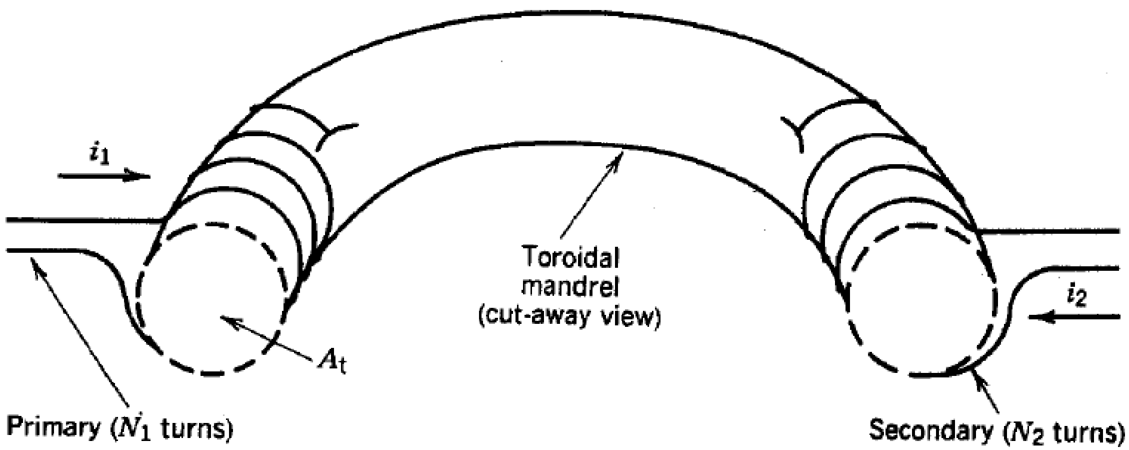




- The L/R time determines how fast current and voltage can be induced in the load.

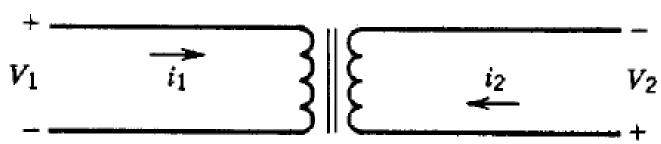
Transformer

- The transformer is a prime component in all high-voltage supplies. It utilizes magnetic coupling to convert a low-voltage ac input to a high-voltage ac output at reduced current.
- The transformer does not produce energy. The product of voltage times current at the output is equal to or less than that at the input.



(a)

$$L_1 = \frac{\mu_0 N_1^2 A_t}{2\pi r_t}$$



$$L_2 = \frac{\mu_0 N_2^2 A_t}{2\pi r_t}$$

(b)

Transformer circuit

- For ideal coupling:

$$N_1 i_1 \cong N_2 i_2$$

$$N_1 \frac{d\Phi}{dt} = V_1 \qquad N_2 \frac{d\Phi}{dt} = V_2$$

$$V_2 = V_1 \frac{N_2}{N_1} = i_2 R_2$$

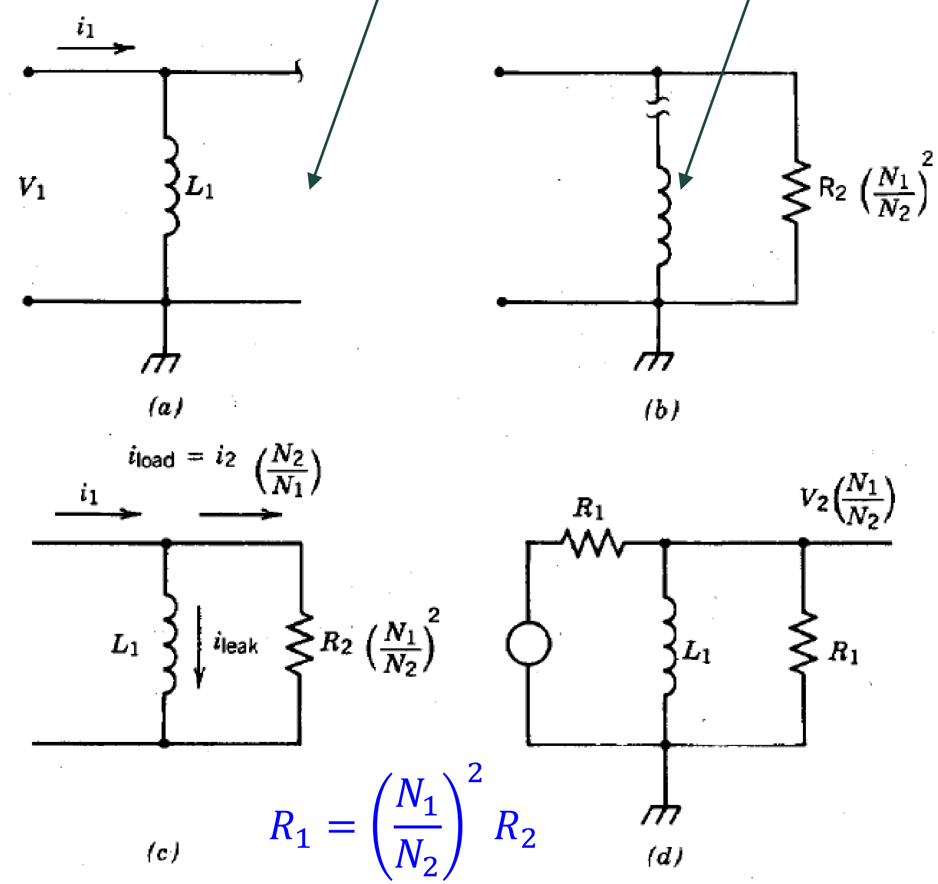
$$\Rightarrow V_1 = i_1 \underbrace{\left(\frac{N_1}{N_2}\right)^2 R_2}$$

Transformed load resistance when viewed from the primary

- To reduce the leakage current, the primary inductance should be high → use of ferromagnetic material as a core

Open load

Infinite L_1



$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2$$

Figure 9.7 Equivalent circuit models for a transformer with ideal coupling. (*a*) Open circuit load on secondary. (*b*) Infinite primary shunt inductance. (*c*) Basic circuit model for an ideal transformer with a resistive load on the secondary. (*d*) Driving a resistive load on the secondary by a pulse modulator with matched impedance at the primary.

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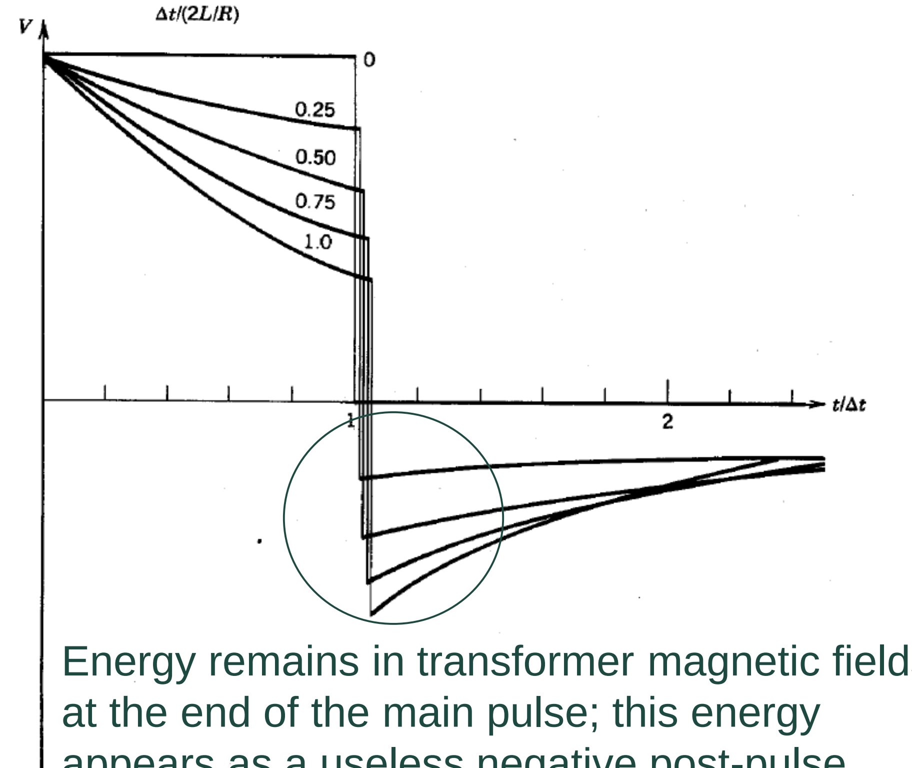
Limitation of pulse transformer

- If the transformer is used to amplify the voltage of a square pulse from a pulsed voltage generator (common accelerator application), then leakage currents contribute to droop of the output voltage waveform.
- The output pulse is a square pulse when $L/R_1 \gg \Delta t$.
- Pulse transformers with low primary inductance have poor energy transfer efficiency and a variable voltage output waveform.
- Volt-sec limitation

$$N_1 \frac{d\Phi}{dt} = V_1$$

⇨ $$N_1 A_1 \big(B(t) - B(0)\big) = \int V(t) dt$$

⇨ $$V_0 \Delta t \text{ [volt} - \text{sec]} \leq 2 N_1 A_1 B_s$$



Energy remains in transformer magnetic fields at the end of the main pulse; this energy appears as a useless negative post-pulse.

Figure 9.8 Pulse waveforms; resistive load driven by a matched pulse modulator through a transformer: Δt is the duration of the voltage pulse, R is the output impedance of the modulator and the load resistance viewed at the primary, and L is the primary shunt inductance.

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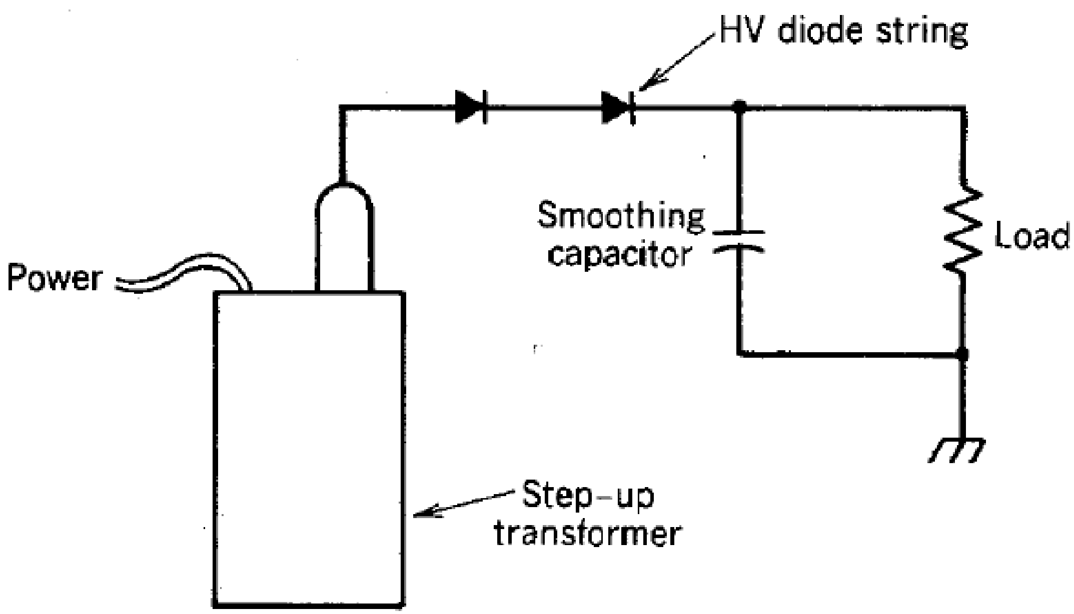
High-voltage dc power supply: half-wave rectifier

- A capacitor is included to reduce ripple in the voltage.
- The fractional drop in voltage during the negative half-cycle is on, the order $(1/2f)/RC$, where R is the load resistance.
- Output voltage is controlled by a variable autotransformer in the primary.
- Because of the core volume and insulation required, transformers are inconvenient to use at voltages above 100 kV. → Use of ladder network for higher voltage.



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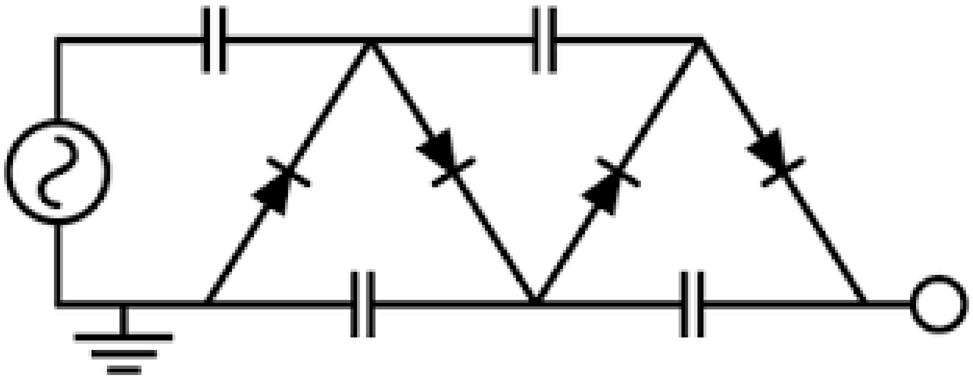
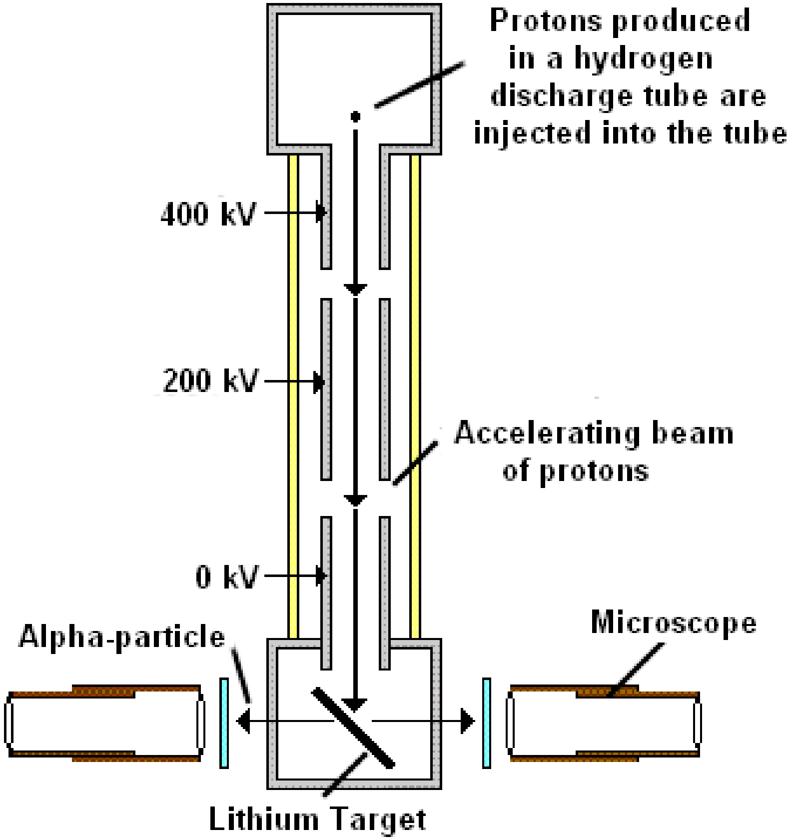
Cockcroft-Walton's voltage multiplier

- The first nuclear disintegration by nuclear projectiles artificially produced in a man-made accelerator (1932).

$^{7}Li + {}^{1}H \rightarrow {}^{4}He + {}^{4}He$ (+17.3 MeV)





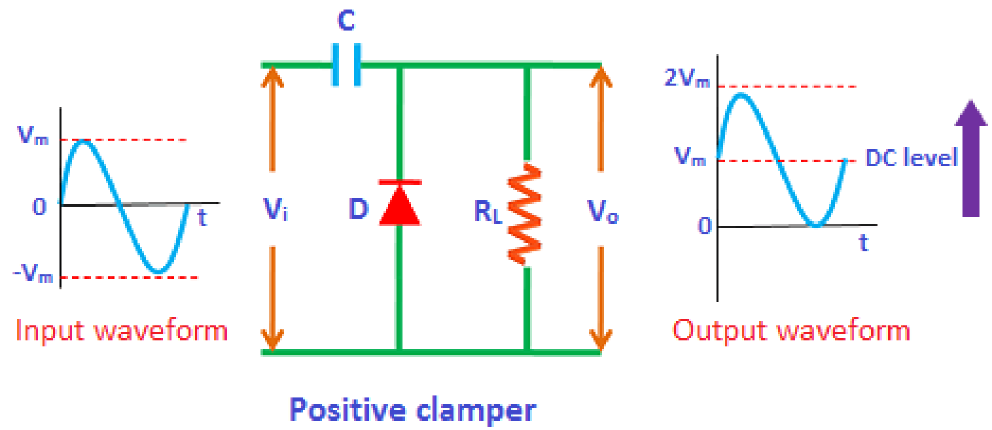
Nobel prize in physics (1951)

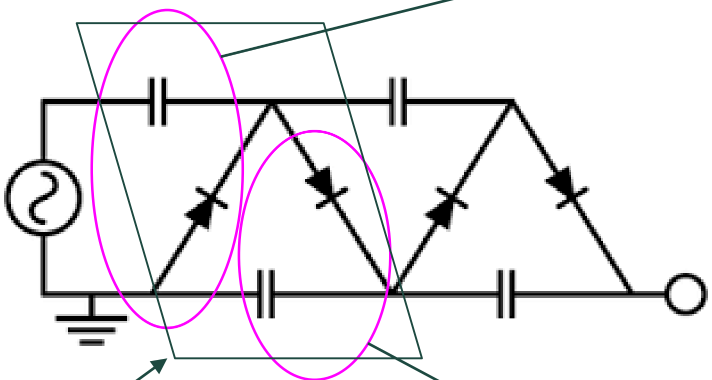
Cockcroft and Walton, Proc. Roy. Soc. A136, 619 (1932)
Cockcroft and Walton, Proc. Roy. Soc. A137, 229 (1932)

Analysis of Cockcroft-Walton's voltage multiplier circuit

- The voltage multiplier circuit is a combination of a clamping circuit and a peak detector circuit (rectifier circuit).

Clamping circuit
(positive clamper)

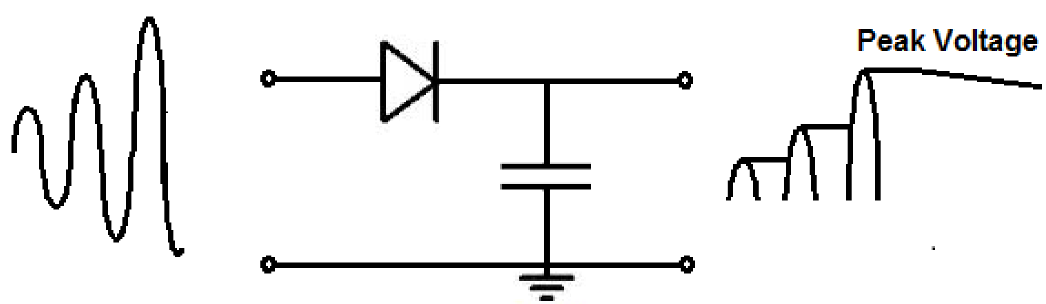




$$V_{out} = 2NV_p = NV_{pp}$$

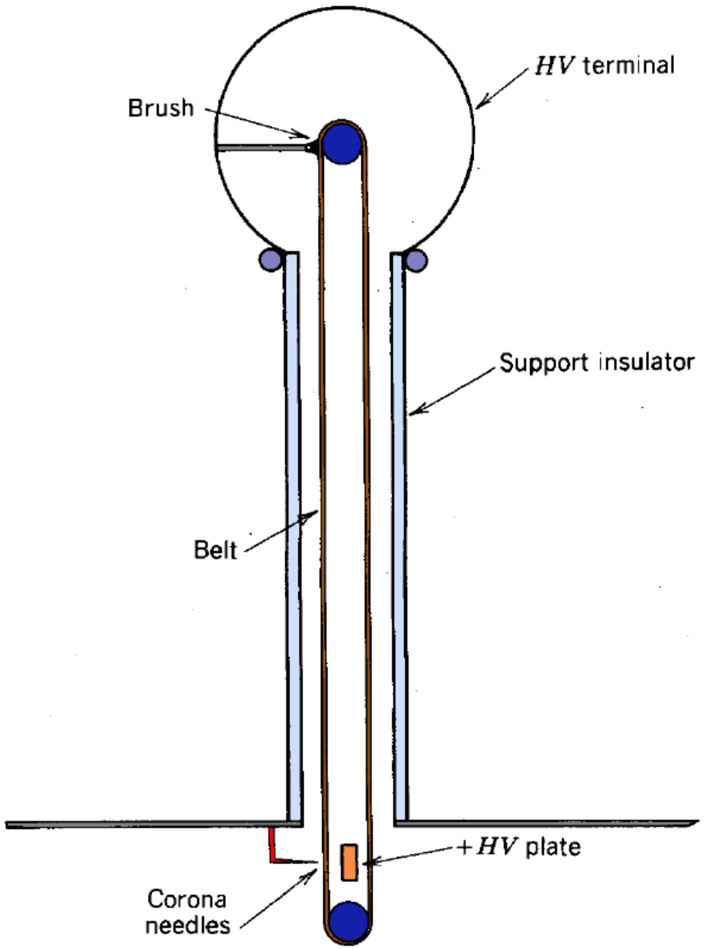
1 stage

Peak detector circuit
(rectifier circuit)



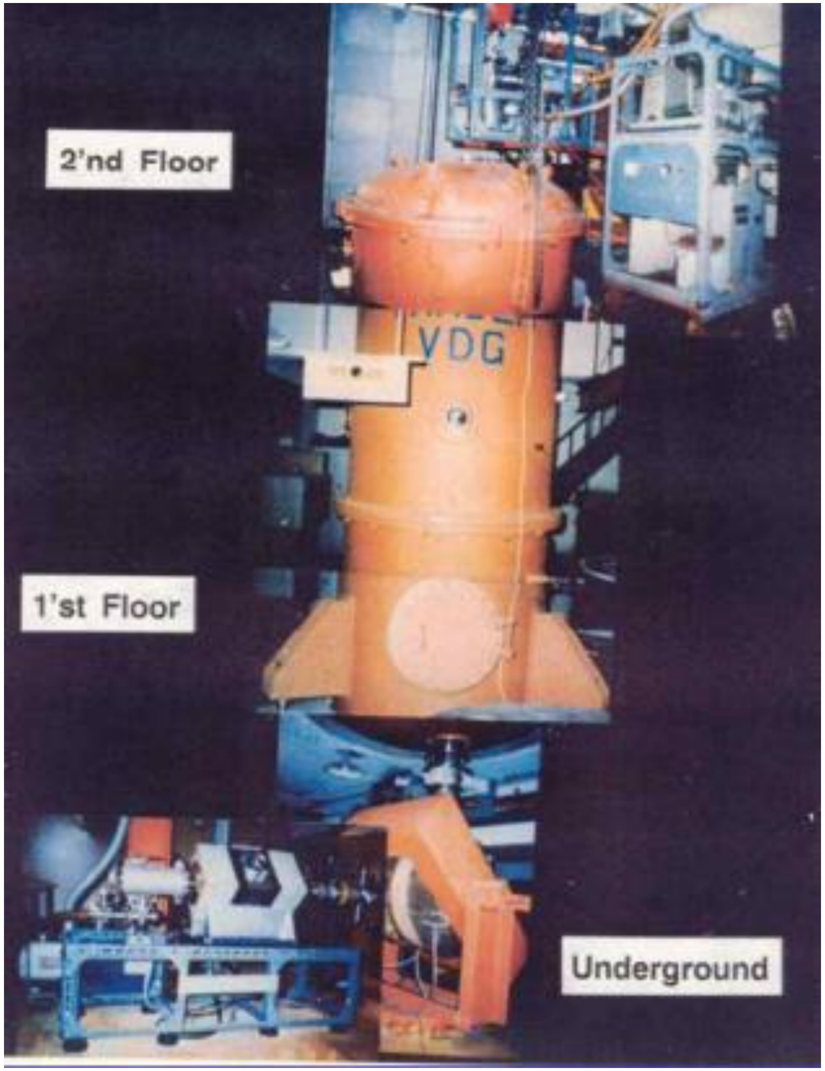
Van de Graaff accelerator

- A Van de Graaff [R. J. Van de Graaff, Phys. Rev. 38, 1919 (1931)] accelerator is a low-current electrostatic MeV-range high-voltage generator using corona discharge from an array of needles in gas.
- The electrons drift toward the positive electrode and are deposited on a moving belt. The belt composed of an insulating material with high dielectric strength, is immersed in insulating gas at high pressure. The attached charge is carried mechanically against the potential gradient into a high-voltage metal terminal.
- The terminal acts as a Faraday cage; there is no electric field inside the terminal other than that from the charge on the belt. The charge flows off the belt when it comes in contact with a metal brush and is deposited on the terminal.



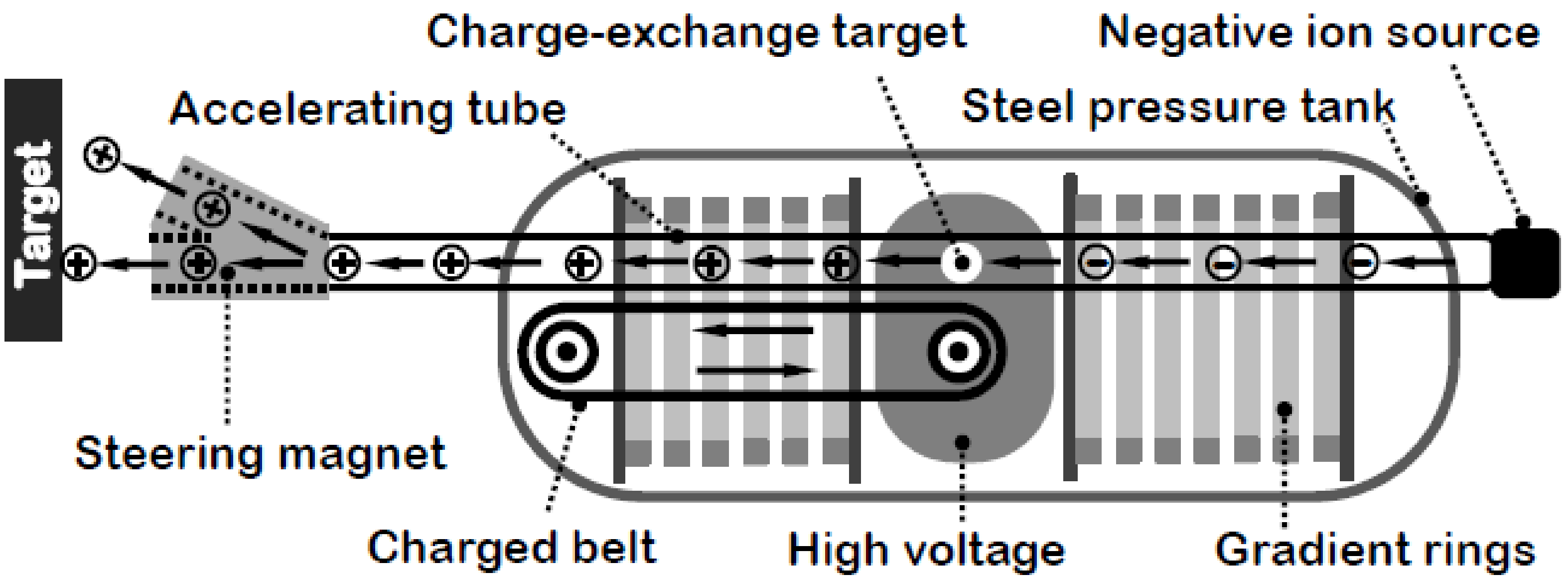
LANL 7 MeV VDG, as injector for 24.5 MeV tandem VDG

1.5 MeV SNU Van de Graaff accelerator



Tandem Van de Graaff accelerator

- The output beam energy of a Van de Graaff accelerator can be extended a factor of 2 through the tandem configuration.
- Negative ions produced by a source at ground potential are accelerated to a positive high-voltage terminal and pass through a stripping cell. Collisions in the cell remove electrons, and some of the initial negative ions are converted to positive ions. They are further accelerated traveling from the high-voltage terminal back to ground.

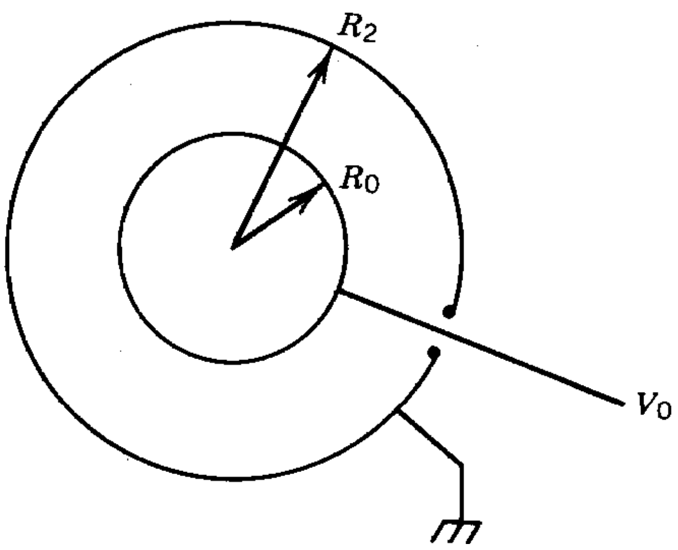


Optimum size of high-voltage terminal

- The solution to the Laplace equation is spherical coordinates gives the radial variation of potential and electric field between the spheres:

$$\varphi(r) = \frac{V_0 R_0}{R_2 - R_0}\left(\frac{R_2}{r} - 1\right)$$

$$E_r(r) = -\frac{\partial \varphi}{\partial r} = \frac{V_0 R_0}{R_2 - R_0}\frac{R_2}{r^2}$$



- The electric field is maximum on the inner sphere

$$E_{r,max} = \frac{V_0}{R_2 - R_0}\frac{R_2}{R_0}$$

- The condition to minimize the peak electric field is $\frac{R_0}{R_2} = \frac{1}{2}$ ⇨ $E_{max} = \frac{4V_0}{R_2}$

- For concentric cylinders, the optimum ratio of inner to outer cylinder radii is

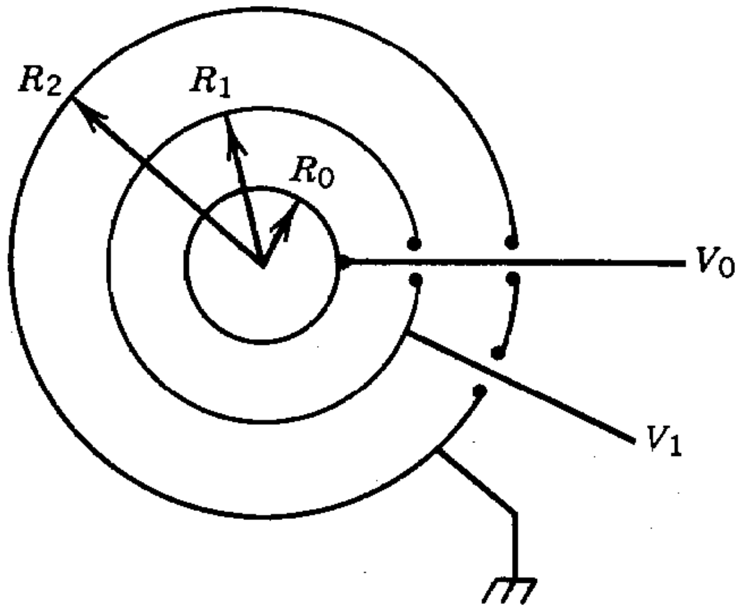
$$\frac{R_0}{R_2} = e^{-1} \approx 0.368$$

Use of equipotential shields between the electrodes

- A better electric field distribution can be obtained through the use of equipotential shields which are biased electrodes located between the high-voltage terminal and ground. Voltage on the shields is maintained at specific intermediate values by a high-voltage resistive divider circuit.
- The fields on the outer surfaces of the electrodes are

$$E_0 = \frac{(V_0 - V_1)/R_0}{1 - R_0/R_1}$$

$$E_1 = \frac{V_1/R_1}{1 - R_1/R_2}$$



- The optimum parameter are found to be

$$R_0 = \frac{1}{2}R_1 \qquad R_1 = \frac{5}{8}R_2 \qquad V_1 = \frac{3}{5}V_0$$

- The peak electric field is reduced to

$$E_{max} = \frac{192}{75}\frac{V_0}{R_2} \approx 2.56\frac{V_0}{R_2}$$

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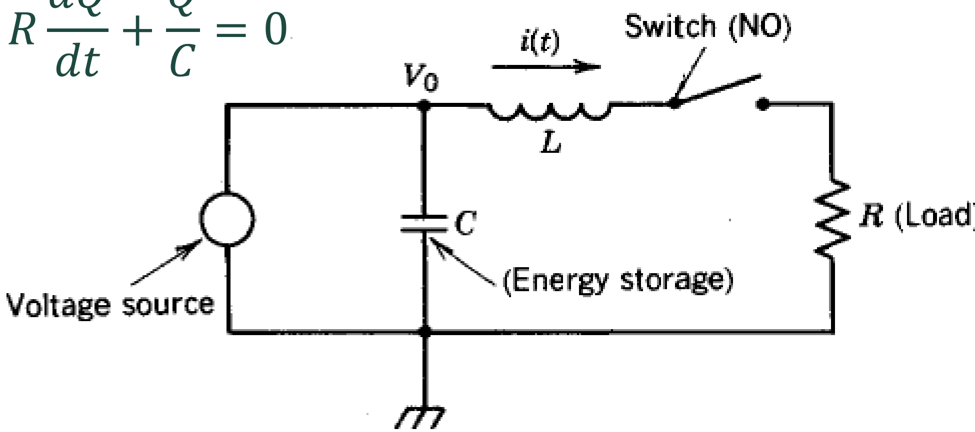
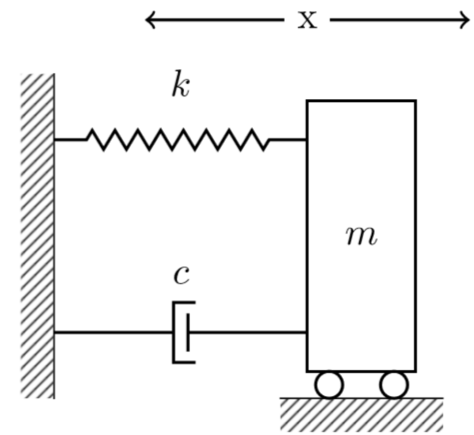
RLC circuit

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$





$$\frac{d^2Q}{dt^2} + 2\alpha\frac{dQ}{dt} + \omega_0^2 Q = 0 \qquad where, \qquad \alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

- Initial conditions: $Q(t=0) = CV_0,\qquad dQ/dt\,(t=0) = 0$

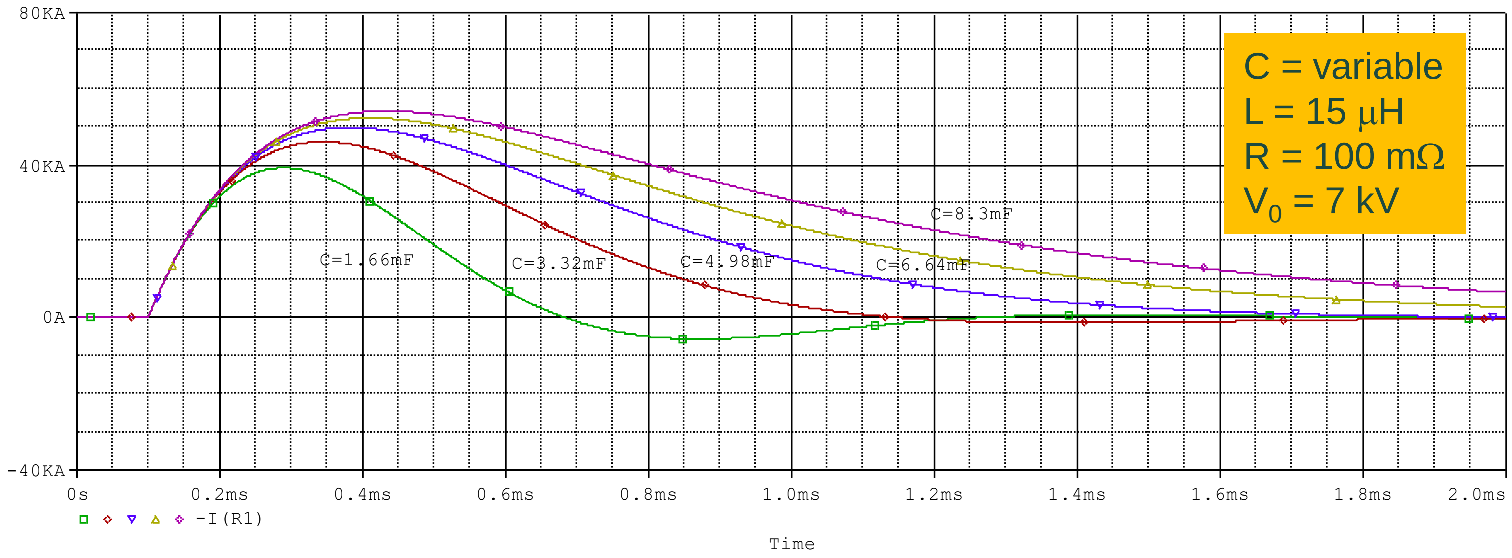
(1) Overdamping $\quad R > 2\sqrt{L/C} \qquad i(t) = \dfrac{V_0}{\beta L} e^{-\alpha t}\sinh\beta t \quad (\beta^2 = \alpha^2 - \omega_0^2)$

(2) Critical damping $\quad R = 2\sqrt{L/C} \qquad i(t) = \dfrac{V_0}{L} t e^{-\alpha t}$

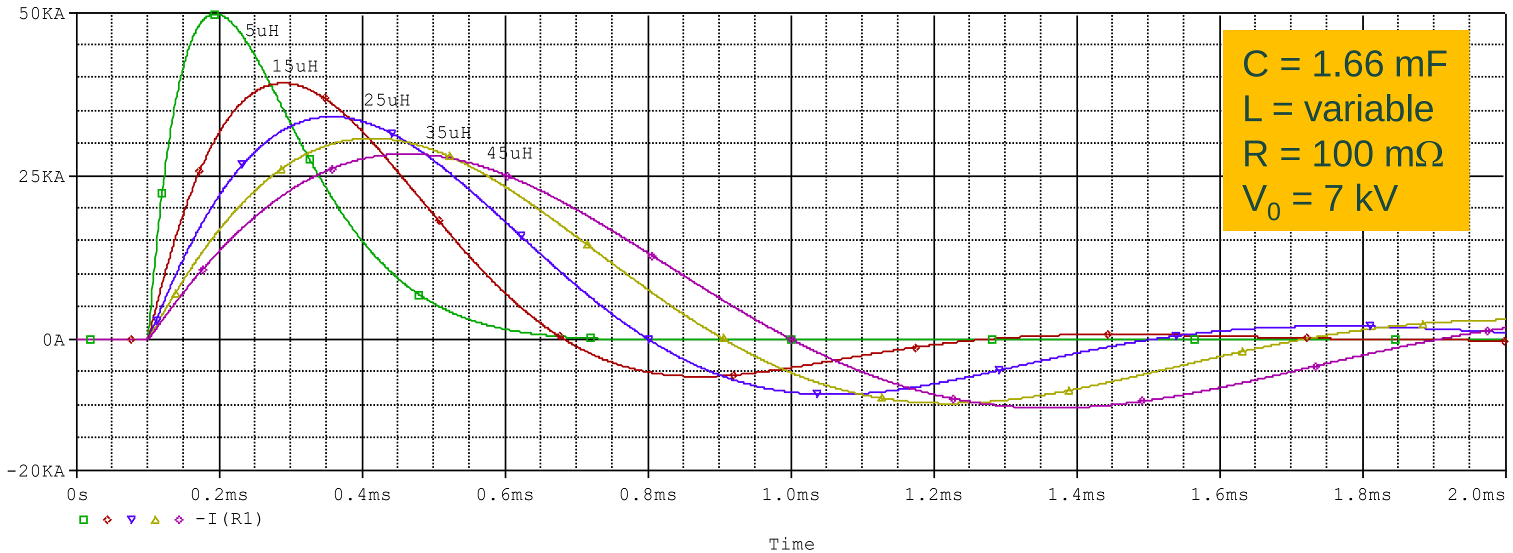
(3) Underdamping $\quad R < 2\sqrt{L/C} \qquad i(t) = \dfrac{V_0}{\omega_d L} e^{-\alpha t}\sin\omega_d t \quad (\omega_d^2 = \omega_0^2 - \alpha^2)$

RLC circuit

- Variable C

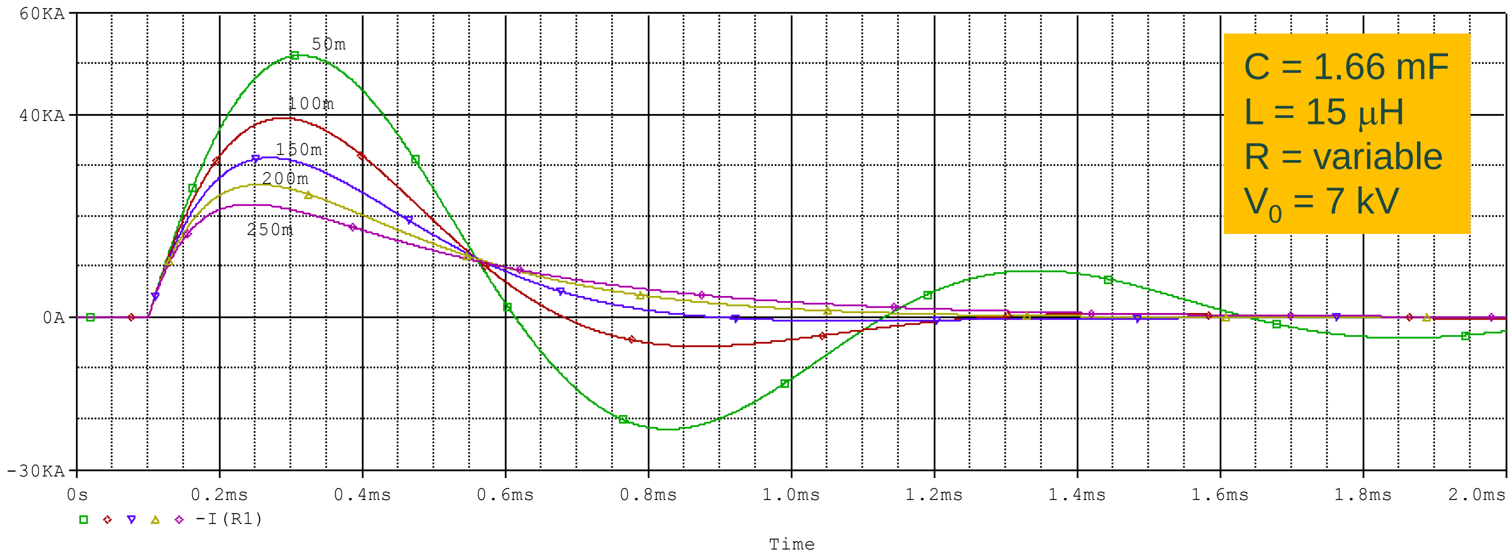


- Variable L

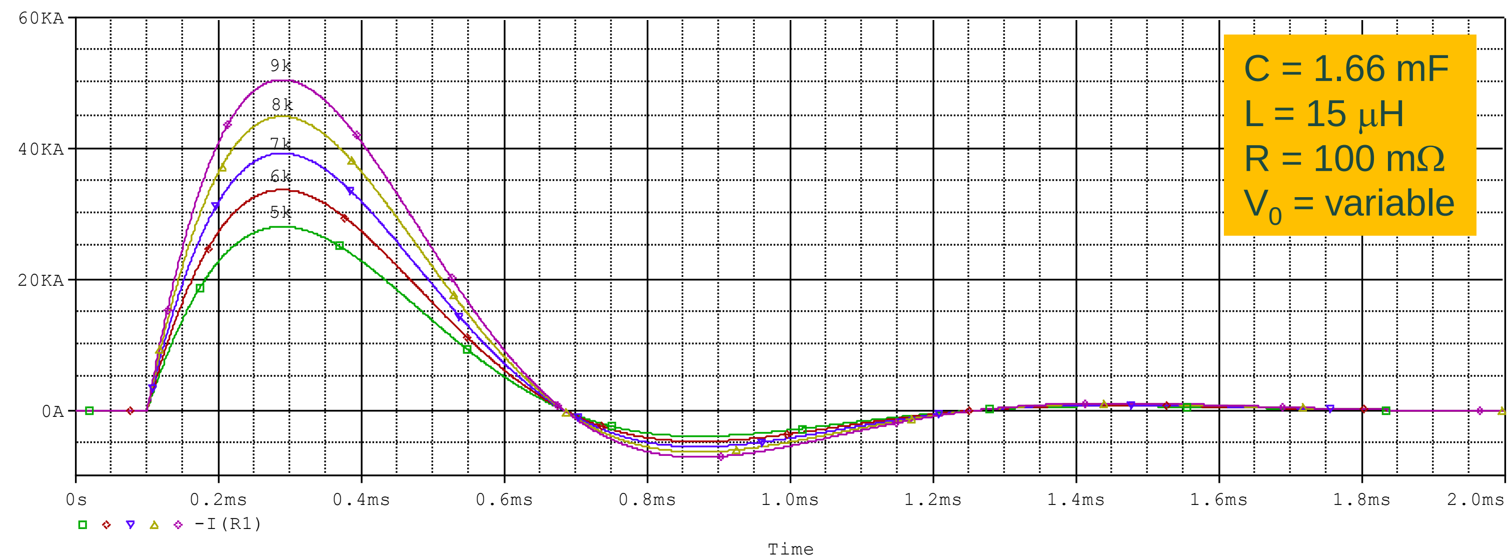


RLC circuit

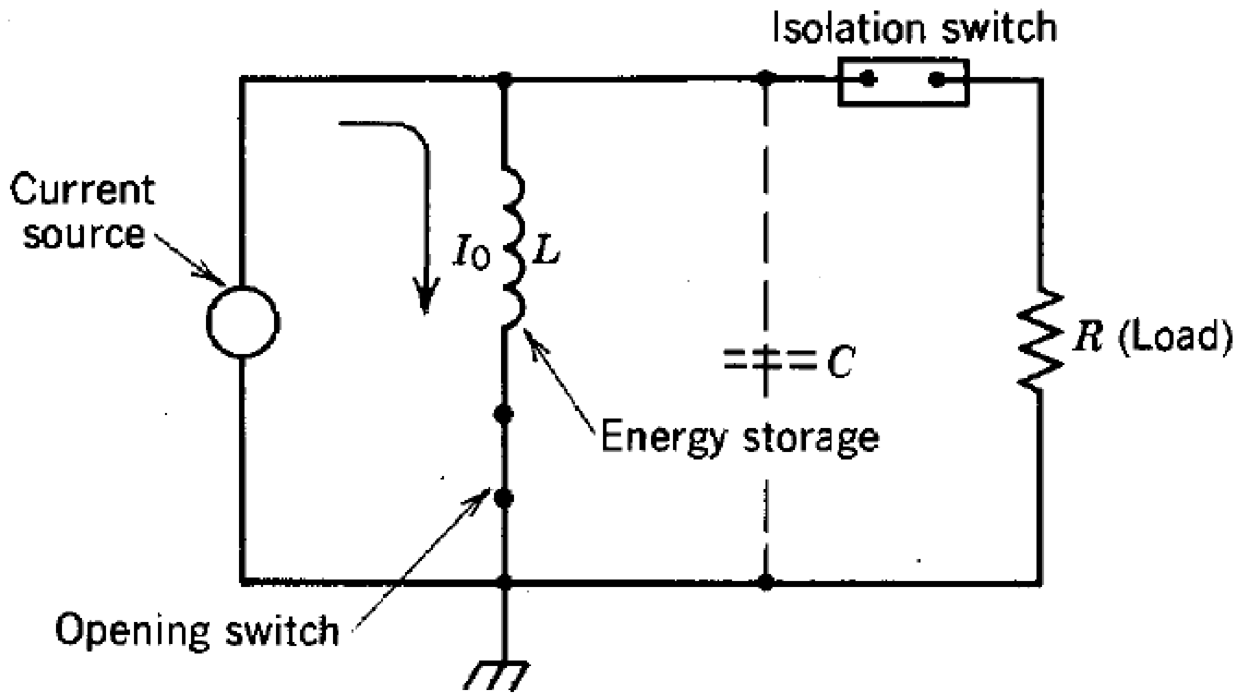
- Variable R



- Variable V_0



Pulse modulator with inductive energy storage and opening switch



- Compare energy density stored in inductor and in capacitor.

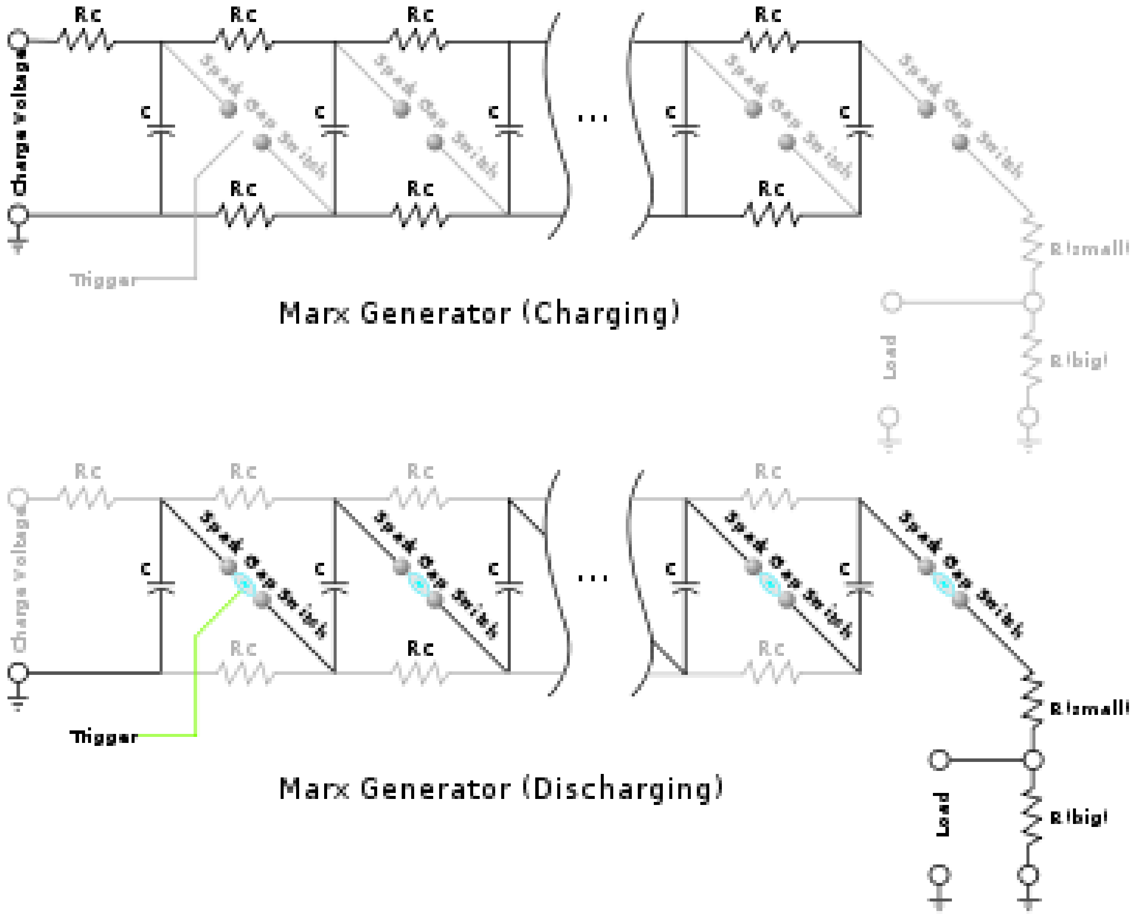
Consider

$$\varepsilon_r \approx 10$$
$$E \approx 10^8 \quad \text{V/m}$$
$$B \approx 10 \text{ T}$$
$$\mu = \mu_o$$

Then

$$\frac{1}{2}\varepsilon E^2 = 4.4 \times 10^5 \quad \text{J/m}^3$$
$$\frac{1}{2}\frac{B^2}{\mu_o} \approx 4 \times 10^7 \quad \text{J/m}^3$$

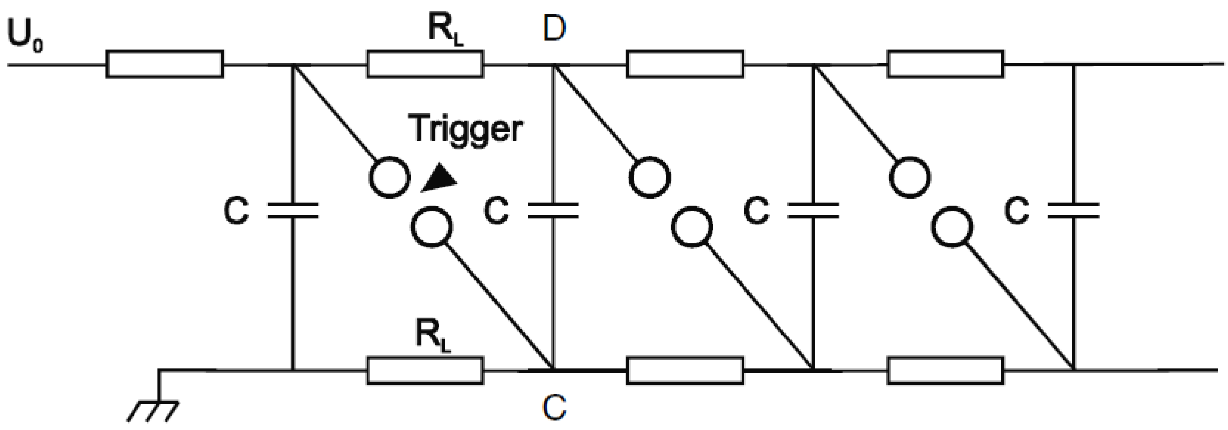
Marx generator (Erwin Marx, 1923)



$$V_M = nV_0$$

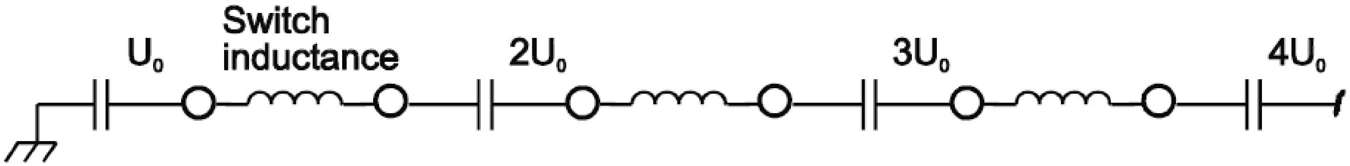
$$C_M = C_0/n$$

Marx generator: equivalent circuit



Self-discharge time:

$$\tau = \frac{1}{2} R_{\rm L} C_0$$



When L is used instead of R:

$$\tau = 2\pi \left(\frac{1}{2} L_{\rm L} C_0 \right)^{1/2}$$

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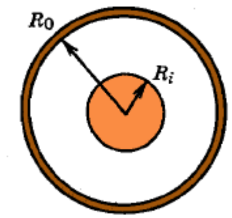
Transmission line

- Most accelerator applications for pulse modulators require a constant-voltage pulse.
- The critically damped waveform is the closest a modulator with a single capacitor and inductor can approach constant voltage.
- Better waveforms can be generated by modulators with multiple elements. Such circuits are called pulse-forming networks (PFNs). The transmission line is the continuous limit of a PFN.
- There are various kinds of transmission line, but coaxial type is the most widely used in the pulsed power system.

TABLE 9.3 Properties of Common Transmission Lines, TEM Modes[a]

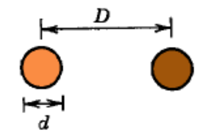
1 Coaxial transmission line:

$$\mathscr{C} = 2\pi\varepsilon/\ln(R_0/R_i)$$
$$\mathscr{L} = (\mu/2\pi)\ln(R_0/R_i)$$
$$Z_0 = (\sqrt{\mu/\varepsilon}/2\pi)\ln(R_0/R_i)$$



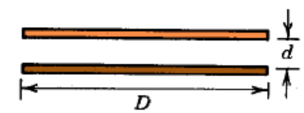
2 Two-wire transmission line

$$\mathscr{C} = \pi\varepsilon/\cosh^{-1}(D/d)$$
$$\mathscr{L} = \mu/\pi \cosh^{-1}(D/d)$$
$$Z_0 = (\sqrt{\mu/\varepsilon}/\pi)\cosh^{-1}(D/d)$$



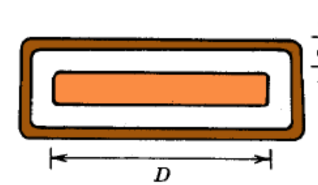
3 Isolated parallel plates ($d \ll D$)

$$\mathscr{C} = \varepsilon D/d$$
$$\mathscr{L} = \mu d/D$$
$$Z_0 = \sqrt{\mu/\varepsilon}(d/D)$$



4 Stripline ($d \ll D$)

$$\mathscr{C} = 2\varepsilon D/d$$
$$\mathscr{L} = \mu d/2D$$
$$Z_0 = \sqrt{\mu/\varepsilon}(d/2D)$$



[a] $\mathscr{C}$ = capacitance per unit length (farads/meter); $\mathscr{L}$ = inductance per unit length (henries/meter); Z_0 = characteristic impedance (ohms).

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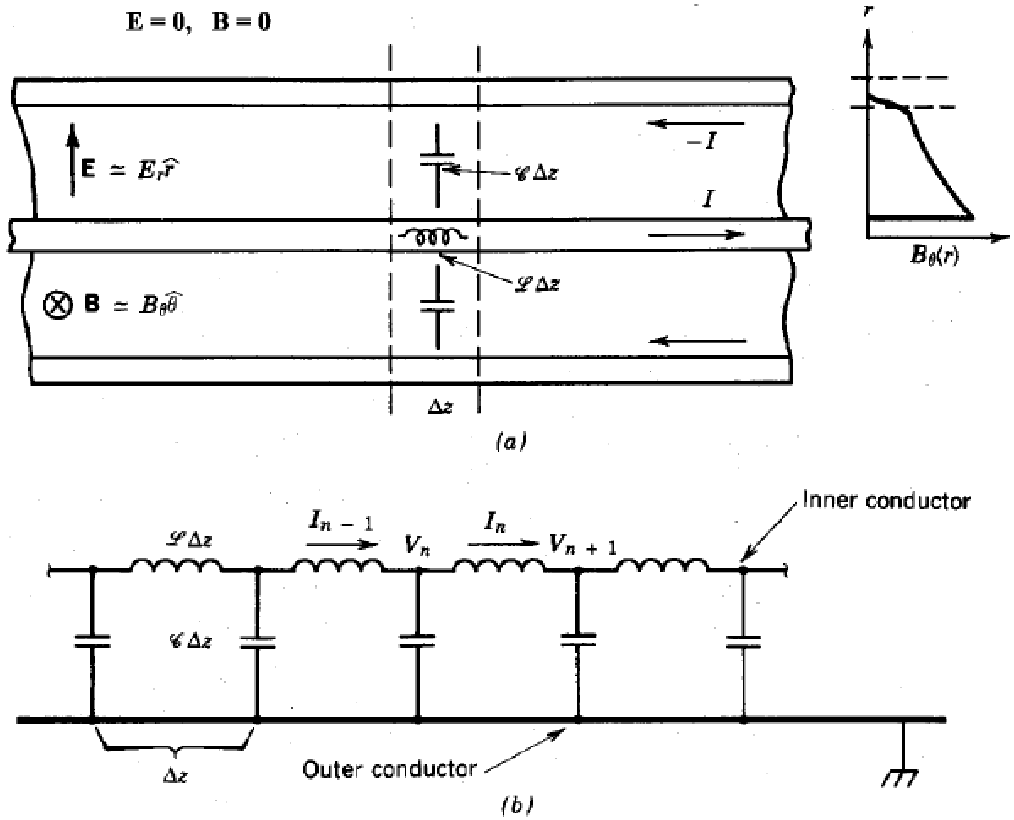
Coaxial transmission line

- Lumped parameter model for a lossless transmission line

$$C' = \frac{2\pi\epsilon}{\ln\left(\frac{R_o}{R_i}\right)} \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$$

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{R_o}{R_i}\right) \left[\frac{\mathrm{H}}{\mathrm{m}}\right]$$



- Kirchhoff's law:

$$-C'\Delta z\frac{\partial V_n}{\partial t} = I_n - I_{n-1}$$

$$L'\Delta z\frac{\partial I_n}{\partial t} = V_n - V_{n+1}$$

- The voltage and current differences are approximated as:

$$V_{n+1} \approx V_n + \frac{\partial V(z_n,t)}{\partial z}\Delta z \qquad I_{n+1} \approx I_n - \frac{\partial I(z_n,t)}{\partial z}\Delta z$$

Coaxial transmission line

- Combining equations, we obtain the continuous partial differential equations:

$$\frac{\partial V}{\partial z} = -L' \frac{\partial I}{\partial t} \qquad \frac{\partial I}{\partial z} = -C' \frac{\partial V}{\partial t}$$

Telegrapher's equation

- Wave equation:

$$\frac{\partial^2 V}{\partial z^2} = L'C' \frac{\partial^2 V}{\partial t^2} \quad \Rightarrow \quad V(z,t) = F\left(t \pm \frac{z}{v}\right)$$

- Phase velocity:

$$v = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\epsilon\mu}} \approx \frac{c}{\sqrt{\epsilon_r}}$$

→ No geometric effects

- Characteristic impedance:

$$Z_0 = \sqrt{\frac{L'}{C'}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{R_o}{R_i}\right) \approx \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{R_o}{R_i}\right) \quad \Rightarrow \quad Z_0^{opt} \approx \frac{60}{\sqrt{\epsilon_r}}$$

(Voltage hold-off is maximized when $R_o/R_i = e$)

Coaxial transmission line

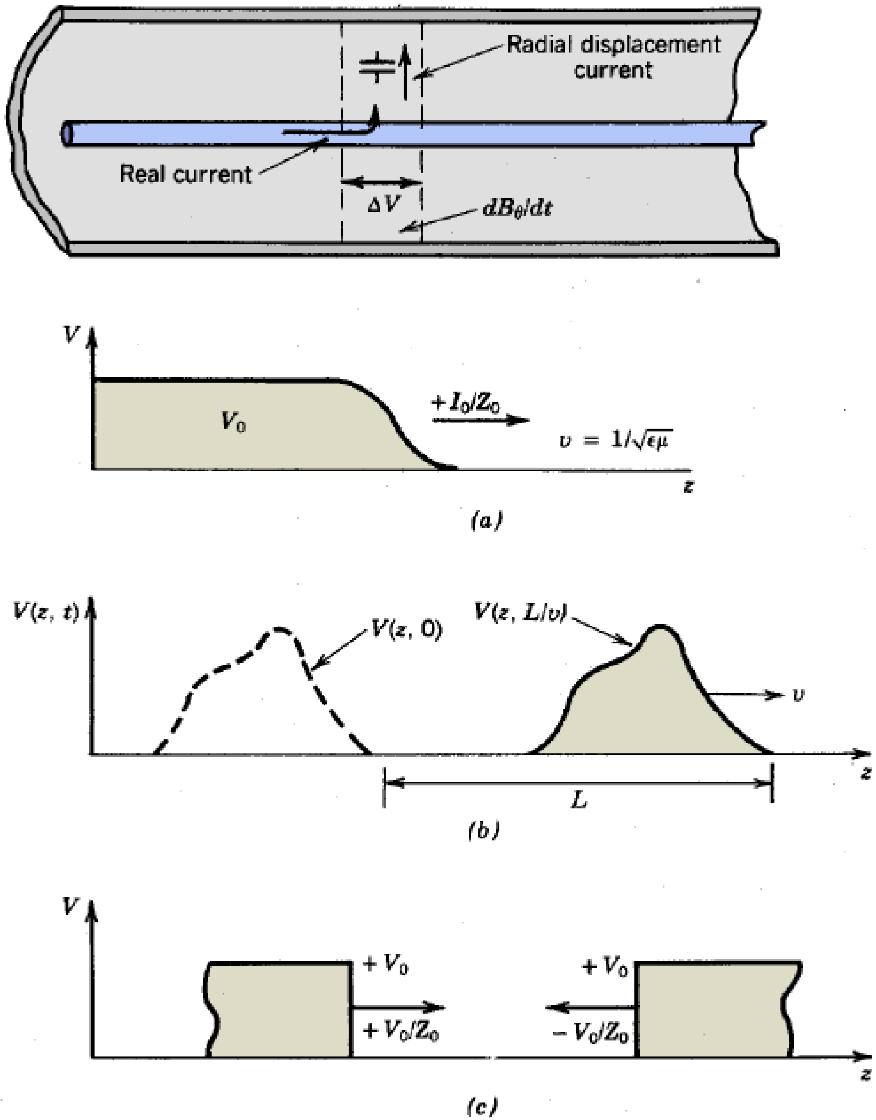
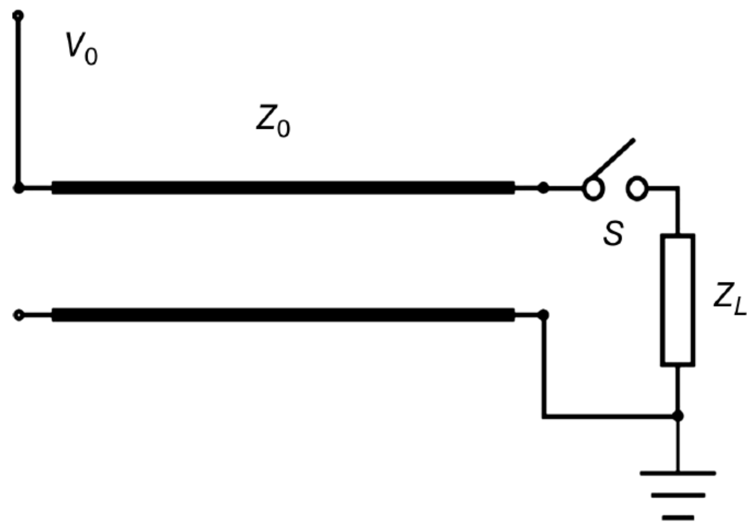


Figure 9.30 Coaxial transmission line. (*a*) Distribution of voltage, real current, and displacement current for a sharp rising step pulse propagating along a transmission line. (*b*) Distributions of voltage on a transmission line at time $t = 0$ (dashed line) and time $t = L/v$ (solid line). (*c*) Polarity conventions for positive voltage impulses traveling in the $+z$ and $-z$ directions.

Pulse forming line (PFL)

- There are numerous applications in both physics and electrical engineering for short ($\sim 10\ ns < t_p < 100\ \mu s$) electrical pulses. These applications often require that the pulses have a "good" square shape.
- Although there are many ways for generating such pulses, the pulse-forming line (PFL) is one of the simplest techniques and can be used even at extremely high pulsed power levels.
- A transmission line of any geometry of length l and characteristic impedance Z_0 makes a pulse forming line (PFL), which when combined with a closing switch S makes the simple transmission line pulser.



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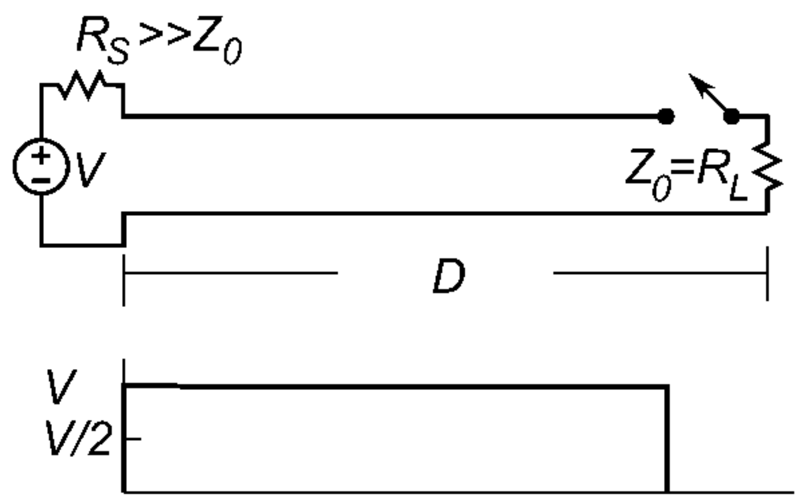
Simple PFL

- When the switch closes, the incident wave V_I, with a peak voltage of $(1/2)V_0$, travels toward the load, while the reverse-going wave V_R, also with a peak voltage of $(1/2)V_0$, travels in the opposite direction.
- The incident wave V_I, then, supplies a voltage of $(1/2)V_0$ for a time determined by the electrical length of the transmission line T_T to the load. The reverse-going wave V_R travels along the transmission line for a duration T_T and then reflects from the high impedance of the voltage source, and becomes a forward-going wave traveling toward the load with peak voltage $(1/2)V_0$ and duration T_T.
- The two waves add at the load to produce a pulse of amplitude $(1/2)V_0$ and pulse duration $T_p = 2T_T$.
- Matching condition: $R_L = Z_0$
- Pulse characteristics

$$V = \frac{V_0}{2}$$

$$T_p = 2T_T = \frac{2l}{v_p} \approx \frac{2l}{c}\sqrt{\epsilon_r}$$



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Coaxial PFL

- Basic parameters

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{R_2}{R_1}\right) \qquad C' = \frac{2\pi\epsilon}{\ln(R_2/R_1)}$$

$$Z_0 = \sqrt{\frac{L'}{C'}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\left(\frac{R_2}{R_1}\right) \approx 60\sqrt{\frac{\mu_r}{\epsilon_r}}\ln\left(\frac{R_2}{R_1}\right)$$

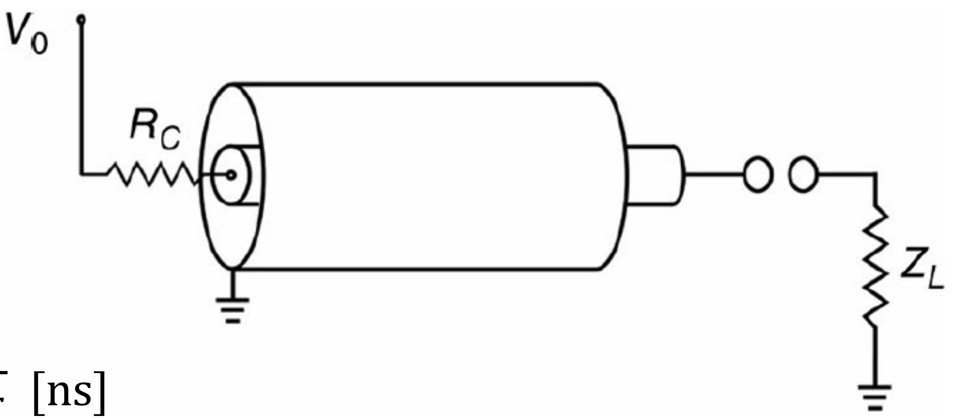
$$v_p = \frac{1}{T_T} = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} \approx \frac{30}{\sqrt{\epsilon_r}}\left[\frac{\text{cm}}{\text{ns}}\right]$$

- Matching condition: $Z_L = Z_0$

- Pulse characteristics

$$V = \frac{V_0}{2}$$

$$T_p = 2T_T = \frac{2l}{v_p} \approx \frac{2l}{c}\sqrt{\epsilon_r} = \frac{l\,[\text{cm}]}{15}\sqrt{\epsilon_r}\ \ [\text{ns}]$$



Coaxial PFL

- Electric field

$$E(r) = \frac{V_0}{r\ln(R_2/R_1)} \qquad E_{max}(r = R_1) = \frac{V_0}{R_1\ln(R_2/R_1)}$$

- Voltage at the maximum electric field

$$V_0 = E_{max} R_1 \ln\left(\frac{R_2}{R_1}\right)$$

- The value of R_2/R_1 that optimizes the inner conductor voltage occurs when $dV_0/dR_1 = 0$, yielding

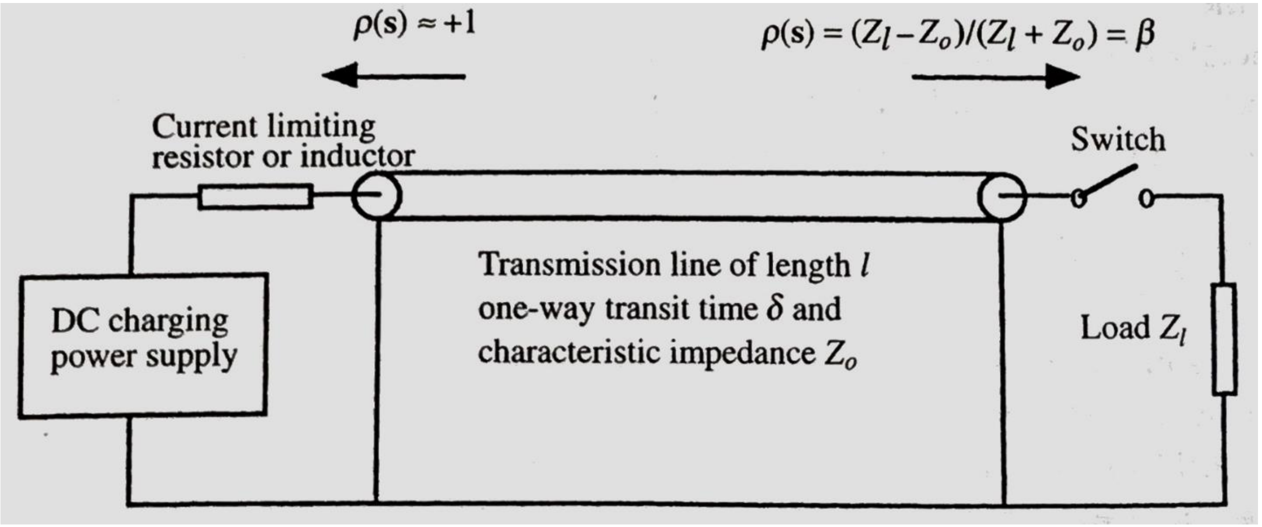
$$\ln\left(\frac{R_2}{R_1}\right) = 1$$

- Optimum impedance for maximum voltage

$$Z_{opt} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}} = 60\sqrt{\frac{\mu_r}{\epsilon_r}} \approx \frac{60}{\sqrt{\epsilon_r}}$$

$$Z_{opt}^{water} = \frac{60}{\sqrt{81}} = 6.7\ \Omega \qquad Z_{opt}^{oil} = \frac{60}{\sqrt{2.4}} = 38.7\ \Omega$$

Analysis of simple PFL



- On closure of the switch, the voltage on the load rises from zero to a value determined by

$$V_L = V\frac{Z_L}{Z_L + Z_0} \qquad\qquad V_L = \frac{V}{2} \quad \text{(matched)}$$

- Simultaneously, a voltage step V_s is propagated away from the load towards the charging end of the line. It takes $\delta = l/v_p$ for the wave to reach the charging end.

$$V_s = V_L - V = V\left(\frac{Z_L}{Z_L + Z_0} - 1\right) = V\left(\frac{-Z_0}{Z_L + Z_0}\right) \qquad\qquad V_s = -\frac{V}{2} \quad \text{(matched)}$$

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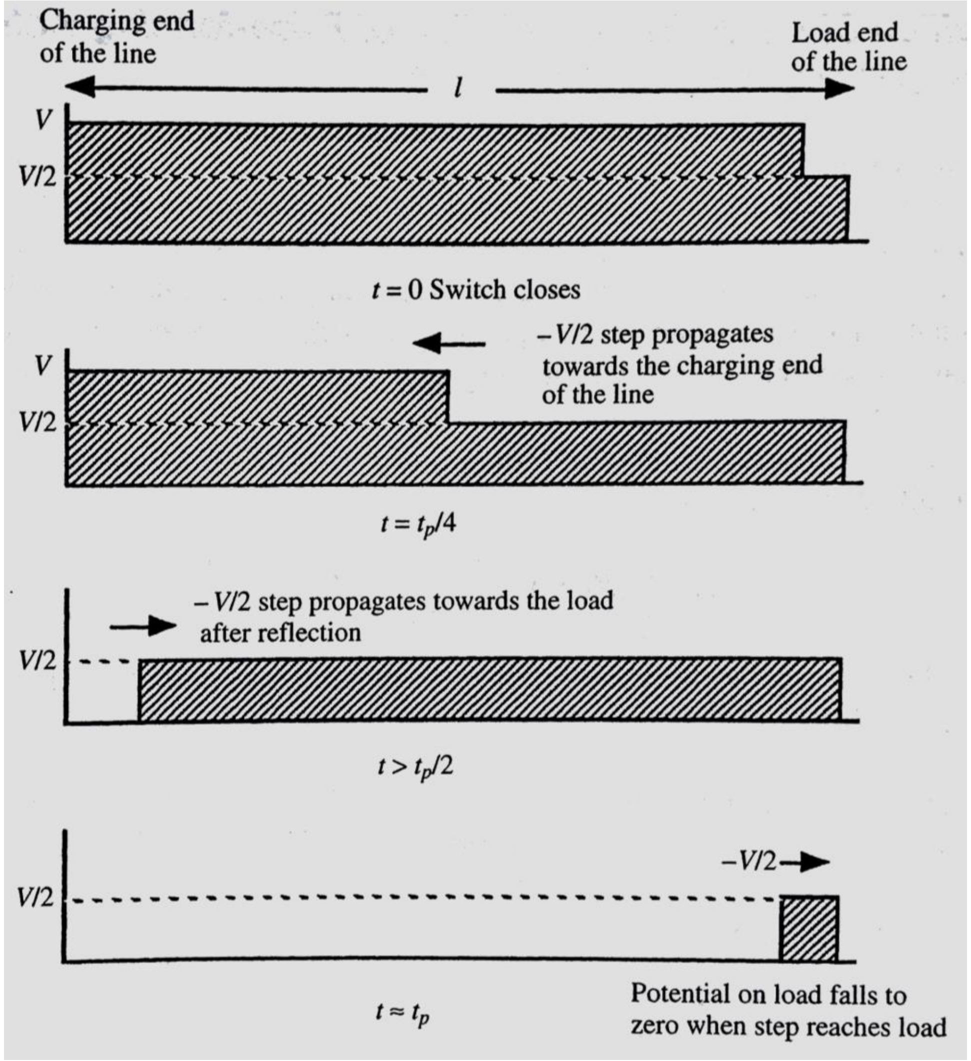
Analysis of simple PFL

- Potential distribution (matched load)



Lattice diagram representation of pulse-forming action

- On closure of the switch, the voltage on the load rises from zero to a value determined by

$$V_L = V\frac{Z_L}{Z_L + Z_0} = \alpha V$$

- The potential on the load is given by

$$V_L = \alpha V \qquad (0 < t < 2\delta)$$

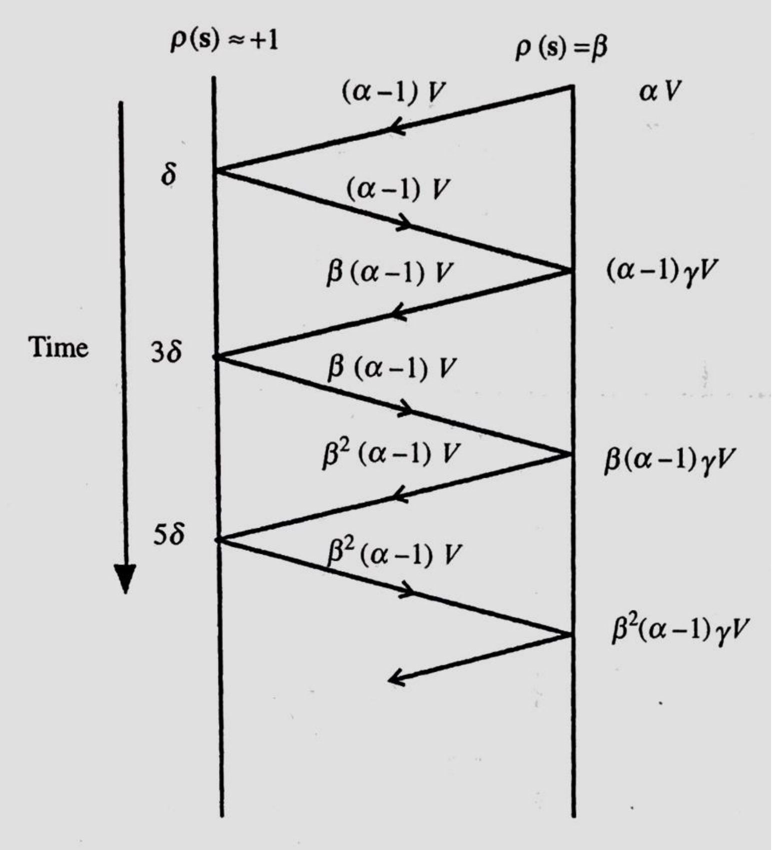
$$V_L = \alpha V + (\alpha - 1)\gamma V \qquad (2\delta < t < 4\delta)$$

$$V_L = \alpha V + (\alpha - 1)\gamma V + \beta(\alpha - 1)\gamma V \qquad (4\delta < t < 6\delta)$$

$$\gamma = \beta + 1$$

- Finally

$$V_L = V[\alpha + (\alpha - 1)\gamma(1 + \beta + \beta^2 + \cdots)]$$



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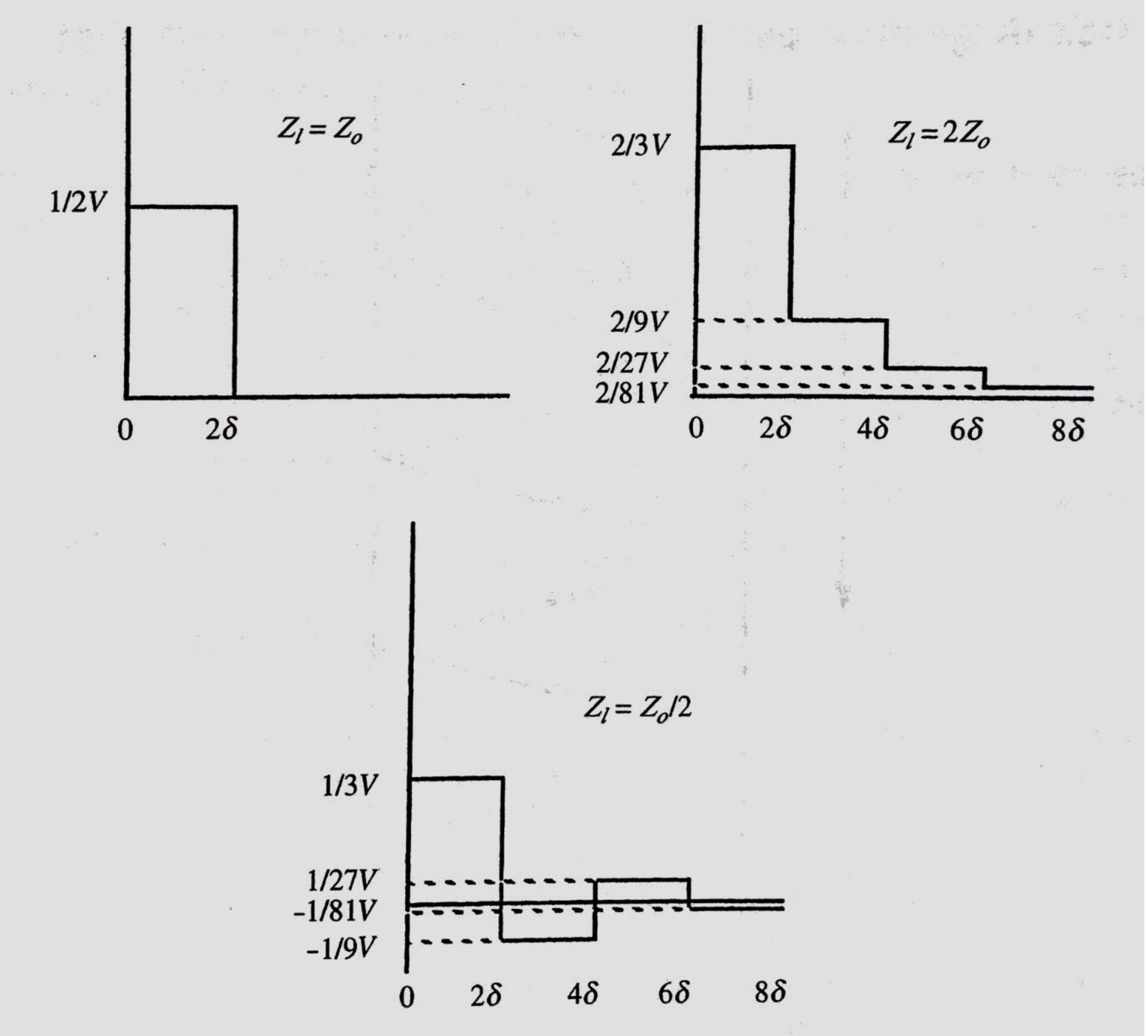
Typical waveforms from PFL under matched and unmatched conditions



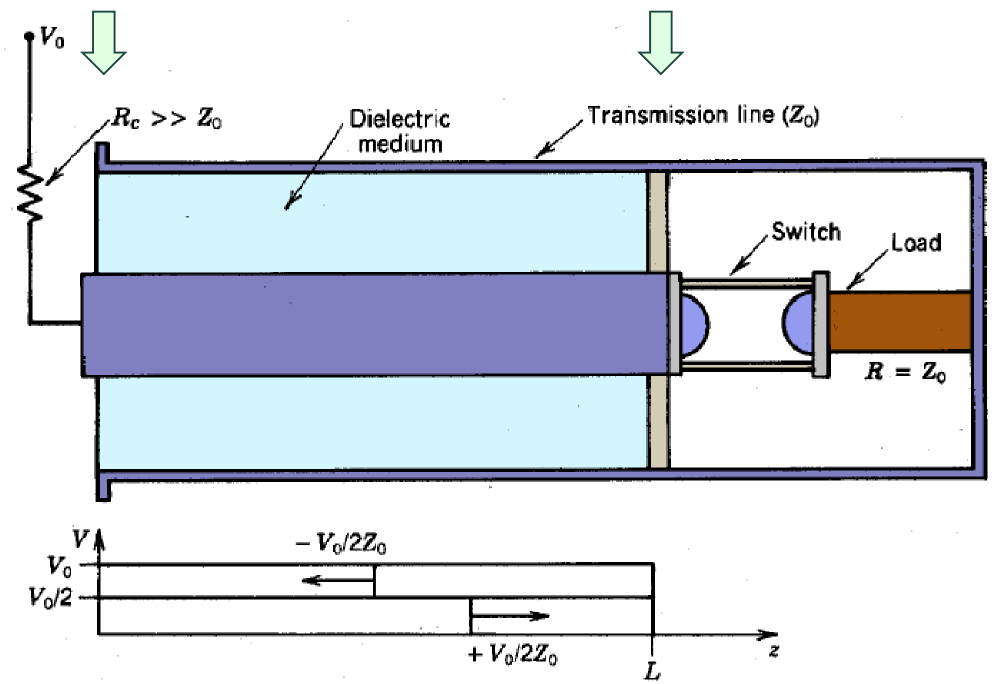
Coaxial simple PFL

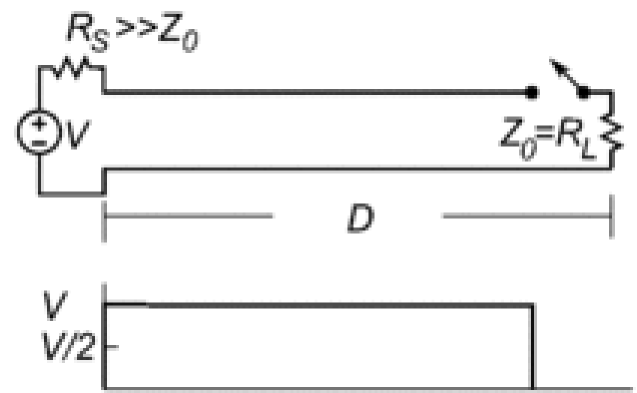
- The discharge of a simple transmission line through a shorting switch into a matched resistive load produces a constant-voltage pulse.
- The magnitude and duration of the pulse:

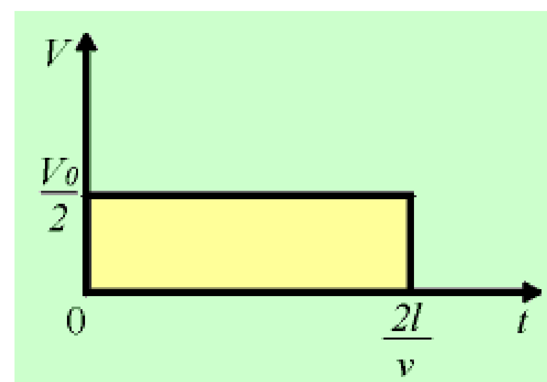
$$V = \frac{V_0}{2} \qquad t_p = \frac{2L}{v}$$

Open

Matched







Blumlein PFL

- An important disadvantage of the simple PFL is that the pulse generated into a matched load is only equal to V/2.
  : This can be serious if one generates pulses at the high voltage, as the power supply used to charge the line must have an output potential which is twice that of the required pulse amplitude.
- This problem can be avoided using the Blumlein PFL invented by A. D. Blumlein.

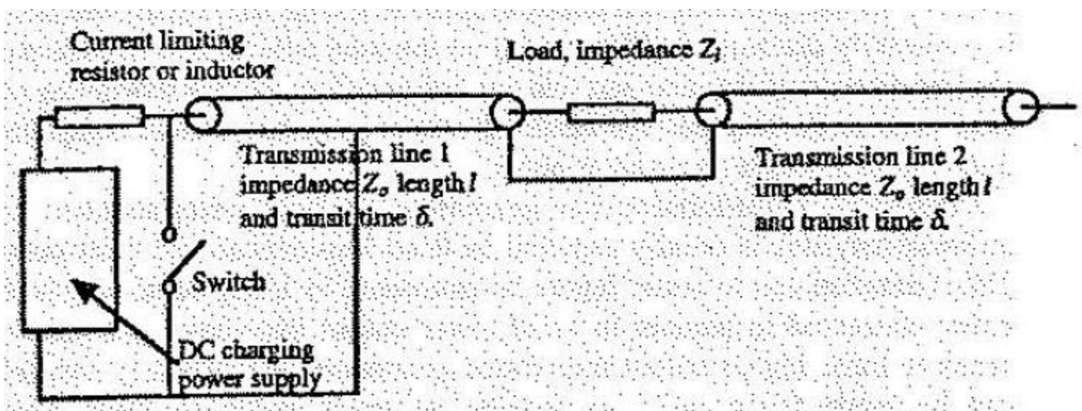
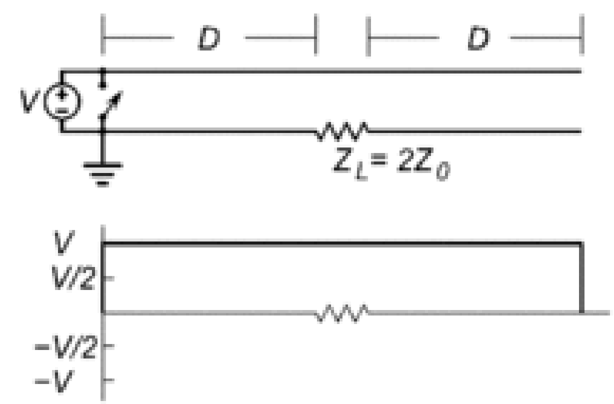
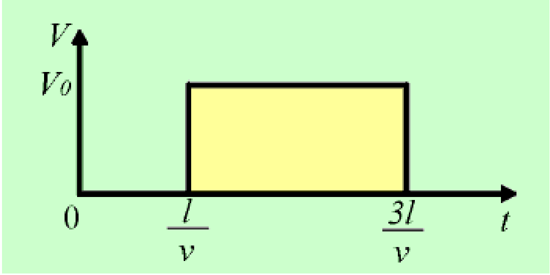


Figure 3.6 Circuit diagram of the basic Blumlein pulse-forming line



- The magnitude and duration of the pulse:

$$V = V_0 \qquad\qquad t_p = \frac{2L}{v}$$



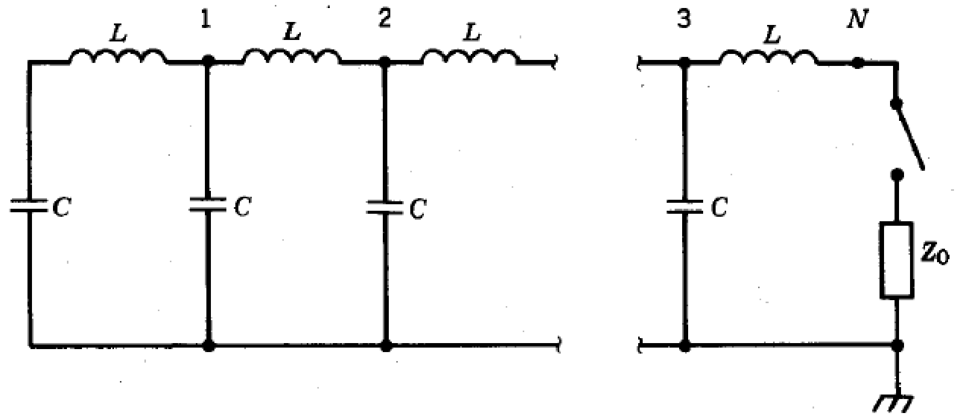
Pulse forming networks (PFNs)

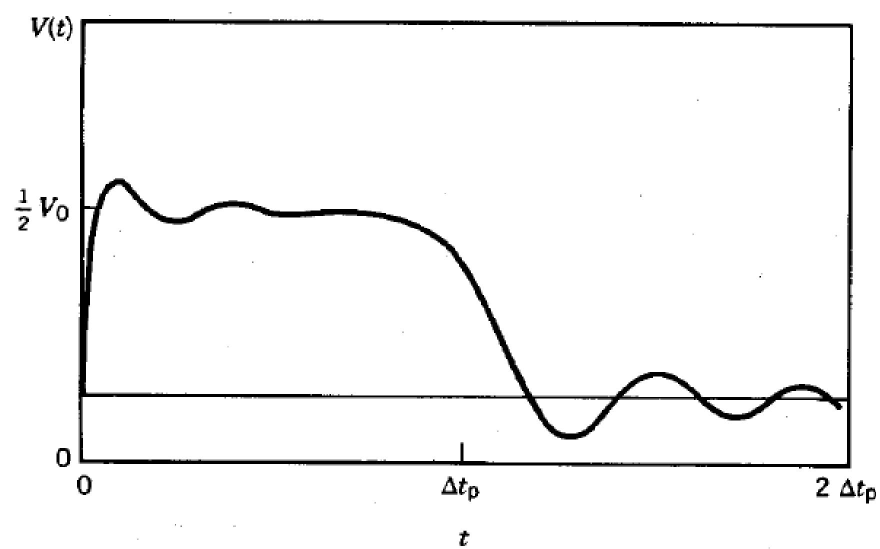
- Transmission lines are well suited for output pulse lengths in the range $5\,ns < t_p < 200\,ns$, but they are impractical for pulse lengths above 1 μs.

- Discrete element circuits that provide a shaped waveform are called pulse-forming networks.

- The magnitude and duration of the pulse:

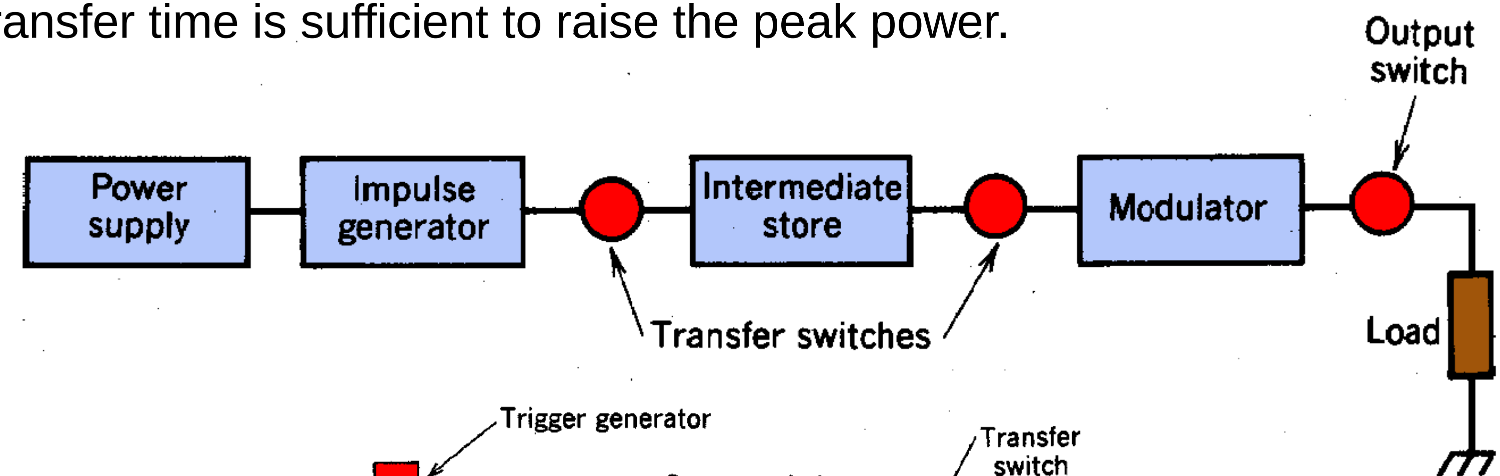
$$V = \frac{V_0}{2} \qquad t_p = \frac{2}{v} = 2N\sqrt{LC}$$

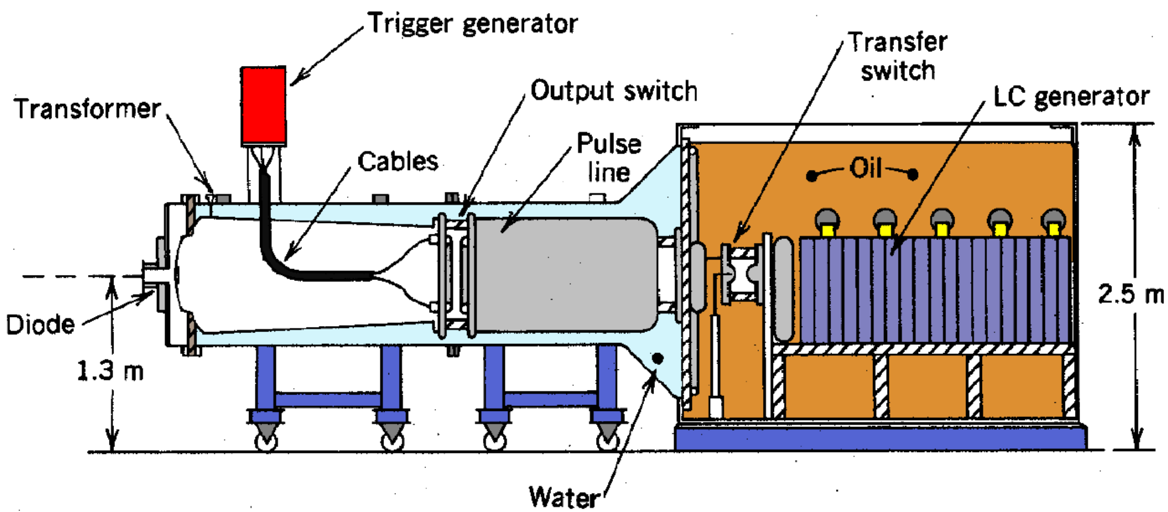




Pulsed power compression

- Power compression is the technique that has allowed the generation of such high output. Energy is transferred from one stage of energy storage to the next in an increasingly rapid sequence. Each storage stage has higher energy density and lower inductance. Even though energy is lost in each transfer, the decrease in transfer time is sufficient to raise the peak power.





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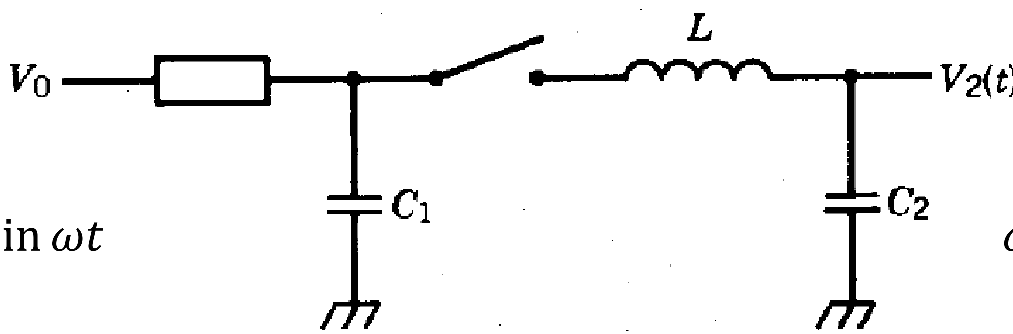
Peaking capacitor circuit: CLC circuit

- Transfer of energy between capacitors forms the basis of most pulsed power generators.

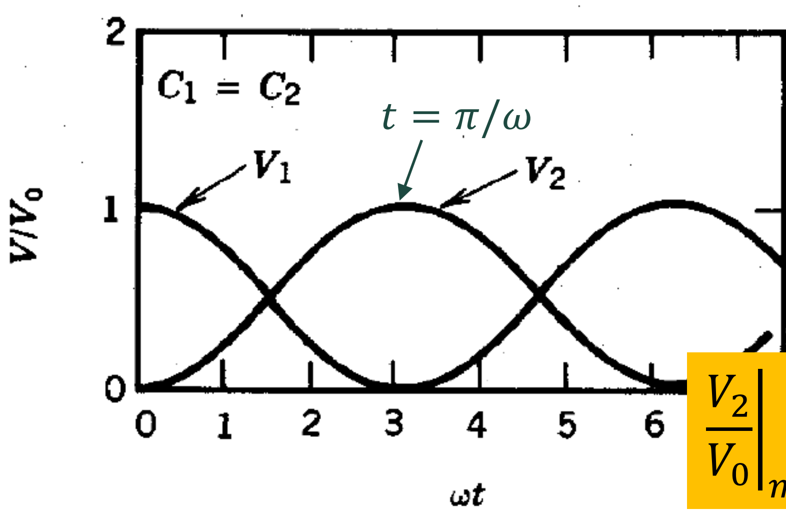


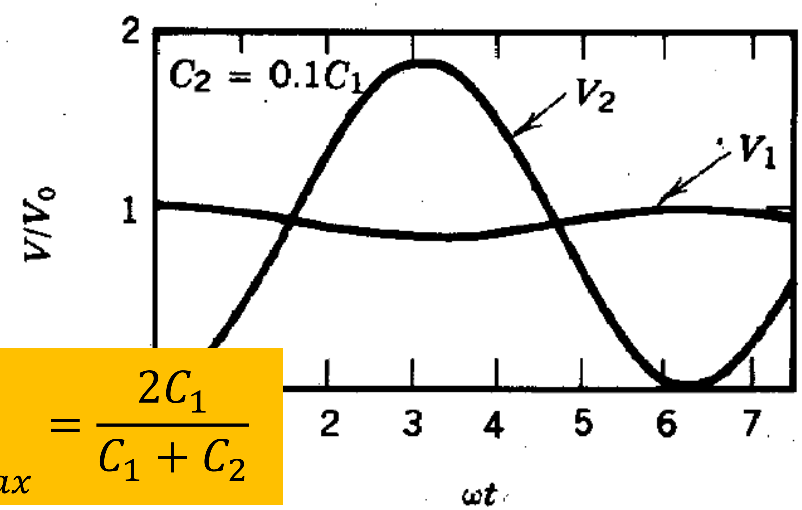
$$i(t) = \frac{V_0}{\omega L}\sin\omega t$$

$$\omega = \frac{1}{\sqrt{LC}} = \sqrt{\frac{C_1 + C_2}{LC_1C_2}}$$

$$V_1(t) = V_0\left[1 - \frac{C_2}{C_1 + C_2}[1 - \cos\omega t]\right]$$

$$V_2(t) = \frac{C_1V_0}{C_1 + C_2}[1 - \cos\omega t]$$

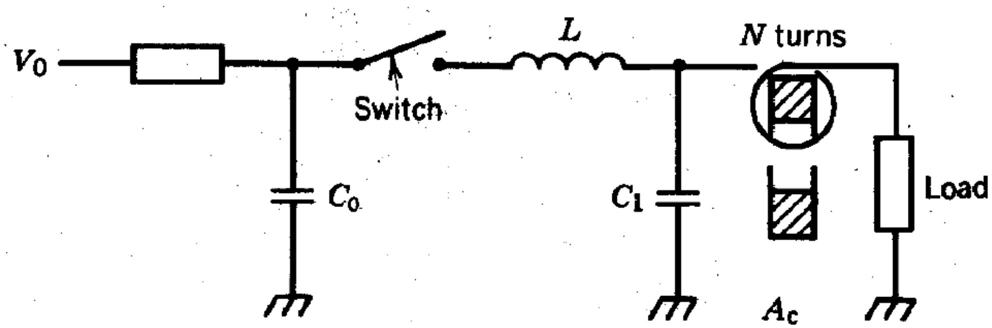


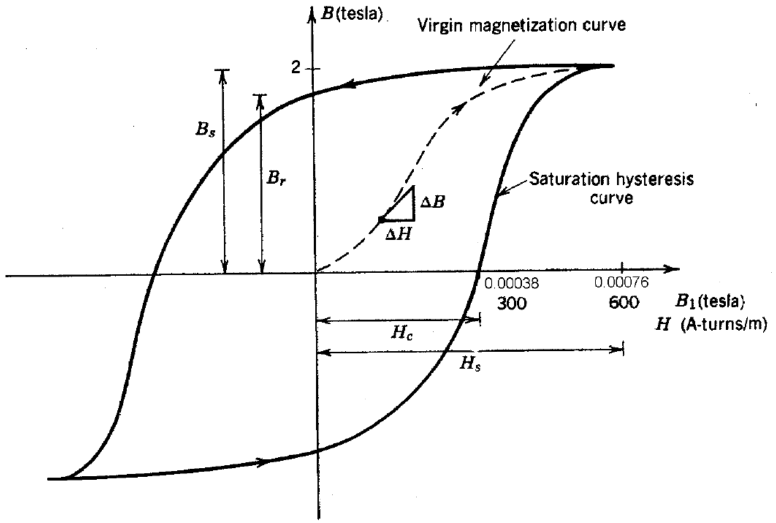


$$\left.\frac{V_2}{V_0}\right|_{max} = \frac{2C_1}{C_1 + C_2}$$

Magnetic switching: saturable core

- A two-stage power compression circuit





- At early times, the right-hand portion of the circuit is approximately an open circuit because of the high inductance of the winding around the high μ core. Energy flows from C_0 to C_1.
- The core reaches saturation when

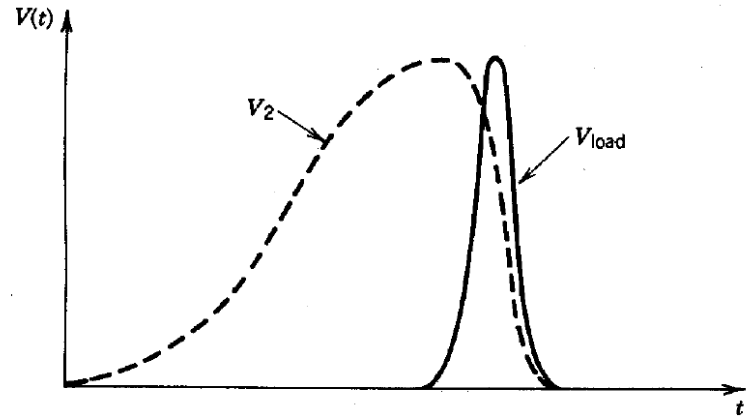
$$\int V_1(t)dt = NA_c(B_s + B_r) \quad \Longleftarrow \quad V_1(t) = NA_c\frac{dB}{dt}$$

- After saturation, the inductance L_2 decreases by a large factor, approaching the vacuum inductance of the winding. The transition from high to low inductance is a bootstrapping process that occurs rapidly.

Magnetic switching: saturable core

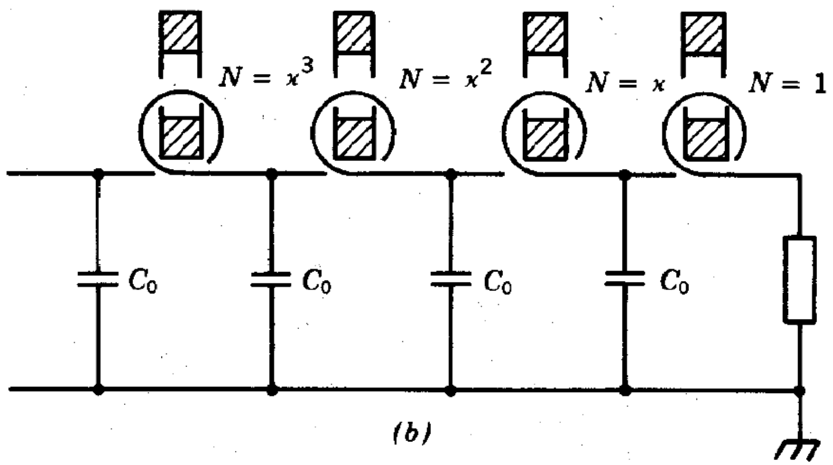
- The optimum core parameter is obtained by integrating for $0 \le t \le \pi/\omega$.

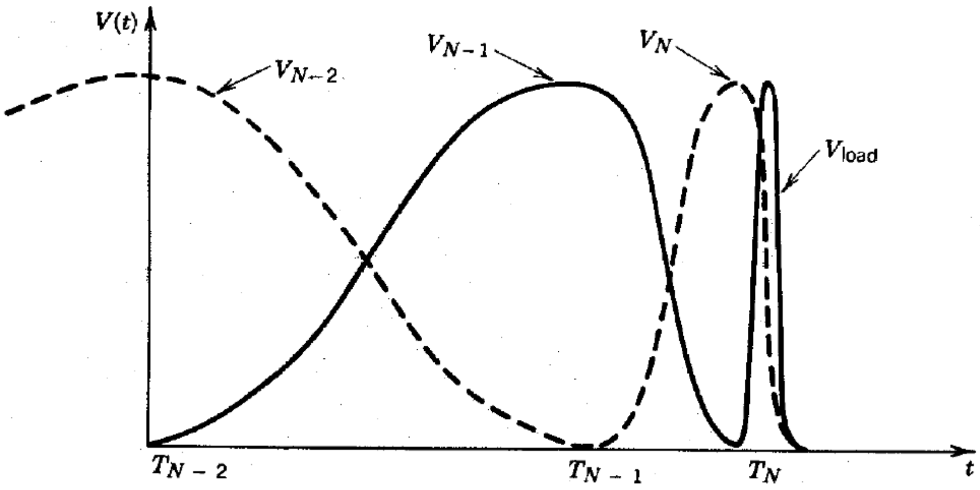
$$NA_c(B_s + B_r) = \frac{V_0 \pi}{2\omega}$$



- For a multiple compression

$$\frac{\tau_n}{\tau_{n-1}} \cong \sqrt{\frac{L_n}{L_{n-1}}} \cong \frac{1}{\kappa} \qquad 1 < \kappa < \sqrt{\frac{\mu}{\mu_0}}$$





Pulse shaping with saturable core magnetic switches

- Saturable core inductors can also be used for pulse length shortening and rise time sharpening if efficiency is not a prime concern.
- The following circuit can produce a short, fast-rising voltage pulse from a slow pulse generator.