Radio-frequency Linear Accelerators

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Concept of synchronous particle

- Accelerating voltage in an array of independently phased cavities:

$$V_n = V_0 \sin[\omega t]$$

$$V_{n+1} = V_0 \sin[\omega t + \Delta\phi_{n+1}]$$

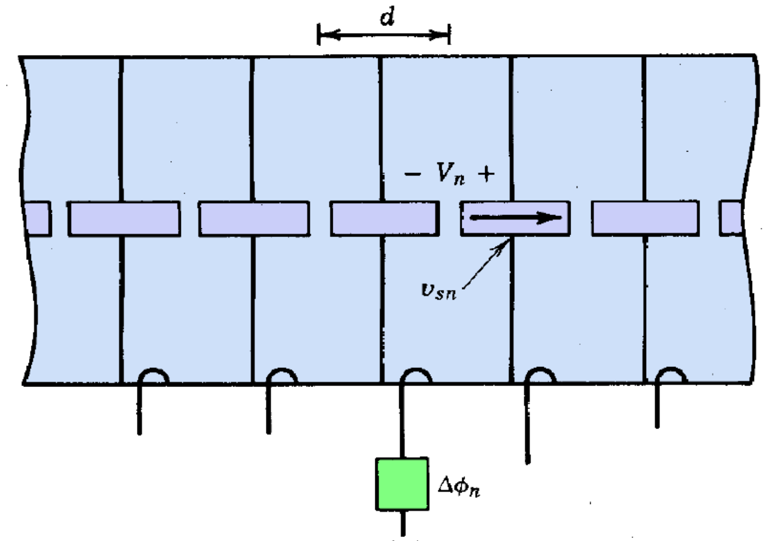
$$V_{n+2} = V_0 \sin[\omega t + \Delta\phi_{n+1} + \Delta\phi_{n+2}]$$

- By the definition of ϕ_s, the synchronous particle crosses cavity n at time $\omega t = 0$.
- Assuming non-relativistic ions, the change in synchronous particle velocity imparted by cavity n is given by

$$\frac{1}{2} m v_{sn}^2 = \frac{1}{2} m v_{sn-1}^2 + q V_0 \sin\phi_s$$

- Phase difference between cavity oscillation

$$\Delta\phi_{n+1} = -\omega \frac{d}{v_{sn}}$$



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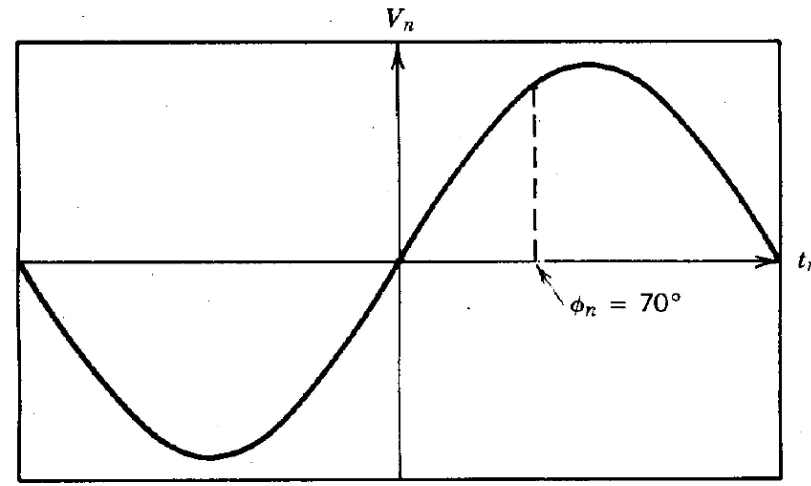

(b)

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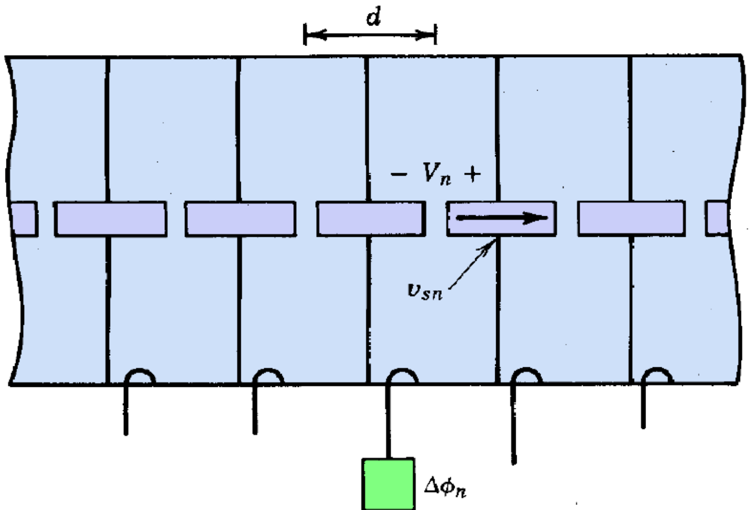
Motion of non-synchronous particles and phase stability



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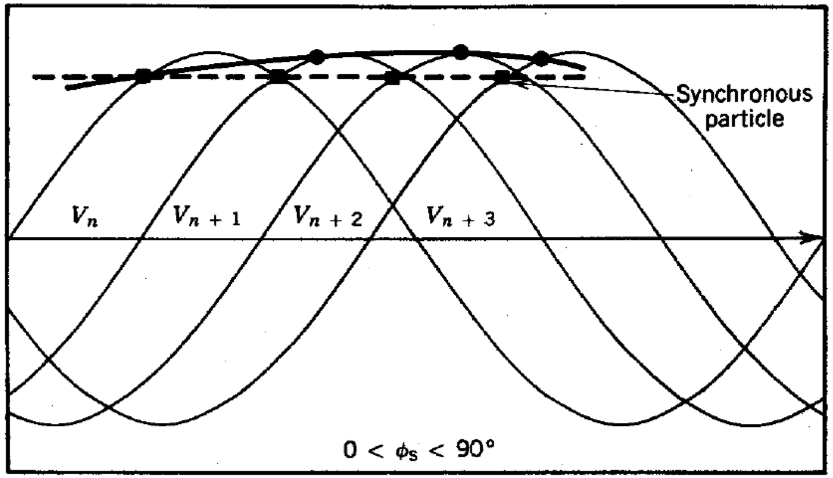
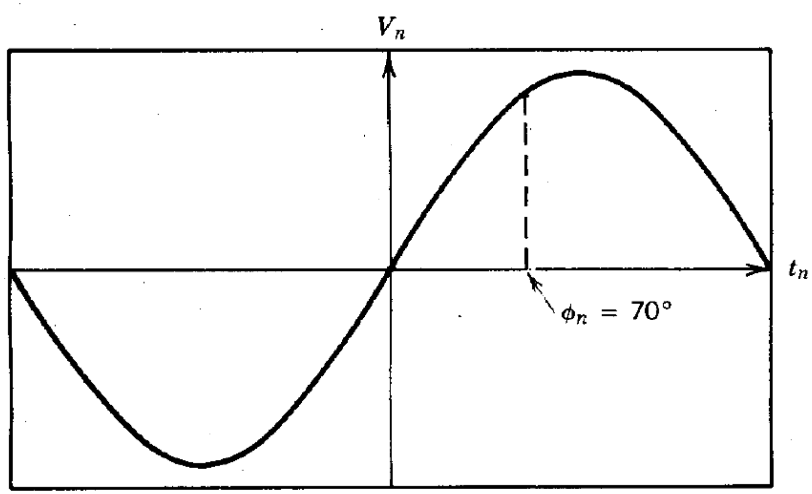


↑ Phase stable



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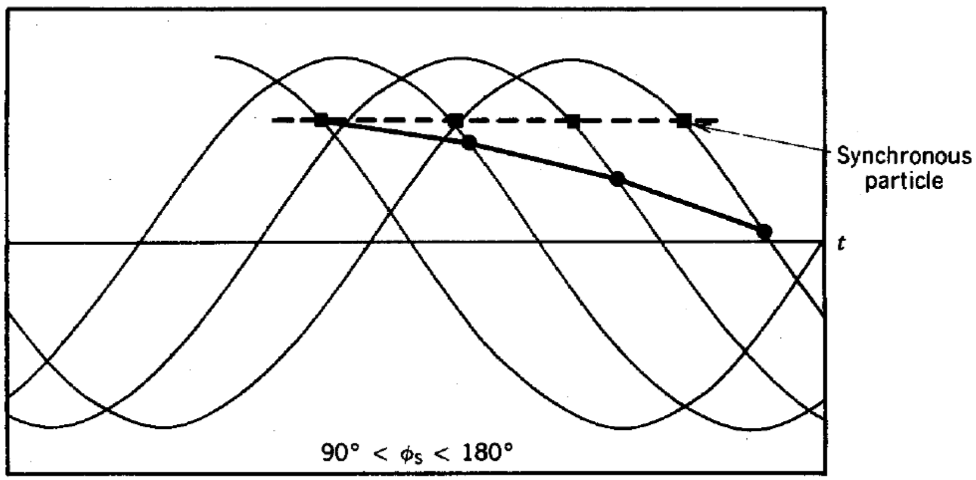

↑ Phase unstable

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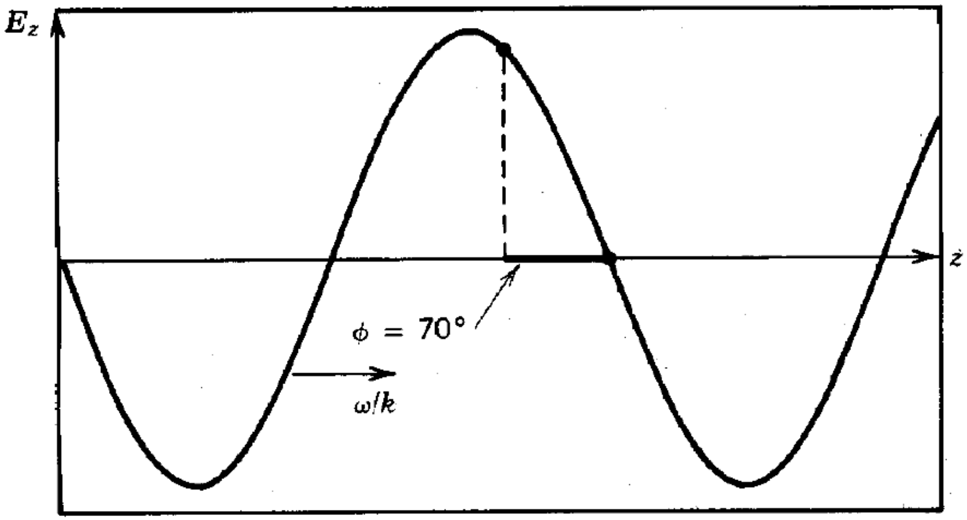
Definition of particle phase moving with a traveling electromagnetic waves

- Figure shows the electric field as a function of position viewed in the rest frame of a slow wave (which shows an electric field variation in space at a constant time. Note that the figure in the previous slide shows an electric field variation in time at a constant position).

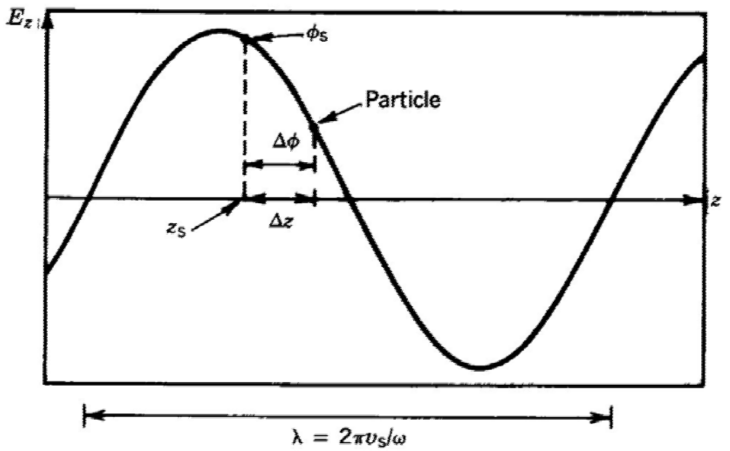


$$E_z(z,t) = E_0(z)\sin[\omega t - k(z)z]$$

- The wave can accelerate particles to high energy only if they stay within the region of accelerating electric field. In other words, the particles must remain at the same phase of the accelerating wave. This means that the wave phase-velocity must increase to match the velocity of the accelerating particles.

Synchronous particle

- The wave accelerates particles with phase in the range $0 < \phi < \pi$ and decelerates particles in the phase range $-\pi < \phi < 0$.
- We can define conditions where a particle stays at a constant phase. A particle with this property is a synchronous particle – its phase is the synchronous phase, ϕ_s.
- Figure shows that the synchronous particle experiences a constant axial electric field, $E_{zs} = E_0 \sin\phi_s$.
- The velocity of the synchronous particle changes as (non-relativistic):



$$\frac{dv_s(z)}{dt} = \frac{qE_0 \sin\phi_s}{m_0}$$

Conditions for having synchronous particles in an accelerator

- The accelerating structure must vary along its length so that the wave number is

$$k(z) = \frac{\omega}{v_s(z)}$$

Phase equation for non-synchronous particle

- Under some conditions, non-synchronous particles have stable oscillations about the synchronous particle position, z_s.
- Let z and v be the axial position and velocity of a non-synchronous particle. We define the small quantities:

$$\Delta z = z - z_s \qquad \Delta v = v - v_s \qquad \Delta\phi = \phi - \phi_s$$

- Then, we obtain

$$\frac{\Delta\phi}{2\pi} = -\frac{\Delta z}{\lambda} = -\frac{k\Delta z}{2\pi} = -\frac{\omega\Delta z}{2\pi v_s}$$

- The instantaneous acceleration of the non-synchronous particle:

$$\frac{d^2 z}{dt^2} = \frac{dv}{dt} = \frac{qE_0 \sin\phi}{m_0}$$

- The general phase equation for non-relativistic particles:

$$\boxed{\frac{d^2\Delta z}{dt^2} = \frac{qE_0 \sin\phi}{m_0} - \frac{d^2 z_s}{dt^2} = \frac{qE_0}{m_0}(\sin\phi - \sin\phi_s)}$$

$$\phi = \phi_s - \omega\Delta z / v_s$$

Phase equation: approximate solution

- If E_0 and v_s are almost constant during an axial oscillation of a non-synchronous particle, then we obtain a nonlinear differential equation as following:

$$\frac{d^2\phi}{dt^2} \cong -\frac{\omega q E_0}{m_0 v_s}(\sin\phi - \sin\phi_s)$$

- For small oscillations about the phase of the synchronous particle, $\Delta\phi \ll \phi_s$, the above equation reduces to:

Phase oscillation frequency

$$\frac{d^2\Delta\phi}{dt^2} \cong -\frac{\omega q E_0}{m_0 v_s}\cos\phi_s\,\Delta\phi \quad \Rightarrow \quad \Delta\phi \cong \Delta\phi_0 \cos\omega_z t \qquad \omega_z = \sqrt{\frac{\omega q E_0}{m_0 v_s}\cos\phi_s}$$

→ If $\cos\phi_s > 0$, the axial oscillations of non-synchronous particles are stable.

- The conditions for synchronized particle acceleration are $\cos\phi_s > 0$ and $\sin\phi_s > 0$, or

$$0 < \phi_s < \pi/2 \qquad \text{(acceleration)}$$

- The conditions for synchronized particle deceleration are $\cos\phi_s > 0$ and $\sin\phi_s < 0$, or

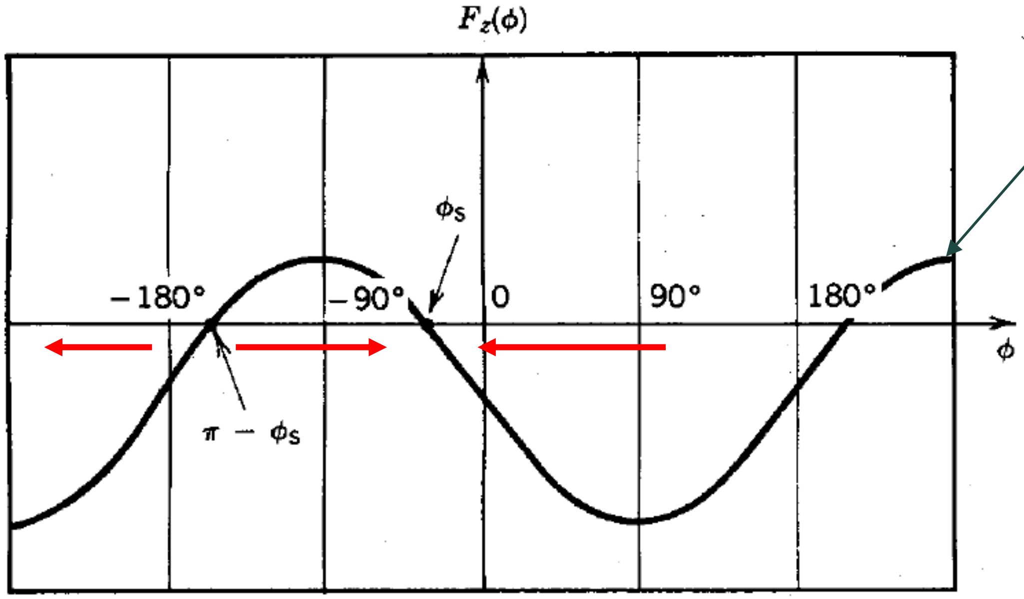
$$-\pi/2 < \phi_s < 0 \quad \text{(deceleration)}$$

Phase equation: general behavior

- The nonlinear phase equation (similar to nonlinear pendulum):

$$\frac{d^2\phi}{dt^2} \cong -\frac{\omega q E_0}{m_0 v_s}(\sin\phi - \sin\phi_s)$$

RHS could be considered as an effective restoring force confining ϕ about ϕ_s



$F_z(\phi) \sim -\sin\phi + \sin\phi_s$

→ Two critical points:
ϕ_s (center), $\pi - \phi_s$ (saddle)

$$U_z(\phi) = -\int F_z(\phi)\,d\phi$$

$$\sim -\cos\phi - \phi\sin\phi_s + K$$

- Integrating the equation after multiplying both sides by $2(d\phi/dt)$:

$$\left(\frac{d\phi}{dt}\right)^2 = \frac{2\omega q E_0}{m_0 v_s}(\cos\phi + \phi\sin\phi_s) + K$$

↖ Integrating constant

Range of stable phase and longitudinal acceptance

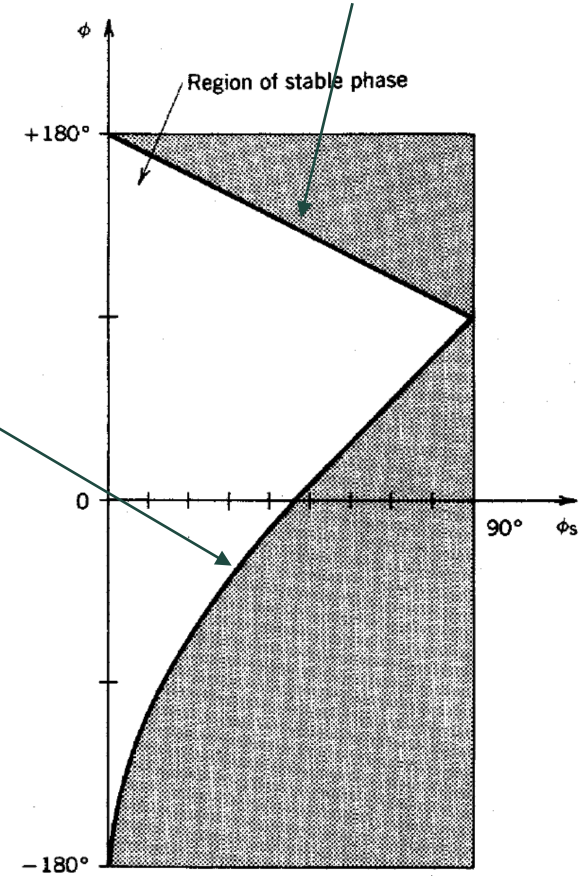
- We shall determine K for the orbit of the oscillating particle with the maximum allowed displacement from ϕ_s: $d\phi/dt = 0$ at $\phi = \pi - \phi_s$:

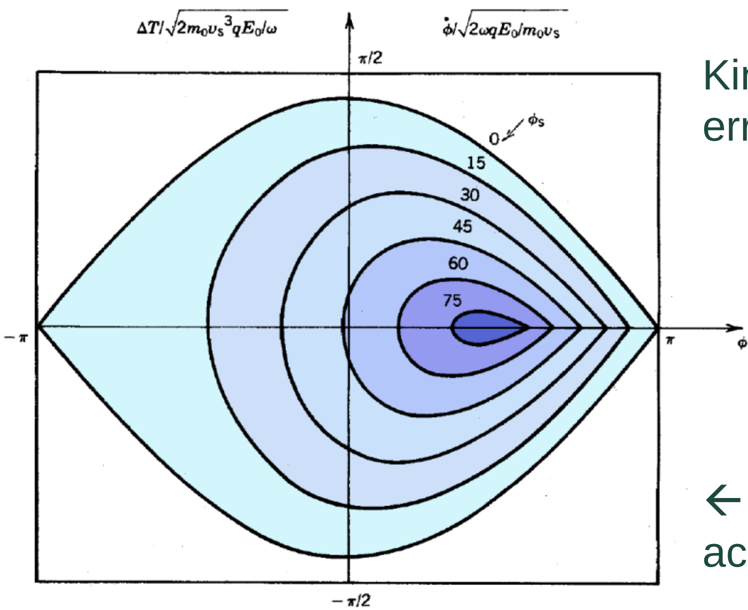
$$\left(\frac{d\phi}{dt}\right)^2 = \frac{2\omega q E_0}{m_0 v_s}\left(\cos\phi + \cos\phi_s + (\phi + \phi_s - \pi)\sin\phi_s\right)$$

- The boundary particle oscillates about ϕ_s with maximum phase excursions given by the solution of

$$\cos\phi + \cos\phi_s = (\pi - \phi - \phi_s)\sin\phi_s$$

$\phi = \pi - \phi_s$



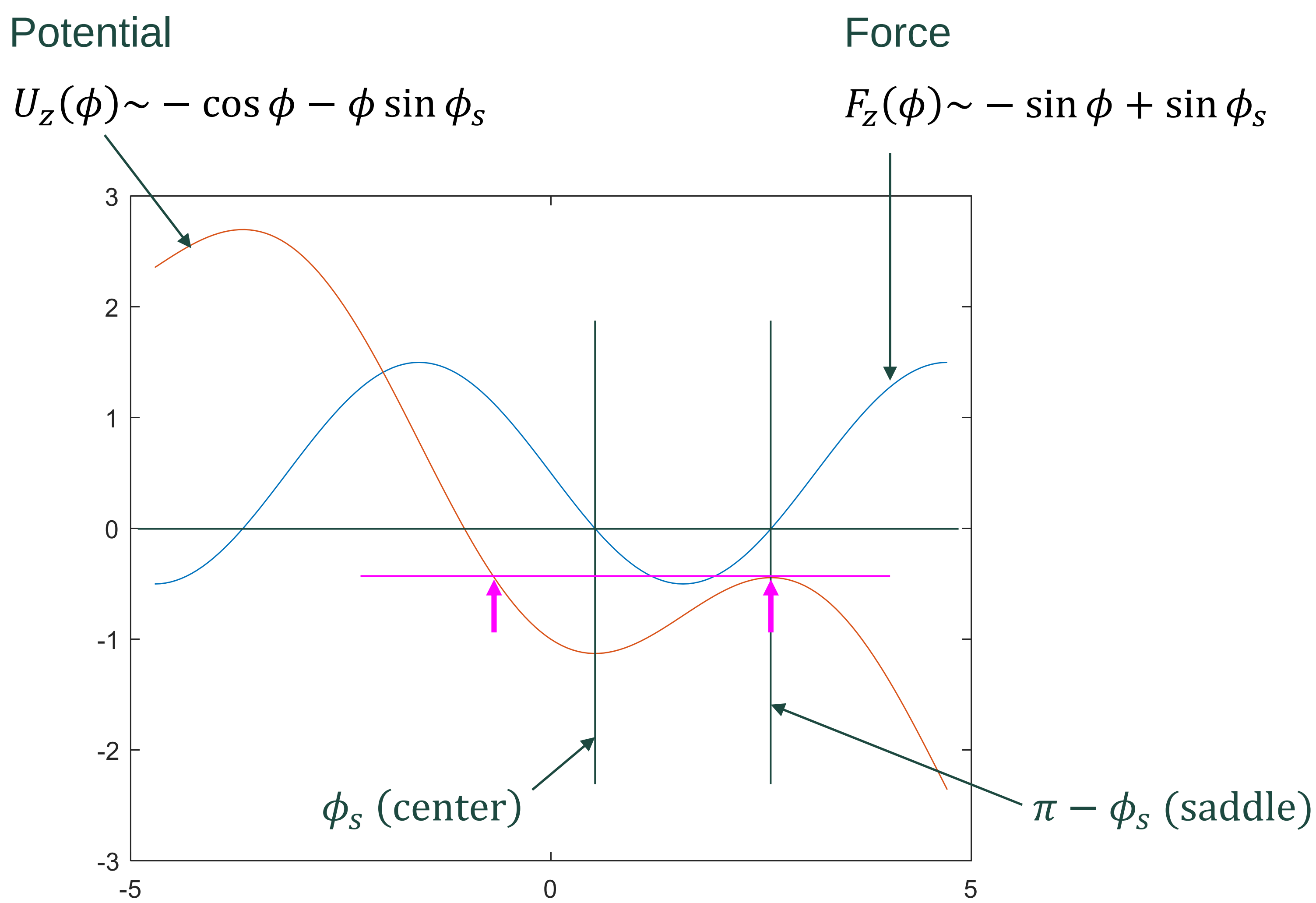


Kinetic energy error: $\Delta T = T - T_s$

Phase

← Longitudinal acceptance diagram

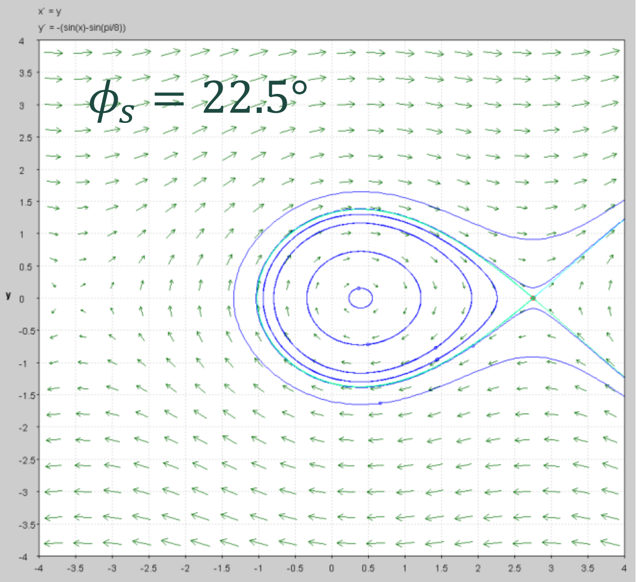
Phase stability

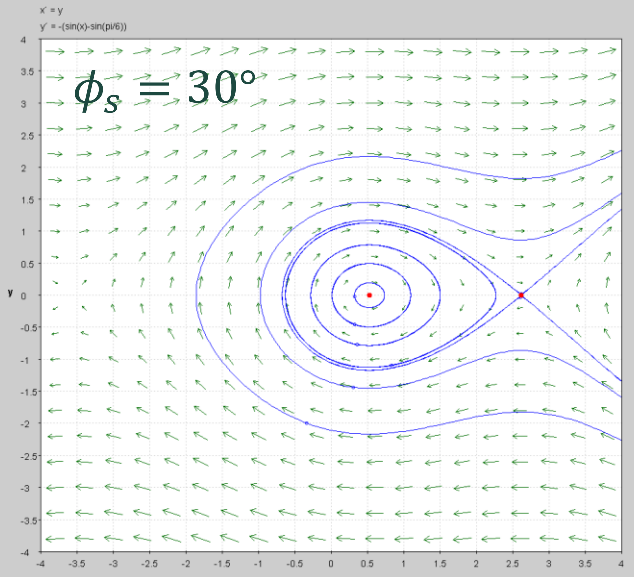


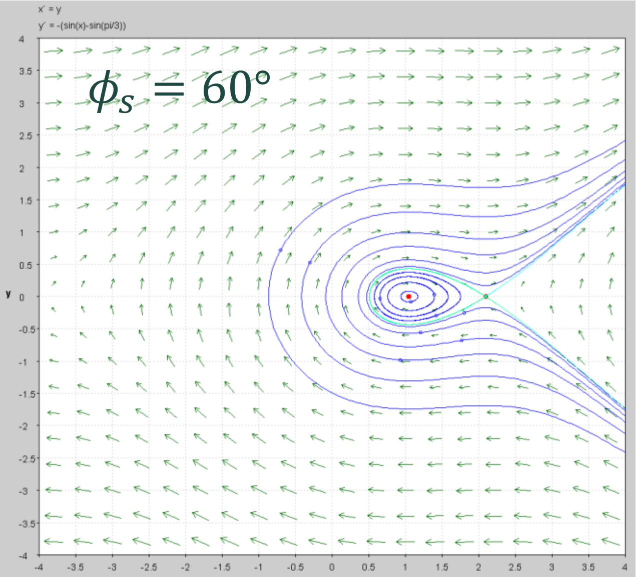
Phase portrait

$$\phi' = \omega$$

$$\omega' = -\frac{\omega q E_0}{m_0 v_s}(\sin\phi - \sin\phi_s)$$







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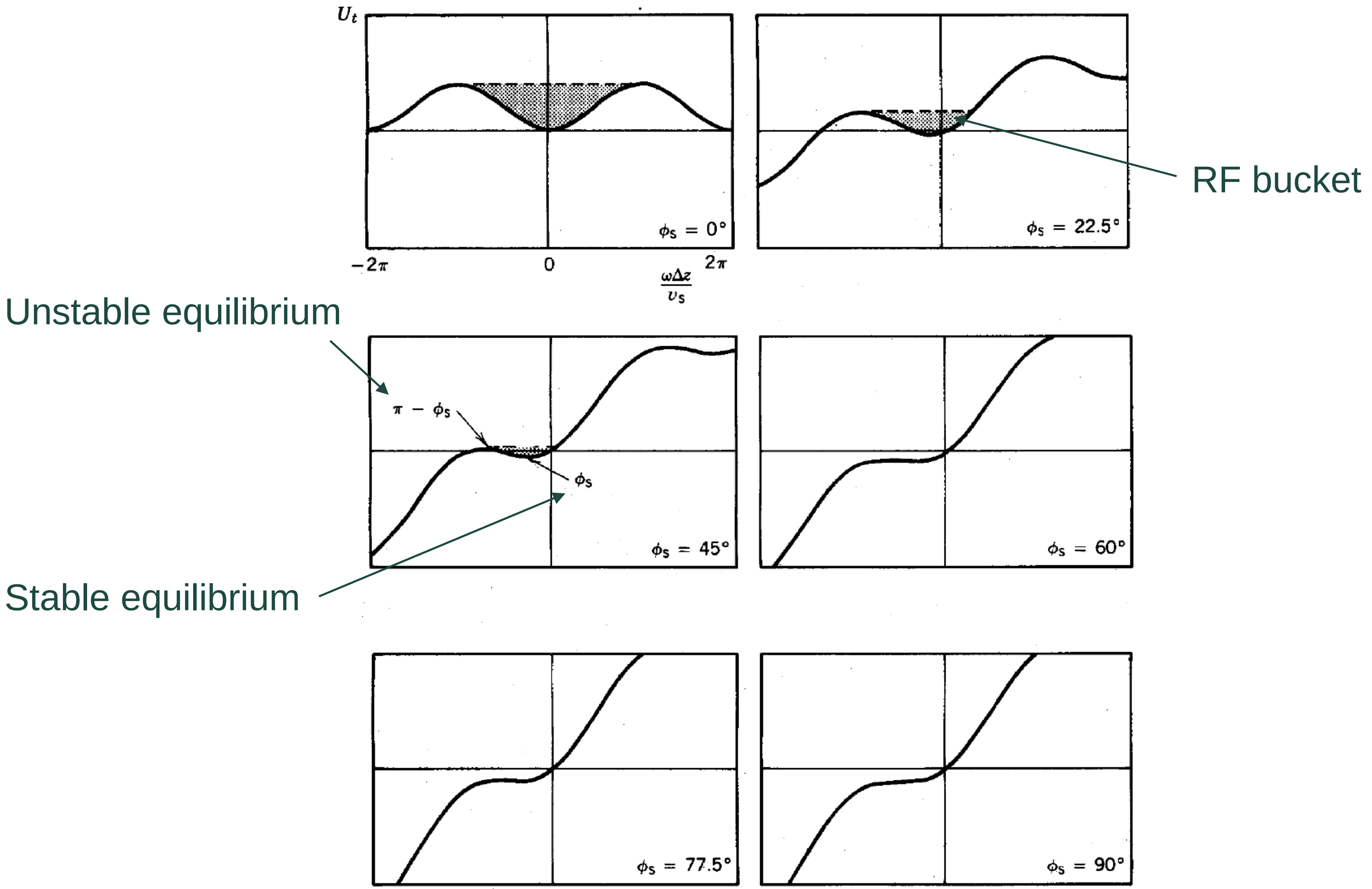
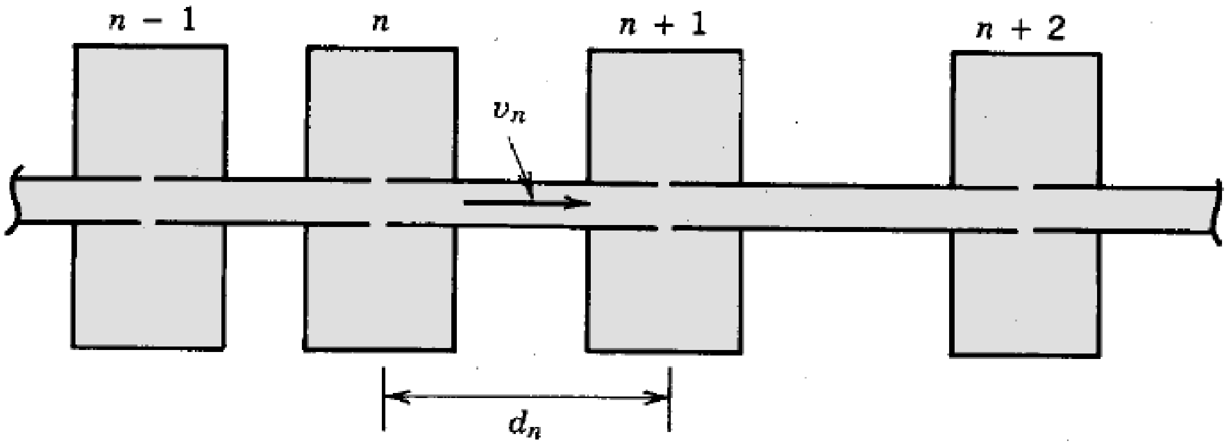
Longitudinal potential energy diagram



Longitudinal space-charge limits in RF accelerators

- Beam-generated axial electric fields can limit the beam current in RF accelerators and induction linacs.



- Ions in RF accelerators must remain in specific phase regions of the accelerating wave. The electric field of a traveling wave can provide stable axial confinement for ions that are localized along z and have a small spread in kinetic energy.
- The wave creates a potential well for ion confinement called an RF bucket. Ions that escape from the bucket quickly lose their synchronization with the wave and are no longer accelerated. Space-charge electric fields can drive ions out of an RF bucket. This process set limits on the current in the accelerator.

Longitudinal space-charge limits in RF accelerators

- The total potential energy for particles in the wave frame:

$$U_t(\Delta z) = \frac{qE_0 v_s}{\omega}\left[1 - \cos\left(\frac{\omega \Delta z}{v_s}\right)\right] + qE_0 \Delta z \sin\phi_s$$

- The depth of the confining potential well:

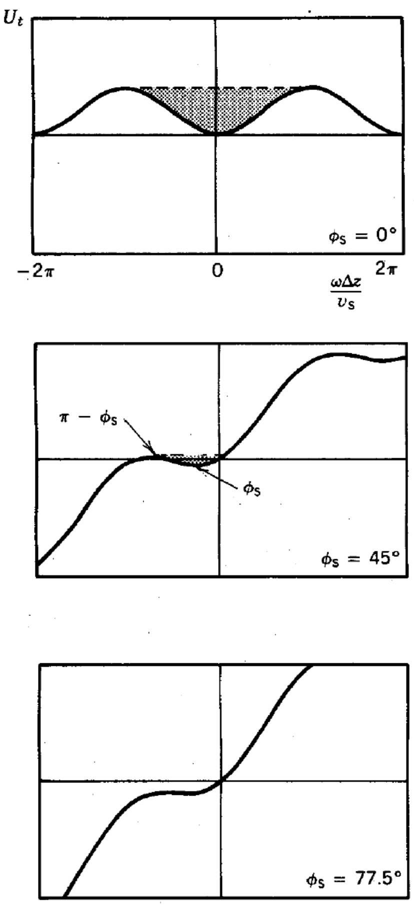
$$\Delta U_c = \frac{2qE_0 v_s}{\omega}\Psi(\phi_s)$$

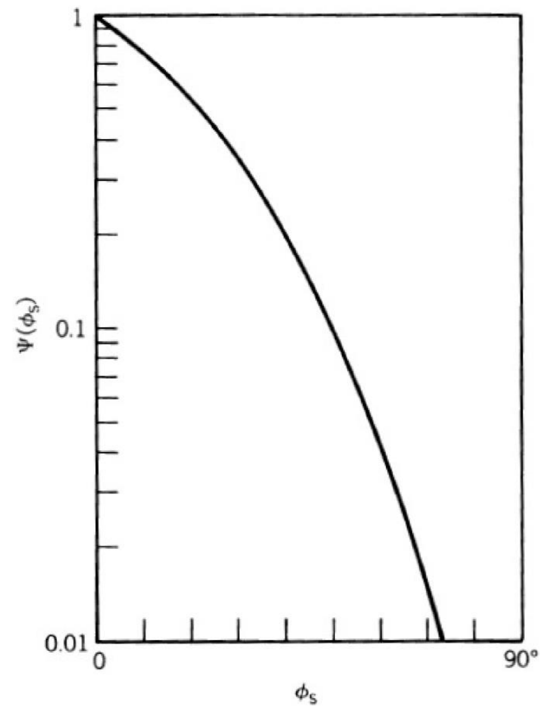
- The beam-generated electric potential:

$$q\Delta\phi = \frac{qI_0}{4\pi\epsilon_0 \beta c}\left[1 + 2\ln\left(\frac{r_w}{r_0}\right)\right]$$

- The beam-generated electric force pushes particles out of the bucket if $q\Delta\phi > \Delta U_c$, giving a peak current:

$$I_0 \leq \frac{8\pi\epsilon_0 \beta c \Psi(\phi_s) E_0 v_s}{\omega[1 + 2\ln(r_w/r_0)]}$$





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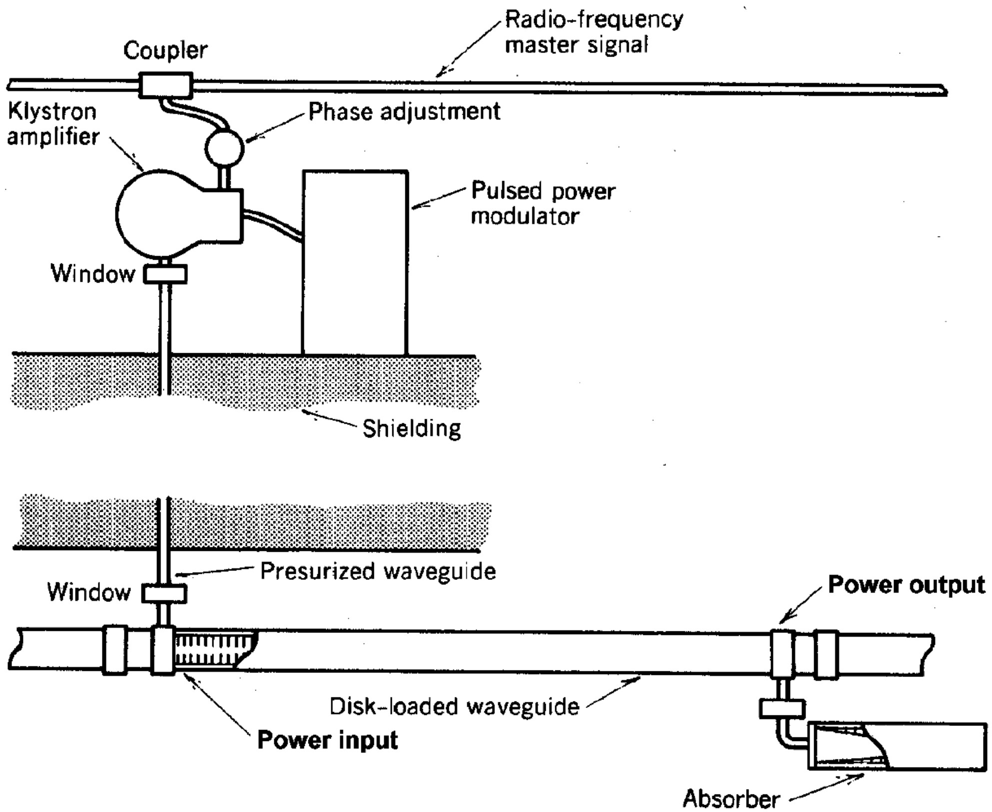
Electron linear accelerators

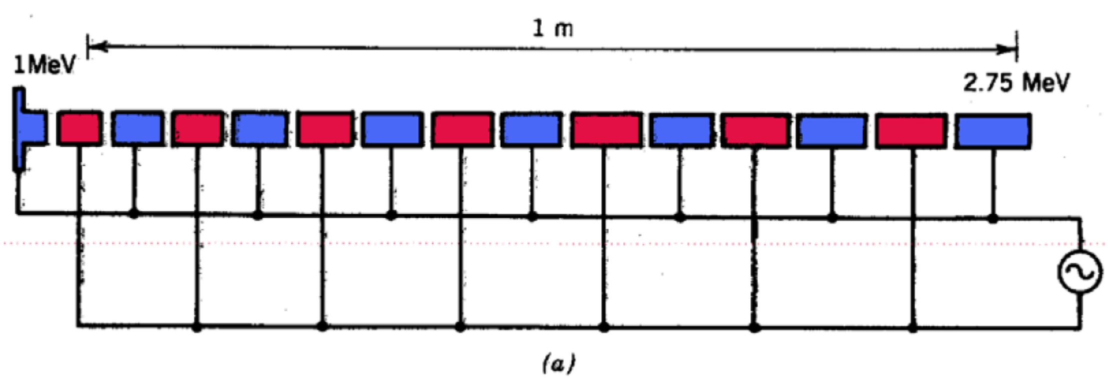
- RF electron linacs are used to generate high-energy electron beams in the range of 2 to 20 GeV. Circular election accelerators cannot reach high output kinetic energy because of the limits imposed by synchrotron radiation.
- Linear accelerators for electrons use high-gradient, travelling wave structures, which are quite different from ion accelerators.



Linear ion accelerator configurations

- In the energy range accessible to linear accelerators, ions are non-relativistic; thus slow-wave structures are not useful for ion acceleration. An iris-loaded waveguide has small apertures for $\omega/k \ll c$. The conduction of electromagnetic energy via slow waves is too small to drive a multi-cavity waveguide.
- An ion linear accelerator typically consists of a sequence of cylindrical cavities supporting standing waves. Cavity oscillations are supported either by individual power feeds or through inter-cavity coupling via magnetic fields.
- Wideröe accelerator: the first successful linear accelerator (1928)

$$L_n = \frac{v_n \pi}{\omega}$$



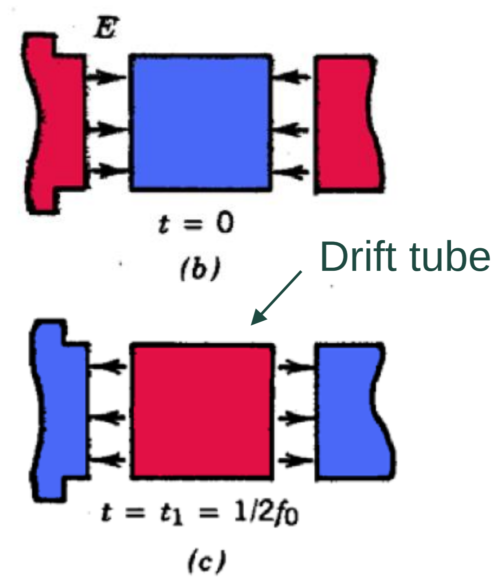
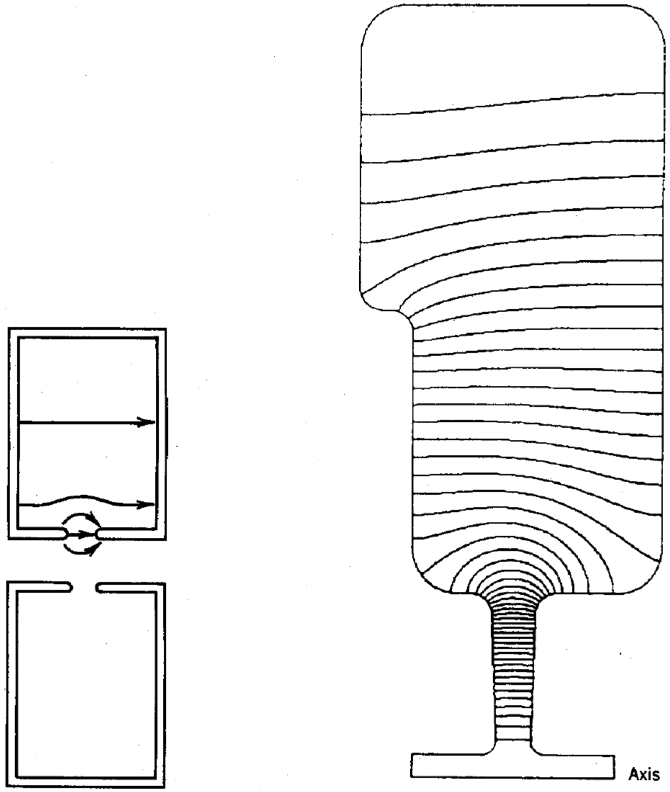


Figure 14.7 Wideröe linear accelerator for heavy ions. (*a*) Scale drawing of accelerator with following parameters: Ion species: Cs-137, $f = 10$ MHz, $T_i = 1$ MeV, $V_0 = 100$ kV, and $\phi_s = 60°$. (*b*) Electric fields in acceleration gaps 1 and 2 at ion injection ($t = 0$). (*c*) Electric fields at time $t = 1/2f$, where f is the rf frequency.

Resonant cavities for particle acceleration

- The Wideröe accelerator is not useful for light-ion acceleration and cannot be extrapolated to produce high-energy heavy ions. At high energy, the drift tubes are unacceptably long, resulting in a low average accelerating gradient.
- The drift tube length is reduced if the rf frequency is increased, but this leads to the following problems:
  - The acceleration gaps conduct large displacement currents at high frequency, loading the rf generator.
  - Adjacent drift tubes act as dipole antenna at high frequency with attendant loss of rf energy by radiation.
- The high-frequency problems are solved if the acceleration gap is enclosed in a cavity with resonant frequency ω. The cavity walls reflect the radiation to produce a standing electromagnetic oscillation.



(a) (b)

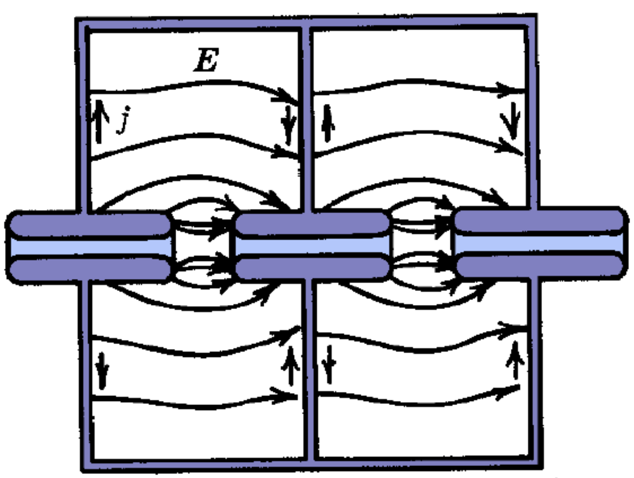
- The TM010 mode produces good electric fields for acceleration.

Array of resonant cavities

- Linear ion accelerators are composed of an array of resonant cavities.
- Two frequently encountered cases of cavity phasing are illustrated in the figure. In the first, the electric fields of all cavities are in phase, while in the second there is a phase change of 180° between adjacent cavities.

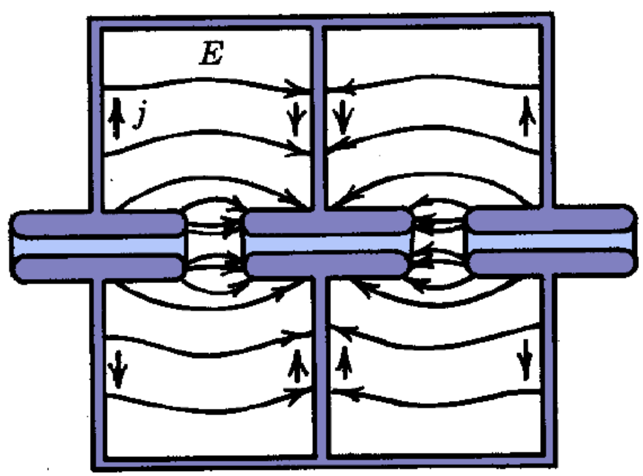
$\beta\lambda$ linac

$$L_n = v_n \frac{2\pi}{\omega} = \beta\lambda$$



$\beta\lambda/2$ linac

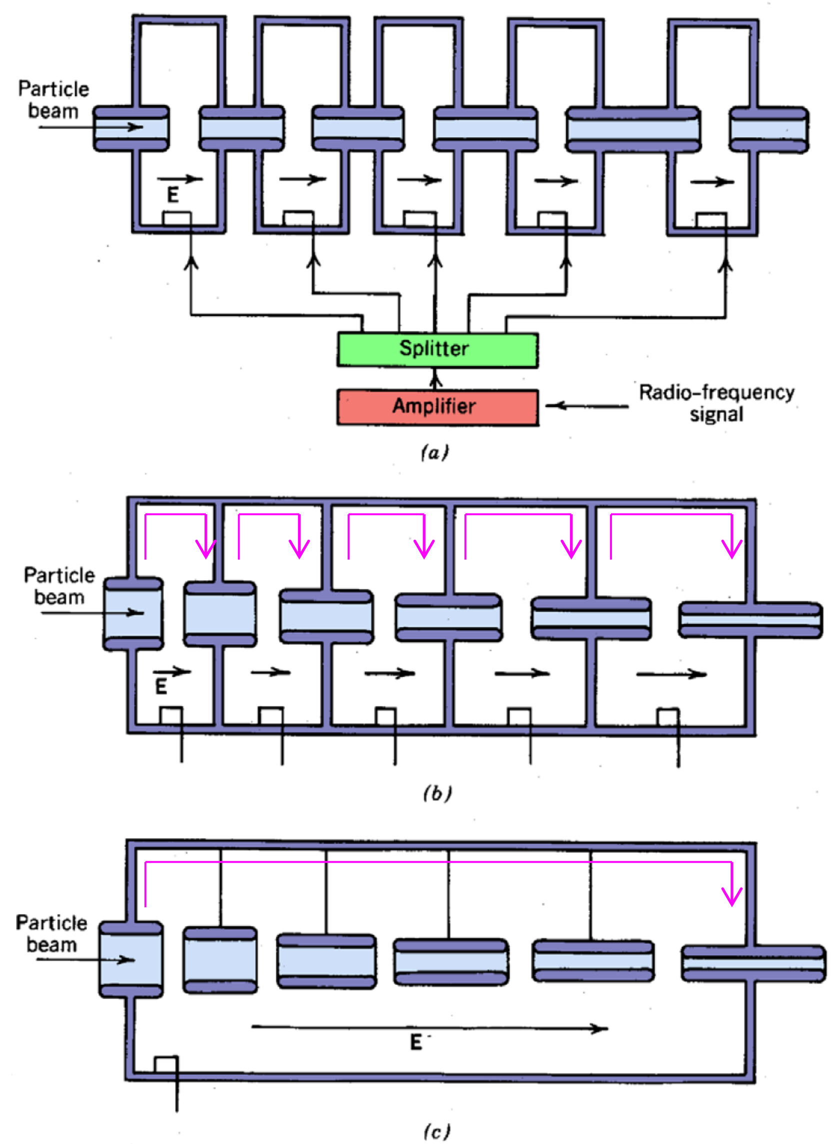
$$L_n = v_n \frac{\pi}{\omega} = \frac{\beta\lambda}{2}$$



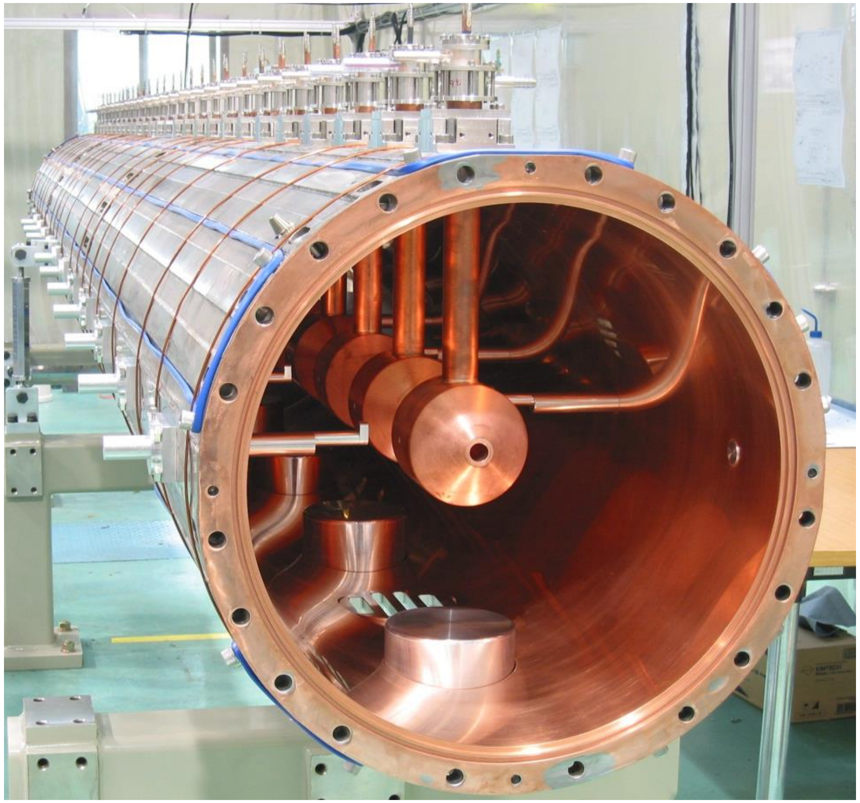
$$\beta = \frac{v_n}{c}$$

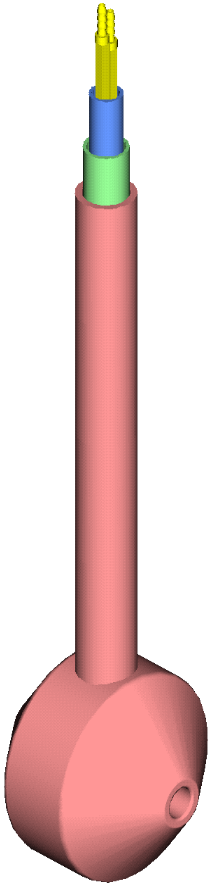
Drift tube or Alvarez linac (1945)

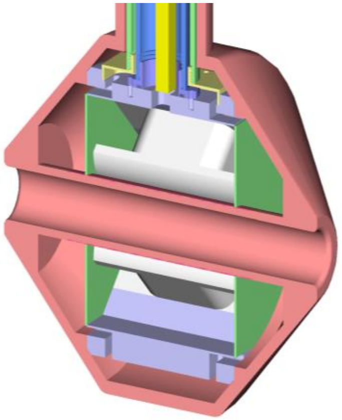
- (a) An improvement over the independently phase array in terms of reduction of microwave hardware. There are separate power feeds but only one amplifier. Synchronization of ion motion to the rf oscillations is accomplished by varying the drift lengths between cavities.
- (b) A mechanically simplified version in which the two walls separating cavities are combined. In the absence of the drift tubes, the cavities have the same resonant frequency because T_{010} does not depend on the cavity length. The additional capacitance of the acceleration gap upsets the balance. It is necessary to adjust the gap geometry in different cavities to maintain a constant resonant frequency. The capacitance is determined by the drift tube diameter and the gap width.

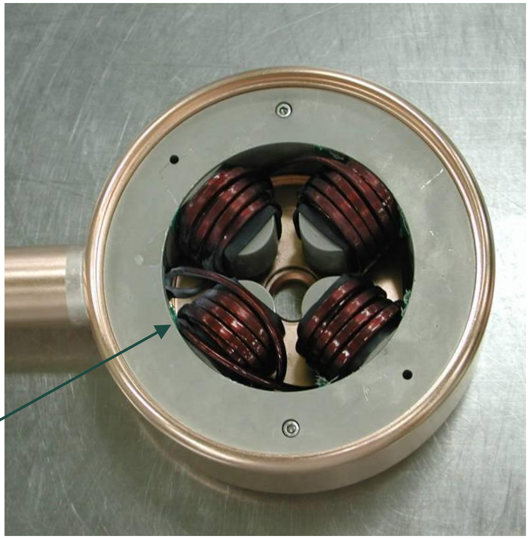


Drift tube linac (KOMAC)





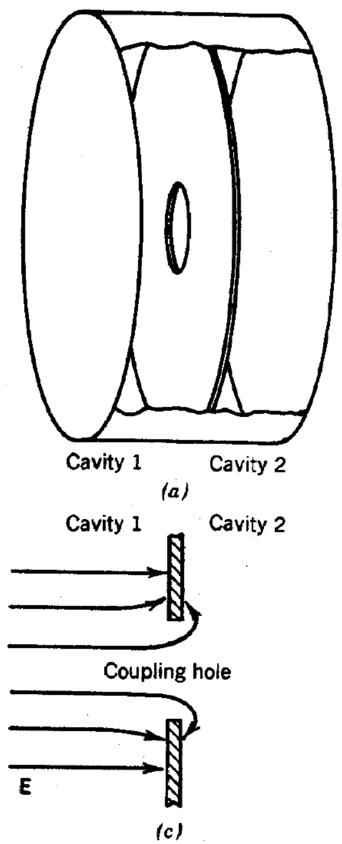


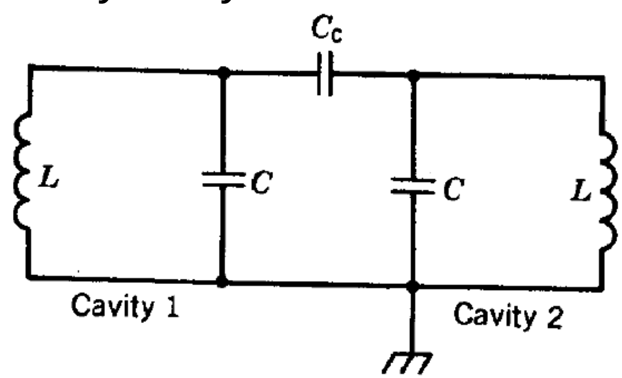


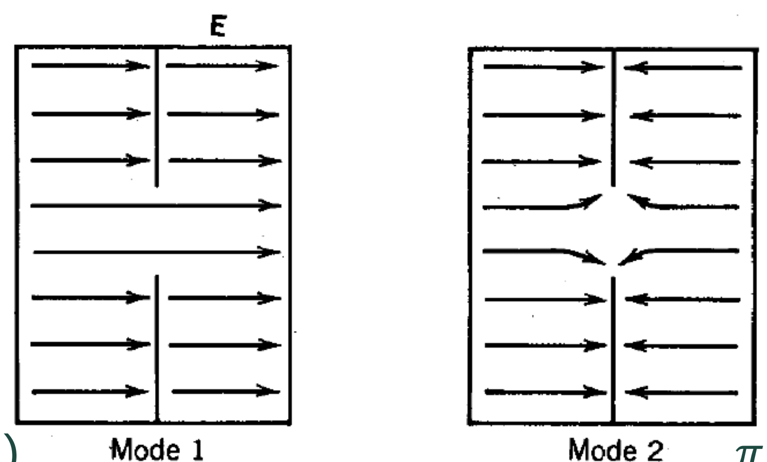
Quadrupole magnet for beam focusing

Coupled cavity

- For a constrained frequency (set by rf power tube technology) and peak electric field (set by breakdown limits), a $\beta\lambda/2$ linac has twice the average accelerating gradient as a $\beta\lambda$ structure such as the drift tube linac. For a given beam output energy, a $\beta\lambda/2$ accelerator is half as long as a $\beta\lambda$ machine. Practical $\beta\lambda/2$ geometries are based on coupled cavity arrays.







0 mode ($\beta\lambda$)

π Mode ($\beta\lambda/2$)

Coupled cavity array

- In a coupled cavity linac, the goal is to drive a large number of cavities from a single power feed. Energy is transferred from the feed cavity to other cavities via magnetic or electric coupling.
- We obtain the solution (similar to thin-lens array)

$$V_n = V_0 \cos(n\mu + \phi)$$

$$\cos\mu = -\frac{1 - \Omega^2 - 2\kappa}{2\kappa}$$

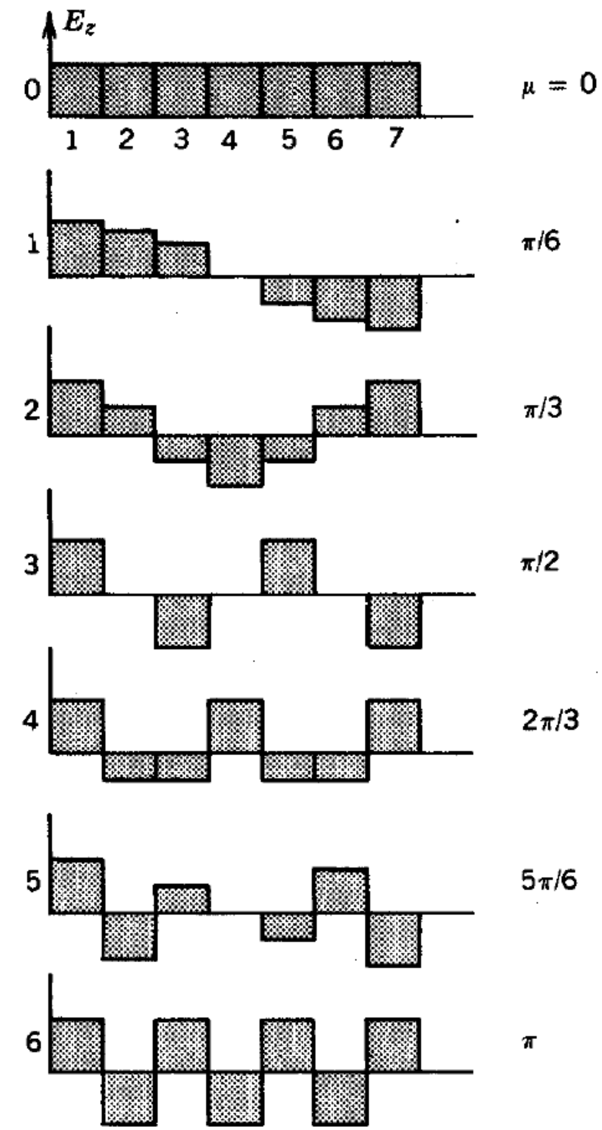
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Omega = \frac{\omega}{\omega_0}$$

$$\kappa = \frac{C_c}{C}$$

- A coupled system of N cavities has N modes of oscillation with frequencies given by

$$\Omega_m = \frac{\omega_m}{\omega_0} = \sqrt{1 - 2\kappa\left(1 - \cos\left(\frac{2\pi m}{N} - 1\right)\right)}$$



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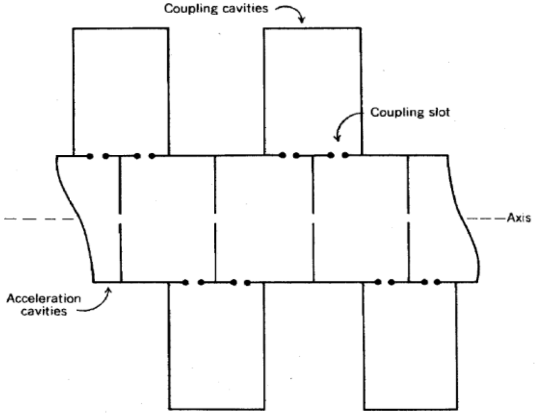
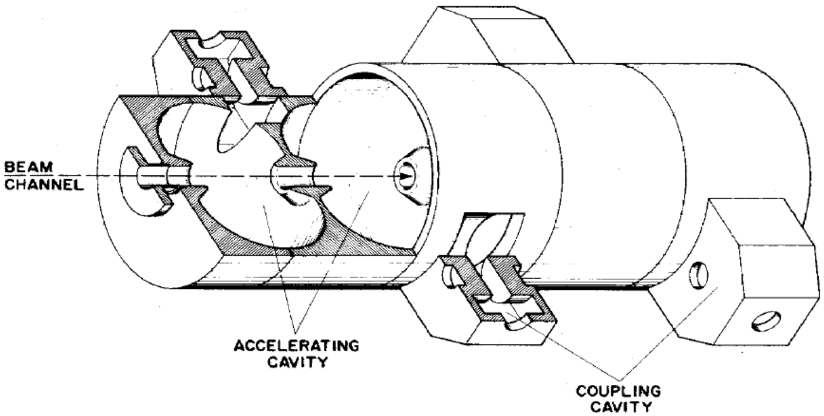
Coupled cavity linac

- It seems that the π mode is the optimal choice for a high-gradient accelerator. Unfortunately, this mode cannot be used because it has a very low energy transfer rate between cavities (zero group velocity for π mode).
- For a positive-going wave $V_+(z,t) = \exp\left[j\left(\frac{\mu z}{d} - \omega t\right)\right]$
- Phase velocity and group velocity

$$v_p = \frac{\omega}{k} = \frac{\omega_0 \Omega d}{\mu} \qquad v_g = \frac{d\omega}{dk} = \omega_0 d \frac{d\Omega}{d\mu} = -\omega_0 d \frac{\kappa \sin\mu}{\sqrt{1 - 2\kappa + 2\kappa\cos\mu}}$$

- The $\pi/2$ mode is the best choice for rf power coupling but it has a relatively low gradient because half of the cavities are unexcited. An effective solution to this problem is to displace the unexcited cavities to the side and pass the ion beam through the even-numbered cavities. → side-coupled cavity





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Transit time factor

- The transit-time factor applies mainly to drift tube accelerators with narrow acceleration gaps. The transit-time factor is important when the time for particles to cross the gap is comparable to or longer than the half-period of an electromagnetic oscillation.

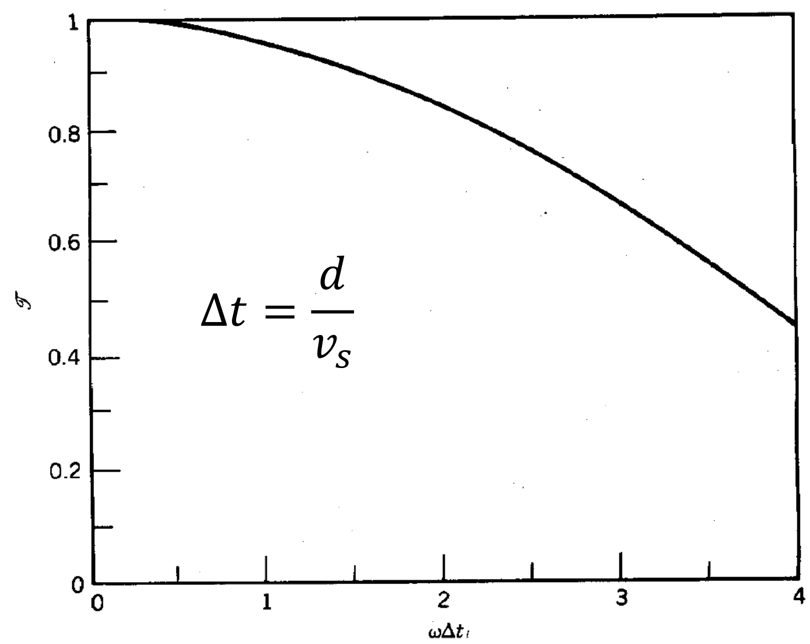
- Transit time factor

$$T = \left(\frac{2}{\omega_0 \Delta t}\right) \sin\left(\frac{\omega_0 \Delta t}{2}\right)$$

- Momentum gain

$$\Delta p_z \cong qE_0 \int_{-d/2v_s}^{+d/2v_s} \cos(\omega t + \phi) dt = T \cdot \Delta p_0$$

$$\cong T \cdot \Delta p_0$$



$$\Delta p_0 = qE_0 \sin\phi \cdot \left(\frac{d}{v_s}\right)$$ when $d \to 0$

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