Basic Concepts of Plasma

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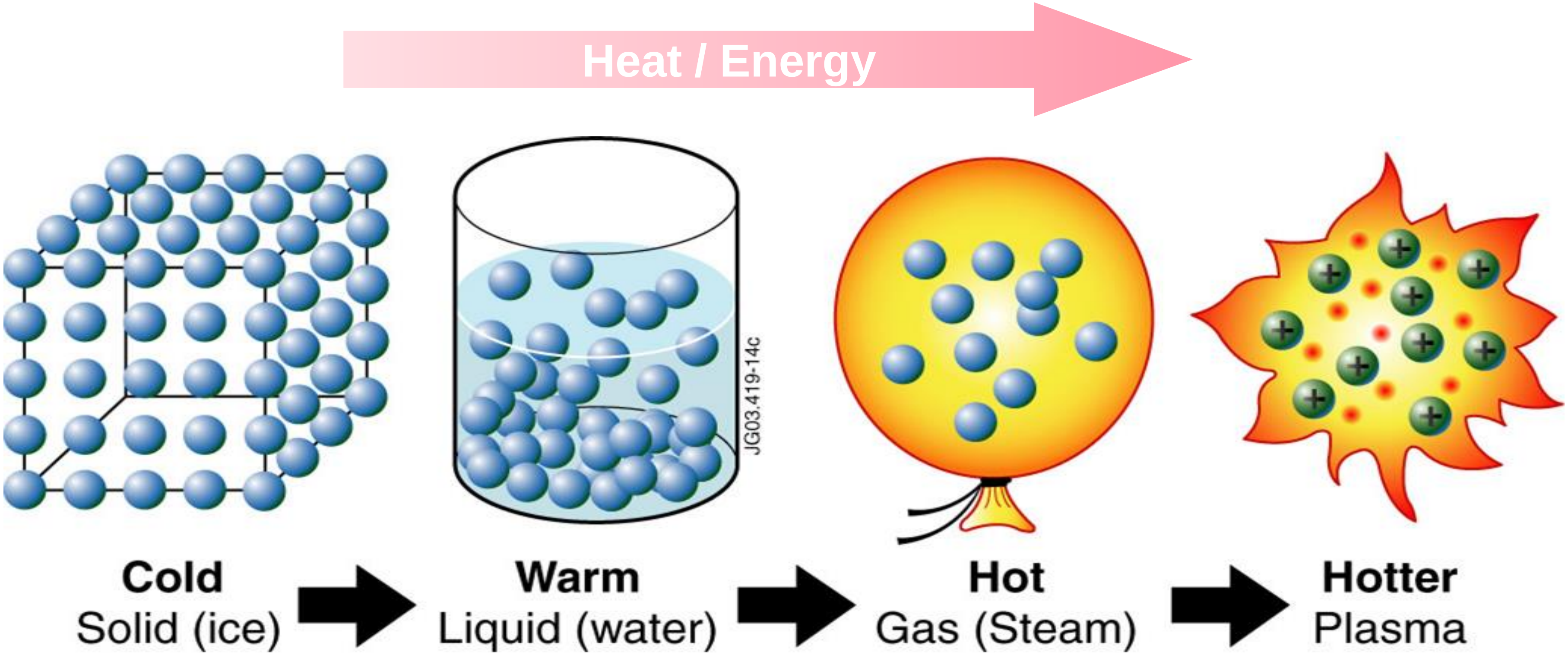
What is a plasma?



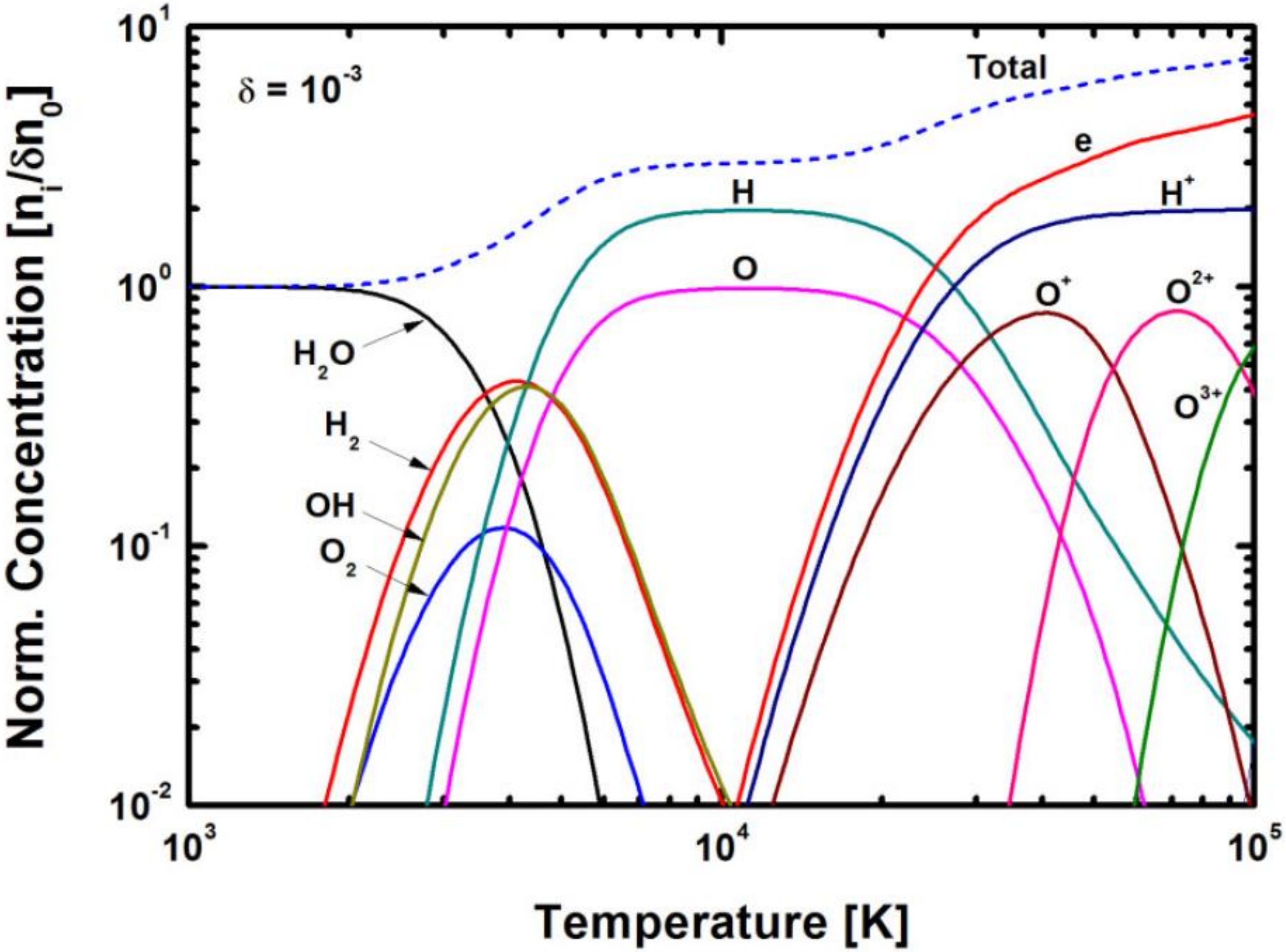
$H_2O \rightarrow H_2 + O_2 + OH$

$\rightarrow H + O$

$\rightarrow H^+ + O^+ + e$

- Plasma is a collection of charged particles (positive ions, negative ions, electrons) and neutral particles (molecules, atoms, radicals).

Composition of plasmas: H_2O example

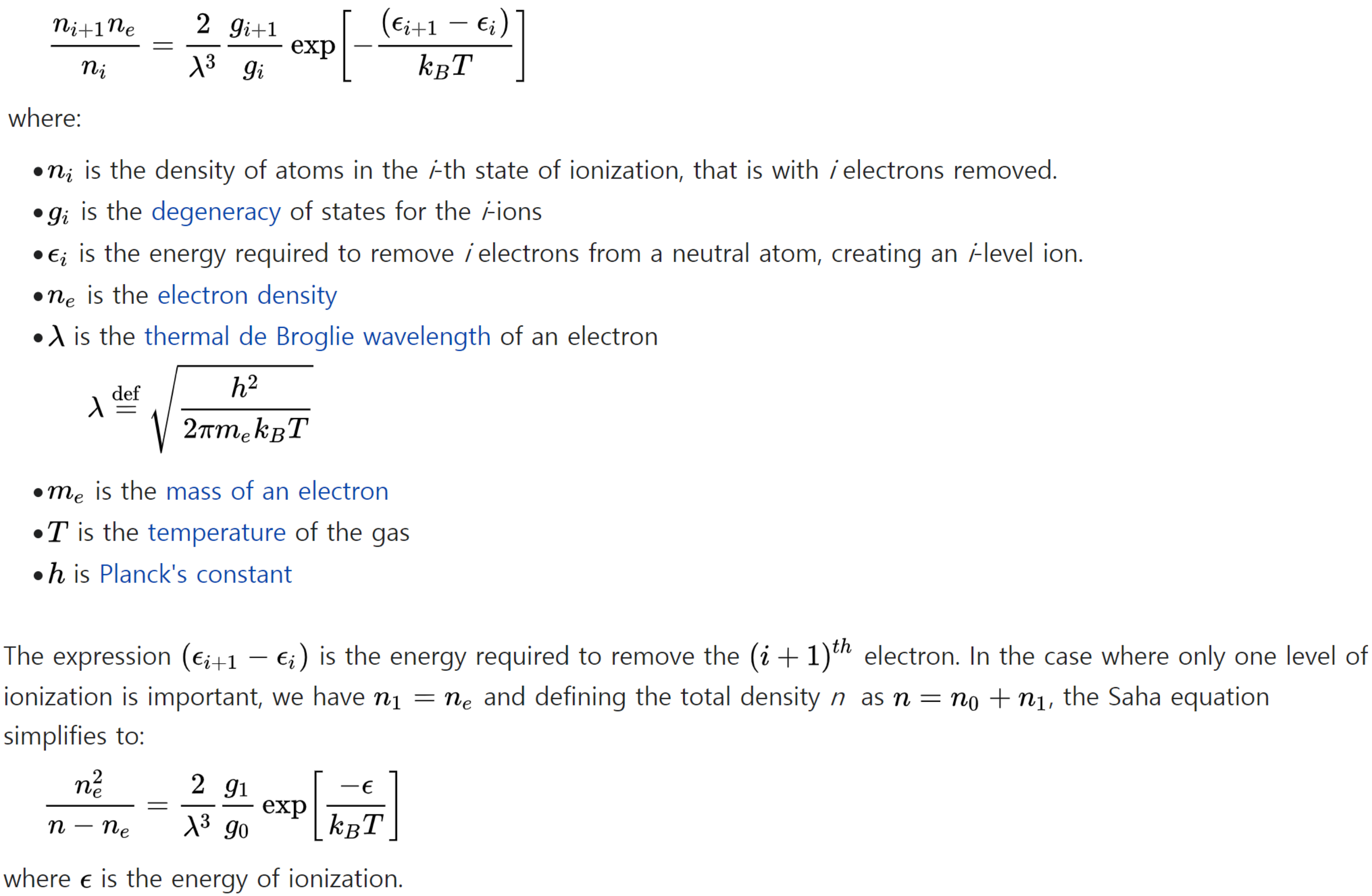


- Thermal equilibrium is assumed.

K. J. Chung, Contrib. Plasma Phys. 53, 330 (2013)

Saha equation: degree of ionization in thermal equilibrium

- The Saha ionization equation is an expression that relates the ionization state of a gas in thermal equilibrium to the temperature and pressure.



$$\frac{n_{i+1}n_e}{n_i} = \frac{2}{\lambda^3}\frac{g_{i+1}}{g_i}\exp\left[-\frac{(\epsilon_{i+1}-\epsilon_i)}{k_B T}\right]$$

where:

- n_i is the density of atoms in the *i*-th state of ionization, that is with *i* electrons removed.
- g_i is the degeneracy of states for the *i*-ions
- ϵ_i is the energy required to remove *i* electrons from a neutral atom, creating an *i*-level ion.
- n_e is the electron density
- λ is the thermal de Broglie wavelength of an electron

$$\lambda \overset{\text{def}}{=} \sqrt{\frac{h^2}{2\pi m_e k_B T}}$$

- m_e is the mass of an electron
- T is the temperature of the gas
- h is Planck's constant

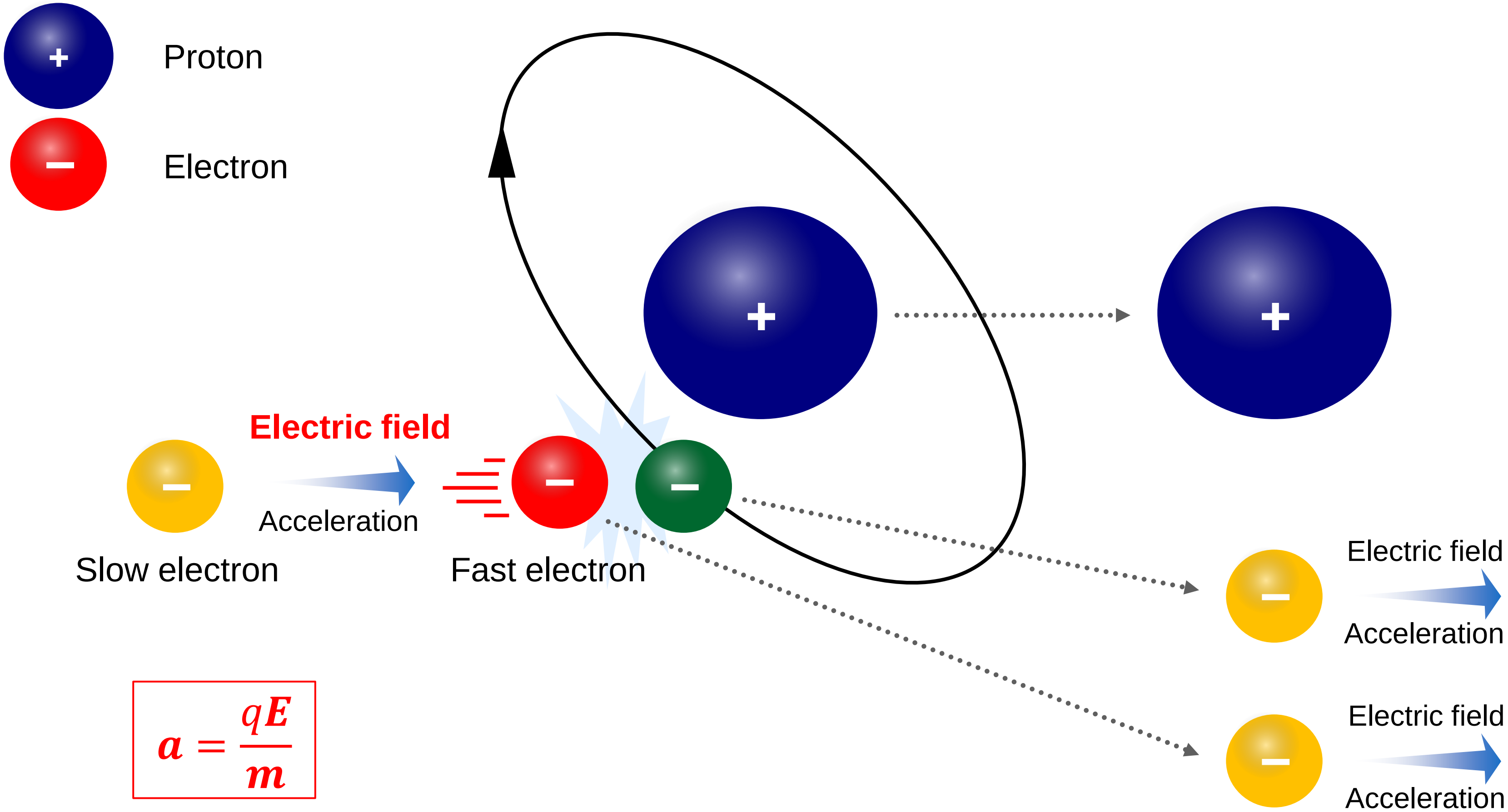
The expression $(\epsilon_{i+1}-\epsilon_i)$ is the energy required to remove the $(i+1)^{th}$ electron. In the case where only one level of ionization is important, we have $n_1 = n_e$ and defining the total density *n* as $n = n_0 + n_1$, the Saha equation simplifies to:

$$\frac{n_e^2}{n-n_e} = \frac{2}{\lambda^3}\frac{g_1}{g_0}\exp\left[\frac{-\epsilon}{k_B T}\right]$$

where ϵ is the energy of ionization.

Generation of charged particles: electron impact ionization

Ionization energy of hydrogen: 13.6 eV (~150,000K)



$$\boxed{a = \frac{qE}{m}}$$

- Electrons are easily accelerated by electric field due to their smaller mass than ions.

Classification of plasmas

플라즈마 온도

- 플라즈마는 기체와 달리 여러 온도가 존재
- T_e (전자온도) , T_i (이온온도) , T_g (중성입자온도)
- 저온 플라즈마 : T_e (>10,000℃) >> $T_i \approx T_g$ (~100℃) → 비평형 플라즈마 (Non-equilibrium plasma)
- 고온(열) 플라즈마 : $T_e \approx T_i \approx T_g$ (>10,000℃) → 평형 플라즈마 (Equilibrium plasma)

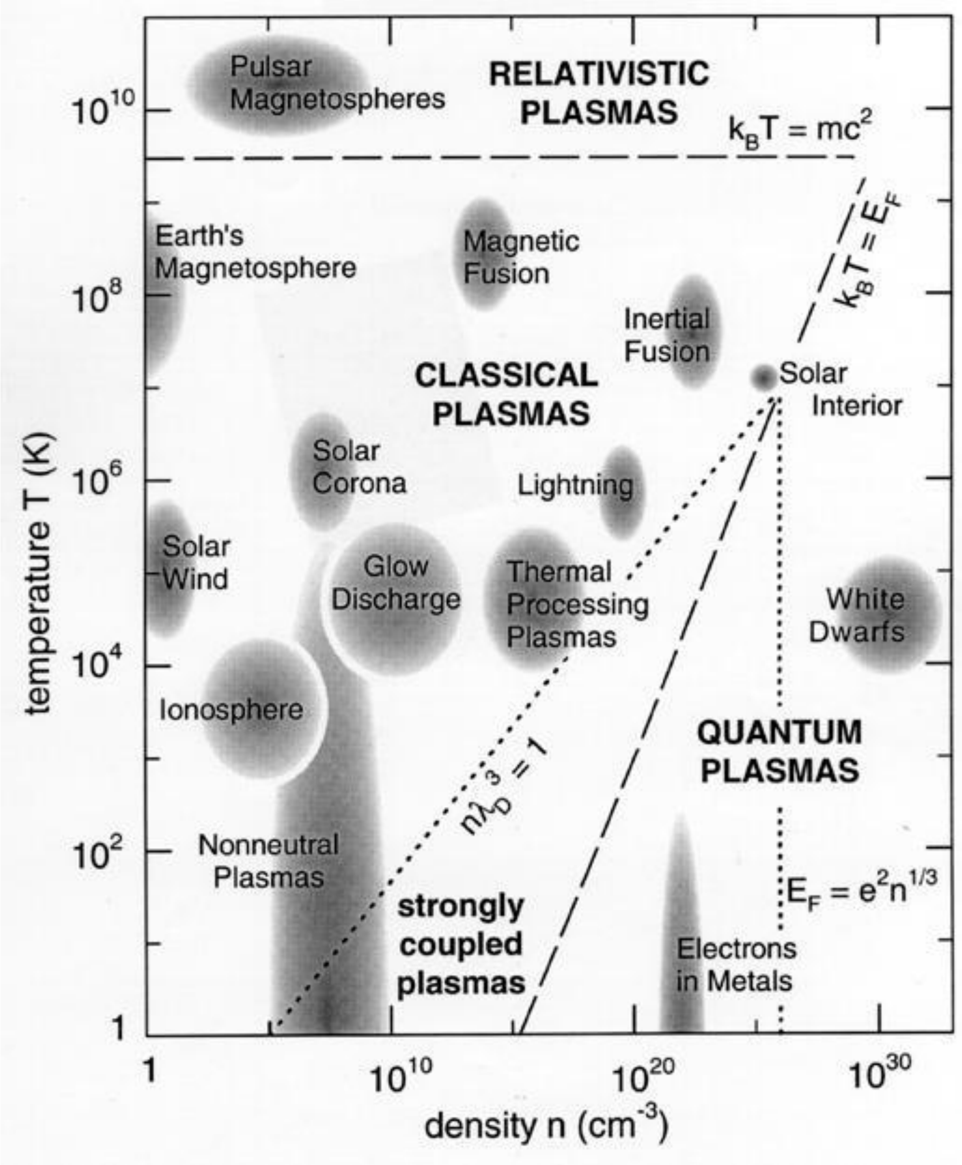
플라즈마 밀도

- 플라즈마는 여러 밀도가 존재
- n_e (전자밀도) , n_i (이온밀도) , n_g (중성입자밀도)
- 저온 플라즈마 : $n_g >> n_i \approx n_e$ → 이온화 정도가 작고 대부분 기체상태(중성)로 존재
- 고온(열) 플라즈마 : $n_g \approx n_i \approx n_e$ → 이온화 정도가 큼

얼마나 뜨거운가?

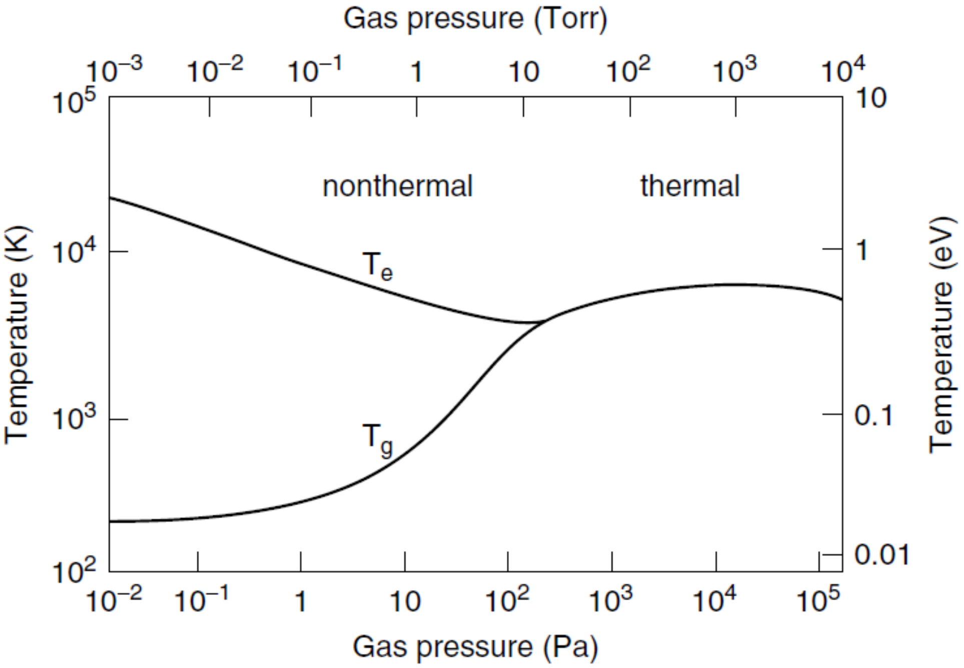
- 저온 플라즈마 : 전자의 온도는 높지만 밀도가 낮고 대부분 저온의 중성입자이므로 뜨겁지 않음.
- 고온(열) 플라즈마 : 전자, 이온, 중성입자 모두 온도가 높아 뜨거움.

Classification of plasmas



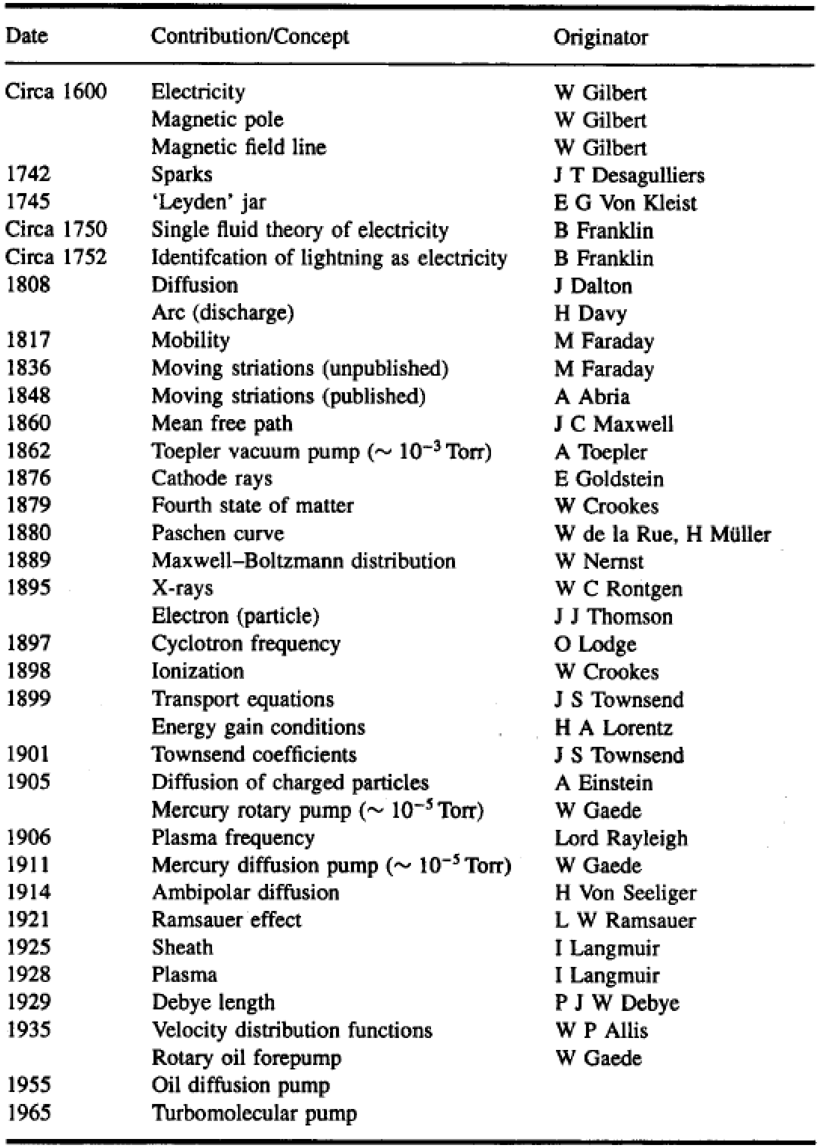
Classification of plasmas

Low-temperature thermal cold plasmas	Low-temperature non-thermal cold plasmas	High-temperature hot plasmas
$T_e \approx T_i \approx T < 2 \times 10^4$ K	$T_i \approx T \approx 300$ K $T_i \ll T_e \leqslant 10^5$ K	$T_i \approx T_e > 10^6$ K
Arcs at 100 kPa	Low pressure ~100 Pa glow and arc	Kinetic plasmas, fusion plasmas



History of plasmas

Irving Langmuir
(1881-1957)
Nobel prize (1932)

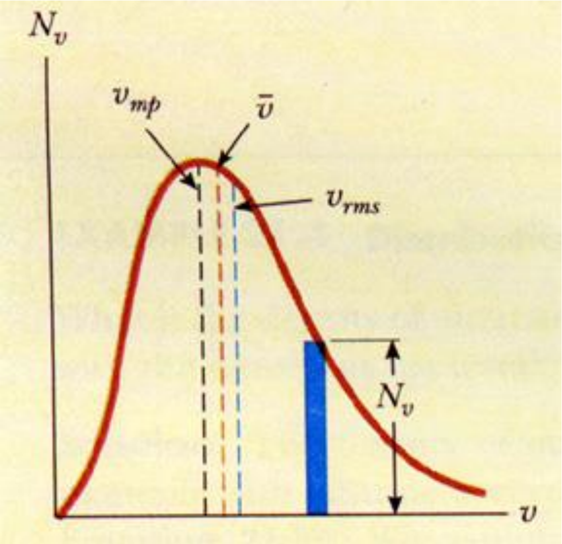


Date	Contribution/Concept	Originator
Circa 1600	Electricity	W Gilbert
	Magnetic pole	W Gilbert
	Magnetic field line	W Gilbert
1742	Sparks	J T Desagulliers
1745	'Leyden' jar	E G Von Kleist
Circa 1750	Single fluid theory of electricity	B Franklin
Circa 1752	Identifcation of lightning as electricity	B Franklin
1808	Diffusion	J Dalton
	Arc (discharge)	H Davy
1817	Mobility	M Faraday
1836	Moving striations (unpublished)	M Faraday
1848	Moving striations (published)	A Abria
1860	Mean free path	J C Maxwell
1862	Toepler vacuum pump ($\sim 10^{-3}$ Torr)	A Toepler
1876	Cathode rays	E Goldstein
1879	Fourth state of matter	W Crookes
1880	Paschen curve	W de la Rue, H Müller
1889	Maxwell–Boltzmann distribution	W Nernst
1895	X-rays	W C Rontgen
	Electron (particle)	J J Thomson
1897	Cyclotron frequency	O Lodge
1898	Ionization	W Crookes
1899	Transport equations	J S Townsend
	Energy gain conditions	H A Lorentz
1901	Townsend coefficients	J S Townsend
1905	Diffusion of charged particles	A Einstein
	Mercury rotary pump ($\sim 10^{-5}$ Torr)	W Gaede
1906	Plasma frequency	Lord Rayleigh
1911	Mercury diffusion pump ($\sim 10^{-5}$ Torr)	W Gaede
1914	Ambipolar diffusion	H Von Seeliger
1921	Ramsauer effect	L W Ramsauer
1925	Sheath	I Langmuir
1928	Plasma	I Langmuir
1929	Debye length	P J W Debye
1935	Velocity distribution functions	W P Allis
	Rotary oil forepump	W Gaede
1955	Oil diffusion pump	
1965	Turbomolecular pump	

Concept of temperature

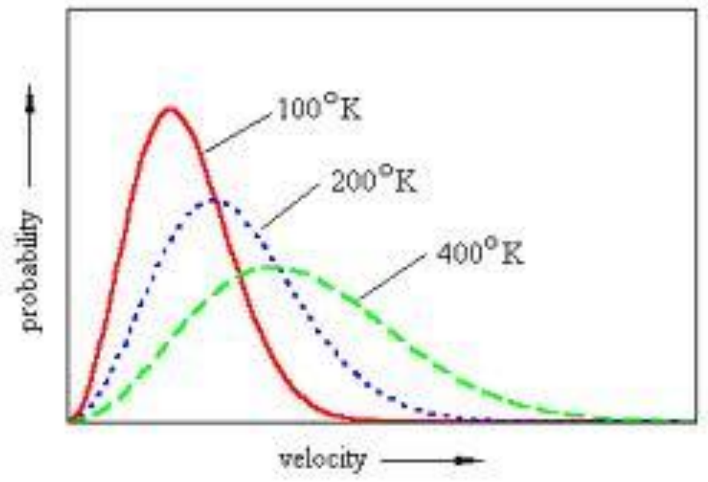
- Maxwell-Boltzmann distribution describes particle speeds in gases, where the particles do not constantly interact with each other but move freely between short collisions.

$$f(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$



- Temperature: average kinetic energy at thermal equilibrium (K)

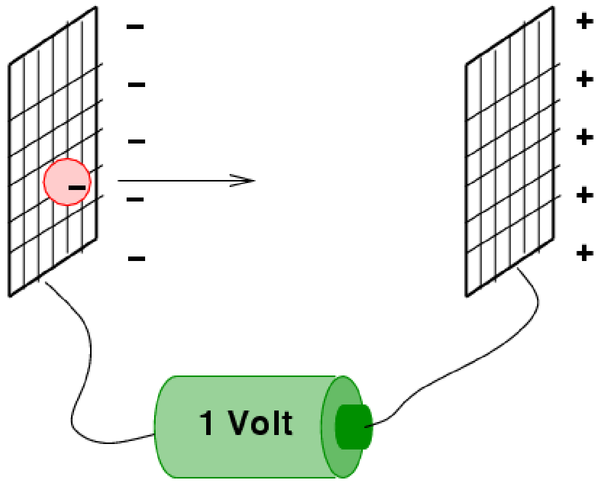
$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$



- Most probable speed $v_p = \sqrt{\frac{2kT}{m}}$
- Mean speed $\langle v \rangle = \sqrt{\frac{8kT}{\pi m}}$
- RMS speed $v_{rms} = \sqrt{\frac{3kT}{m}}$

Electron-volt unit

- The electron-volt (eV) unit is widely used for presenting the plasma temperature instead of Kelvin.



- Unit conversion between K and eV

$$1\ \text{eV} = (1.6\text{x}10^{-19}\ \text{C})\ \text{x}\ (1\ \text{V}) = (1.6\text{x}10^{-19}\ \text{J}) / (1.38\text{x}10^{-23}\ \text{J/K})$$

$$= 11{,}600\text{K}$$

- Fluorescent lamp: a few eV
  - ➢ Too hot? How can we touch such a hot plasma?
- What is the difference between energy and temperature?

Boltzmann constant k

- The Boltzmann constant is a bridge between macroscopic and microscopic physics.

$$PV = nRT \Longrightarrow PV = NkT$$

R (gas constant) = 8.31 JK^{-1}mol^{-1}

k (Boltzmann constant) = 1.38x10^{-23} JK^{-1}

$$\boxed{k = \frac{R}{N_A}}$$

- Entropy

$$S = k \ln W$$

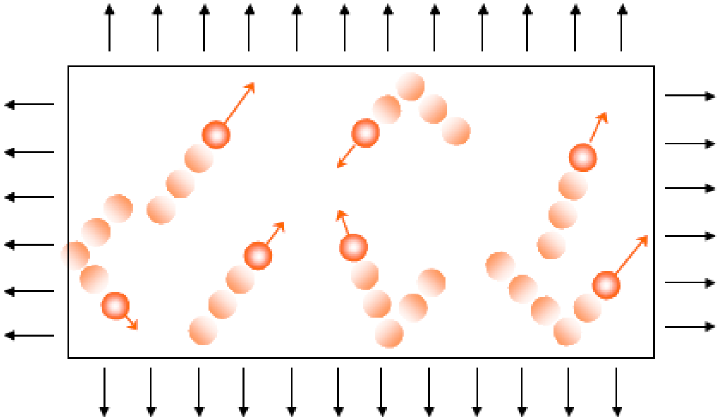
Macroscopic state

No. of microscopic states

Pressure

- Pressure = the force per unit area applied in a direction perpendicular to the surface of an object (energy density).

$$p = nkT$$

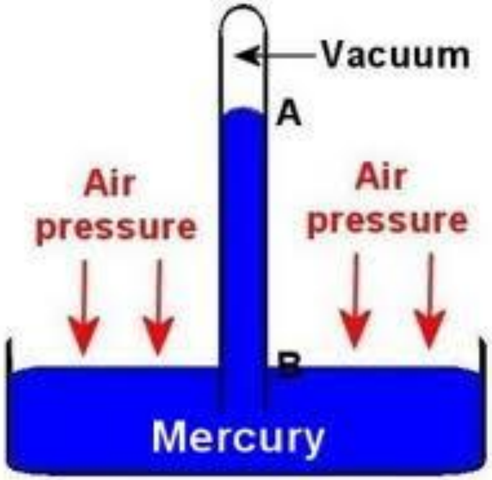


- Units
  - mmHg (millimeter of Hg, Torr)

    1 atm = 760 mmHg = 760 Torr
  - Pa (SI) = N/m^2

    1 atm = 101,325 Pa = 1,013.25 hPa

    1 bar = 10^5 Pa
  - 1 Pa = 7.5 mTorr



- Loschmidt number: No. of particles at 0°C and 1 atm: 2.7×10^{25} m^{-3}

Vacuum

- A volume of space that is essentially empty of matter, such that its gaseous pressure is much less than atmospheric pressure.

Pressure ranges of each quality of vacuum in different units

Vacuum quality	Torr	Pa	Atmosphere
Atmospheric pressure	760	1.013×10^{5}	1
Low vacuum	760 to 25	1×10^{5} to 3×10^{3}	9.87×10^{-1} to 3×10^{-2}
Medium vacuum	25 to 1×10^{-3}	3×10^{3} to 1×10^{-1}	3×10^{-2} to 9.87×10^{-7}
High vacuum	1×10^{-3} to 1×10^{-9}	1×10^{-1} to 1×10^{-7}	9.87×10^{-7} to 9.87×10^{-13}
Ultra high vacuum	1×10^{-9} to 1×10^{-12}	1×10^{-7} to 1×10^{-10}	9.87×10^{-13} to 9.87×10^{-16}
Extremely high vacuum	$< 1 \times 10^{-12}$	$< 1 \times 10^{-10}$	$< 9.87 \times 10^{-16}$
Outer space	1×10^{-6} to $< 1 \times 10^{-17}$	1×10^{-4} to $< 3 \times 10^{-15}$	9.87×10^{-10} to $< 2.96 \times 10^{-20}$
Perfect vacuum	0	0	0

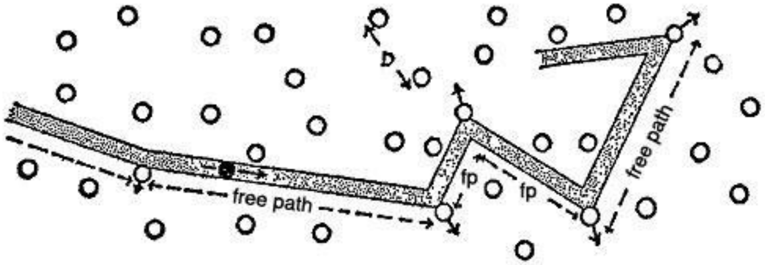
Turbomolecular pump (TMP)

- No. of particles

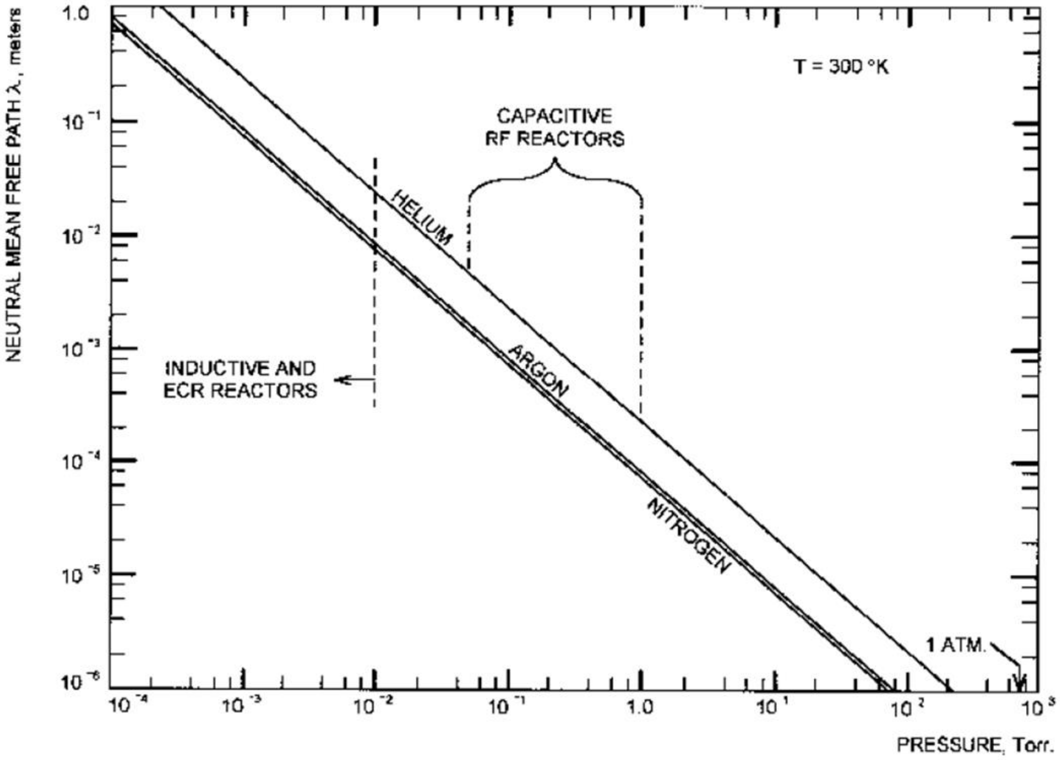
$$n = p/kT$$

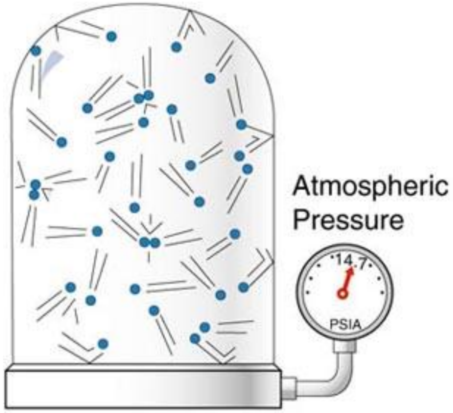
- Calculate the number of particles at 1 mTorr.

Neutral mean free path

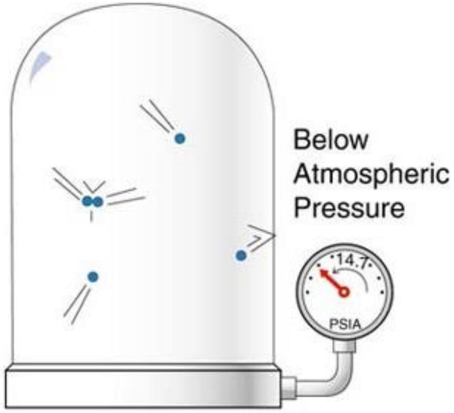


Mean free path of a gas molecule





Short Mean Free Path
(Atmospheric Pressure)



Long Mean Free Path
(Low Pressure)

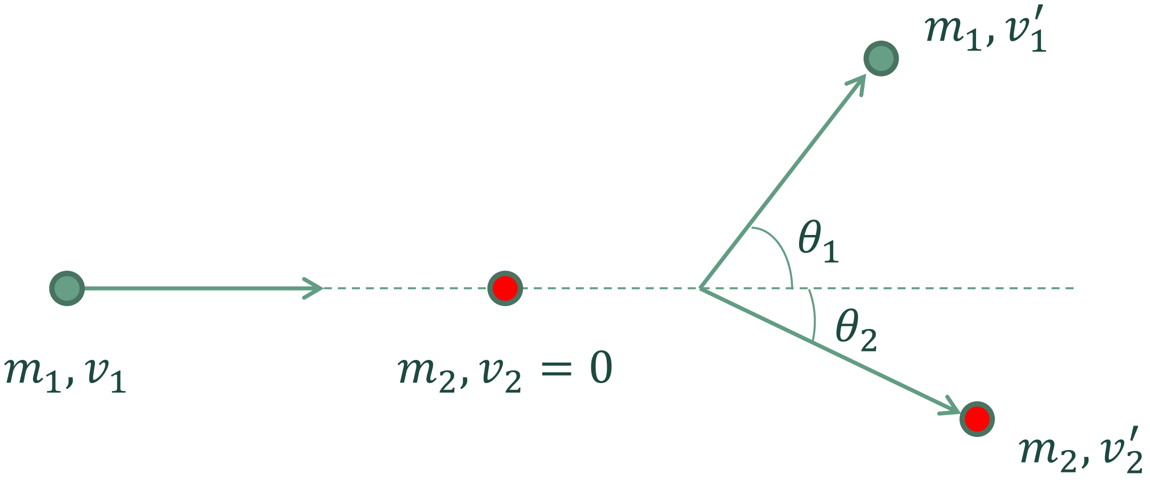
Collision

- Collisions conserve momentum and energy: the total momentum and energy of the colliding particles after collision are equal to that before collision.
- Electrons and fully stripped ions possess only kinetic energy. Atoms and partially stripped ions have internal energy level structures and can be excited, de-excited, or ionized, corresponding to changes in potential energy.
- It is the total energy, which is the sum of the kinetic and potential energy, that is conserved in a collision.

- Elastic: the sum of kinetic energies of the collision partners are conserved.
- Inelastic: the sum of kinetic energies are not conserved. ionization and excitation. the sum of kinetic energies after collision is less than that before collision.
- Super-elastic: the sum of kinetic energies are increased after collision. de-excitation.

Elastic collision

- Conservation of momentum and energy



- Energy transfer rate

$$\zeta_L = \frac{\frac{1}{2}m_2 v_2'^2}{\frac{1}{2}m_1 v_1^2} = \frac{4m_1 m_2}{(m_1+m_2)^2}\cos^2\theta_2$$

Energy transfer rate by a single collision

$$\overline{\zeta_L} = \frac{4m_1 m_2}{(m_1+m_2)^2}\overline{\cos^2\theta_2} = \frac{2m_1 m_2}{(m_1+m_2)^2}$$

Average energy transfer rate by many collisions

Average energy transfer by collisions

- $m_1 = m_2$ (electron-electron, ion-ion, neutral-neutral, ion-neutral)

$$\overline{\zeta_L} = \frac{2m_1 m_2}{(m_1+m_2)^2} = \frac{1}{2}$$

⇨ Effective energy transfer (quick thermalization)

- $m_1 \ll m_2$ (electron-ion, electron-neutral)

$$\overline{\zeta_L} = \frac{2m_1 m_2}{(m_1+m_2)^2} \approx \frac{2m_1}{m_2} \approx 10^{-5} \sim 10^{-4}$$

⇨ Hard to be thermalized

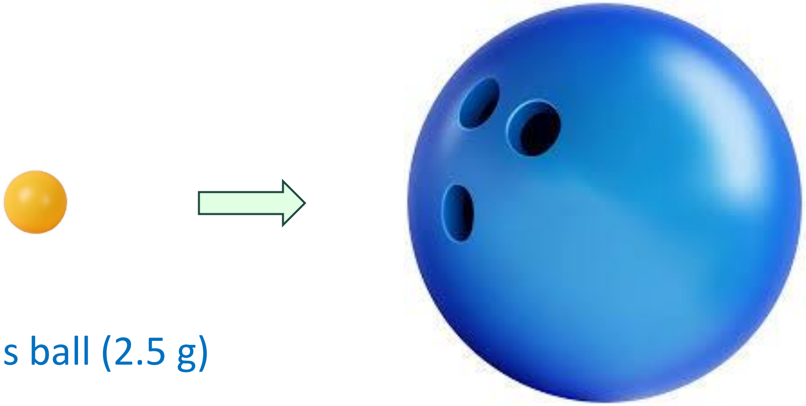


Table tennis ball (2.5 g)

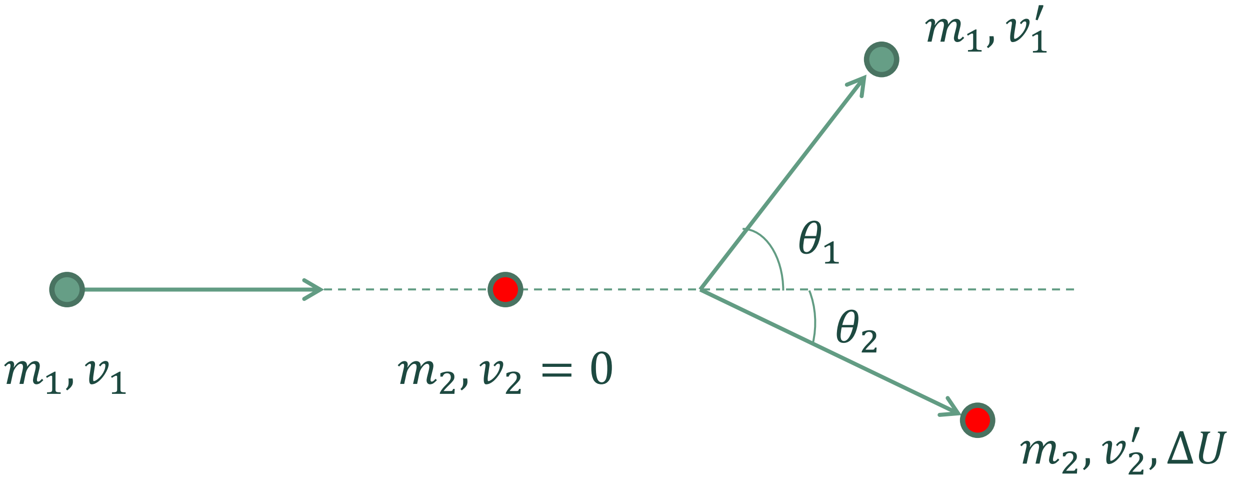
Bowling ball (10 lb.)

- Therefore, in weakly ionized plasma

$$\boxed{T_e \gg T_i \approx T_n}$$ **: Non-equilibrium**

Inelastic collision

- The sum of kinetic energies are not conserved. Where is the lost energy?
  - ➢ transferred to internal energy
    - ✓ Atomic gases : electronic transition
    - ✓ Molecules : excitation of rotational and vibrational states



- Energy transfer rate to internal energy ΔU

$$\zeta_L = \frac{\Delta U_{max}}{\frac{1}{2}m_1 v_1^2} = \frac{m_2}{m_1 + m_2}\cos^2\theta_2$$

✓ Hint: $\frac{\Delta U}{dv_2'} = 0 \ \ at \ \ \Delta U_{max}$

Average energy transfer to internal energy by collisions

- $m_1 = m_2$ (electron-electron, ion-ion, neutral-neutral, ion-neutral)

$$\bar{\zeta}_L = \frac{m_2}{2(m_1 + m_2)} = \frac{1}{4}$$

- $m_1 \gg m_2$ (ion-electron, neutral-electron)

$$\bar{\zeta}_L = \frac{m_2}{2(m_1 + m_2)} \approx \frac{m_2}{2m_1} \approx 10^{-5} \sim 10^{-4}$$

- $m_1 \ll m_2$ (electron-ion, electron-neutral)

$$\bar{\zeta}_L = \frac{m_2}{2(m_1 + m_2)} \approx \frac{1}{2}$$

- Ionization (plasma generation)
- Excitation (light emission)
- Dissociation (radical production)

Various inelastic collisions in plasmas

- Photon-induced reactions
  - photo-excitation : $h\nu + N = N^*$
  - photo-ionization : $h\nu + N = N^+ + e$

- Electron-induced reactions
  - electron impact excitation : $e + N = N^* + e$
  - electron impact ionization : $e + N = N^+ + 2e$
  - electron impact dissociation : $e + N_2 = 2N + e$
  - super-elastic collision : $N^{**} + e = N + e$ (fast)
  - radiative recombination : $N^+ + e = h\nu + N$
  - dissociative recombination : $e + N_2{}^+ = 2N$

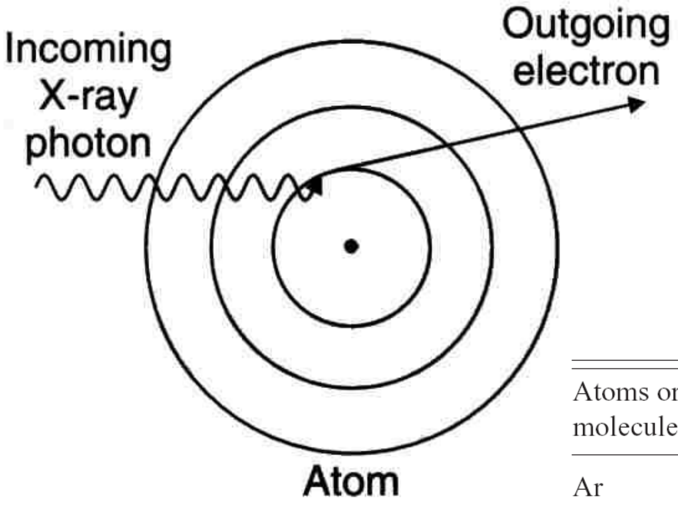
- Ion-induced reactions
  - charge exchange : $N^+(1) + N(2) = N(1) + N^+(2)$

Photon-induced reactions

- Photo-ionization

$$\boxed{h\nu + A \rightarrow e + A^{+}}$$

The physical process in which an incident photon ejects one or more electrons from an atom, ion or molecule. This is essentially the same process that occurs with the photoelectric effect with metals. To provide the ionization, the photo wavelength should be usually less than 1000 Å, which is ultraviolet radiation.



$$\lambda < \frac{12{,}400}{I(eV)} \text{Å}$$

$$K.E. = h\nu - I$$

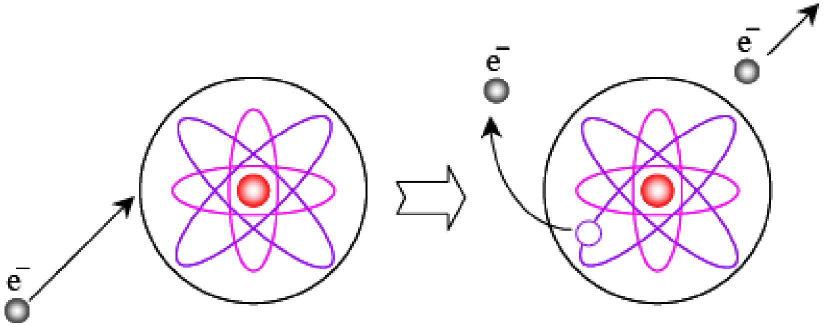
Atoms or molecules	Wavelength λ, Å	Cross sections, cm^2	Atoms or molecules	Wavelength λ, Å	Cross sections, cm^2
Ar	787	$3.5 \cdot 10^{-17}$	Ne	575	$0.4 \cdot 10^{-17}$
N_2	798	$2.6 \cdot 10^{-17}$	O	910	$0.3 \cdot 10^{-17}$
N	482	$0.9 \cdot 10^{-17}$	O_2	1020	$0.1 \cdot 10^{-17}$
He	504	$0.7 \cdot 10^{-17}$	Cs	3185	$2.2 \cdot 10^{-19}$
H_2	805	$0.7 \cdot 10^{-17}$	Na	2412	$1.2 \cdot 10^{-19}$
H	912	$0.6 \cdot 10^{-17}$	K	2860	$1.2 \cdot 10^{-20}$

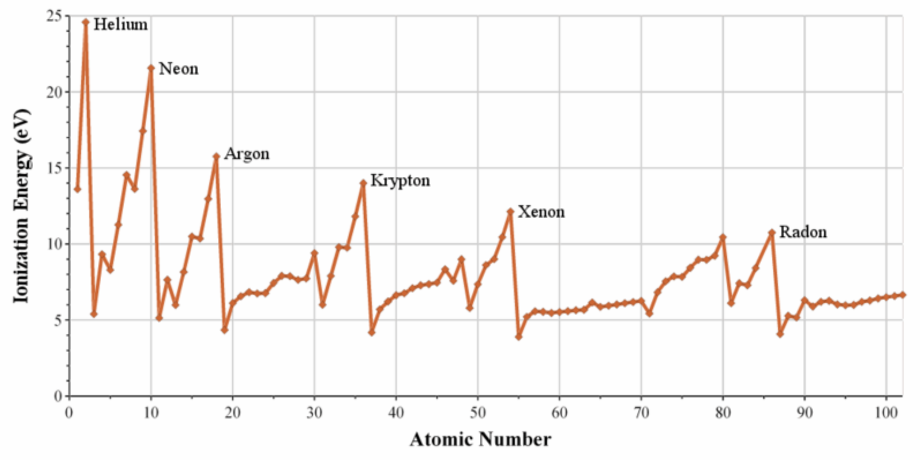
Electron-induced reactions

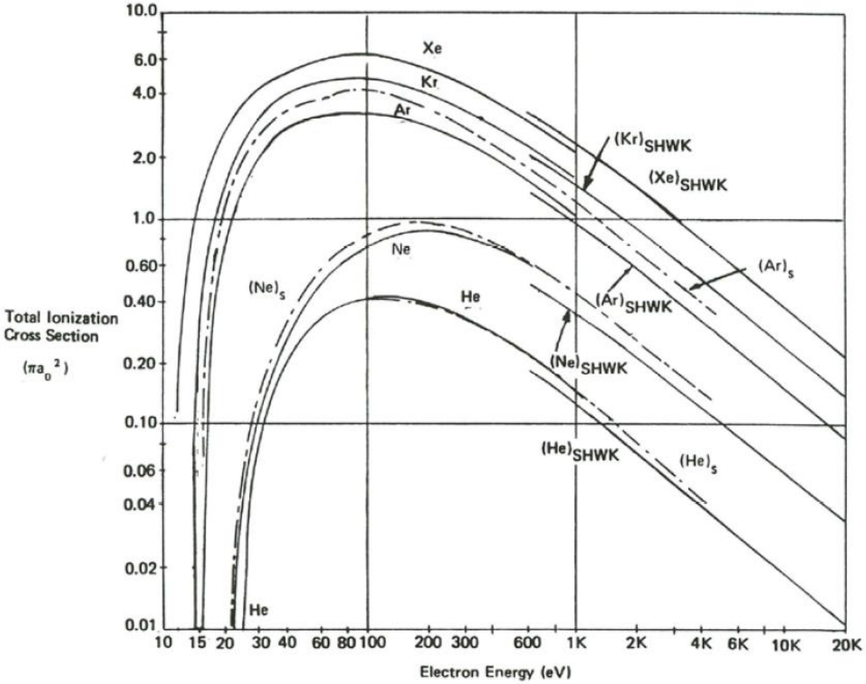
- Electron impact ionization

$$\boxed{e + A \rightarrow 2e + A^{+}}$$

Electrons with sufficient energy (> 10 eV) can remove an electron from an atom and produce one extra electron and an ion.







Electron-induced reactions

- Electron impact excitation

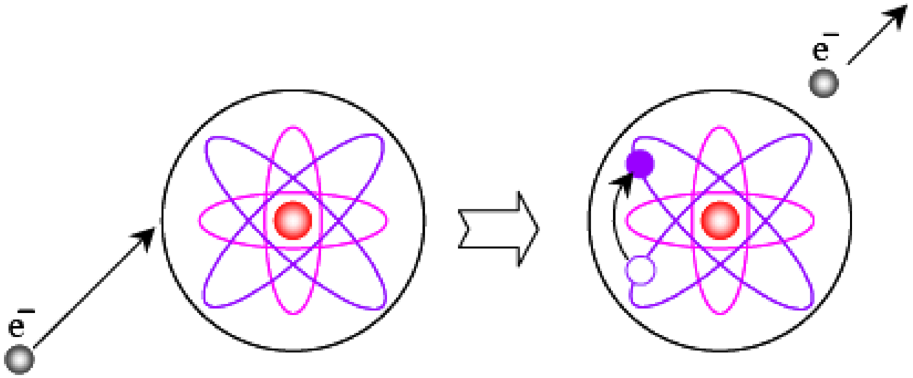
$$e + A \rightarrow e + A^*$$

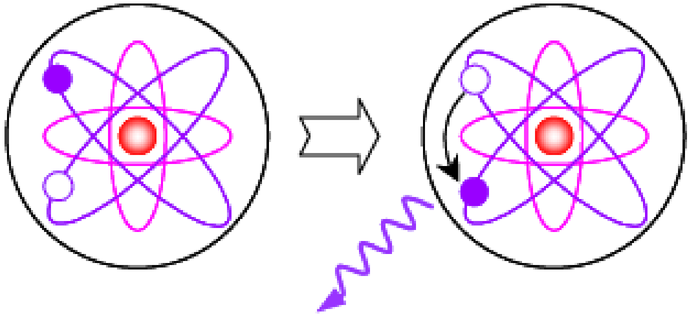
Electrons with sufficient energy can also excite the electrons of an atom from the lower energy level to a higher energy level.

- De-excitation

$$A^* \rightarrow A + h\nu$$

The excited states of atoms are usually unstable and the electron configuration can soon return to its original ground sate, accompanied by the emission of a photon with a specific energy that equals the energy difference between the two quantum levels.



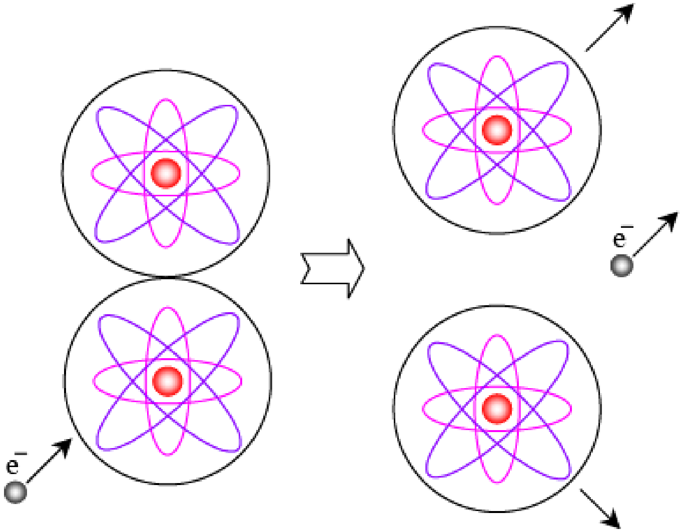


Electron-induced reactions

- Electron impact dissociation

$$\boxed{e + A_2 \rightarrow e + 2A}$$

Most responsible for the production of chemically active radicals in most of the plasmas.



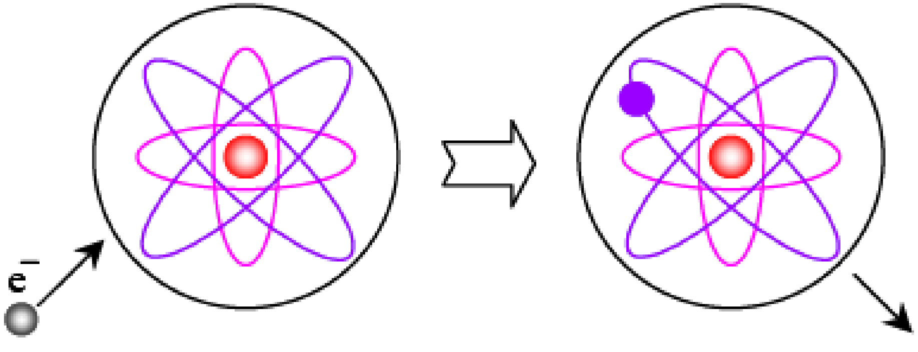
- Radical generation reactions
  - $e + O_2 \rightarrow 2O + e$ (Chemically active O radical generation)
  - $e + CF_4 \rightarrow 2e + CF_3^+ + F$ (Dissociative ionization)

Electron-induced reactions

- Electron attachment

$$\boxed{e + A \rightarrow A^-}$$

Electron can attach to an electronegative atom to form a negative ion, for example, a halogen atom or an oxygen atom (electron-capture ionization).



- Negative ion generation
  - $e + SF_6 \rightarrow SF_6^-$ (Electronegative plasma, electron loss)
  - $e + SF_6 \rightarrow SF_5^- + F$ (Dissociative attachment, negative ion, radical)

Recombination processes

- Radiative recombination

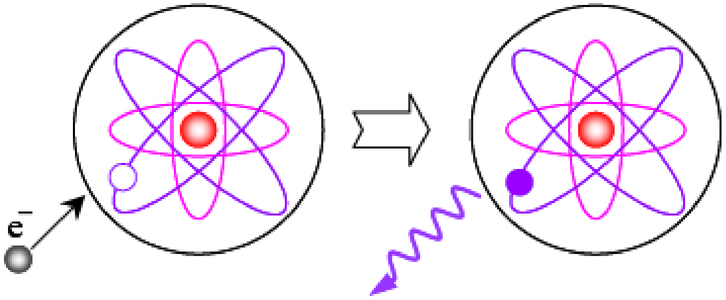
$$\boxed{e + A^{+} \rightarrow A + h\nu}$$

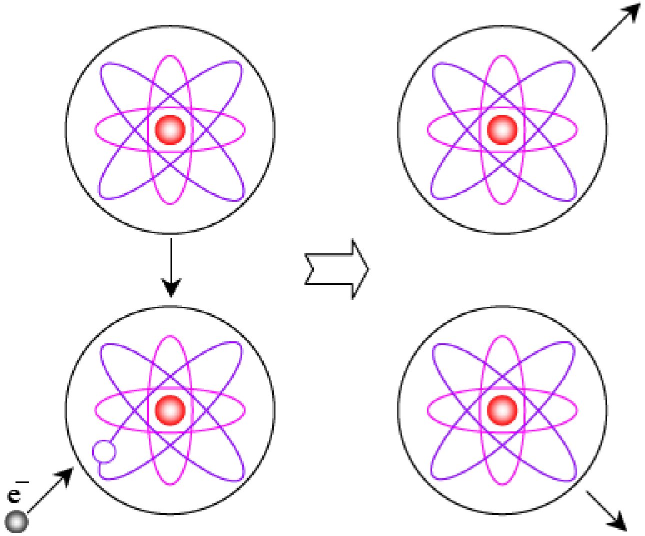
Positive ions capture a free (energetic) electrons and combine with electrons to form new neutral atoms, radiating a photon.

- Electron-ion recombination

$$\boxed{e + A^{+} + A \rightarrow A^{*} + A}$$

For electron-ion recombination, a third-body must be involved to conserve the energy and momentum. Abundant neutral species or reactor walls are ideal third-bodies. This recombination process typically results in excited neutrals.



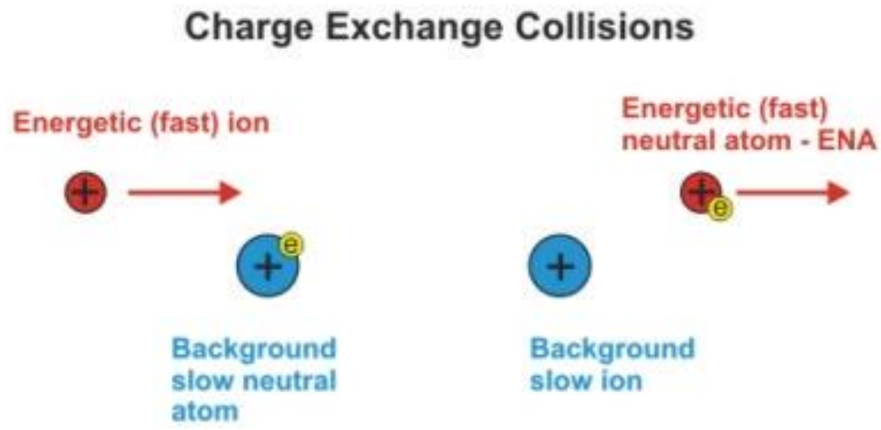


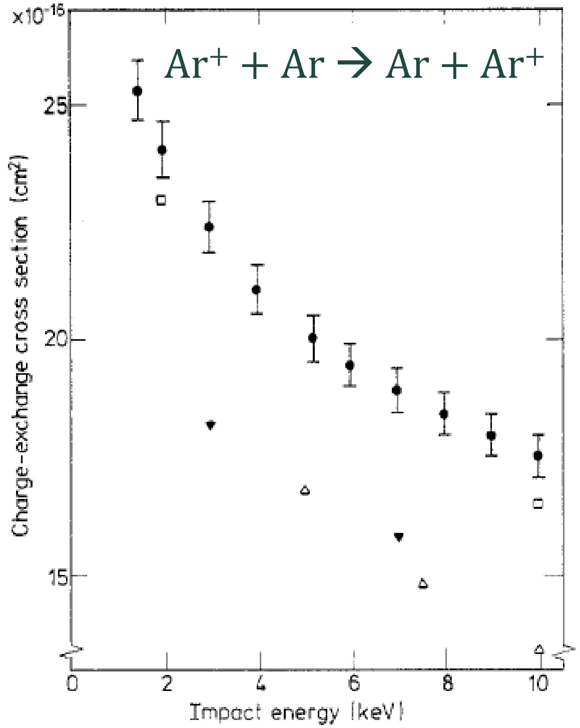
Ion-induced reactions

- (resonant) Charge exchange (or charge transfer)

The cross section for resonant charge transfer is large at low collision energies, making this an important process in weakly ionized plasmas.

$$\boxed{A^{+}(\text{fast}) + A(\text{slow}) \rightarrow A(\text{fast}) + A^{+}(\text{slow})}$$

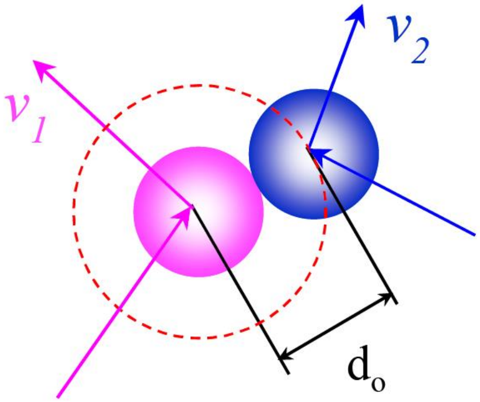




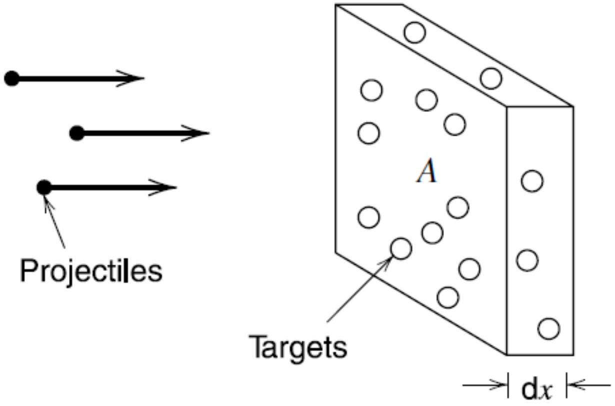
Collision parameters

- Let $\mathrm{d}n$ be the number of incident particles per unit volume at x that undergo an "interaction" with the target particles within a differential distance $\mathrm{d}x$, removing them from the incident beam. Clearly, $\mathrm{d}n$ is proportional to n, n_g, and $\mathrm{d}x$ for infrequent collisions within $\mathrm{d}x$.

$$\mathrm{d}n = -\sigma n n_g \mathrm{d}x \qquad \mathrm{d}\boldsymbol{\Gamma} = -\sigma \boldsymbol{\Gamma} n_g \mathrm{d}x$$

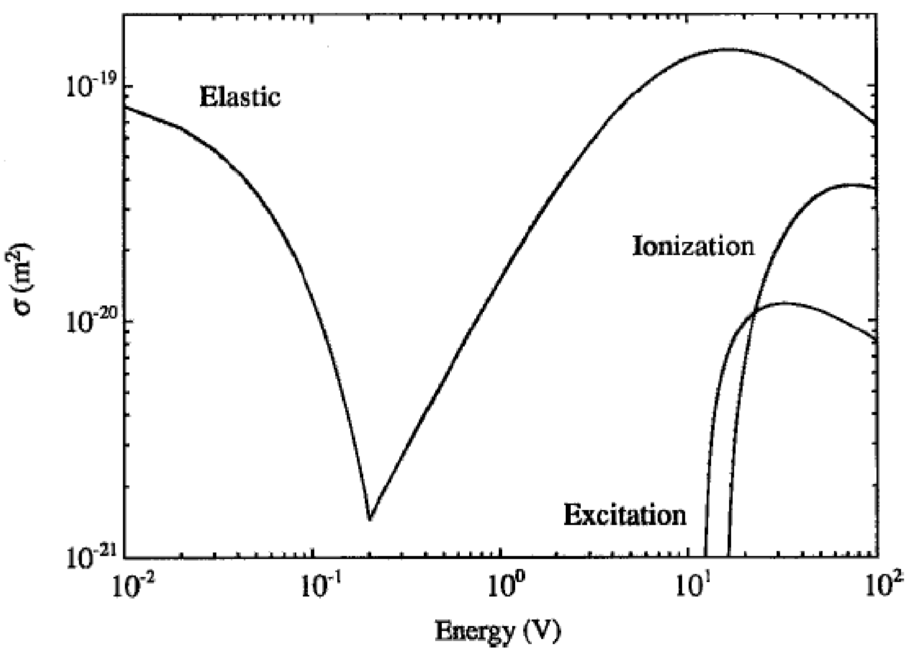


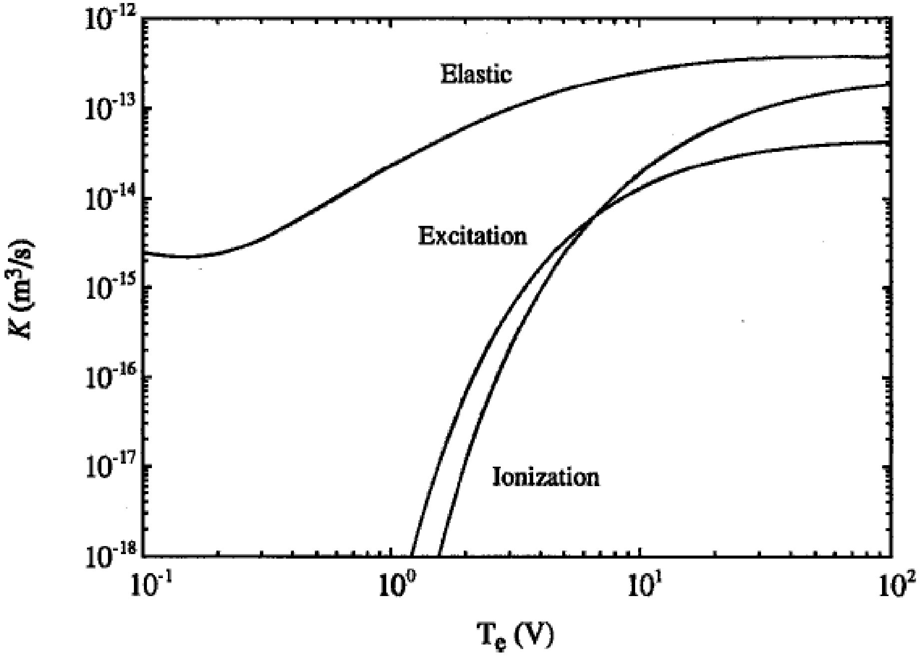
- The collided flux: $\Gamma(\mathrm{x}) = \Gamma_0\left(1 - e^{-x/\lambda}\right)$
- Mean free path: $\boxed{\lambda = \dfrac{1}{n_g \sigma}}$
- Mean collision time: $\tau = \dfrac{\lambda}{v}$
- Collision frequency: $\nu = \tau^{-1} = n_g \sigma v = n_g K$
- Rate constant: $K = \sigma v$



Reaction rate

- Cross section vs Maxwellian-averaged rate constant





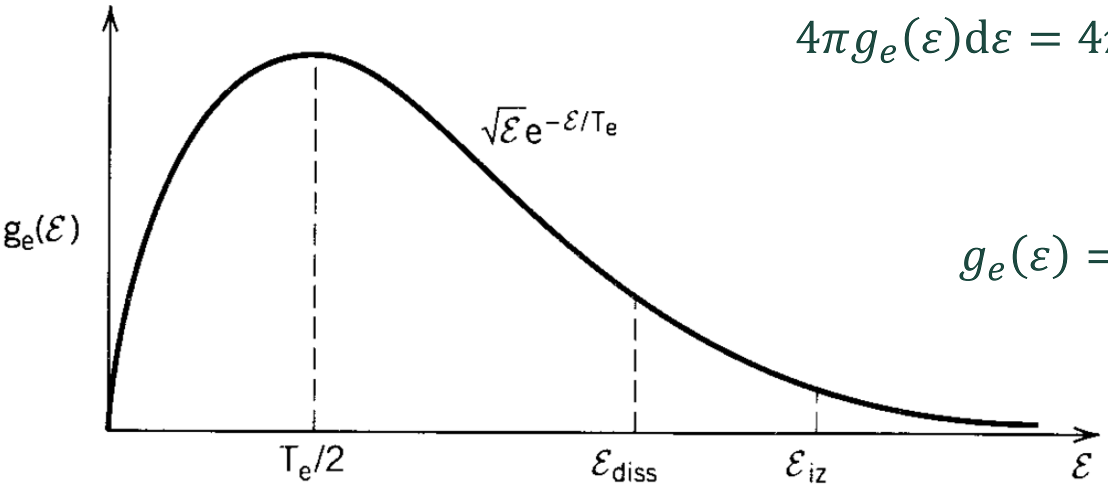
- Reaction rate [reactions per volume and second, reactions/m^{3}s]

$$R_i = n_e n_g < \sigma_i(v_e) v_e >_M = n_e n_g K$$

❖ Note that $R_i = n_g \sigma_i(v) n_e v_e = \Sigma_i \cdot \phi$ in nuclear reactor physics

EEDF (electron energy distribution function)

- It has a nearly Maxwellian energy distribution (not always)



$$4\pi g_e(\varepsilon)\mathrm{d}\varepsilon = 4\pi v^2 f_e(v)\mathrm{d}v$$

+

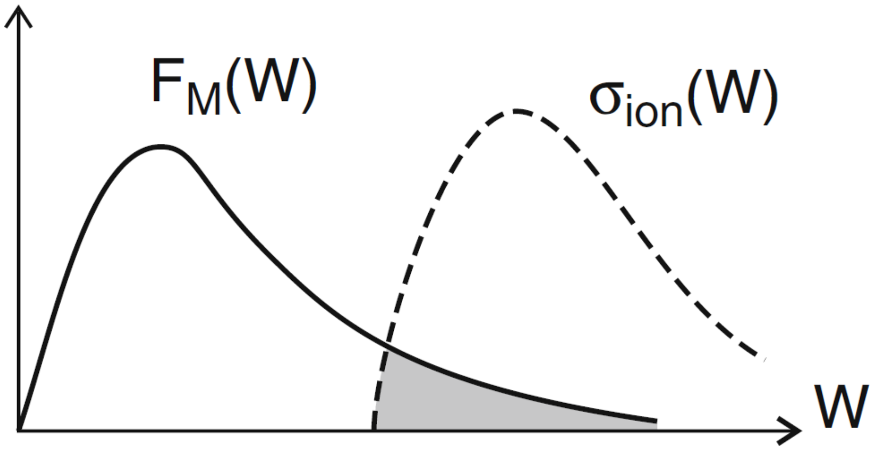
$$\varepsilon = \frac{1}{2}mv^2$$

⇩

$$g_e(\varepsilon) = \frac{2}{\sqrt{\pi}}\left(\frac{1}{kT_e}\right)^{3/2} \varepsilon^{1/2}\exp\left(-\frac{\varepsilon}{kT_e}\right)$$

- For non-Maxwellian electron distribution, the reaction rate is obtained by

$$R_i = n_e n_g \int_0^\infty f_e(v_e)\sigma_i(v_e)v_e 4\pi v_e^2 dv_e$$



Homework

- Derive the energy transfer rates for both elastic and inelastic collisions.