Fundamentals of Plasma Physics

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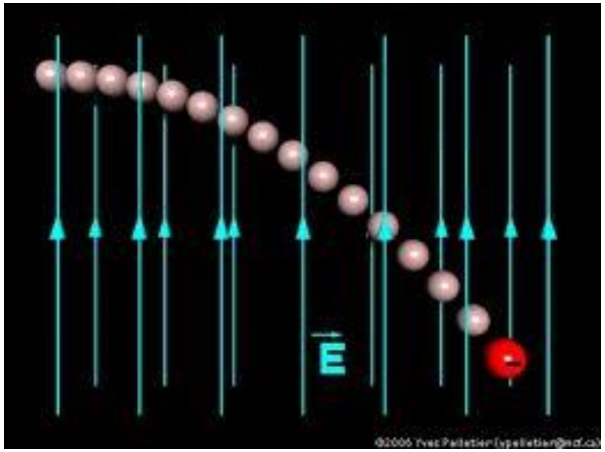
Motions of a charged particle in uniform electric field

- Equation of motion of a charged particle in fields

$$m\frac{d\boldsymbol{v}}{dt} = q[\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{v}\times\boldsymbol{B}(\boldsymbol{r},t)], \qquad \frac{d\boldsymbol{r}}{dt} = \boldsymbol{v}(t)$$

- Motion in constant electric field
  - ✓ For a constant electric field $\boldsymbol{E} = \boldsymbol{E_0}$ with $\boldsymbol{B} = 0$,

$$\boldsymbol{r}(t) = \boldsymbol{r_0} + \boldsymbol{v_0}t + \frac{q\boldsymbol{E_0}}{\boldsymbol{2}m}t^2$$



  - ✓ Electrons are easily accelerated by electric field due to their smaller mass than ions.
  - ✓ Electrons (Ions) move against (along) the electric field direction.
  - ✓ The charged particles get kinetic energies.

Motions of a charged particle in uniform magnetic field

- Motion in constant magnetic field

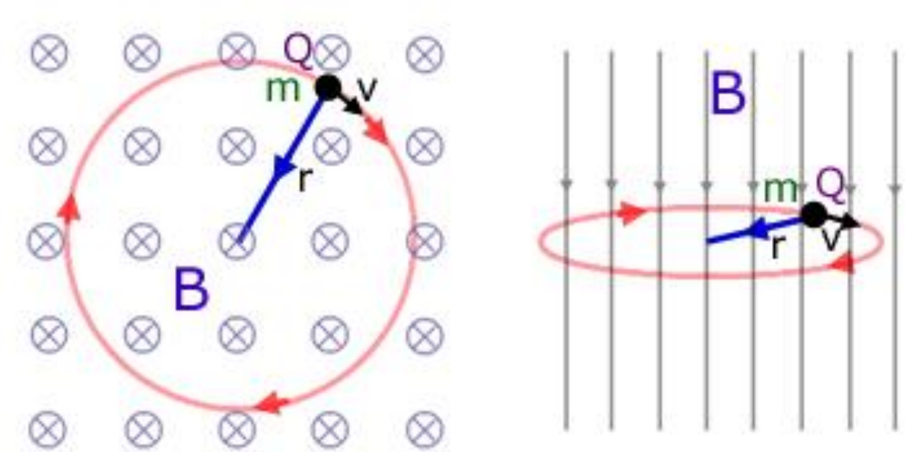
$$m\frac{d\boldsymbol{v}}{dt} = q\boldsymbol{v} \times \boldsymbol{B}$$

- For a constant magnetic field $\boldsymbol{B} = B_0\boldsymbol{z}$ with $\boldsymbol{E} = 0$,

$$m\frac{dv_x}{dt} = qB_0 v_y$$

$$m\frac{dv_y}{dt} = -qB_0 v_x$$

$$m\frac{dv_z}{dt} = 0$$



- Cyclotron (gyration) frequency

$$\frac{d^2 v_x}{dt^2} = -\omega_c^2 v_x$$

$$\boxed{\omega_c = \frac{|q|B_0}{m}}$$

Motions of a charged particle in uniform magnetic field

- Particle velocity

$$v_x = v_\perp \cos(\omega_c t + \phi_0)$$
$$v_y = -v_\perp \sin(\omega_c t + \phi_0)$$
$$v_z = v_{z0}$$

- Particle position

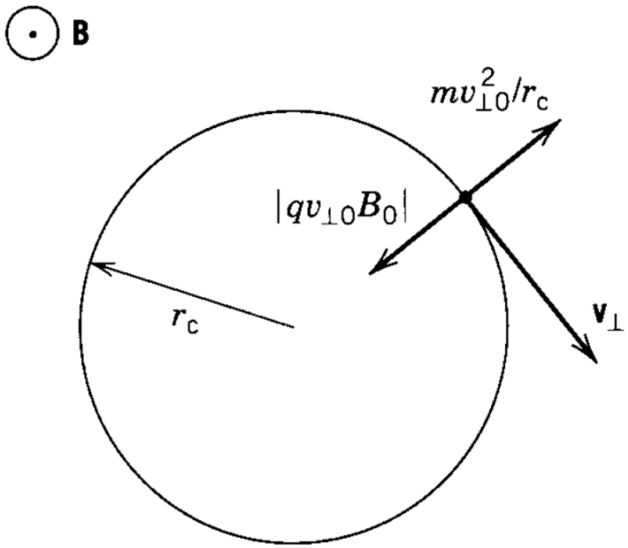
$$x = x_0 + r_c \sin(\omega_c t + \phi_0)$$
$$y = y_0 + r_c \cos(\omega_c t + \phi_0)$$
$$z = z_0 + v_{z0} t$$

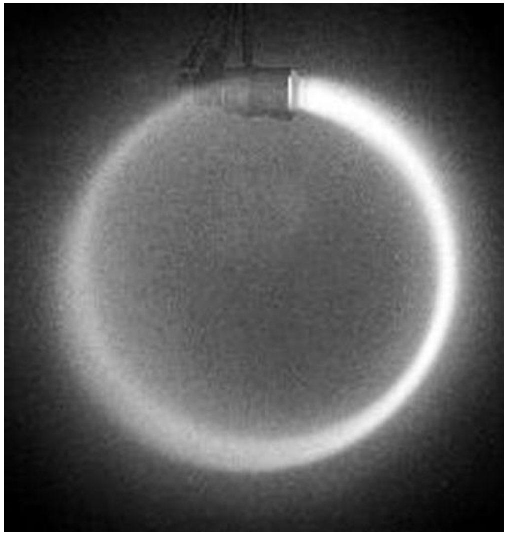
- Guiding center

$$(x_0, y_0, z_0 + v_{z0} t\,)$$

- Larmor (gyration) radius

$$\boxed{r_c = r_\mathrm{L} = \frac{v_\perp}{\omega_c} = \frac{m v_\perp}{|q| B_0}}$$





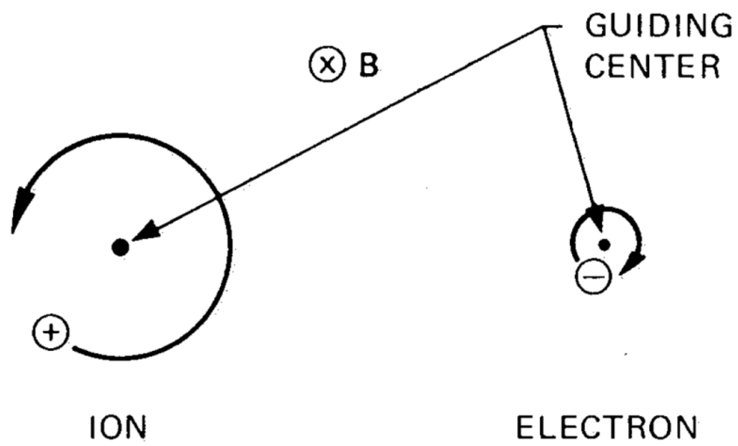
Gyro-frequency and gyro-radius

- The direction of gyration is always such that the magnetic field generated by the charged particle is opposite to the externally imposed field. → diamagnetic

- For electrons

$$f_{ce} = 2.80 \times 10^6 \, B_0 \; [\mathrm{Hz}] \quad (B_0 \text{ in gauss})$$

$$r_{ce} = \frac{3.37\sqrt{E}}{B_0} \; [\mathrm{cm}] \quad (E \text{ in volts})$$



- For singly charged ions

$$f_{ci} = 1.52 \times 10^3 \, B_0/M_A \; [\mathrm{Hz}] \quad (B_0 \text{ in gauss})$$

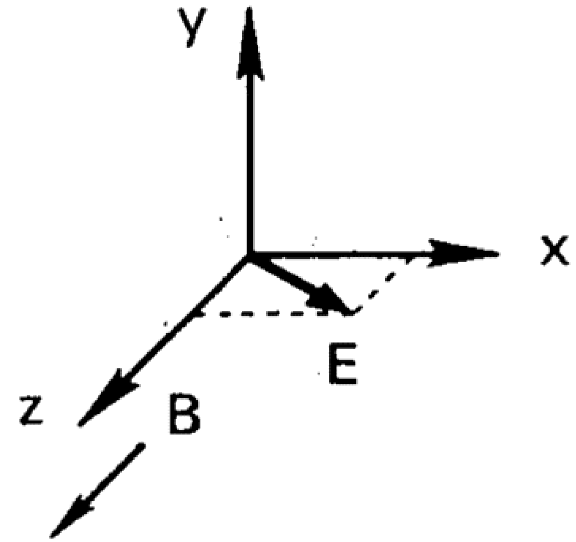
$$r_{ci} = \frac{144\sqrt{EM_A}}{B_0} \; [\mathrm{cm}] \quad (E \text{ in volts}, M_A \text{ in amu})$$

- Energy gain?

Motions of a charged particle in uniform E and B fields

- Equation of motion

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$



- Parallel motion: $\boldsymbol{B} = B_0\boldsymbol{z}$ and $\boldsymbol{E} = E_0\boldsymbol{z}$,

$$m\frac{dv_z}{dt} = qE_z$$

$$v_z = \frac{qE_z}{m}t + v_{z0}$$

→ Straightforward acceleration along B

$\boldsymbol{E}\times\boldsymbol{B}$ drift

- Transverse motion: $\boldsymbol{B} = B_0\boldsymbol{z}$ and $\boldsymbol{E} = E_0\boldsymbol{x}$,

$$m\frac{dv_x}{dt} = qE_0 + qB_0v_y$$
$$m\frac{dv_y}{dt} = -qB_0v_x$$

- Differentiating,

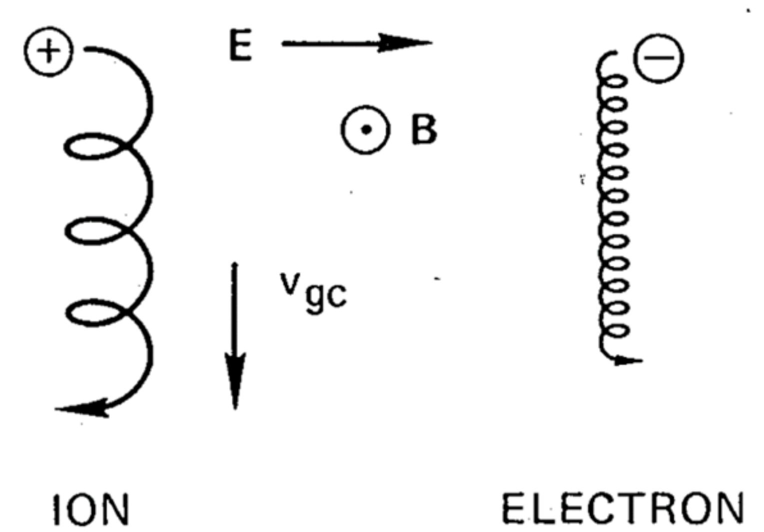
$$\frac{d^2v_x}{dt^2} = -\omega_c^2 v_x$$
$$\frac{d^2v_y}{dt^2} = -\omega_c^2\left(\frac{E_0}{B_0} + v_y\right)$$

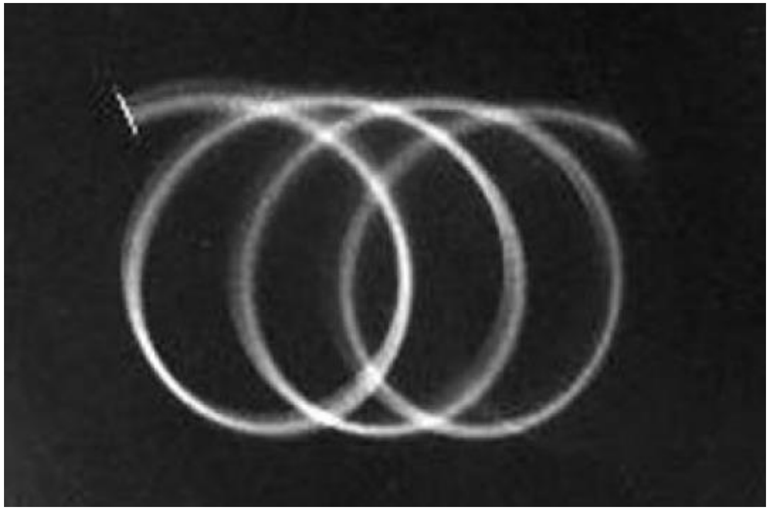
- Particle velocity

$$v_x = v_\perp\cos(\omega_c t + \phi_0)$$
$$v_y = -v_\perp\sin(\omega_c t + \phi_0) - \frac{E_0}{B_0}$$

v_{gc}

$$\boxed{\boldsymbol{v}_E = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}}$$





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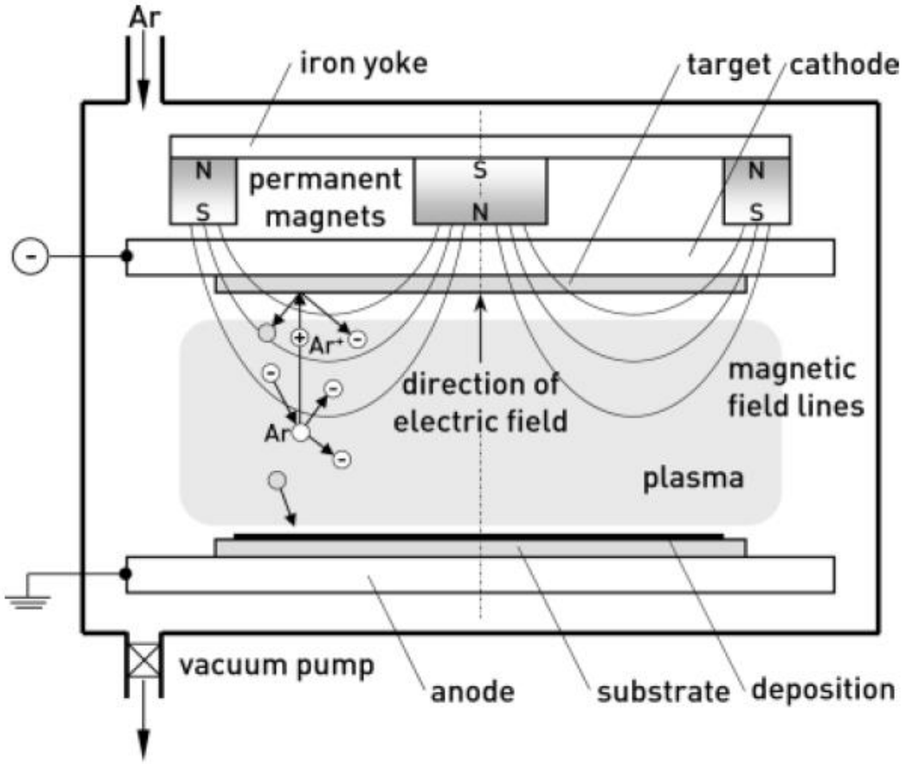
DC magnetron

- A magnetron which is widely used in the sputtering system uses the $\boldsymbol{E}\times\boldsymbol{B}$ drift motion for plasma confinement.



- What is the direction of $\boldsymbol{E}\times\boldsymbol{B}$ drift motion?

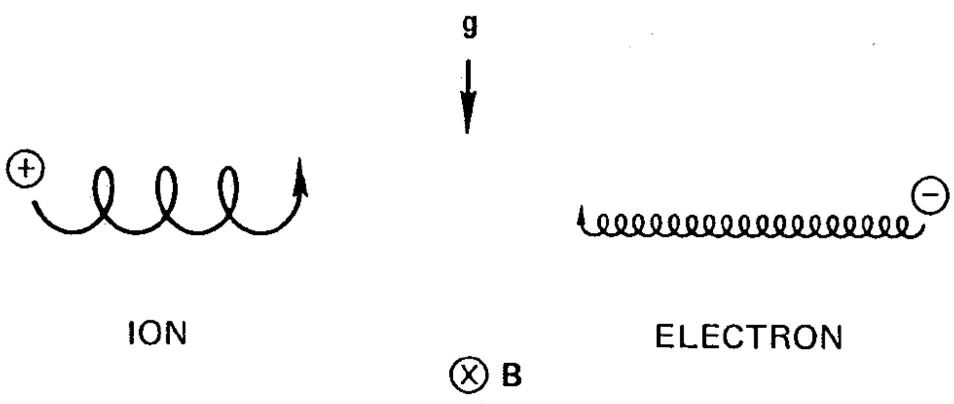
Motions of a charged particle in gravitational field

- Generally, the guiding center drift caused by general force $\boldsymbol{F}$

$$\boxed{\boldsymbol{v}_f = \frac{1}{q}\frac{\boldsymbol{F}\times\boldsymbol{B}}{B^2}}$$

- If $\boldsymbol{F}$ is the force of gravity $m\boldsymbol{g}$,

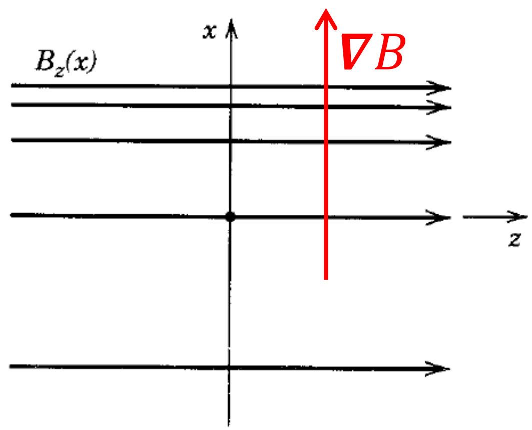
$$\boxed{\boldsymbol{v}_g = \frac{m}{q}\frac{\boldsymbol{g}\times\boldsymbol{B}}{B^2}}$$

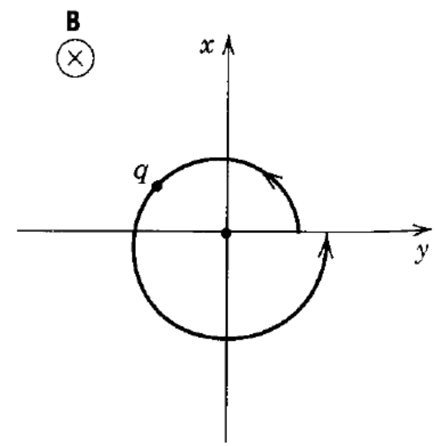


- What is the difference between $\boldsymbol{v}_E$ and $\boldsymbol{v}_g$?

$\nabla B \perp \boldsymbol{B}$: Grad-B drift

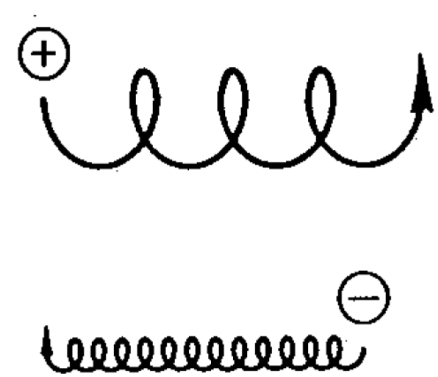
- The gradient in $|\boldsymbol{B}|$ causes the Larmor radius to be larger at the bottom of the orbit than at the top, and this should lead to a drift, in opposite directions for ions and electrons, perpendicular to both $\boldsymbol{B}$ and ∇B.





- Guiding center motion

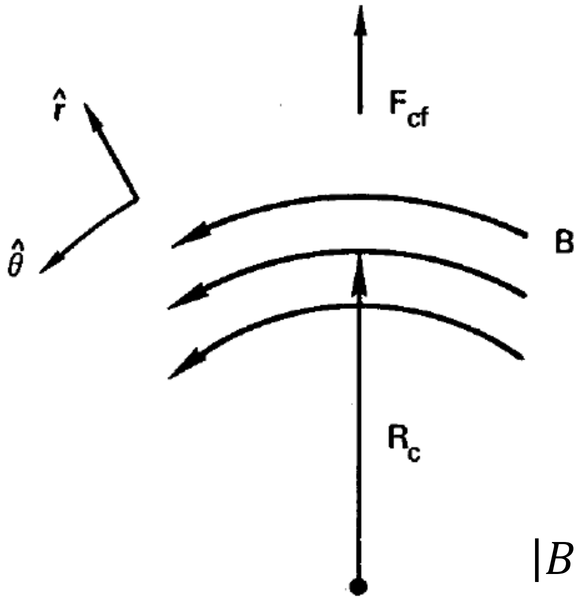
$$\boxed{\boldsymbol{v}_{\nabla B} = \pm \frac{1}{2} v_\perp r_c \frac{\boldsymbol{B} \times \nabla B}{B^2}}$$



Curved B: Curvature drift

- The average centrifugal force

$$\boldsymbol{F}_{cf} = \frac{m v_{\parallel}{}^{2}}{R_c}\hat{\boldsymbol{r}} = m v_{\parallel}{}^{2}\frac{\boldsymbol{R}_c}{R_c{}^{2}}$$



- Curvature drift

$$\boxed{\boldsymbol{v}_R = \frac{1}{q}\frac{\boldsymbol{F}_{cf}\times\boldsymbol{B}}{B^{2}} = \frac{m v_{\parallel}{}^{2}}{qB^{2}}\frac{\boldsymbol{R}_c\times\boldsymbol{B}}{R_c{}^{2}}}$$

$$|B| \propto \frac{1}{R_c}$$

- Total drift in a curved vacuum field (curvature + grad-B)

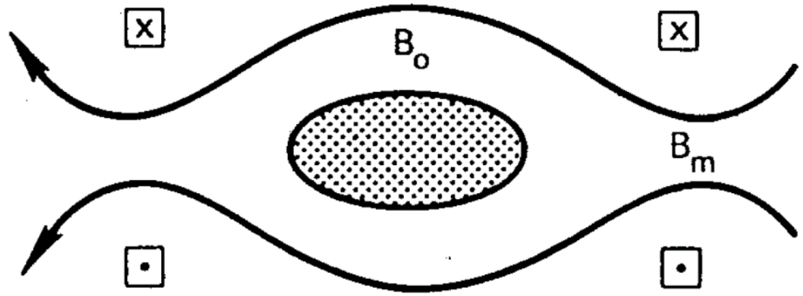
$$\boxed{\boldsymbol{v}_R + \boldsymbol{v}_{\nabla B} = \frac{m}{q}\frac{\boldsymbol{R}_c\times\boldsymbol{B}}{R_c{}^{2}B^{2}}\left(v_{\parallel}{}^{2} + \frac{1}{2}v_{\perp}{}^{2}\right)}$$

$$\frac{\boldsymbol{\nabla}|B|}{|B|} = -\frac{\boldsymbol{R}_c}{R_c{}^{2}}$$

$\nabla B \parallel \boldsymbol{B}$: Magnetic mirror

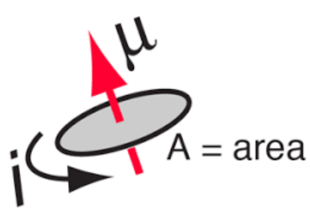
- Adiabatic invariant: Magnetic moment

$$\boxed{\mu = IA = \frac{q}{2\pi/\omega_c}\cdot \pi r_c^2 = \frac{\frac{1}{2}m{v_\perp}^2}{B}}$$



- As the particle moves into regions of stronger or weaker B, its Larmor radius changes, but μ remains invariant.

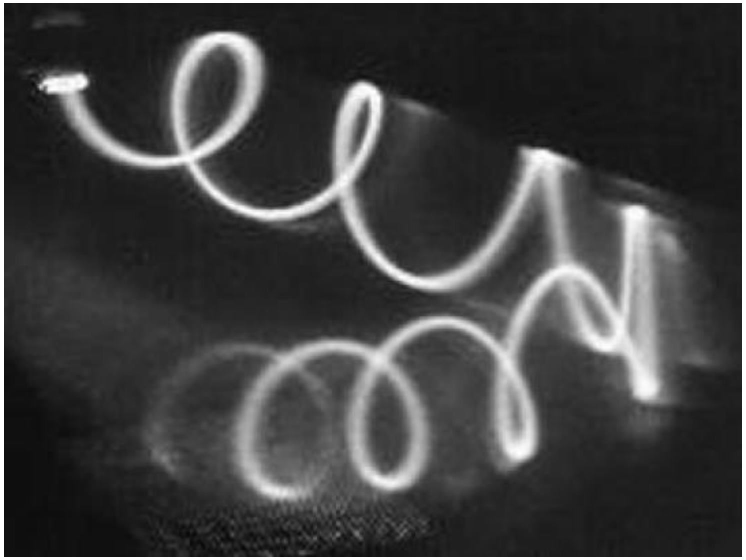
- Magnetic mirror



$$\frac{\frac{1}{2}m{v_{\perp 0}}^2}{B_0} = \frac{\frac{1}{2}m{v_{\perp m}}^2}{B_m}$$

$${v_{\perp m}}^2 + {v_{\parallel m}}^2 = {v_{\perp 0}}^2 + {v_{\parallel 0}}^2 \equiv {v_0}^2$$

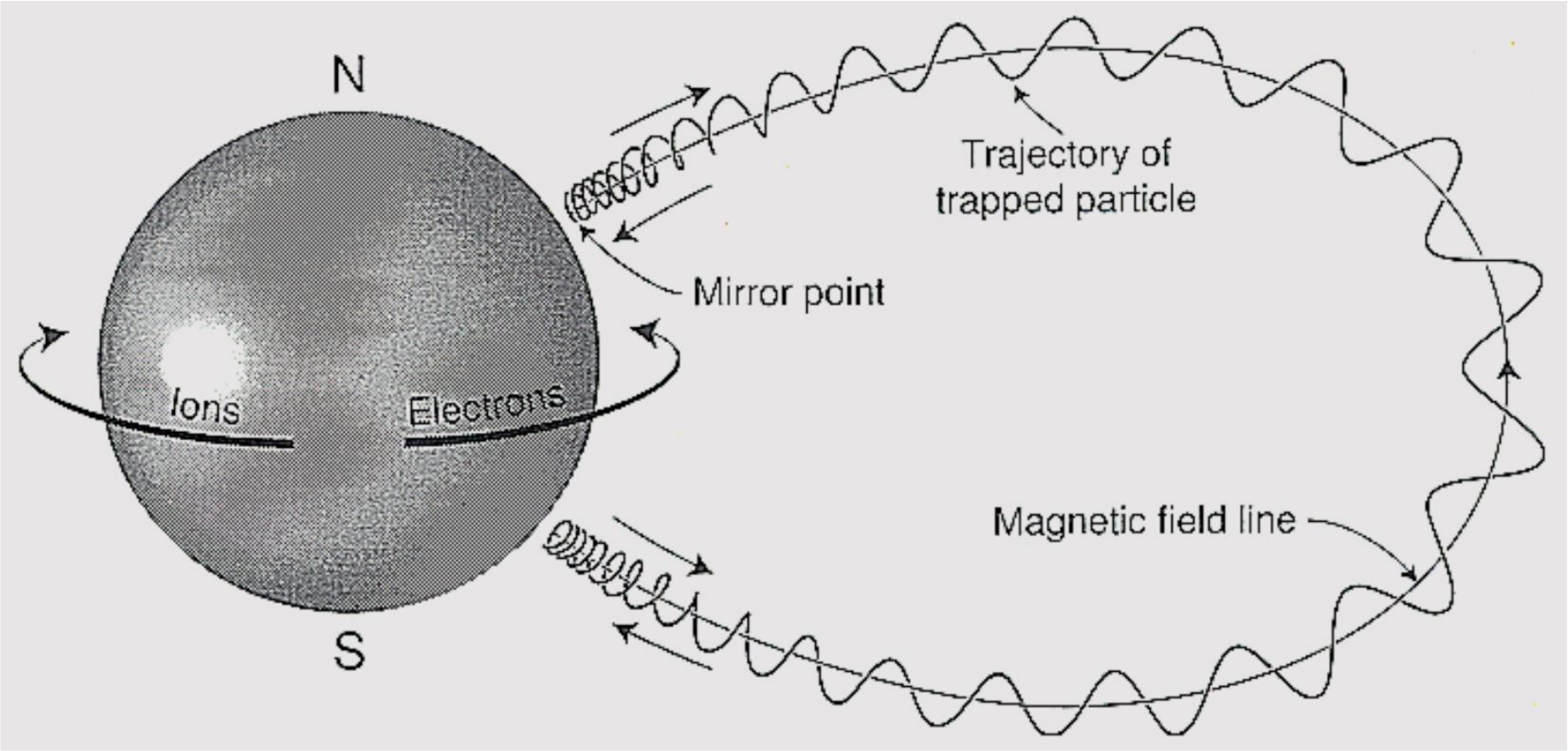
$$\frac{B_0}{B_m} = \frac{{v_{\perp 0}}^2}{{v_{\perp m}}^2} = \frac{{v_{\perp 0}}^2}{{v_0}^2} = \sin^2\theta$$



Motions of a charged particle in a dipole magnetic field

- Trajectories of particles confined in a dipole field

  → Particles experience gyro-, bounce- and drift- motions



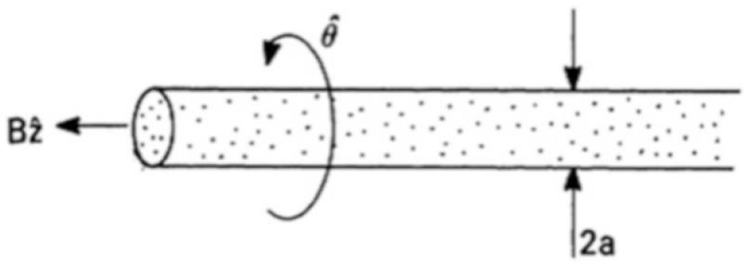
Homework

- F. Chen, Introduction to Plasma Physics and Controlled Fusion, Springer (2016), chapter 2   Problems: 2.1, 2.7, 2.21(a)

2.1. Compute r_L for the following cases if $v_{||}$ is negligible:

(a) A 10-keV electron in the earth's magnetic field of 5×10^{-5} T.
(b) A solar wind proton with streaming velocity 300 km/s, $B = 5 \times 10^{-9}$ T.
(c) A 1-keV He^+ ion in the solar atmosphere near a sunspot, where $B = 5 \times 10^{-2}$ T.
(d) A 3.5-MeV He^{++} ash particle in an 8-T DT fusion reactor.

2.7. An unneutralized electron beam has density $n_e = 10^{14}$ m^{-3} and radius $a = 1$ cm and flows along a 2-T magnetic field. If **B** is in the $+z$ direction and **E** is the electrostatic field due to the beam's charge, calculate the magnitude and direction of the $\mathbf{E} \times \mathbf{B}$ drift at $r = a$ (See Fig. P2.7).



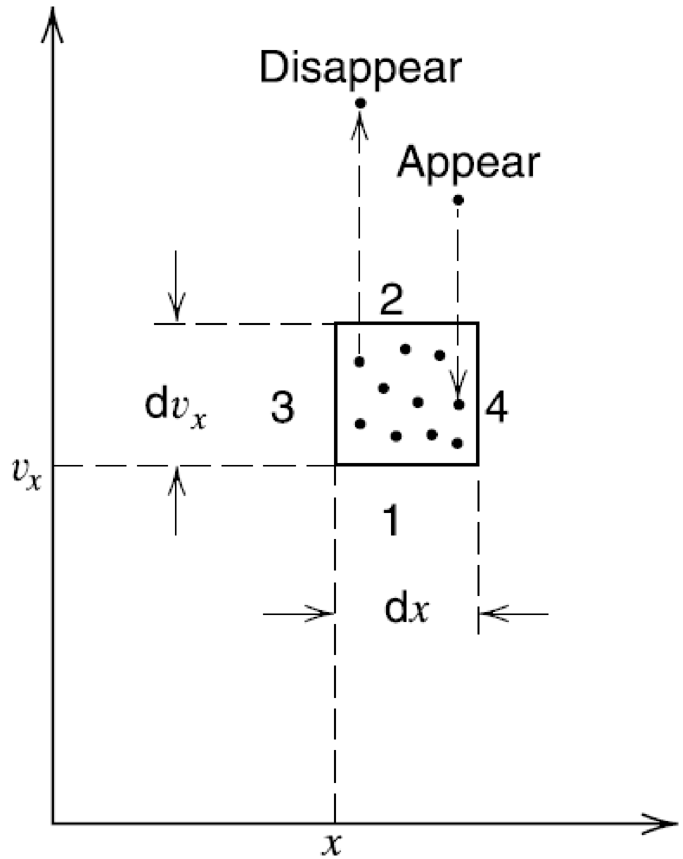
2.21. An infinite straight wire carries a constant current I in the $+z$ direction. At $t = 0$, an electron of small gyroradius is at $z = 0$ and $r = r_0$ with $v_{\perp 0} = v_{||0}$. ($\perp$ and $||$ refer to the direction relative to the magnetic field.)

(a) Calculate the magnitude and direction of the resulting guiding center drift velocity.

Boltzmann's equation & macroscopic quantities

$f(\mathbf{r}, \mathbf{v}, t)\mathrm{d}^3 r\, \mathrm{d}^3 v$ = number of particles inside a six-dimensional phase space volume $\mathrm{d}^3 r\, \mathrm{d}^3 v$ at $(\mathbf{r}, \mathbf{v})$ at time t

Distribution function



One-dimensional phase space

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}} f = \left.\frac{\partial f}{\partial t}\right|_{\mathrm{c}}$$

- Particle density

$$n(\mathbf{r}, t) = \int f\, \mathrm{d}^3 v$$

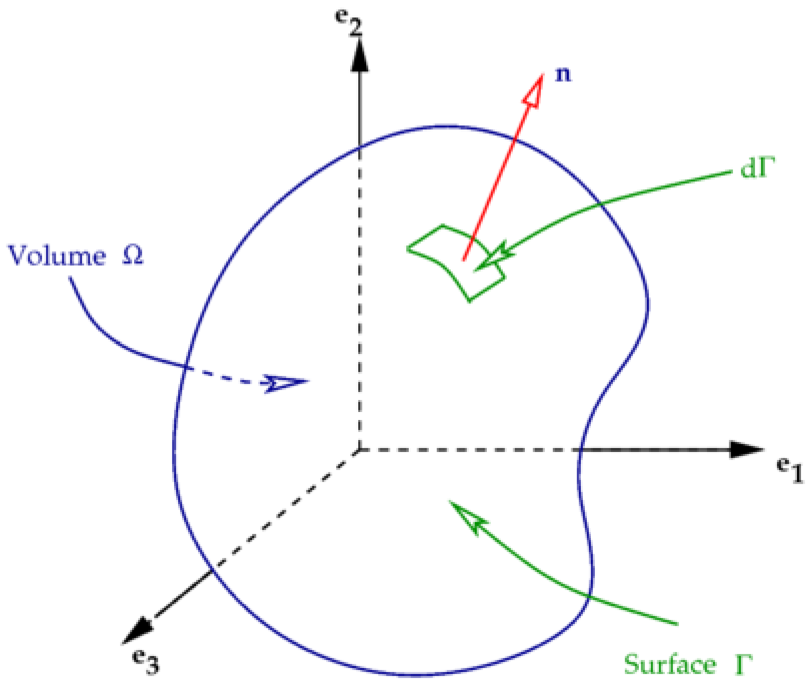
- Particle flux

$$\Gamma(\mathbf{r}, t) = n\mathbf{u} = \int \mathbf{v} f\, \mathrm{d}^3 v$$

- Particle energy density

$$w = \frac{3}{2} p + \frac{1}{2} m u^2 n = \frac{1}{2} m \int v^2 f\, \mathrm{d}^3 v$$

Particle conservation



$$\boxed{\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot (n\boldsymbol{u}) = G - L}$$

The net number of particles per second generated within Ω either flows across the surface Γ or increases the number of particles within Ω.

For common low-pressure discharges in steady-state:

$$G = \nu_{iz} n_e, \qquad L \approx 0$$

Hence, $\quad \boldsymbol{\nabla} \cdot (n\boldsymbol{u}) = \nu_{iz} n_e$

- The continuity equation is clearly not sufficient to give the evolution of the density n, since it involves another quantity, the mean particle velocity $\boldsymbol{u}$.

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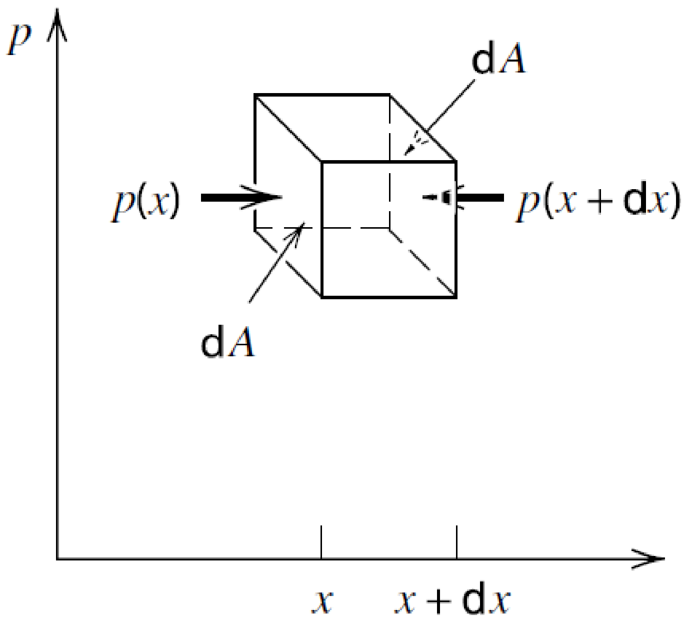
Momentum conservation

$$mn\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right] = qn(\mathbf{E} + \mathbf{u}\times\mathbf{B}) - \nabla\cdot\mathbf{\Pi} + \mathbf{f}\Big|_{\mathrm{c}}$$

Convective derivative



- Pressure tensor
  → isotropic for weakly ionized plasmas

$$\Pi_{ij} = mn\langle (v_i - u)(v_j - u)\rangle_{\mathbf{v}}$$

$$\nabla\cdot\mathbf{\Pi} = \nabla p$$

- The time rate of momentum transfer per unit volume due to collisions with other species

$$\mathbf{f}|_{\mathrm{c}} = -\sum_\beta mn\nu_{\mathrm{m}\beta}(\mathbf{u} - \mathbf{u}_\beta). - m(\mathbf{u} - \mathbf{u}_{\mathrm{G}})G + m(\mathbf{u} - \mathbf{u}_{\mathrm{L}})L$$

$$\boxed{mn\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right] = qn\boldsymbol{E} - \boldsymbol{\nabla}p - mn\nu_m\boldsymbol{u}}$$

Equation of state (EOS)

- An equation of state is a thermodynamic equation describing the state of matter under a given set of physical conditions.

$$p = p(n, T), \qquad \varepsilon = \varepsilon(n, T)$$

- Isothermal EOS for slow time variations, where temperatures are allowed to equilibrate. In this case, the fluid can exchange energy with its surroundings.

$$\boxed{p = nkT, \qquad \nabla p = kT\nabla n} \qquad n_g\ (\mathrm{cm}^{-3}) \approx 3.250 \times 10^{16}\, p\,(\mathrm{Torr})$$

→ The energy conservation equation needs to be solved to determine *p* and *T*.

- Adiabatic EOS for fast time variations, such as in waves, when the fluid does not exchange energy with its surroundings

$$\boxed{p = Cn^{\gamma}, \qquad \frac{\nabla p}{p} = \gamma \frac{\nabla n}{n}} \qquad \gamma = \frac{C_p}{C_v} \quad \text{(specific heat ratio)}$$

→ The energy conservation equation is not required.

- Specific heat ratio vs degree of freedom (f) $\qquad \gamma = 1 + \frac{2}{f}$

Equilibrium: Maxwell-Boltzmann distribution

- For a single species in thermal equilibrium with itself (e.g., electrons), in the absence of time variation, spatial gradients, and accelerations, the Boltzmann equation reduces to

$$\left.\frac{\partial f}{\partial t}\right|_c = 0$$

- Then, we obtain the Maxwell-Boltzmann velocity distribution

$$f(v) = n\left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$

- The mean speed

$$\bar{v} = (m/2\pi kT)^{3/2}\int_0^\infty v\left[\exp\left(-\frac{v^2}{2v_{\mathrm{th}}^2}\right)\right]4\pi v^2\,\mathrm{d}v \qquad v_{th} = \left(\frac{kT}{m}\right)^{1/2}$$

$$\bar{v} = \left(\frac{8kT}{\pi m}\right)^{1/2}$$

Particle and energy flux

- The directed particle flux: the number of particles per square meter per second crossing the $z = 0$ surface in the positive direction

$$\Gamma_z = n\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi/2} \sin\theta\,\mathrm{d}\theta \int_0^{\infty} v\cos\theta \exp\left(-\frac{v^2}{2v_{\mathrm{th}}^2}\right) v^2\,\mathrm{d}v$$

$$\Gamma_z = \frac{1}{4} n\bar{v}$$

- The average energy flux: the amount of energy per square meter per second in the $+z$ direction

$$S_z = n\left\langle \frac{1}{2} m v^2 v_z \right\rangle_v = 2kT\Gamma_z$$

- The average kinetic energy W per particle crossing $z = 0$ in the positive direction

$$W = 2kT$$

Diffusion and mobility

- The fluid equation of motion including collisions

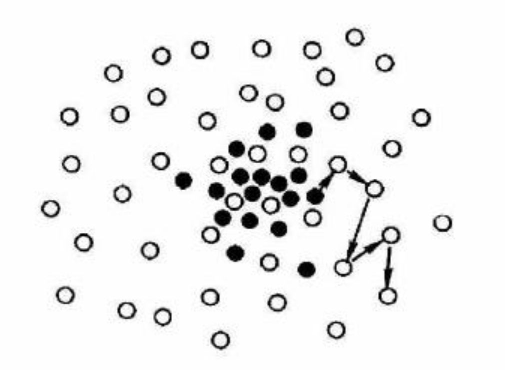
$$mn\frac{d\boldsymbol{u}}{dt} = mn\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right] = qn\boldsymbol{E} - \boldsymbol{\nabla}p - mn\nu_m\boldsymbol{u}$$

- In steady-state, for isothermal plasmas

$$\boldsymbol{u} = \frac{1}{mn\nu_m}(qn\boldsymbol{E} - \boldsymbol{\nabla}p) = \frac{1}{mn\nu_m}(qn\boldsymbol{E} - kT\boldsymbol{\nabla}n)$$

$$= \frac{q}{m\nu_m}\boldsymbol{E} - \frac{kT}{m\nu_m}\frac{\boldsymbol{\nabla}n}{n} = \pm\mu\boldsymbol{E} - D\frac{\boldsymbol{\nabla}n}{n}$$

Drift Diffusion



Diffusion is a random walk process.

- In terms of particle flux

$$\boldsymbol{\Gamma} = n\boldsymbol{u} = \pm n\mu\boldsymbol{E} - D\boldsymbol{\nabla}n$$

$$\boxed{\mu = \frac{|q|D}{kT} \quad : \quad \text{Einstein relation}}$$

$$\boxed{\mu = \frac{|q|}{m\nu_m} \quad : \quad \text{Mobility}}$$

$$\boxed{D = \frac{kT}{m\nu_m} \quad : \quad \text{Diffusion coefficient}}$$

Ambipolar diffusion

- The flux of electrons and ions out of any region must be equal such that charge does not build up. Since the electrons are lighter, and would tend to flow out faster in an unmagnetized plasma, an electric field must spring up to maintain the local flux balance.

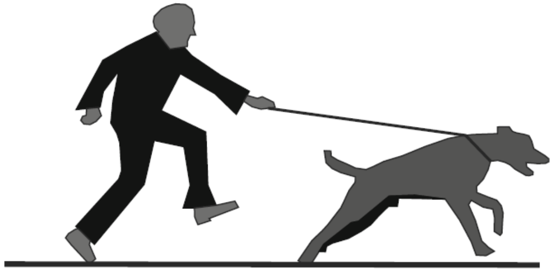
$$\Gamma_i = +n\mu_i E - D_i \nabla n \qquad \Gamma_e = -n\mu_e E - D_e \nabla n$$

- Ambipolar electric field for $\Gamma_i = \Gamma_e$

$$E = \frac{D_i - D_e}{\mu_i + \mu_e} \frac{\nabla n}{n}$$

- The common particle flux

$$\Gamma = -\frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e} \nabla n = -D_a \nabla n$$



- The ambipolar diffusion coefficient for weakly ionized plasmas

$$D_a = \frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e} \approx D_i + \frac{\mu_i}{\mu_e} D_e \approx D_i \left(1 + \frac{\mathrm{T_e}}{\mathrm{T_i}}\right) \sim \mu_i \mathrm{T_e}$$

Decay of a plasma by diffusion in a slab

- Diffusion equation (w/o source term)

$$\frac{\partial n}{\partial t} - D_a \nabla^2 n = 0 \quad \Leftarrow \quad n_i \approx n_e = n, \quad D = D_a$$

- In Cartesian coordinates,

$$\boxed{\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2}}$$

- Find $n(x,t)$ under the boundary conditions [H/W]

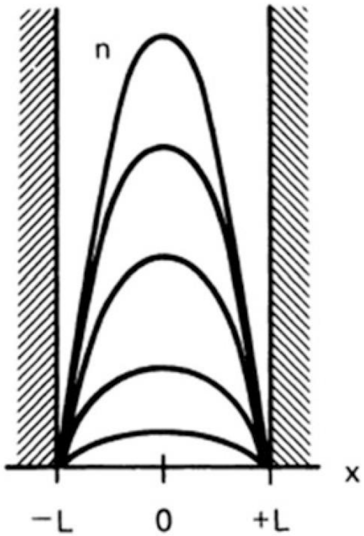
$$n(x = \pm L, t) = 0$$

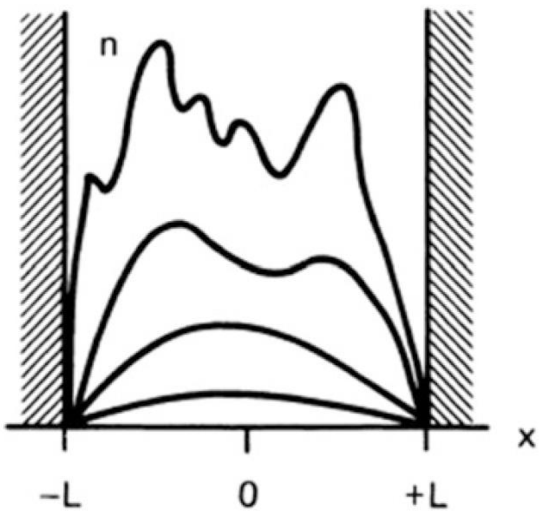
$$n(x, t = 0) = n_0 (1 - (x/L)^2)$$

- In general

$$n = n_0 \left(\sum_l a_l e^{-t/\tau_l} \cos \frac{(l + \frac{1}{2})\pi x}{L} + \sum_m b_m e^{-t/\tau_m} \sin \frac{m\pi x}{L} \right)$$

$$\tau_l = \left[\frac{L}{(l + \frac{1}{2})\pi} \right]^2 \frac{1}{D}$$





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Steady-state solution in cylindrical geometry

- Diffusion equation

volume source and sink

$$\frac{\partial n}{\partial t} - D_a \nabla^2 n = G - L = \nu_{iz} n - \alpha n^2 \qquad \Leftarrow \qquad n_i \approx n_e = n$$

- In steady-state, ignoring volume recombination

$$\boxed{\nabla^2 n + \frac{\nu_{iz}}{D} n = 0}$$

where, $D = D_a$ and ν_{iz} is the ionization frequency

- In cylindrical coordinates,

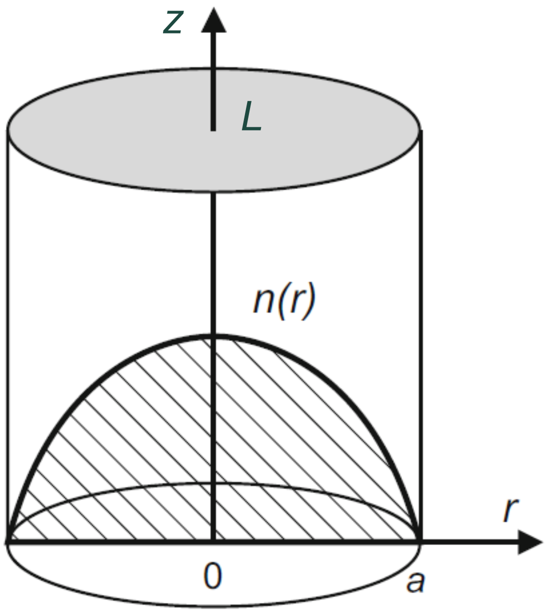
$$\frac{\partial^2 n}{\partial r^2} + \frac{1}{r}\frac{\partial n}{\partial r} + \frac{\partial^2 n}{\partial z^2} + \frac{\nu_{iz}}{D} n = 0$$

- Find $n(r, z)$ under the boundary conditions [H/W]

$$n(r = R, z) = 0$$

$$n(r, z = 0) = 0$$

$$n(r, z = L) = 0$$



Boltzmann's relation

- The density of electrons in thermal equilibrium can be obtained from the electron force balance in the absence of the inertial, magnetic, and frictional forces.

$$mn_e\left[\cancel{\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}}\right]=-en_e\boldsymbol{E}-\boldsymbol{\nabla}p_e-\cancel{mn_e\nu_m\boldsymbol{u}}$$

Setting $\boldsymbol{E}=-\boldsymbol{\nabla}\Phi$ and assuming $p_e=n_ekT_e$

$$en_e\boldsymbol{\nabla}\Phi-kT_e\boldsymbol{\nabla}n_e=n_e\boldsymbol{\nabla}(e\Phi-kT_e\ln n_e)=0$$

Integrating, we have

$kT_e=e\mathrm{T_e}$

$$n_e(\boldsymbol{r})=n_0\exp\left(\frac{e\Phi(\boldsymbol{r})}{kT_e}\right)=n_0\exp\left(\frac{\Phi(\boldsymbol{r})}{\mathrm{T_e}}\right)$$

Boltzmann's relation for electrons

- For positive ions in thermal equilibrium at temperature $\mathrm{T_i}$ $\quad n_i(\boldsymbol{r})=n_0\exp\left(-\frac{\Phi(\boldsymbol{r})}{\mathrm{T_i}}\right)$

- However, positive ions are almost never in thermal equilibrium in low pressure discharges because the ion drift velocity u_i is large, leading to inertial or frictional forces comparable to the electric field or pressure gradient forces.

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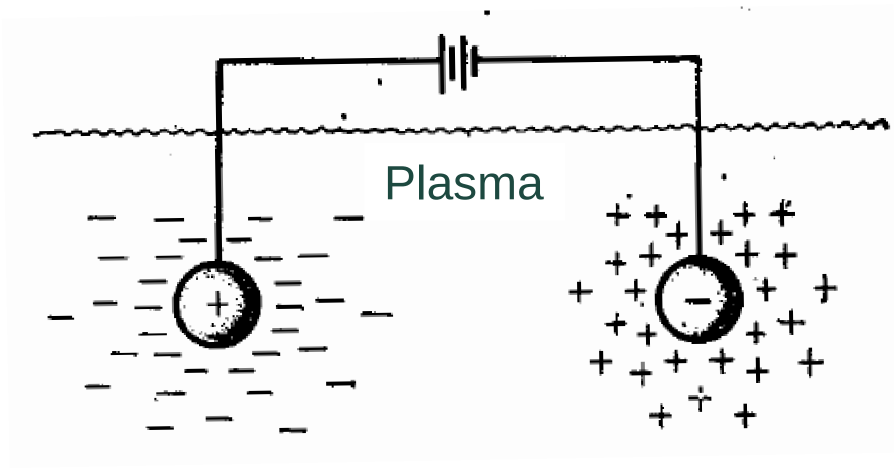
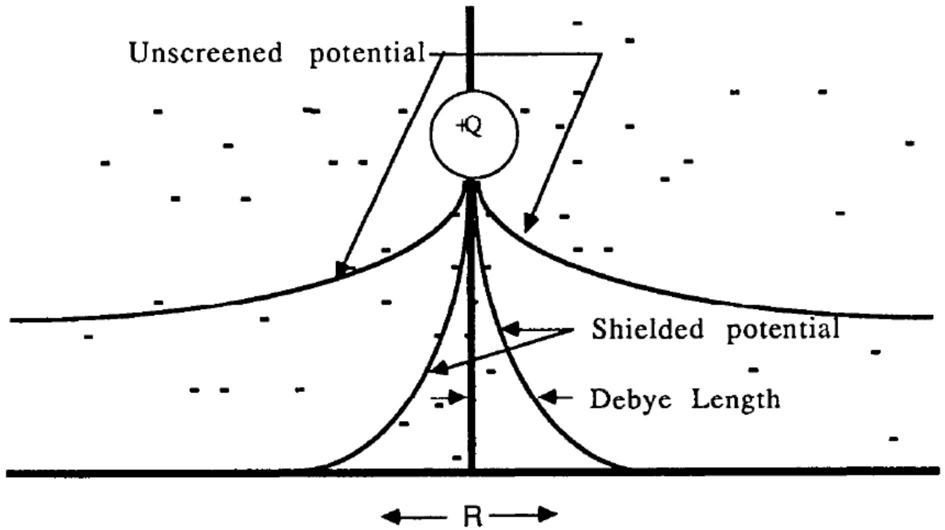
Debye shielding (screening)





- Coulomb potential

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0 r}$$

- Screened Coulomb potential

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right)$$

- Poisson's equation

$$\nabla^2\Phi = -\frac{en_0}{\epsilon_0}\left(1 - e^{e\Phi/kT_e}\right) \approx \frac{e^2 n_0}{\epsilon_0 kT_e}\Phi = \frac{\Phi}{\lambda_D^2}$$

- Debye length

$kT_e = e\mathrm{T_e}$

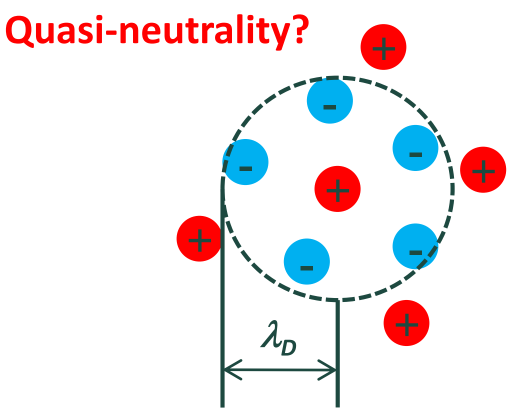
$$\boxed{\lambda_D = \left(\frac{\epsilon_0 kT_e}{e^2 n_0}\right)^{1/2} = \left(\frac{\epsilon_0 \mathrm{T_e}}{e n_0}\right)^{1/2}}$$

Debye length

- The electron Debye length λ_{De} is the characteristic length scale in a plasma.
- The Debye length is the distance scale over which significant charge densities can spontaneously exist. For example, low-voltage (undriven) sheaths are typically a few Debye lengths wide.
- Typical values for a processing plasma (n_e = 10^{10} cm^{-3}, T_e = 4 eV)

$$\lambda_D[\mathrm{cm}] = \left(\frac{\epsilon_0 \mathrm{T_e}}{e n_0}\right)^{1/2} = 743\sqrt{\frac{\mathrm{T_e[eV]}}{n_0[\mathrm{cm}^{-3}]}} = 743\sqrt{\frac{4}{10^{10}}} \approx 0.14\ \mathrm{mm}$$

- It is on space scales larger than a Debye length that the plasma will tend to remain neutral.
- The Debye length serves as a characteristic scale length to shield the Coulomb potentials of individual charged particles when they collide.



Quasi-neutrality

- The potential variation across a plasma of length $l \gg \lambda_{De}$ can be estimated from Poisson's equation

$$\nabla^2\Phi \sim \frac{\Phi}{l^2} \sim \left|\frac{e}{\epsilon_0}(Zn_i - n_e)\right|$$

We generally expect that

$$\Phi \lesssim \mathrm{T_e} = \frac{n_e e}{\epsilon_0}\lambda_{De}^2$$

Then, we obtain

$$\frac{|Zn_i - n_e|}{n_e} \sim \frac{\Phi}{l^2}\frac{\epsilon_0}{n_e e} \lesssim \frac{\lambda_{De}^2}{l^2} \ll 1$$

Plasma approximation

$$Zn_i = n_e$$

- The plasma approximation is violated within a plasma sheath, in proximity to a material wall, either because the sheath thickness $s \approx \lambda_{De}$, or because $\Phi \gg \mathrm{T_e}$.

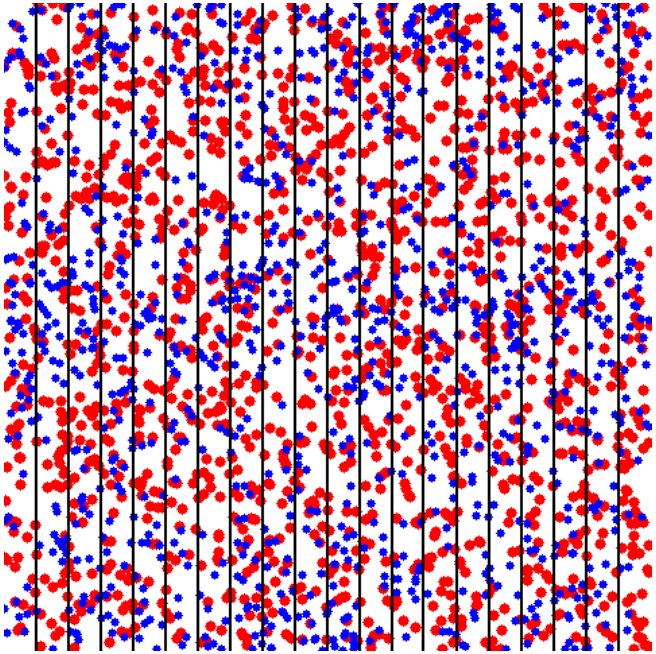
Plasma oscillations

- Electrons overshoot by inertia and oscillate around their equilibrium position with a characteristic frequency known as plasma frequency.

- Equation of motion (cold plasma)

$$m\frac{d^2\Delta x}{dt^2} = -eE_x = -e\frac{n_0 e\Delta x}{\epsilon_0}$$

$$\frac{d^2\Delta x}{dt^2} + \frac{n_0 e^2}{m\epsilon_0}\Delta x = 0$$ ← Harmonic oscillator



- Electron plasma frequency

$$\boxed{\omega_{pe} = \left(\frac{n_0 e^2}{m\epsilon_0}\right)^{1/2}}$$

- If the assumption of infinite mass ions is not made, then the ions also move slightly and we obtain the natural frequency

$$\omega_p = \left(\omega_{pe}^2 + \omega_{pi}^2\right)^{1/2}$$

where, $\omega_{pi} = (n_0 e^2/M\epsilon_0)^{1/2}$ (ion plasma frequency)

Plasma frequency

- Plasma oscillation frequency for electrons and ions

$$f_{pe} = \frac{\omega_{pe}}{2\pi} = 8980\sqrt{n_0}\ [\mathrm{Hz}] \quad (n_0 \text{ in cm}^{-3})$$

$$f_{pi} = \frac{\omega_{pi}}{2\pi} = 210\sqrt{n_0/M_A}\ [\mathrm{Hz}] \quad (n_0 \text{ in cm}^{-3}, M_A \text{ in amu})$$

- Typical values for a processing plasma (Ar)

$$f_{pe} = \frac{\omega_{pe}}{2\pi} = 8980\sqrt{10^{10}}\ [\mathrm{Hz}] = 9 \times 10^{8}\ [\mathrm{Hz}]$$

$$f_{pi} = \frac{\omega_{pi}}{2\pi} = 210\sqrt{10^{10}/40}\ [\mathrm{Hz}] = 3.3 \times 10^{6}\ [\mathrm{Hz}]$$

- Collective behavior

$\boxed{\omega_{pe}\tau_c > 1}$ Plasma frequency > collision frequency

Criteria for plasmas

- The picture of Debye shielding is valid only if there are enough particles in the charge cloud. Clearly, if there are only one or two particles in the sheath region, Debye shielding would not be a statistically valid concept. We can compute the number of particles in a "Debye sphere":

$$N_D = n\frac{4}{3}\pi\lambda_D^3$$

Wigner-Seitz radius $a = \sqrt[3]{4\pi n_e/3}$

- Plasma parameter

$$\Lambda = 4\pi n\lambda_D^3 = 3N_D$$

- Coupling parameter

$$\Gamma = \frac{\text{Coulomb energy}}{\text{Thermal energy}} = \frac{q^2/(4\pi\epsilon_0 a)}{kT_e} \sim \Lambda^{-2/3}$$

- Criteria for plasmas:

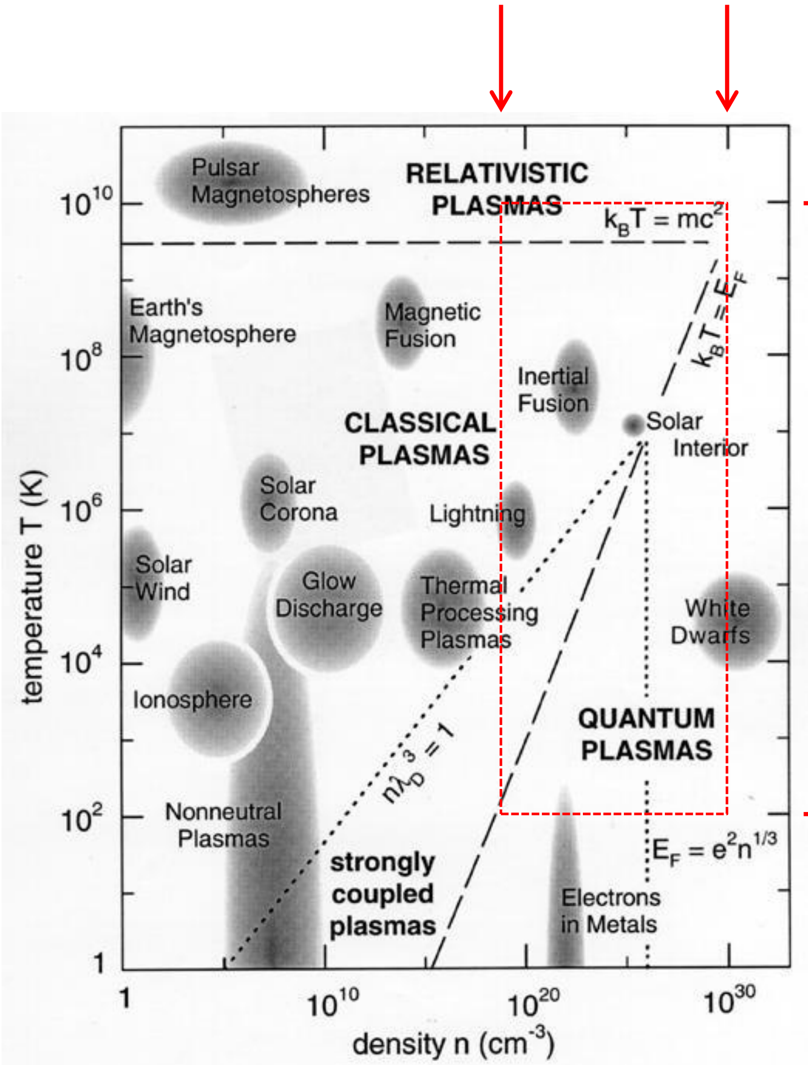
$$\boxed{\lambda_D \ll L}$$

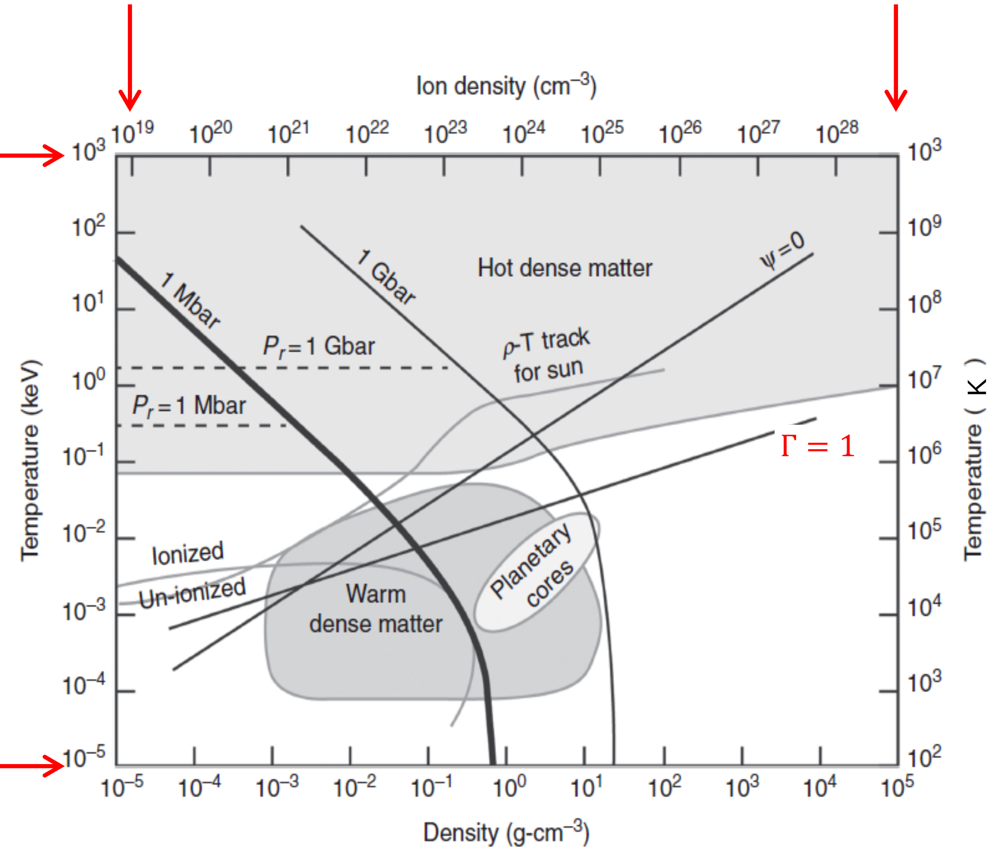
$$\boxed{N_D \gg 1}$$

$$\boxed{\omega_{pe}\tau_c > 1}$$

Description	Plasma parameter magnitude	
	$\Lambda \ll 1$ ($\Gamma \gg 1$)	$\Lambda \gg 1$ ($\Gamma \ll 1$)
Coupling	Strongly coupled plasma	Weakly coupled plasma
Debye sphere	Sparsely populated	Densely populated
Electrostatic influence	Almost continuously	Occasional
Typical characteristic	Cold and dense	Hot and diffuse
Examples	Solid-density laser ablation plasmas Very "cold" "high pressure" arc discharge Inertial fusion experiments White dwarfs / neutron stars atmospheres	Ionospheric physics Magnetic fusion devices Space plasma physics Plasma ball

Various kinds of plasmas

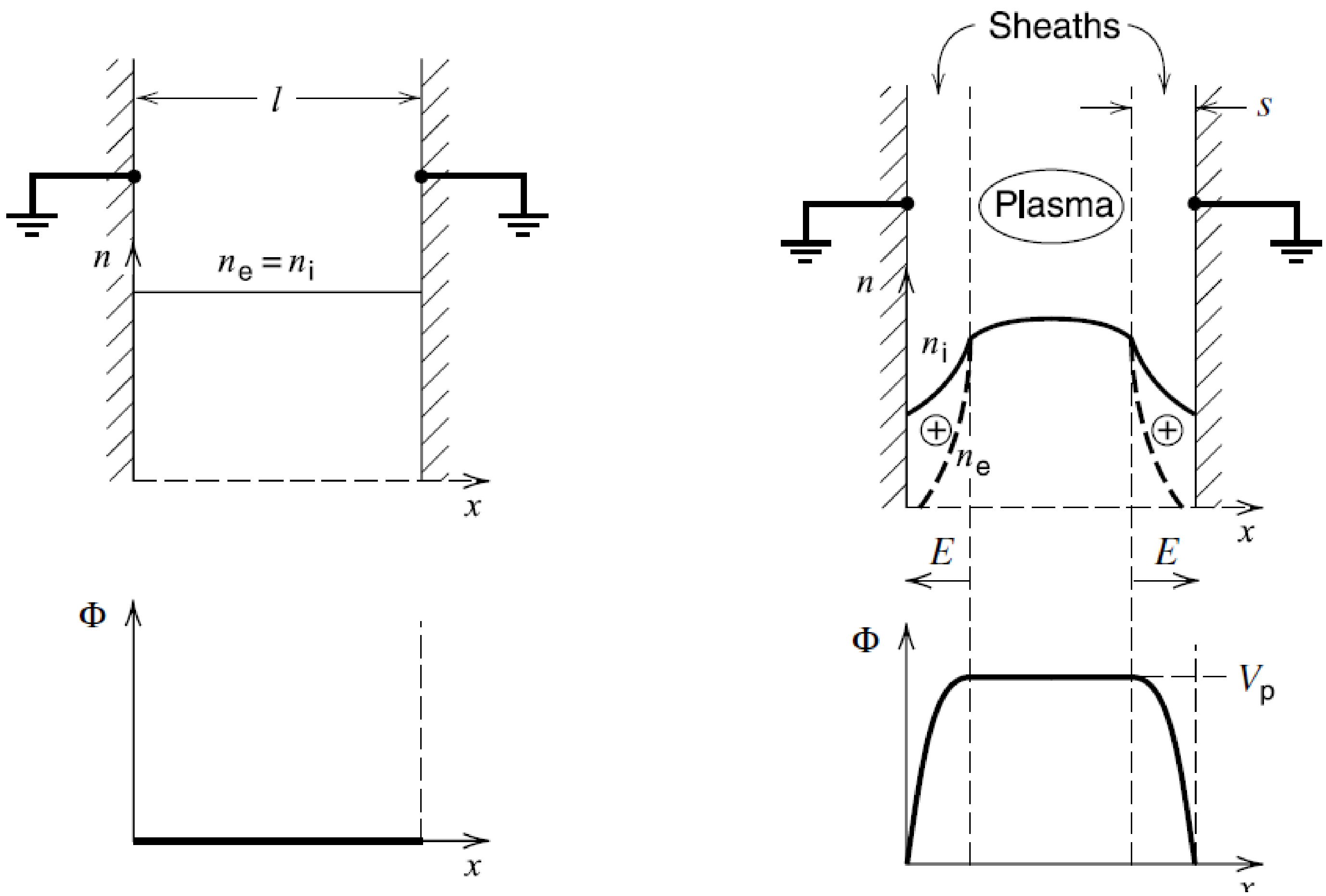




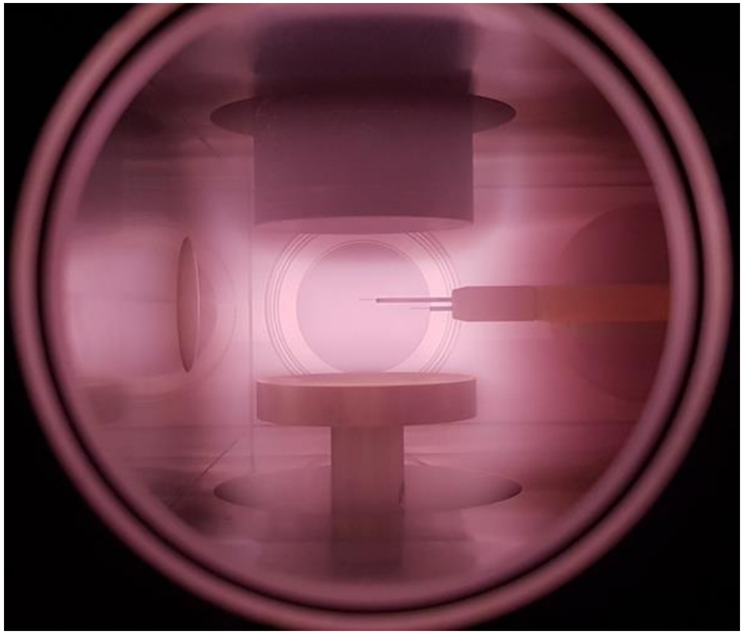
High-energy density regime

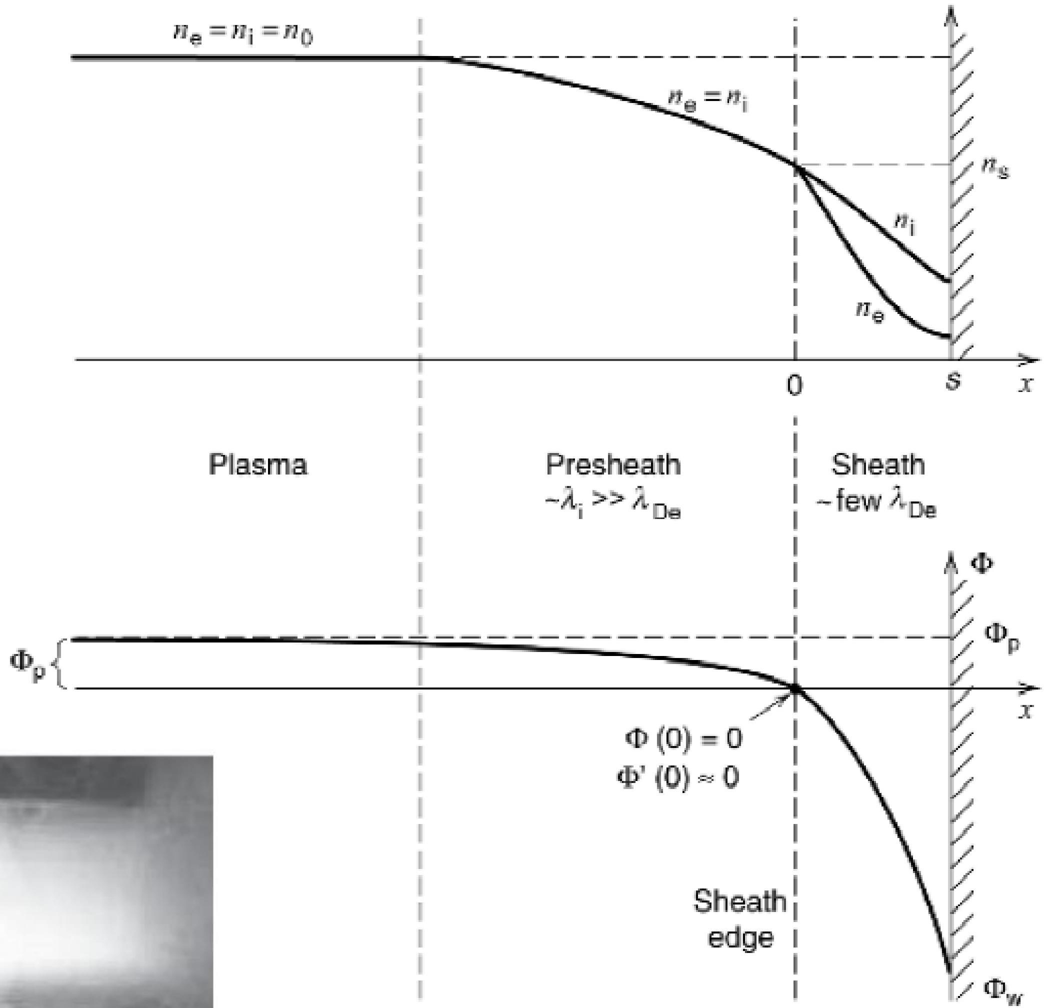
Formation of plasma sheaths

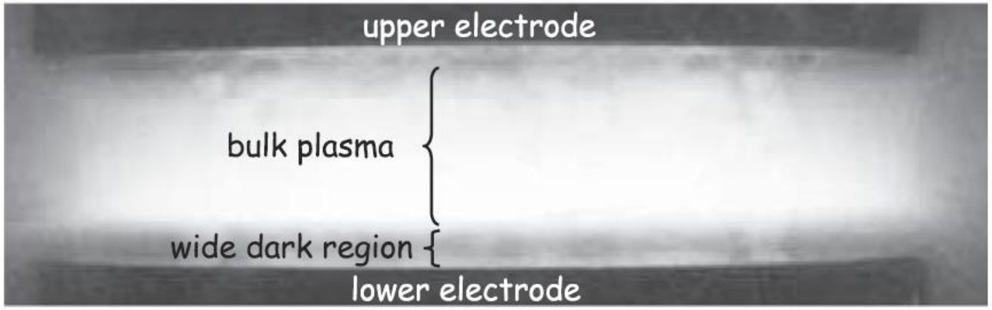
- Plasma sheath: the non-neutral potential region between the plasma and the wall caused by the balanced flow of particles with different mobility such as electrons and ions.



Plasma-sheath structure







Homework

- F. Chen, Introduction to Plasma Physics and Controlled Fusion, Springer (2016), chapter 5 Problems: 5.2, 5.6

5.2. A weakly ionized plasma slab in plane geometry has a density distribution

$$n(x) = n_0 \cos(\pi x/2L) \qquad -L \leq x \leq L$$

The plasma decays by both diffusion and recombination. If $L = 0.03$ m, $D = 0.4$ m^2/s, and $\alpha = 10^{-15}$ m^3/s, at what density will the rate of loss by diffusion be equal to the rate of loss by recombination?

5.6. You do a recombination experiment in a weakly ionized gas in which the main loss mechanism is recombination. You create a plasma of density 10^{20} m^{-3} by a sudden burst of ultraviolet radiation and observe that the density decays to half its initial value in 10 ms. What is the value of the recombination coefficient α? Give units.