Vector Mechanics: Dynamics

CHAPER 11.
Kinematics of Particles

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11.0 Introduction

Statics: analysis of bodies at rest

Dynamics: mechanics that deals with the analysis of bodies in motion

Dynamics

- ✓ Kinematics: study of the geometry of motion relate displacement, velocity, acceleration and time without reference to the cause of the motion

- ✓ Kinetics: study of the relation existing between the forces acting in the body, the mass of the body and the motion of the body.
  predict the motion caused by given forces or to determine the forces required to produce a given motion

11.0 Introduction

Contents

a. Dynamic of particles: particle → motion as an entire unit. Any rotation about their own mass will be neglected

b. Dynamic of rigid bodies: such a rotation is not negligible

c. Rectilinear motion: Moves along a straight line
- Uniform motion
- Uniformly accelerated motion of a particle
- Simultaneous motion of particles

Curvilinear motion: Moves along a curved line
- Derivative of a vector function
- Rectangular component for velocity and acceleration
- Motion of a particle relative to reference frame in translation
- Other than rectangular coordinate
- Tangential and normal component of velocity, acceleration

11.1 Rectilinear Motion of Particles

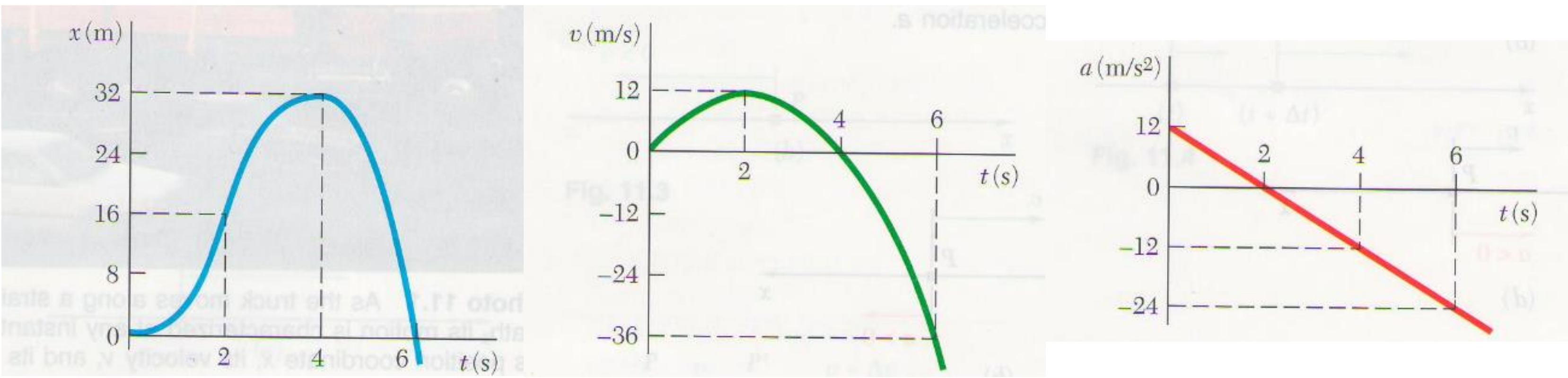
RECTILINEAR MOTION (straight line)

- Position coordinate: the distance measured from a fixed origin (positive or negative)

when position coordinate x of a particle is known for every value of time t, motion of the particle is known.

$$x = 6t^2 - t^3$$

x: meter (m), millimeter(mm) t: seconds(s)



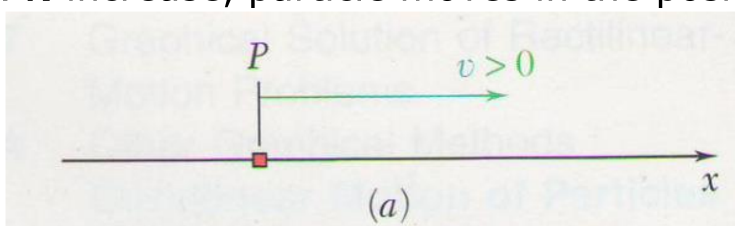
11.1 Rectilinear Motion of Particles

- Average velocity: $\dfrac{\Delta x}{\Delta t}$ (m/s)

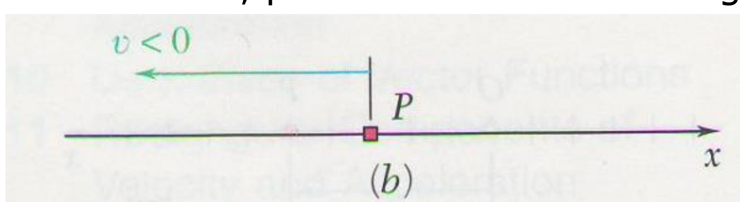
  x: meter (m) t : seconds (s)

- Instantaneous velocity: $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t}$ (m/s) $\to \dfrac{dx}{dt}$

  velocity v
  - (+): x increase, particle moves in the positive direction



  - (-) : x decreases, particle moves in the negative direction

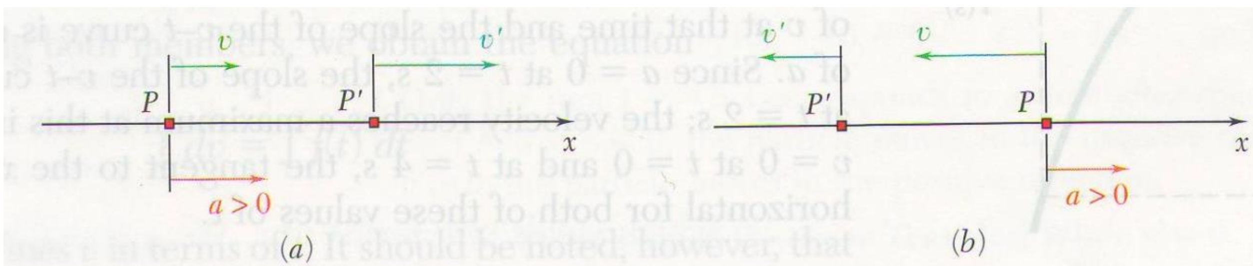


- Magnitude of v: speed

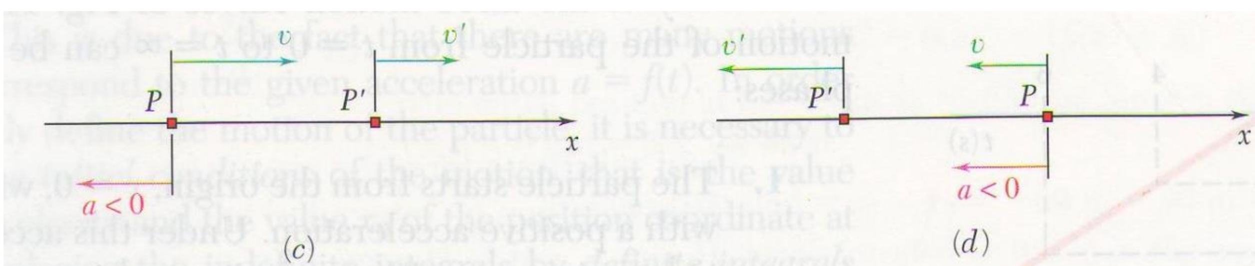
11.1 Rectilinear Motion of Particles

- Average acceleration: $\dfrac{\Delta v}{\Delta t}$ (m/s²)
- Instantaneous velocity: $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t}$ (m/s²) $\rightarrow \dfrac{dv}{dt} = \dfrac{d^2 x}{dt^2}$

  - (+): velocity increases → (a) particle is moving faster in the positive dir.
    (b) moving more slowly in the negative dir.



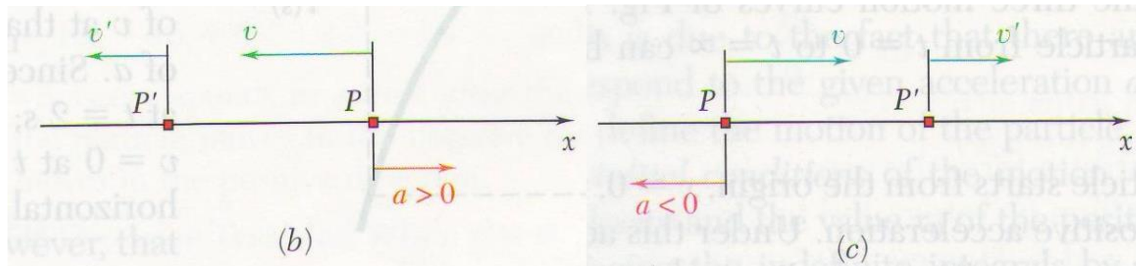
  - (-): velocity decreases → (c) moving more slowly in the positive dir.
    (d) moving faster in the negative dir.



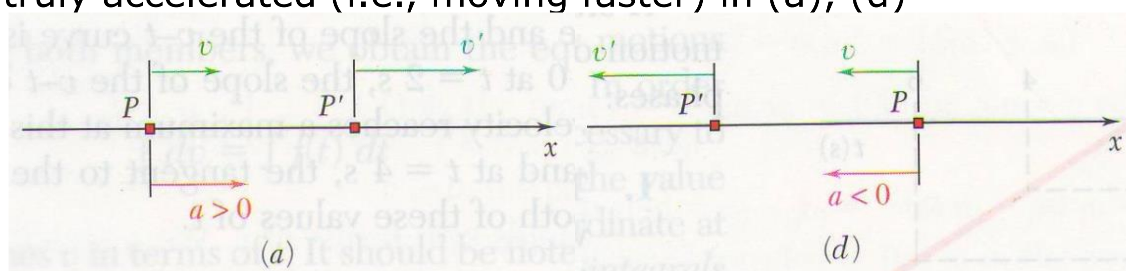
11.1 Rectilinear Motion of Particles

- Deceleration: speed of particle (i.e., magnitude of v) decreases.

  decelerated in Fig 11.5 (b), (c)



  truly accelerated (i.e., moving faster) in (a), (d)



- Another expression of acceleration:

$$v = \frac{dx}{dt}, \quad dt = \frac{dx}{v}, \quad a = \frac{dv}{dt} = v\frac{dv}{dx}$$

[Example] position is defined by an eqn: $x = 6t^2 - t^3$
velocity: $v = dx/dt = 12t - 3t^2$
acceleration: $a = dv/dt = 12 - 6t$
motion curves: Fig.11.6

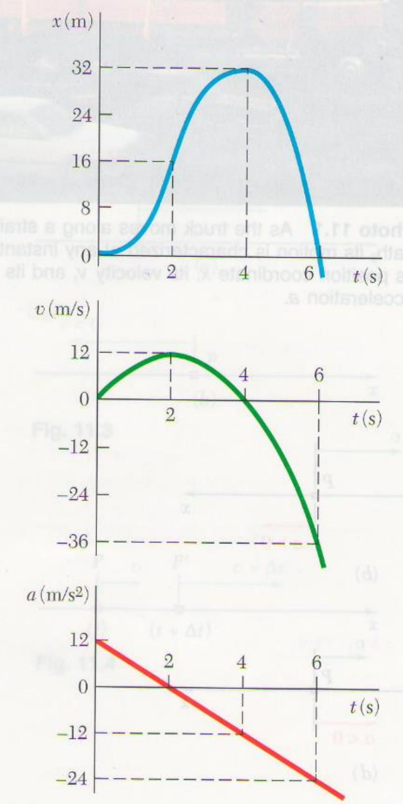


Fig. 11.6

- Study of three motion curves in Fig. 11.6

  i) Particle start at the origin (x=0) with no velocity
  but with a positive acceleration
  gains a positive velocity, moves in positive direction.
  t =0~2s: x, v, a are all (+)

  ii) t =2s, acceleration 0. velocity reaches to its maximum
  t =2~4s, v(+), but a(-) particle still moves in the positive direction, but more and more slowly

  iii) t=4s, v=0, x reaches its maximum. And v, a (-)
  particle is accelerating and moves in the negative dir. with increasing speed

  iv) t=6s. Particle passes the origin x=0 the total distance traveled in 64m
  t>6s. x, v, a are all (-)
  particle keeps moving in the negative dir. away from 0, faster and faster

11.1B Determination of a Motion

In practice, the motion is specified by the type of acceleration that the particle possesses, rather than the relation of x and t

[Example] freely falling body: constant acceleration 9.81m/s^2 downward

mass attached to a spring: acceleration proportional to instantaneous elongation of the spring

In general, a can be expressed as a function of x, v and t
→ need two successive integration to obtain x

(i) $a = f(t)$ (acceleration is a given function of t)

$$dv = a\,dt = f(t)\,dt \;\rightarrow\; \int dv = \int f(t)\,dt$$ (indefinite integral)

→ definite integrals by providing initial condition $t = 0,\ v = v_0$

$$\int_{v_0}^{v} dv = \int_0^t f(t)\,dt \;\rightarrow\; v - v_0 = \int_0^t f(t)\,dt$$

$$dx = v\,dt \;\rightarrow\; \int_{x_0}^{x} dx = \int_0^t v\,dt$$

a=0: uniform motion
a=constant: uniformly accelerated motion

11.1B Determination of a Motion

(ii) $a = f(x)$ (acceleration is a given function of x)

$$v\,dv = a\,dx = f(x)\,dx$$

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx \rightarrow \frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \int_{x_0}^{x} f(x)\,dx \rightarrow v = function(x)$$

$$\underbrace{dt = \frac{dx}{v}}$$

usually, use numerical integration

(iii) $a = f(v)$ (acceleration is a given function of v)

$$f(v) = \frac{dv}{dt} \rightarrow dt = \frac{dv}{f(v)} \rightarrow v = function(t)$$

$$f(v) = v\frac{dv}{dx} \rightarrow dx = \frac{v\,dv}{f(v)} \rightarrow v = function(x)$$

$$\downarrow$$

$$v = \frac{dx}{dt} \rightarrow x = function(t)$$

11.2A Uniform Rectilinear Motion

- Straight-line motion

acceleration a is zero for every value of t

$$\frac{dx}{dt} = v \text{ : constant}$$

$$\int_{x_0}^{x} dx = v\int_0^t dt$$

$$x - x_0 = vt$$

$$\boxed{x = x_0 + vt}$$: only if the velocity of the particle is known to be constant

11.2B Uniformly Accelerated Rectilinear Motion

- acceleration a = constant

$$\frac{dv}{dt} = a \quad \text{: constant}$$

$$\int_{v_0}^{v} dv = a\int_0^t dt \;\rightarrow\; v - v_0 = at \rightarrow \boxed{v = v_0 + at}$$

$$\frac{dx}{dt} = v_0 + at$$

$$\int_{x_0}^{x} dx = \int_0^t (v_0 + at)\,dt \;\rightarrow\; x - x_0 = v_0 t + \frac{1}{2}at^2 \rightarrow \boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

$$v\frac{dv}{dx} = a \quad \text{: constant} \qquad v\,dv = a\,dx$$

$$\int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \;\rightarrow\; \frac{1}{2}\left(v^2 - v_0^2\right) = a\left(x - x_0\right) \rightarrow \boxed{v^2 = v_0^2 + 2a\left(x - x_0\right)}$$

- Application of uniformly accelerated motion : freely falling body. $g=9.81\text{m/s}^2$

11.2C Motion of Several Particles

- several particles move independently

  : common clock and measuring tape should be used

- relative motion of two particles

  i) relative position coordinate: $x_{B/A} = x_B - x_A$ or $x_B = x_A + x_{B/A}$

  (+): B is to the right of A

  (-): B is to the left of A

  ii) relative velocity: $v_{B/A} = v_B - v_A$ (differentiating) or $v_B = v_A + v_{B/A}$

  (+): B is observed from A to move in the positive direction

  (-): B is observed from A to move in the negative direction

  iii) relative acceleration: $a_{B/A} = a_B - a_A$ or $a_B = a_A + a_{B/A}$

11.2C Motion of Several Particles

- dependent motions:
  position of a particle depends upon the position of another particle or of several other particles → dependent

  Fig. 11.8 → $x_A + 2x_B$ = constant

  only one of x_A and x_B can be chosen arbitrary
  → system has one d.o.f

  if x_A is given an increment Δx_A,
  x_B will be receive an increment $\Delta x_B = -\frac{1}{2}\Delta x_A$

  Fig. 11.9 → $2x_A + 2x_B + x_C$ = constant
  → system has two d.o.f's

$$\begin{cases} 2\dfrac{dx_A}{dt} + 2\dfrac{dx_B}{dt} + \dfrac{dx_c}{dt} = 0 & 2v_A + 2v_B + v_C = 0 \\ 2\dfrac{dv_A}{dt} + 2\dfrac{dx_B}{dt} + \dfrac{dx_c}{dt} = 0 & 2a_A + 2a_B + a_C = 0 \end{cases}$$

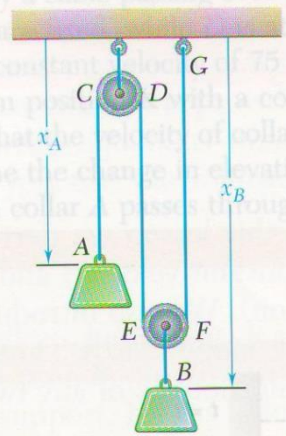


Fig. 11.8

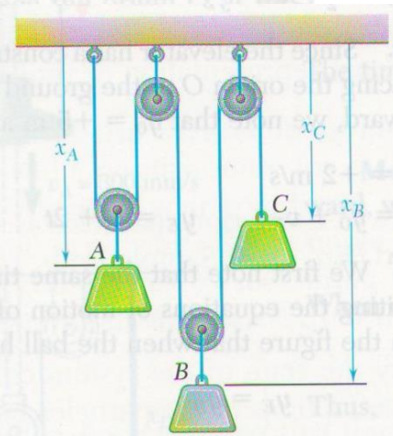


Fig. 11.9

11.4A Position vector, velocity, Acceleration

moves along a curve other than straight line → curvilinear motion

- Position vector: vector $\vec{r}$ joining the origin O and point P
  magnitude r and its direction
  $\vec{r}'$ defining the position P' at $t+\Delta t$
  $\rightarrow \Delta\vec{r}'$ represents change in direction as well as magnitude
- Average velocity: $\dfrac{\Delta\vec{r}}{\Delta t}$, vector attached at P, the same direction as $\Delta\vec{r}'$
  magnitude equal to $\left|\Delta\vec{r}\right|/\Delta t$
- Instantaneous velocity: $\vec{v} = \lim\limits_{\Delta t\to 0}\dfrac{\Delta\vec{r}}{\Delta t}$, tangent to the path of the particle

position vector $\vec{r}(t)$: vectors function of scalar variable t

$$\vec{v} = \frac{d\vec{r}}{dt}$$

magnitude v : speed

length of the segment $\overline{PP'}$ approaches the length $\overset{\frown}{PP'}$ of the arc

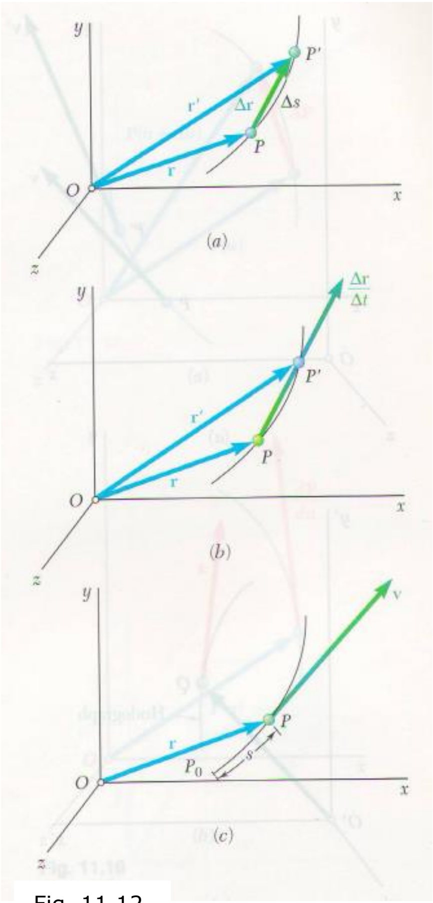


Fig. 11.12

11.4B Derivatives of Vector Functions

- Fig. 11.13

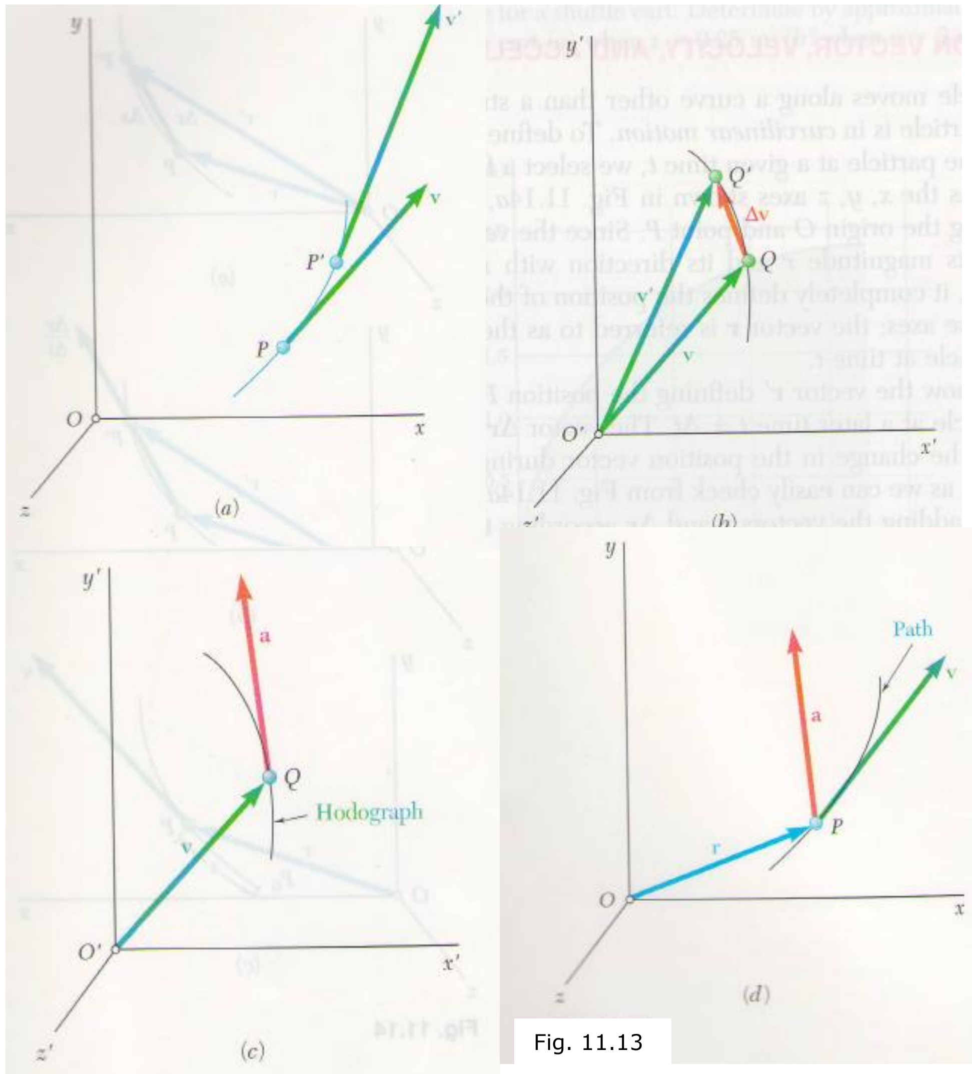


Fig. 11.13

governing rules of the differentiation of sums and products of vector function

$\vec{P}(u)$: vector function of the scalar variable u

$\Delta\vec{P}$: vector joining the tips of the two given vectors

$$\Delta\vec{P} = \vec{P}\left(u+\Delta u\right) - \vec{P}(u)$$

derivative of the vector function $\vec{P}(u)$

$$\frac{d\vec{P}}{du} = \lim_{\Delta u \to 0}\frac{\Delta\vec{P}}{\Delta u} = \lim_{\Delta u \to 0}\frac{\vec{P}\left(u+\Delta u\right) - \vec{P}(u)}{\Delta u}$$

└ tangent to the curve described by the tip of $\vec{P}(u)$

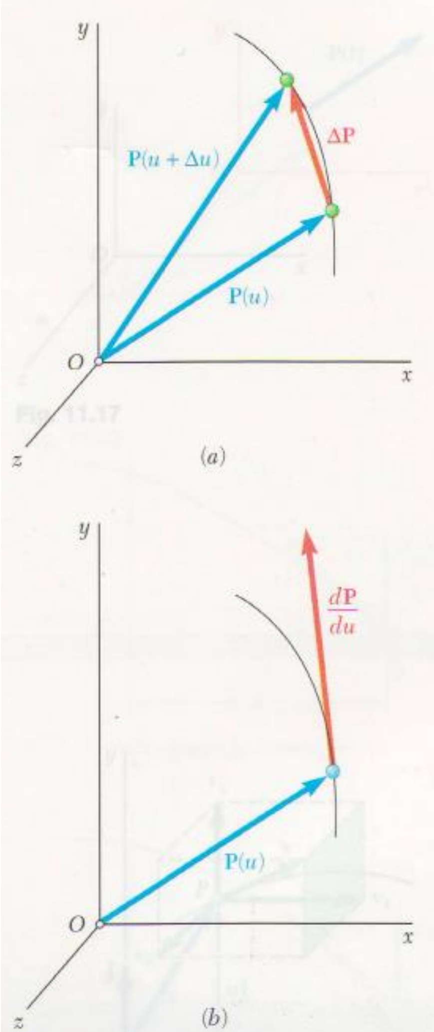


Fig. 11.14

- sum of two vector functions

$$\frac{d(\vec{P}+\vec{Q})}{du} = \frac{d\vec{P}}{du} + \frac{d\vec{Q}}{du}$$

- product of scalar function $f(u)$ and vector function $\vec{P}(u)$

$$\frac{d(f\vec{P})}{du} = \lim_{\Delta u \to 0}\frac{(f+\Delta f)(\vec{P}+\Delta\vec{P}) - f\vec{P}}{\Delta u} = \lim_{\Delta u \to 0}\left(\frac{df}{\Delta u}\vec{P} + f\frac{\Delta\vec{P}}{\Delta u}\right)$$

$$\frac{d(f\vec{P})}{du} = \frac{df}{du}\vec{P} + f\frac{d\vec{P}}{du}$$

11.4B Derivatives of Vector Functions

- scalar / vector product of two vector functions

$$\frac{d(\vec{P}\cdot\vec{Q})}{du}=\frac{d\vec{P}}{du}\cdot\vec{Q}+\vec{P}\cdot\frac{d\vec{Q}}{du}$$

$$\frac{d(\vec{P}\times\vec{Q})}{du}=\frac{d\vec{P}}{du}\times\vec{Q}+\vec{P}\times\frac{d\vec{Q}}{du}$$

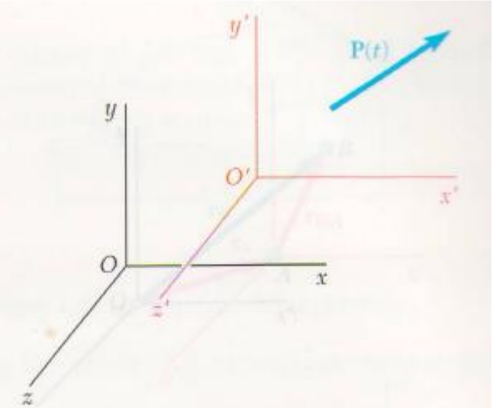


Fig. 11.15

- Rectangular component

$$\vec{P}=P_x\vec{i}+P_y\vec{j}+P_z\vec{k}$$

P_x, P_y, P_z : rectangular scalar component of the vector $\vec{P}$
$\vec{i}, \vec{j}, \vec{k}$: unit vector corresponding to x, y, z
└ constant magnitude (equal to 1) and fixed direction → derivatives are zero

∴ differentiating the corresponding scalar components of $\vec{P}$

11.4B Derivatives of Vector Functions

- Rate of change of vector

  $\vec{P}$: function of time t

  $\dfrac{d\vec{P}}{dt}$: rate of change of $\vec{P}$ with respect to the frame O_{xyz}

$$\dot{\vec{P}} = \dot{P}_x\vec{i} + \dot{P}_y\vec{j} + \dot{P}_z\vec{k}$$

- Fixed frame and moving frame

  In general, rate of change of a vector as observed from a moving frame of reference is different from that in fixed frame

  However, if the moving frame is in translation

  → the same unit vector $\vec{i}, \vec{j}, \vec{k}$ are used in both frames
  at any given instant, the same scalar components P_x, P_y, P_z in both frames

  → rate of change $\vec{P}$ is the same with respect to O_{xyz} and $O_{x'y'z'}$

  Rate of change of a vector is the same w.r.t. a fixed frame and w.r.t. a frame in translation

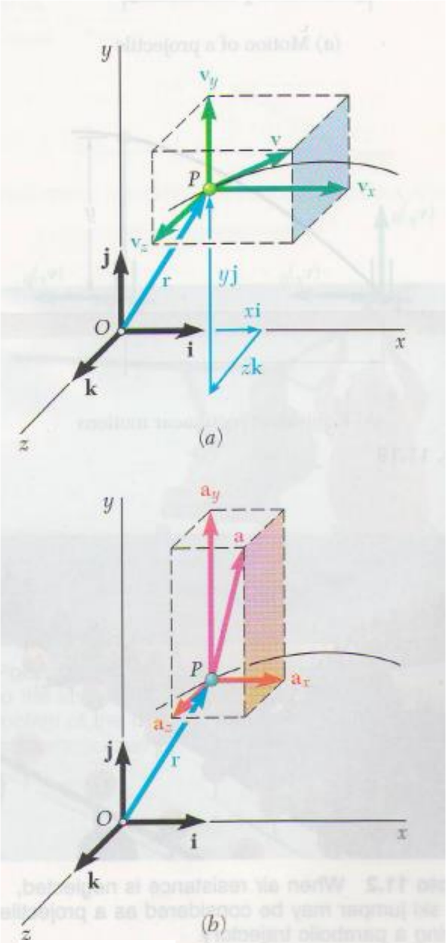


Fig. 11.16

11.4C Rectangular components of Velocity and Acceleration

Position vector $\vec{r}$

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$$

$\vec{a} = \dfrac{d\vec{v}}{dt} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$: First and second derivate of x, y, z

v_x : (+) Vector component $\vec{v}_x$ is directed to the right

(-) Vector component $\vec{v}_x$ is directed to the left

Rectangular component description

- a_x depends only on t, x, and/or v_x
- a_y depends only on t, y, and/or v_y
- a_z depends only on t, y, and/or v_z

  - Easy to integrate $a_x = \ddot{x}, \quad a_y = \ddot{y}, \quad a_z = \ddot{z}$
  - Independently $v_x = \dot{x}, \quad \mathrm{v}_y = \dot{y}, \quad \mathrm{v}_z = \dot{z}$

11.4C Rectangular components of Velocity and Acceleration

Motion of a projectile (1)

$$a_x = \ddot{x} = 0, \quad a_y = \ddot{y} = -g, \quad a_z = \ddot{z} = 0$$

(air resistance neglected)

- By integration

$$v_x = \dot{x} = (v_x)_0, \quad \mathrm{v}_y = \dot{y} = (\mathrm{v}_y)_0 - gt, \quad \mathrm{v}_z = \dot{z} = (\mathrm{v}_z)_0$$

$$x = x_0 + (v_x)_0 t, \quad y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2, \quad z = z_0 + (v_z)_0 t$$

- If fired in x-y plane,

$$x_0 = y_0 = z_0 = 0, \quad (v_z)_0 = 0$$

$$v_x = (v_x)_0, \quad \mathrm{v}_y = (\mathrm{v}_y)_0 - gt, \quad \mathrm{v}_z = 0$$

$$x = (v_x)_0 t, \quad y = (v_y)_0 t - \frac{1}{2} g t^2, \quad z = 0$$

(1) Projectile remains in the x-y plane
(2) Motion in horizontal direction: uniform
(3) Motion in vertical direction: uniformly accelerated

11.4C Rectangular components of Velocity and Acceleration

Motion of a projectile (2)

- Two independent rectilinear motion
  - projectile fired vertically with $(\mathrm{v}_y)_0$ from a platform moving with $(\mathrm{v}_x)_0$
  - Coordinate x = distance traveled by the platform
  - Coordinate y = computed as if the projectile were moved along a vertical line
- Equation defining x and y of the projectile
  - parametric eqns. of a parabola $\longrightarrow$ trajectory is parabolic

However, air resistance

variation of gravitational acceleration with altitude will make the above result invalid.

11.4D Motion Relative to a Frame in Translation

- Fixed frame of reference: attached to the earth, but its selection is purely arbitrary

- Two particles A, B
  - Position vectors $\vec{r}_A, \vec{r}_B$ with respect to the fixed frame O_{xyz}
  - New frame of reference $A_{x'y'z'}$ in translation with respect to O_{xyz}

  $\vec{r}_{B/A}$: position of B relative to the moving frame $A_{x'y'z'}$
  (or position of B relative to A)

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

Differentiating

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

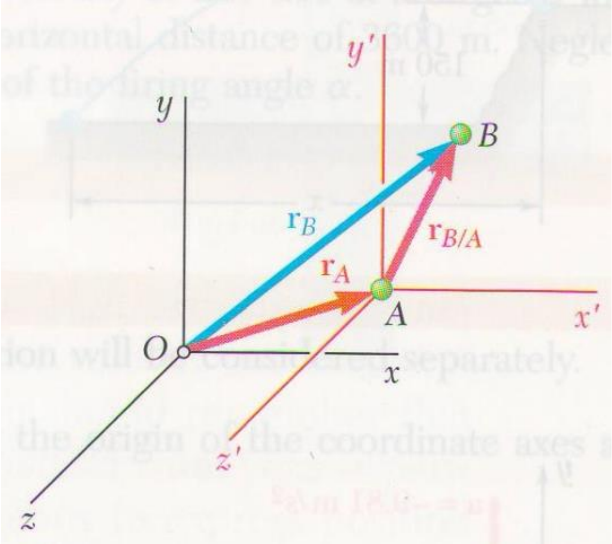


Fig. 11.18

- Absolute motion of B: B w.r.t. the fixed frame O_{xyz}
  - motion of A + relative motion of B w.r.t. the moving frame attached to A.
  - Only if the frame $A_{x'y'z'}$ is in translation
  - In the case of a rotating frame of reference, different relations must be used.

11.5A Tangential and normal component

- Acceleration is NOT tangential to the path
  ⋯ Sometimes convenient to resolve the acceleration into $\begin{cases} \text{tangent component} \\ \text{normal component} \end{cases}$

Plane Motion (1)

$\vec{e}_t$: unit vector tangent to the path of the particle attached at P

$\vec{e}_{t'}$: unit vector tangent to the path of the particle attached at P'

$$\Delta\vec{e}_t = \vec{e}_{t'} - \vec{e}_t$$

$\vec{e}_t, \vec{e}_{t'}$ are of unit length. Their tips lie on a circle of radius 1

$\Delta\theta$: angle formed by $\vec{e}_t$ and $\vec{e}_{t'}$

magnitude of $\Delta\vec{e}_t = 2\sin\left(\dfrac{\Delta\theta}{2}\right)$

- $\Delta\vec{e}_t / \Delta\theta$: tangent to the unit circle as $\Delta\theta$ approaches to zero
  - Perpendicular to $\vec{e}_{t'}$

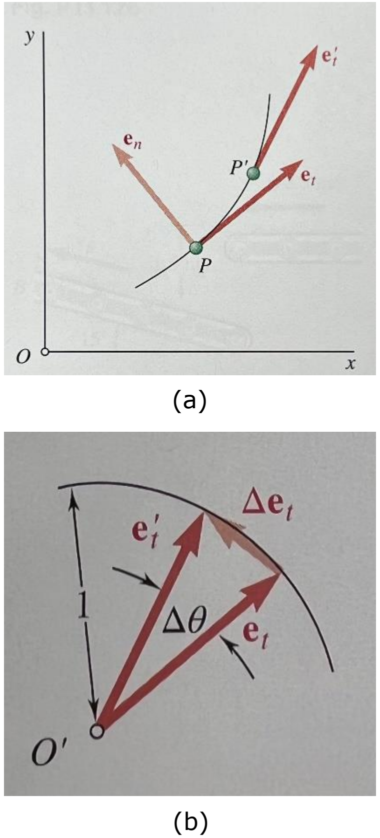


Fig. 11.19

11.5A Tangential and normal component

Plane Motion (2)

- Magnitude:

$$\lim_{\Delta\theta->0}\frac{2\sin\left(\frac{\Delta\theta}{2}\right)}{\Delta\theta}=\lim_{\Delta\theta->0}\frac{\sin\left(\frac{\Delta\theta}{2}\right)}{\Delta\theta/2}=1$$

  - Unit vector along the normal line to the path, in the direction toward which $\vec{e}_t$ turns

$$\vec{e}_n=\lim_{\Delta\theta\to0}\frac{\Delta\vec{e}_t}{\Delta\theta},\quad \mathrm{e}_n=\frac{d\vec{e}_t}{d\theta}$$

- $\vec{v}$: tangent to the path
  - $\vec{v}=v\vec{e}_t$

- Acceleration: $\vec{a}=\frac{d\vec{v}}{dt}=\frac{dv}{dt}\vec{e}_t+v\frac{d\vec{e}_t}{dt},\quad \frac{d\vec{e}_t}{dt}=\frac{d\vec{e}_t}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt}$

$\frac{ds}{dt}=v$: speed, $\frac{d\vec{e}_t}{d\theta}=\vec{e}_n$

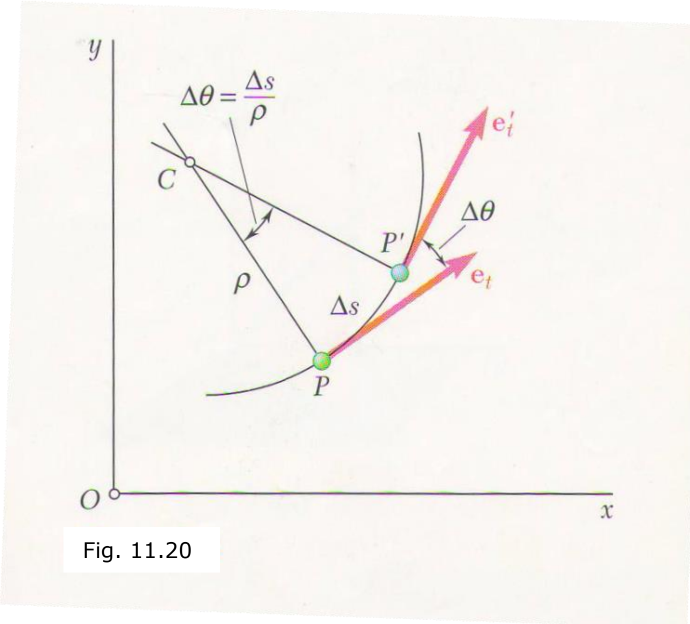


Fig. 11.20

From elementary calculus, $\frac{d\theta}{ds}=\frac{1}{\rho},\quad \rho$: radius of curvature

Plane Motion (3)

$$\frac{d\vec{e}_t}{d\theta} = \frac{v}{\rho}\vec{e}_n$$

$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n \qquad a_t = \frac{dv}{dt}\vec{e}_t, \quad a_n = \frac{v^2}{\rho}\vec{e}_n$$

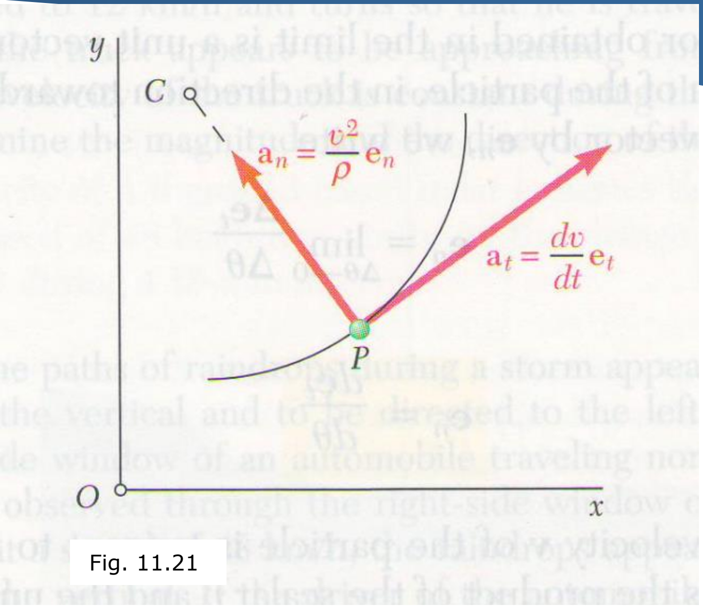


Fig. 11.21

- Tangential component: rate of change of speed
- Normal component: $(speed)^2 / (radius\ of\ \ curvature)$

Speed increases, $\vec{a}_t$ (+), points in the direction of motion

Speed decreases, $\vec{a}_t$ (-), points against the direction of motion

- $\vec{a}_n$: always directed toward the center of curvature C of the path

- Particle moving with constant speed
  - acceleration is zero if the particle happens to pass through the point of inflection (radius of curvature = ∞)
  - if the curve is a straight line

11.5A Tangential and normal component

Plane Motion (4)

- Example of avoiding sudden changes in acceleration

  i) Airplane wings: wing profiles designed without any sudden change in curvature to avoid sudden change in acceleration of air particles

  ii) Railroad curves: straight infinite radius of curvature section ~ special transition section ~ circular finite radius of curvature section

  iii) Motion of a particle in space: There are infinite number of straight lines perpendicular to the tangent

  - more precise definition needed for the direction of $\vec{e}_n$

- Osculating plane at P : P' approaches to P, the plane which fits the curve most closely in the neighborhood of P.
  - contains $\vec{e}_n$: principal normal at P
  - $\vec{e}_b = \vec{e}_t \times \vec{e}_n$: binormal at P, perpendicular to the osculating plane P

- Resolving acceleration: $\left\{ \begin{array}{l} \text{tangent component} \\ \text{principal normal component} \\ \text{no component along the binormal} \end{array} \right.$

- Certain plane motion: polar coordinate (r, θ)
  - $\vec{e}_r$: radial direction, in which P would move if r were increased and θ were kept constant
  - $\vec{e}_\theta$: transverse direction, in which P would move if θ were increased and r were kept constant

$$\frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta, \qquad \frac{d\vec{e}_\theta}{d\theta} = -\vec{e}_r$$

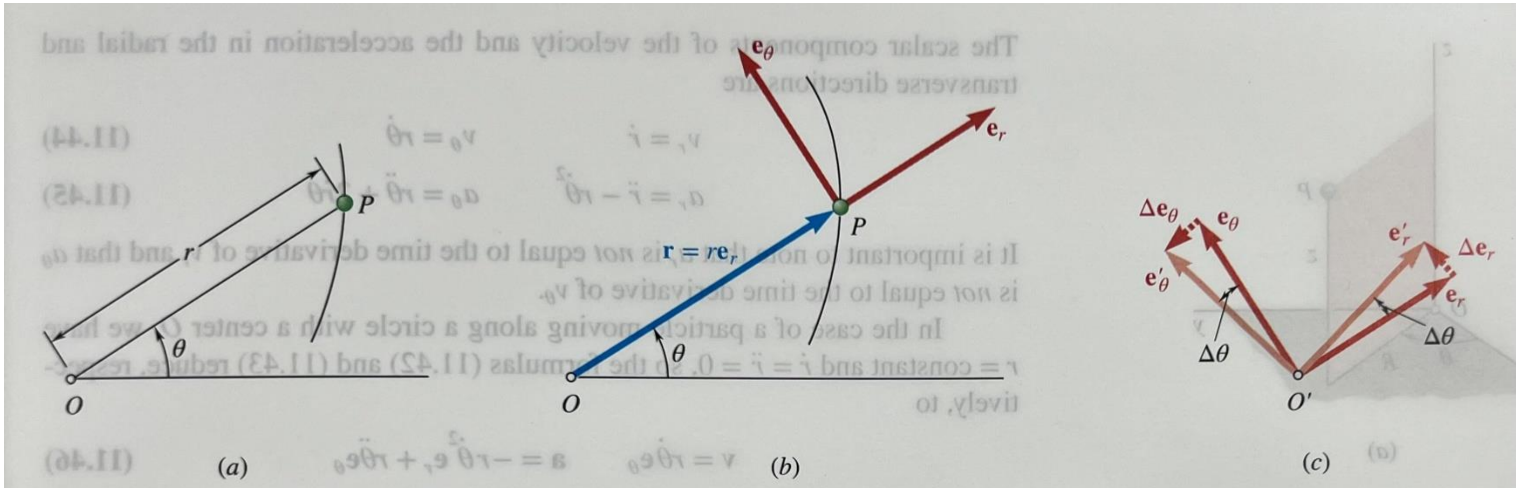


Fig. 11.23 (*a*) Polar coordinates *r* and *θ* of a particle at *P*; (*b*) radial and transverse unit vectors; (*c*) changes of the radial and transverse unit vectors resulting from a change in angle Δ*θ*.

11.5B Radial and transverse components

- Certain plane motion: polar coordinate (r,θ)
  - $\vec{e}_r$: radial direction, in which P would move if r were increased and θ were kept constant
  - $\vec{e}_\theta$: transverse direction, in which P would move if θ were increased and r were kept constant

Using the chain rule,

$$\frac{d\vec{e}_r}{dt}=\frac{d\vec{e}_r}{d\theta}\frac{d\theta}{dt}=\vec{e}_\theta\frac{d\theta}{dt},\qquad \frac{d\vec{e}_\theta}{dt}=\frac{d\vec{e}_\theta}{d\theta}\frac{d\theta}{dt}=-\vec{e}_r\frac{d\theta}{dt}$$

$$\dot{\vec{e}}_r=\dot{\theta}\vec{e}_\theta,\qquad \dot{\vec{e}}_\theta=-\dot{\theta}\vec{e}_r$$

- Velocity :

$$\vec{v}=\frac{d}{dt}\left(r\vec{e}_r\right)=\dot{r}\vec{e}_r+r\dot{\vec{e}}_r=\dot{r}\vec{e}_r+r\dot{\theta}\vec{e}_\theta$$

11.5B Radial and transverse components

- Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{r}\vec{e}_r + \dot{r}\dot{\vec{e}}_r + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}\dot{\vec{e}}_\theta$$

$$= \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

$$\Uparrow \qquad\qquad \Uparrow$$

$$r\dot{\theta}\dot{\vec{e}}_\theta \quad + \quad \dot{r}\dot{\vec{e}}_r$$

Not equal to the time derivative

- In case of a particle moving along a circle

$$r = \text{constant}, \qquad \dot{r} = \ddot{r} = 0$$

$$\vec{v} = r\dot{\theta}\vec{e}_\theta, \qquad \vec{a} = -r\dot{\theta}^2\vec{e}_r + r\ddot{\theta}\vec{e}_\theta$$

- Particle in space (Cylindrical coordinate)
  - cylindrical coordinate R, θ, z
  - position vector $\vec{r} = R\,\mathrm{e}_R + z\mathbf{k}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{R}\vec{e}_R + R\dot{\theta}\vec{e}_\theta + \dot{z}\vec{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\ddot{R} - R\dot{\theta}^2\right)\vec{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\vec{e}_\theta + \ddot{z}\vec{k}$$

Q & A