Ch. 12 Kinetics of Particles

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12.0 Introduction

Newton's { 1^{st} law → / 3^{rd} law → } motion of bodies with no acceleration

2^{nd} law → accelerated : magnitude or direction of the velocity changes

Resultant of the forces are not zero, particle will have an acceleration
ratio of the resultant force and the acceleration → mass

Linear momentum : $\vec{L} = m\vec{v}$, alternative form of Newton's 2^{nd} law

International System of Units (SI) vs. US customary units

Rectangular components vs. tangential/normal component

Radial/transverse component
angular momentum $\overrightarrow{H_0} = \vec{r} \times m\vec{v}$, another Newton's 2^{nd} law

Under central force → angular momentum about O is conserved
orbit motion under gravitational attraction

12.1A Newton's 2nd Law of Motion

If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force

Fig. 12.1 (a)~(c)

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \cdots = \text{const.} \longrightarrow \text{mass } m$$

$$\vec{F} = m\vec{a} \qquad (12.1)$$

- $\vec{F}$ and $\vec{a}$ are proportional
- $\vec{F}$ and $\vec{a}$ have the same direction

: still holds if $\vec{F}$ varies with time

they will not, in general, be tangent to the path.

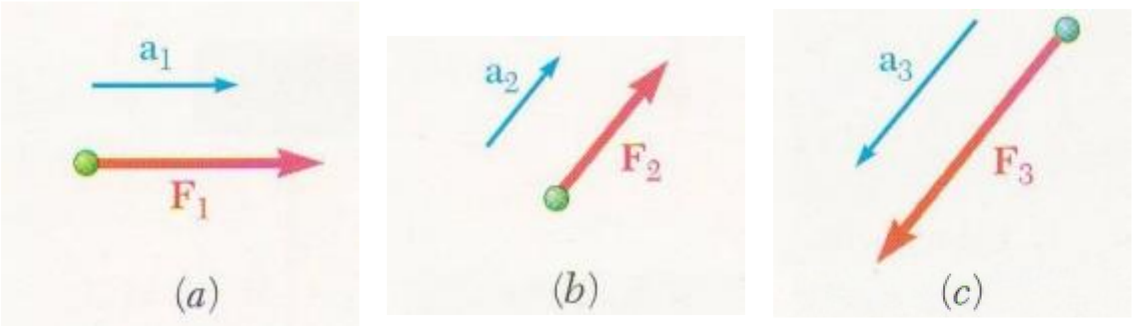

Fig. 12.1

12.1A Newton's 2nd Law of Motion

Several forces $$\sum \vec{F} = m\vec{a} \qquad (12.2)$$

↑ sum, or resultant, of all the forces

Frame of reference

: system of axes with respect to which $\vec{a}$ is determined is not arbitrary.

- must have a constant orientation with respect to the stars
- their origin must either be attached to the sun

  or move with a constant velocity w. r. t. the sun

  (sun ← mass center of the solar system)

→ Newtonian frame of reference (inertial system)

Precisely, axes attached to the earth →X Newtonian frame of reference

However, in most engineering applications, enough for (12.1) and (12.2) without any applicable error.

If $\vec{a}$ represents a relative acceleration w. r. t. moving axes, (12.1) do not hold. (e.g. attached to an accelerated car rotating piece of machinery)

$\sum \vec{F} = 0$ → if initially at rest $(\vec{v}_0 = 0)$, remain at rest originally moving with $\vec{v}_0$, maintain constant velocity (in a straight line)

→ Newton's 1st law : particular case of the 2nd law.

12.1B Linear Momentum, Rate of change of Linear Momentum

$$\vec{a} = \frac{d\vec{v}}{dt} \quad , \quad \sum \vec{F} = m\frac{d\vec{v}}{dt}$$

m is constant,

$$\sum \vec{F} = \frac{d}{dt}(m\vec{v}) \qquad (12.3)$$

$m\vec{v}$: (linear) momentum, same direction with velocity

(12.3) : the resultant of the forces acting on the particle is equal to the rate of change of the linear momentum of the particle.
(Newton's original statement regarding 2nd law)

$\vec{L}$: linear momentum

$$\vec{L} = m\vec{v}$$

$$\sum \vec{F} = \dot{\vec{L}} \qquad (12.5)$$

mass is assumed to be constant

12.1B Linear Momentum, Rate of change of Linear Momentum

(12.3), (12.5) → not applicable to the rockets, which gain or lose mass
However, applicable to relativistic mechanics, where mass is assumed to vary with the particle speed

If $\sum \vec{F} = \mathbf{0}$, rate of change of $m\vec{v}$ is zero

→ Principle of conservation of linear momentum
(alternative statement of Newton's 1st law)

12.1C System of Units

$\vec{F} = m\vec{a}$, units of force, mass, length and time cannot be chosen arbitrary

can choose three of four units arbitrarily but must choose the fourth unit

so that $\vec{F} = m\vec{a}$ is satisfied.
→ coherent set of kinetic units

International System of Units (SI Units)

length → metre (m)
mass → kilogram (kg)
time → second (s)

} arbitrarily defined

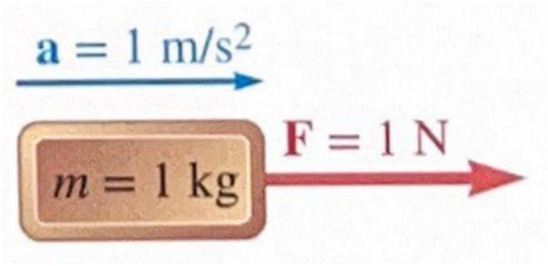

Fig. 4 A force of 1 newton gives a 1-kilogram mass an acceleration of 1 m/s^2

force → derived unit, newton (N)

$$1N = (1kg)(1m/s^2) = 1kg \cdot m/s^2$$

SI base units: chosen and defined to be independent of the location,
can be used anywhere on earth

12.1C System of Units

Weight $\vec{W}$: $W = mg$

acceleration of gravity g

$$W = (1kg)(9.81m/s^2) = 9.81N$$

Multiple/submultiples of units

$$1\ km = 1000\,m, \qquad 1\,mm = 0.001\,m$$

$$1\,Mg = 1000\,kg = 1t, \qquad 1\,g = 0.001\,kg$$

$$1\,kN = 1000\ N$$

Unit of linear momentum

$$mv = (kg)(m/s) = kg \cdot m/s$$

U.S. customary units: foot (ft), pound (lb), second (s)

$$1\ ft = 0.3048\,m, \qquad 1\,lb = 0.4536\,kgf$$

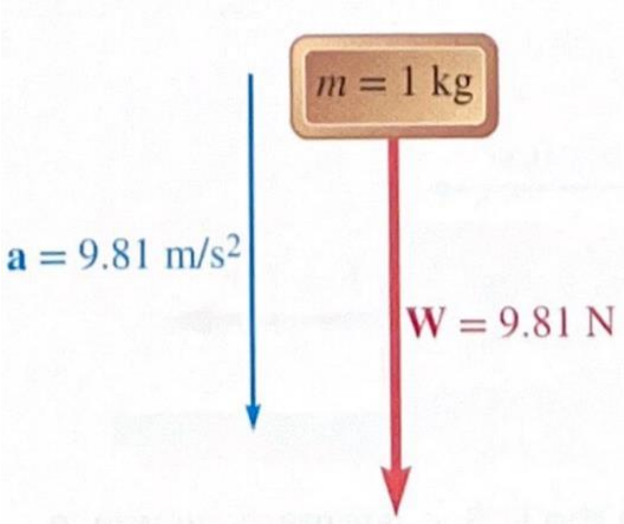

Fig. 5 In the SI system, a block with mass 1kg has a weight of 9.81N

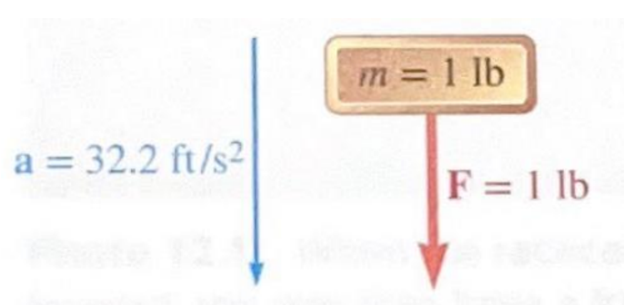

Fig. 6 In the U.S. system, a block with a weight of 1 lb in free fall has an acceleration of 32.2ft/s^2

12.1C System of Units

Unit of mass consistent with ft, lb, s

$$f = ma, \qquad 1\,lb = (1 slug)(1\,ft/s^2)$$

$$1 slug = \frac{1 lb}{1\,ft/s^2} = 1 lb \cdot s^2/\,ft$$

Slug is a mass 32.2 times larger than the mass of the standard pound.

$$m = \frac{W}{g}, mv = (slug)(\,ft/s) = (lb \cdot s^2/\,ft)(\,ft/s) = lb \cdot s$$

Conversion from one system of units to another

$$1\ \ ft = 0.3048\,m, \qquad 1\,lb = 4.448\,N$$

$$1 slug = 1 lb \cdot s^2/\,ft = 14.59 kg$$

$$1\,pound - mass = 0.4536 kg$$

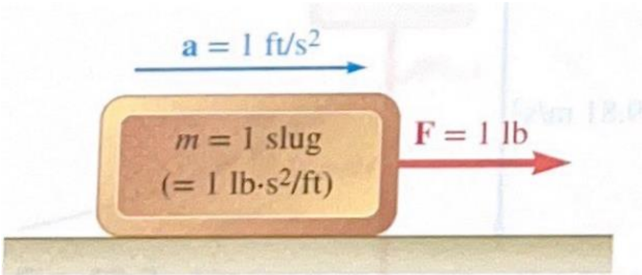

Fig. 7 In the U.S. customary system, a force of 1 lb applied to a block with a mass of 1 slug produces an acceleration of 1 ft/s^2

12.1D Eqn. of Motion

$$\vec{F} = m\vec{a}$$

- 2 important graphical tools to solve the dynamics problem using Newton's 2nd law:
  free-body diagram (FBD), kinematic diagram (KD)

FBD: - body: multiple diagrams when necessary
- axes: Cartesin, normal/tangential, radial/transverse
- support forces: 2 perpendicular forces for a pin, normal force, friction force
- applied forces and body forces: weight, magnetic forces, known pulling force
- dimension: angle, distance

KD: - body: same in FBD
- inertial term: $m\vec{a}$ consistent with the coord. System or components

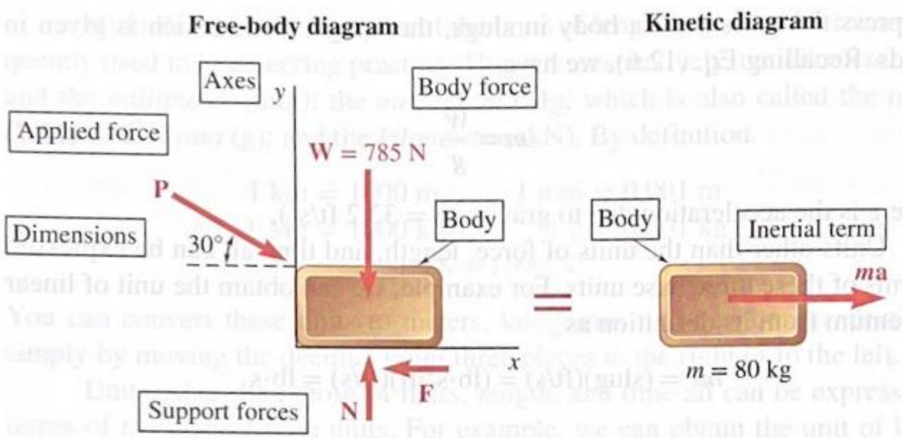

Fig. 9 Steps in drawing a free-body diagram and a kinetic diagram for solving dynamics problems

12.1D Eqn. of Motion

More convenient to replace $\vec{F} = m\vec{a}$ by equivalent equations
Involving scalar quantities

i) Rectangular components

: resolving $\vec{F}$ and $\vec{a}$ into rectangular components

$$\sum (F_x \vec{i} + F_y \vec{j} + F_z \vec{k}) = m(a_x \vec{i} + a_y \vec{j} + a_z \vec{k})$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

$$= m\ddot{x} \qquad = m\ddot{y} \qquad = m\ddot{z}$$

: example of projectile $\vec{w} = -w\vec{j}$

$$m\ddot{x} = 0, \qquad m\ddot{y} = -w, \qquad m\ddot{z} = 0$$

$$\ddot{x} = 0, \quad \ddot{y} = -\frac{w}{m} = -g, \quad \ddot{z} = 0$$

↳ can be integrated independently

: two or more bodies, → eqn. of motion should be written for each body
all accelerations should be measured w. r. t. a Newtonian frame of reference

12.1D Eqn. of Motion

ii) Tangential and Normal Components

$$\sum F_t = ma_t, \qquad \sum F_n = ma_n$$

$$= m\frac{dv}{dt} \qquad = m\frac{v^2}{\rho} \tag{12.9}$$

↳ solve for two unknowns

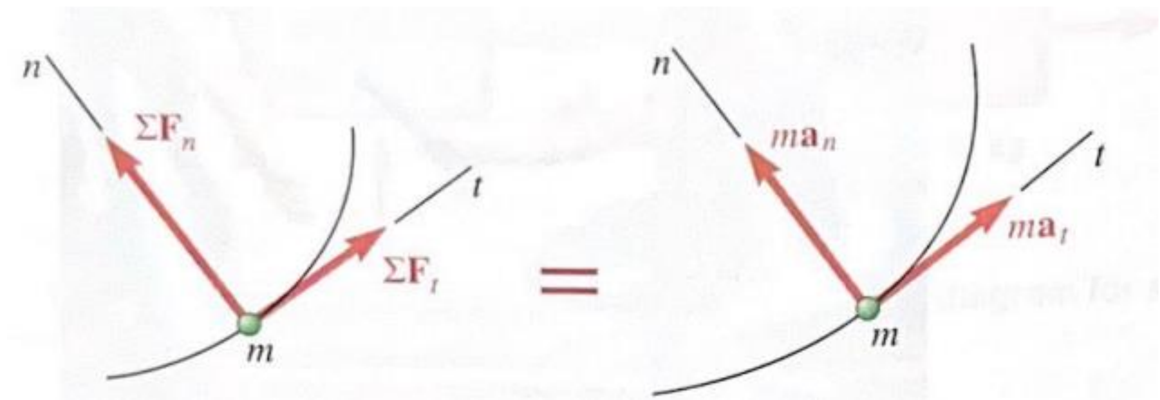

Fig. 10 The net force acting on a particle moving in a curvilinear path can be resolved into components tangent to the path and normal to the path, producing tangential and normal components of acceleration

12.1D Eqn. of Motion

iii) Radial and Transverse Components

$$\sum F_r = ma_r, \qquad \sum F_\theta = ma_\theta$$

$$= m\left(\ddot{r} - r\dot{\theta}^2\right) \qquad = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) \qquad (12.10\text{-}12)$$

→ solve for two unknowns

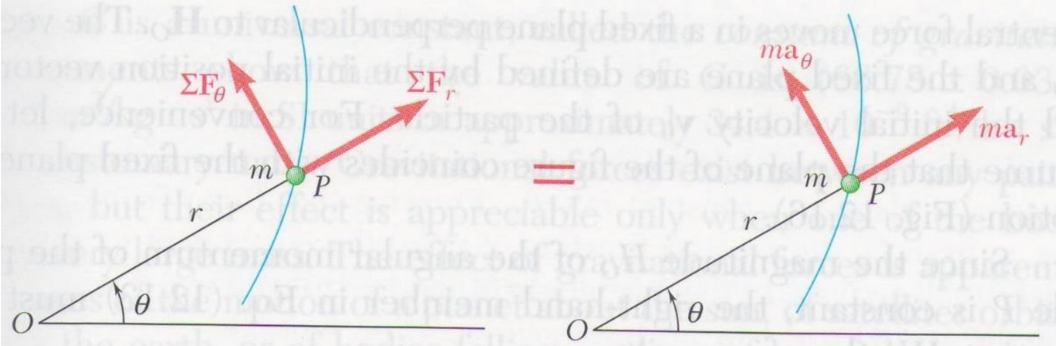

Fig. 12.14

12.1 Dynamic Equilibrium

Alternative form of Newton's 2nd Law

$$\sum \vec{F} - m\vec{a} = 0 \qquad (12.10)$$

If we add the vector $-m\vec{a}$, we obtain a system of vectors equivalent to zero (Fig. 12.8)

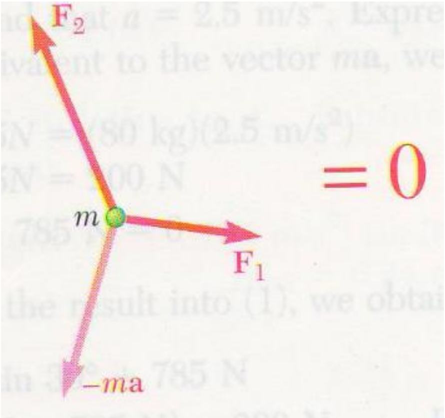

Fig. 12.8

$-m\vec{a}$: inertia vector

Dynamic equilibrium : equilibrium under the given forces and inertia vector

- Closed-vector polygon : coplanar forces

$\sum F_x = 0, \ \sum F_y = 0$ including inertia vector

- tangential and normal components :

inertia vector $\begin{cases} -m\vec{a}_t \\ -m\vec{a}_n \end{cases}$

12.1 Dynamic Equilibrium

Tangential component : measure of the resistance to a change in speed
Normal component : tendency to leave its curved path(centrifugal force)

→ Either = 0 under special conditions

i) Start from rest, initial velocity =0 → normal component of inertia vector =0
ii) Constant speed → tangential component =0

Inertia vectors is often called "inertia forces" : measure of resistance
when we set them in a motion or we try to change the conditions of motion
Inertia forces ≠ forces found in statics (e.g. contact forces, gravitational forces)

12.2A Angular Momentum

Angular momentum, moment of momentum : moment about O of the vector $m\vec{v} \rightarrow \vec{H}_o$

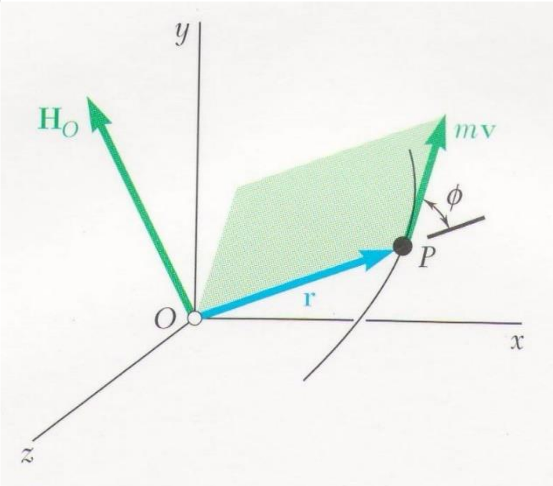

Fig. 12.12

$$\vec{H}_o = \vec{r} \times m\vec{v}$$

↳ perpendicular to the plane containing $\vec{r}$ and $m\vec{v}$

$$\left|\vec{H}_o\right| = rmv\sin\phi$$

↳ angle between $\vec{r}$ and $m\vec{v}$

unit : $(m)(kg \cdot m/s) = kg \cdot m^2/s$

Resolving into component

$$\vec{H}_o = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

$$H_x = m(yv_x - zv_y)$$
$$H_y = m(zv_x - xv_z)$$
$$H_z = m(xv_y - yv_x)$$

12.2A Angular Momentum

In case of a particle moving in the xy-plane

$$z = v_z = 0, \quad H_y = H_z = 0$$

$$H_0 = H_z = m(xv_y - yv_x)$$

↳ perpendicular to the xy-plane

(+) or (-) according to the sense in which the particle is observed to move from ***O***

Polar coordinate : $H_0 = rmv\sin\phi = rmv_\theta$ (12.18)

$$= mr^2\dot{\theta} \quad (v_\theta = r\dot{\theta})$$

Derivative w. r. t. t

$$\dot{\vec{H}}_0 = \dot{\vec{r}} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}} = \underbrace{\vec{v} \times m\vec{v}}_{0} + \vec{r} \times \underbrace{m\vec{a}}_{\sum \vec{F}}$$

$\therefore \vec{v}$ *and* $m\vec{v}$ are collinear

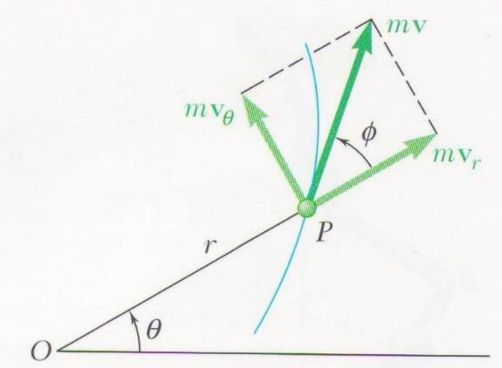

Fig. 12.13

$$\dot{\vec{H}}_0 = \vec{r} \times \sum \vec{F} = \sum \vec{M}_0$$

: sum of the moments about ***O*** of the force
= rate of change of angular momentum about ***O***

12.1D Eqn. of Motion in terms of Radial and Transverse Components

Polar coordinate r, θ

$$\sum F_r = ma_r, \qquad \sum F_\theta = ma_\theta$$

$$= m(\ddot{r} - r\dot{\theta}) \qquad = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

or recalling $\sum \vec{M}_0 = \dot{\vec{H}}_0, \quad H_0 = mr^2\dot{\theta}$

$$\sum M_0 = r\sum F_\theta$$

$$r\sum F_\theta = \frac{d}{dt}\left(mr^2\dot{\theta}\right)$$

$$= m(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta})$$

$$\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

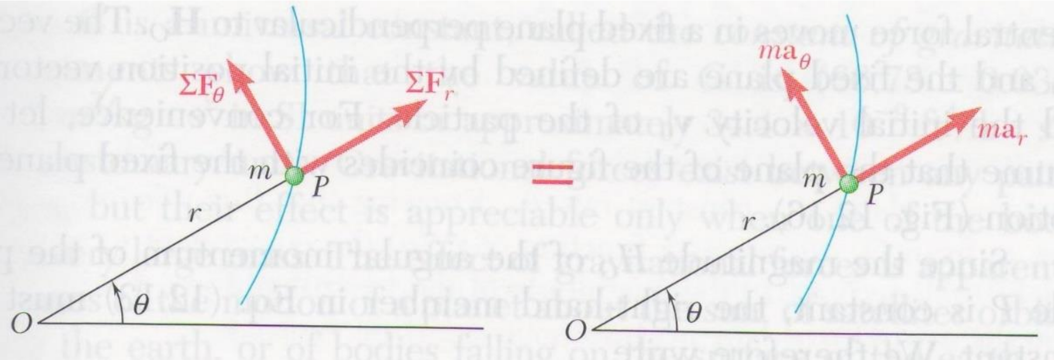

Fig. 12.14

12.2B Motion under a Central Force

Moving under a central force : force $\vec{F}$ directed toward or away from a fixed point O(center of force)

$$\rightarrow \sum \vec{M}_0 = 0$$

$$\rightarrow \dot{\vec{H}}_0 = 0, \quad \vec{H}_0 = const \qquad (12.23)$$

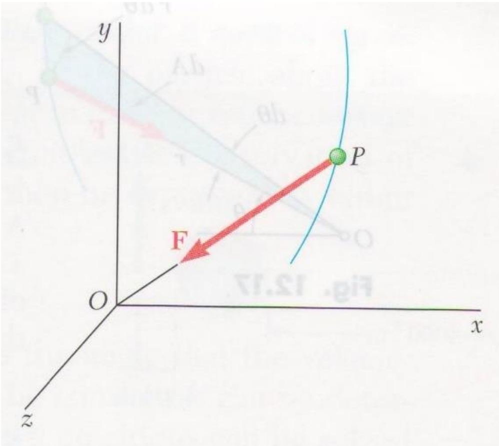

Fig. 12.15

Angular momentum of a particle moving under a central force is constant, in both magnitude and direction

$$\rightarrow \vec{r} \times m\vec{v} = \vec{H}_0 = const.$$

$\rightarrow \vec{r}$ must be perpendicular to the constant vector $\vec{H}_0$, moves in a fixed plane Perpendicular to $\vec{H}_0$

$\vec{H}_0$ and the fixed plane are defined by the initial vector $\vec{r}_0$ and $\vec{v}_0$

Since $|\vec{H}_0|$ is constant, $rmv\sin\phi = r_0 m v_0 \sin\phi_0$

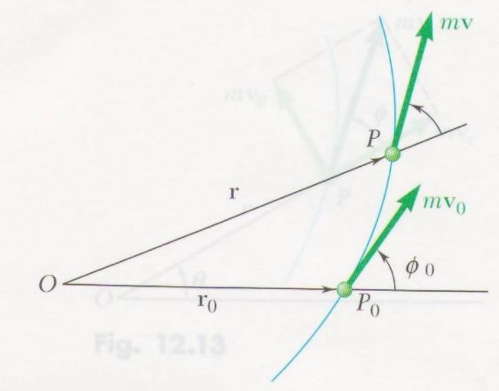

Fig. 12.16

Application : planetary motion, space vehicles in orbit about the earth

Polar coordinate

$$mr^2\dot{\theta} = H_0 = const.$$

Divide by m, h : angular momentum per unit mass H_0 / m

$$r^2\dot{\theta} = h \qquad (12.25)$$

Fig. 12. 15, radius vector ***OP*** sweeps an infinitesimal area

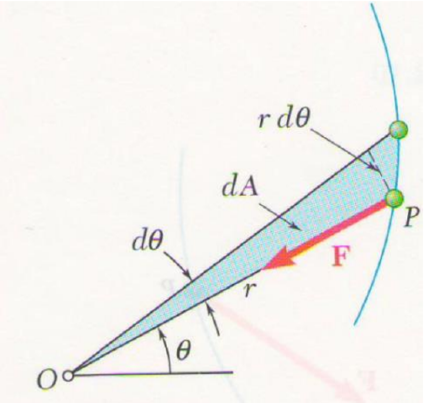

Fig. 12.15

$$dA = \frac{1}{2} r^2 d\theta$$

areal velocity $\dfrac{dA}{dt} = \dfrac{1}{2} r^2 \dfrac{d\theta}{dt}$

twice of areal velocity

- Moving under a central force, areal velocity is constant.

12.2C Newton's Law of Gravitation

Example of central force : { force exerted by the sun or the planet
force exerted by the earth or orbiting satellite }

→ how to determine the magnitude of gravitational force

Law of universal Gravitation : two particles of masses M and m at a distance r attract each other with equal and opposite forces $\vec{F}$ and $-\vec{F}$ directed along the line joining the particles.

$$F = G\frac{Mm}{r^2}$$

G : universal constant, "constant of gravitation"

$(66.73 \pm 0.03) \times 10^{-12} m^3 / kg \cdot s^2$

- Affect becomes appreciable only when one of the bodies has a very large mass

{ a planet about the sun
satellite orbiting the earth
bodies falling on the surface of the earth }

12.2C Newton's Law of Gravitation

Weight $\vec{W}$: force exerted by the earth on a body located on or near its surface

$$W = mg = \frac{GM}{R^2}m \qquad or \qquad g = \frac{GM}{R^2}$$

Earth is not truly spherical → the value of W and g vary with the altitude and latitude

Reason #2 of varying W and g : system of axes attached to the earth is NOT a Newtonian frame of reference

- include centrifugal force due to the earth rotation
  $g \simeq 9.781 m/s^2$ (at the equator), $g \simeq 9.833 m/s^2$ (at the poles)

- $g = 9.7807(1 + 0.0053 \sin^2 \phi) m/s^2$, ϕ : latitude (위도) longitude (경도)

Force exerted by the earth on a body of mass m located in space at a distance r

$$F = G\frac{Mm}{r^2}$$

Can be simplified by $GM = gR^2$

$$\left.\begin{array}{l} g : 9.81 m/s^2 \\ R : 6.37 \times 10^6 m \end{array}\right\} \leftarrow \text{average values}$$