Ch. 15 Kinematics of Rigid Bodies

Prof. SangJoon Shin

15.0 Introduction

Categories of rigid-body motion

i) Translation --- if any straight line inside the body keeps the same direction during motion

- All the particles forming the body move along parallel paths.
  straight lines → rectilinear translation (Fig. 15.1.)
  curved lines → curvilinear translation (Fig. 15.2.)

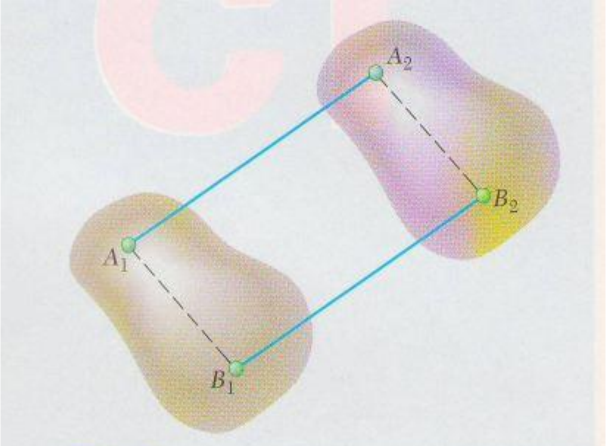


Fig. 15.1

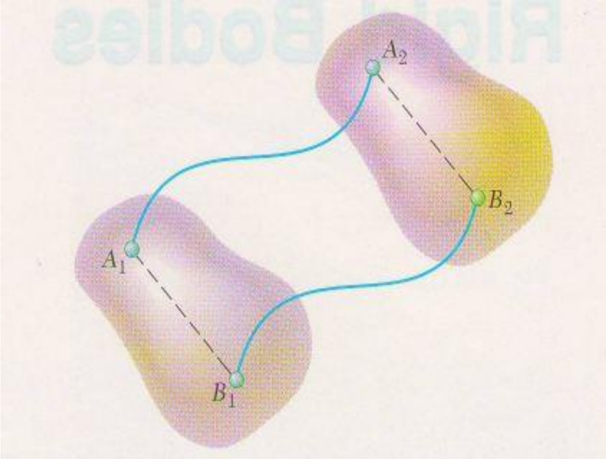


Fig. 15.2

15.0 Introduction

Categories of rigid-body motion

ii) Rotation about a fixed axis --- particles move in parallel planes along circles centered on the same axis (Fig 15.3.)

→ axis of rotation, the particles on the axis have zero velocity / acceleration.

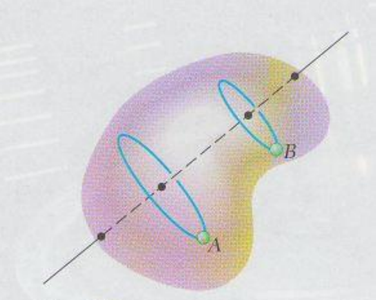


Fig. 15.3

Fig. 15.4.(a) --- curvilinear translation, all particles moving along parallel circles

(b) ---rotation, all particles moving along concentric circles.

↪ each particle moves in a given plane → plane motion

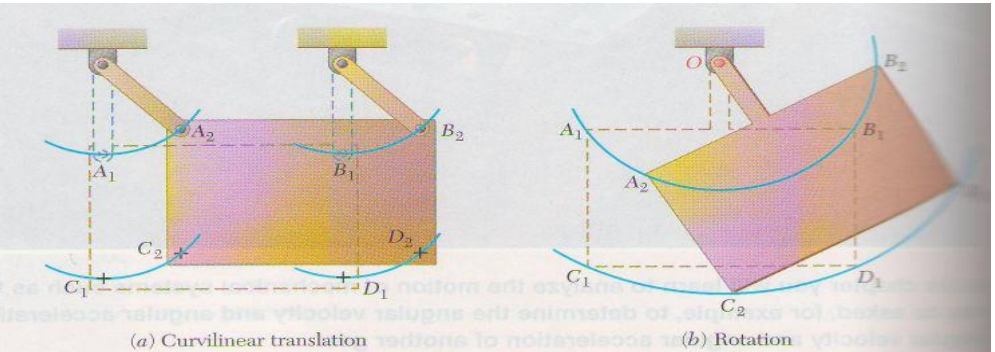


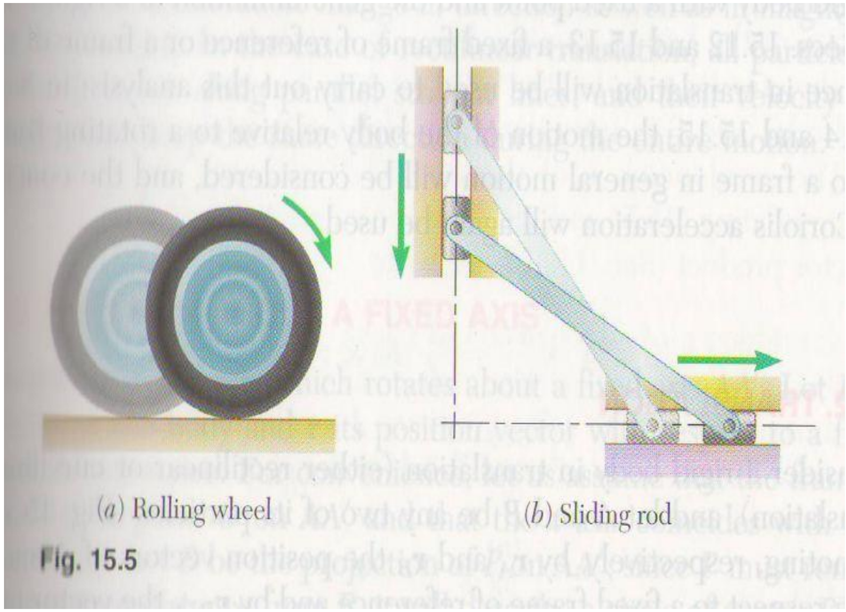
Fig. 15.4

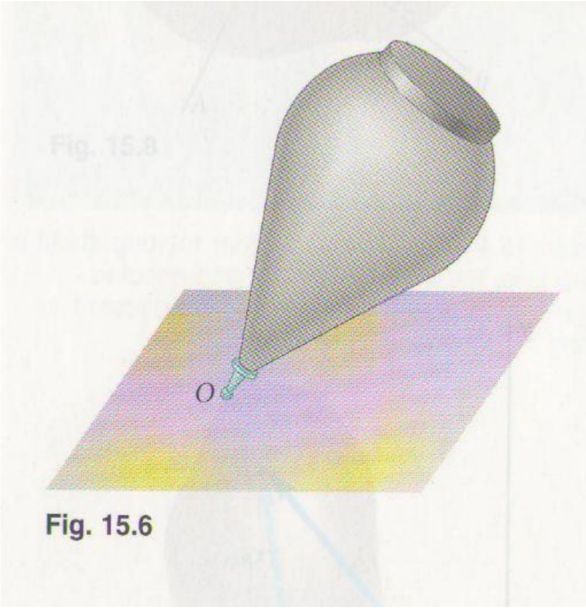
15.0 Introduction

Categories of rigid-body motion

iii) General plane motion--- all the particles move in parallel planes.
neither a rotation nor a translation
two examples ---- Fig. 15.5.

iv) Motion about a fixed point --- three-dimensional motion of a rigid body attached at a fixed point (Fig. 15.6.)





15.0 Introduction

Categories of rigid-body motion

v) General motion ---- which does not fall in any of the categories above

Rotation about a fixed axis → angular velocity, angular acceleration velocity, acceleration of a given point ---- position vector + angular { velocity / acceleration }

General plane motion --- gears, connecting rods, pin-connected linkages.

velocity of a point B of the slab --- sum of { the velocity of the ref. point A. / the velocity of B relative to a frame of ref. translating with A (Moving with A, but not rotating) }

---- same approach used for acceleration

Alternative methods --- { instantaneous center of rotation / use of parametric expressions }

Motion of a particle relative to a rotating frame of ref. , Coriolis acceleration

15.1A Translation

Rigid body in translation. A,B ; two particles in it, (Fig. 15.7.(a))

$\vec{r}_A , \vec{r}_B$ - - - position vectors of A and B with respect to fixed frame of ref.

$\vec{r}_{B/A}$ - - - vector joining A and B

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad (15.1.)$$

Differentiate w.r.t.

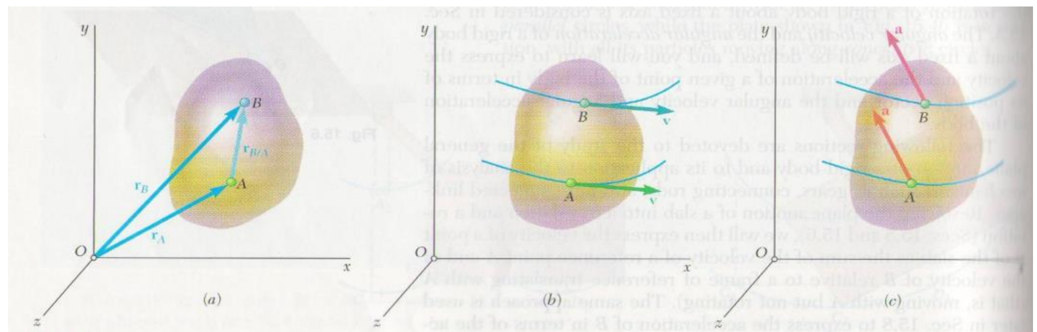


Fig. 15.7

Translation ---- > $\vec{r}_{B/A}$ must maintain a constant direction, also const. magnitude

=> $\vec{r}_{B/A} = 0$

$$\vec{v}_B = \vec{v}_A \qquad (15.2.)$$

Differentiate once more

$$\vec{a}_B = \vec{a}_A \qquad (15.3.)$$

All particles have the same velocity / acceleration at any given instant.

15.1A Translation

Curvilinear translation - - - velocity / acceleration change direction / magnitude

Rectilinear translation - - - all particles move along a straight line, velocity / acceleration keep the same direction.

15.1B Rotation about a Fixed Axis

Rigid body rotating about a fixed axis AA′

P · · · · · · point of the body, $\vec{r}$ = position vector w.r.t. a fixed frame of ref.
frame is centered at *O* on AA′, Z axis coincides with AA′ (Fig. 15.8.)

B : projection of P on AA′ , P will describe a circle of center B, radius of $r\sin\phi$, ϕ : angle formed by $\vec{r}$ and AA′.

Angular coordinate θ · · · · completely defines the position of P and the entire body positive when viewed as counterclockwise from A′

→ unit · · · radians(rad), degrees(°), revolutions(r)

$$1r = 2\pi \ rad = 360^{\circ}$$

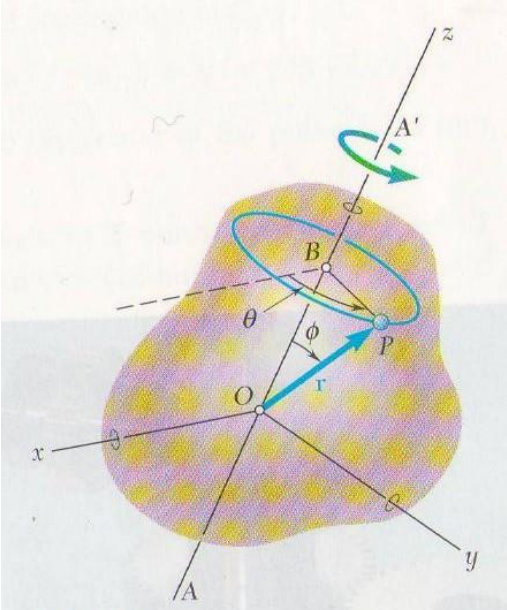


Fig. 15.8

Length of the arc Δs

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

$$v = \frac{ds}{dt} = r\dot{\theta}\sin\phi \qquad (15.4.)$$

→ Independent of P

→ Vector perpendicular to the plane containing AA′ and $\vec{r}$

$$=> \quad \vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega}\times\vec{r} \qquad (15.5.)$$

Angular velocity $\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k}$ (15.6.)

right-hand rule from the rotation of a body

obeys the parallelogram law of addition(vector quantities)

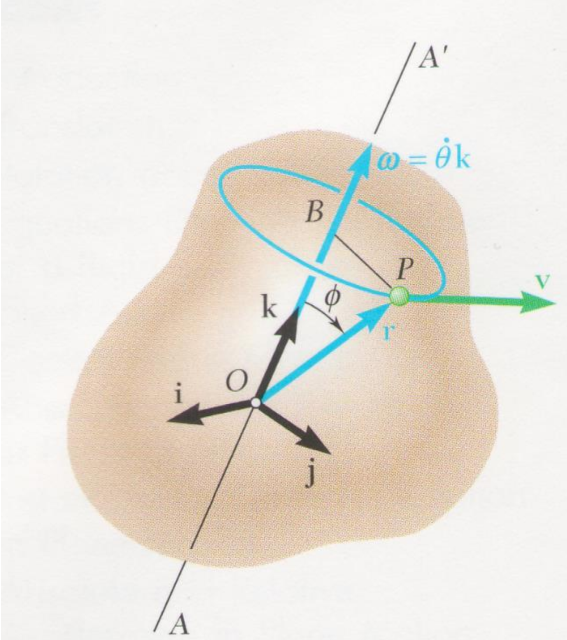


Fig. 15.9

15.1B Rotation about a Fixed Axis

Acceleration · · · · differentiate Eq.(15.5.)

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \vec{v}$$

$\vec{\alpha}$: *angular acceleration*

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) \qquad (15.8.)$$

$\vec{\alpha} \times \vec{r}$: Tangential component

$\vec{\omega} \times (\vec{\omega} \times \vec{r})$: Normal component

Angular acceleration $\vec{a} = \alpha \vec{k} = \dot{\omega} \vec{k} = \ddot{\theta} \vec{k}$ (15.9.)

along the axis of rotation

15.1B Rotation about a Fixed Axis

Rotation of a Representative slab

xy plane · · · · reference plane, z-axis - - - - axis of rotation, $\vec{\omega} = \omega\vec{k}$

(+) : counterclockwise, (-) : clockwise

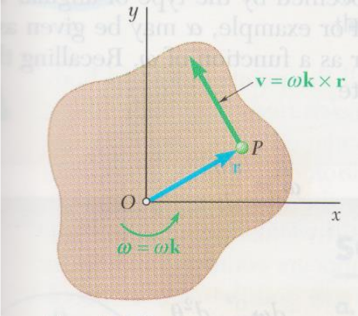


Fig. 15.10

- velocity of any given point --- $\vec{v} = \omega\vec{k}\times\vec{r}$ (15.10.)

magnitude - - - $v = r\omega$

direction - - - by rotating $\vec{r}$ through 90° in the sense of rotation

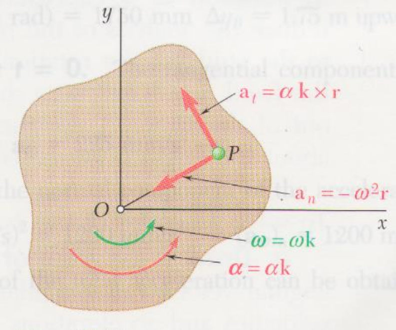


Fig. 15.11

- acceleration - - - $\vec{a} = \alpha\vec{k}\times\vec{r} - \omega^2\vec{r}$ (15.11.)

$$\vec{a}_t = \alpha\vec{k}\times\vec{r},\ \left|\vec{a}_t\right| = r\alpha$$

counterclockwise if $\alpha(+)$, *clockwise if* $\alpha(-)$

$$\vec{a}_n = -\omega^2\vec{r},\ \left|\vec{a}_t\right| = r\omega^2$$

always opposite to $\vec{r}$, *i.e., toward O*

15.1C Equations defining the Rotation about a Fixed Axis

More often, motion is specified by angular acceleration α as a function of t, or a function of θ, or ω

$$\omega = \frac{d\theta}{dt} \qquad (15.12.)$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad (15.13.)$$

(15.12.) → (15.13.)

$$\alpha = \omega \frac{d\omega}{dt} \qquad (15.14.)$$

Integration →

i) Uniform rotation --- angular acceleration = 0

$$\theta = \theta_o + \omega t \qquad (15.15.)$$

ii) Uniformly accelerated rotation --- α = constant

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad (15.16.)$$

$$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$$

15.2A General Plane Motion

General plane motion ---- always sum of a translation and a rotation.

[example]

i) Rolling wheel --- rolling motion = combination of simultaneous translation and rotation

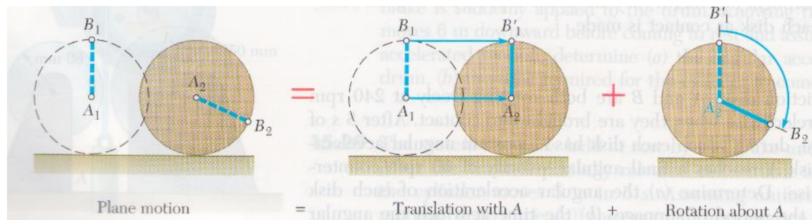


Fig. 15.12

ii) Sliding rod--- motion = translation(horizontal) + rotation about A(Fig.15.13(a)) or

translation(vertical) + rotation about B(Fig. 15.13(b))

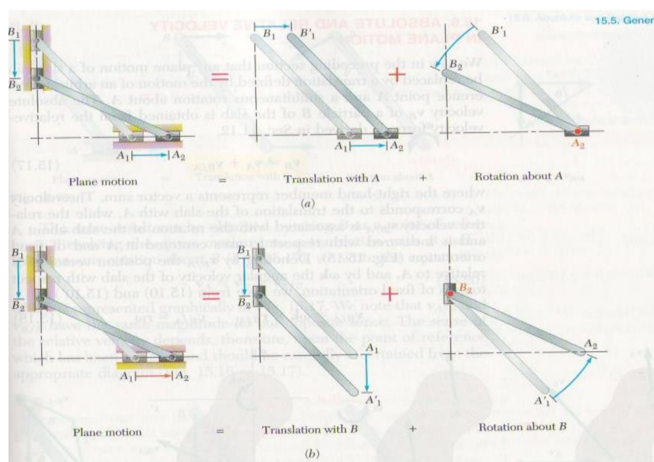


Fig. 15.13

15.2A General Plane Motion

Displacement of two particles A and B(Fig.15.14.)

two parts ---- { A1, B1 → A2, B1′ : translation
A2, B1′ → A2, B2 : rotation about A

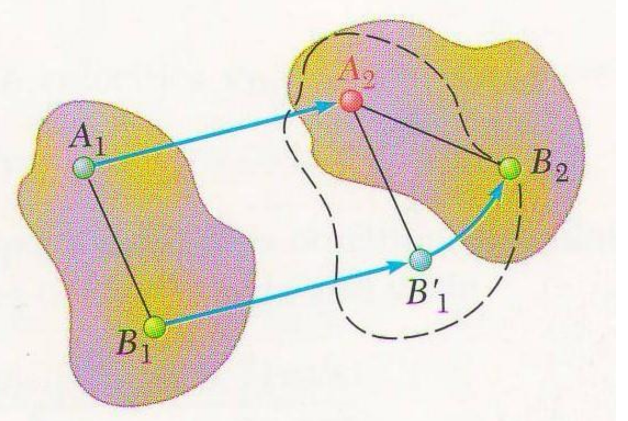


Fig. 15.14

Relative motion of B w.r.t. a frame attached at A, of fixed orientation

→ rotation, B will appear to describe an arc of circle centered at A.

15.2B Absolute velocity of a particle of the slab

$$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{v_{B/A}} \qquad (15.17)$$

$\overrightarrow{v_A}$ → translation of the slab

$\overrightarrow{v_{B/A}}$ → rotation of the slab about A (Fig.15.15)

$$\overrightarrow{v_{B/A}} = \omega\vec{k} \times \overrightarrow{r_{B/A}}, \quad \left|\overrightarrow{v_{B/A}}\right| = r\omega \qquad (15.18)$$

$\omega\vec{k}$ → angular velocity of the slab w.r.t. axes of fixed orientation

$\overrightarrow{r_{B/A}}$ → position vector of B relative to A

(15.15) → (15.17) = $\overrightarrow{v_B} = \overrightarrow{v_A} + \omega\vec{k} \times \overrightarrow{r_{B/A}}$ (15.17')

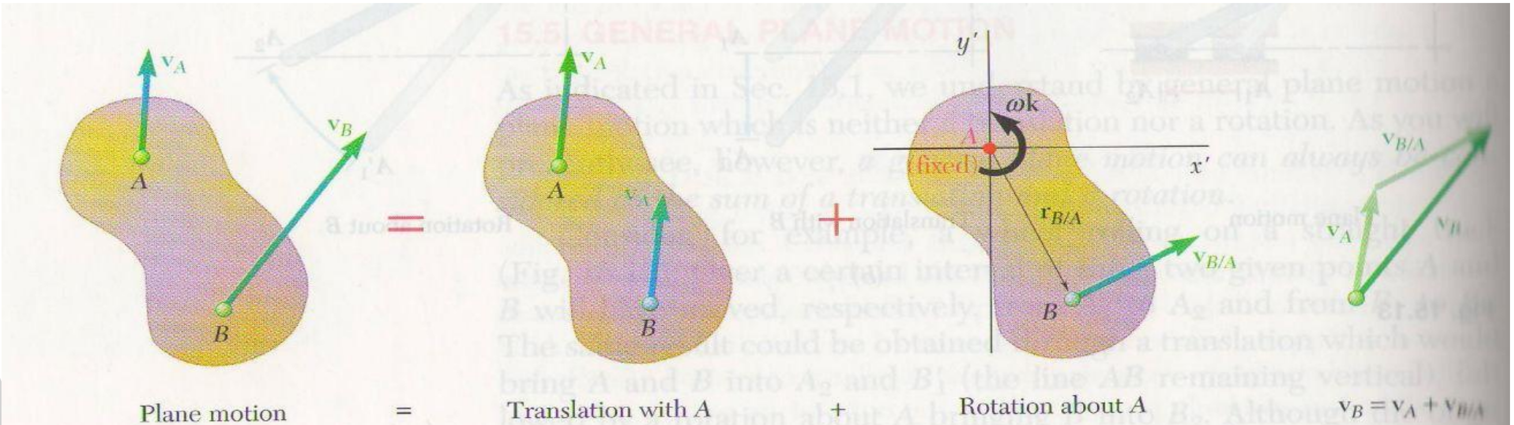


Fig. 15.15

15.2B Absolute velocity of a particle of the slab

Sliding rod of Fig. 15.13

--- $\vec{v}_A$ known, fixed $\vec{v}_B$, $\vec{\omega}$ in terms of $\vec{v}_A$, l, θ

motion = translation with A + simultaneous rotation about A (Fig.15.16.)

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad (15.17)$$

$\vec{v}_{B/A}$ direction is known, but its magnitude $l\omega$ is unknown.

$\leftarrow \vec{v}_B$ direction is known

$$v_B = v_A \tan\theta, \qquad \omega = \frac{v_{B/A}}{l} = \frac{v_A}{l\cos\theta} \quad (15.19)$$

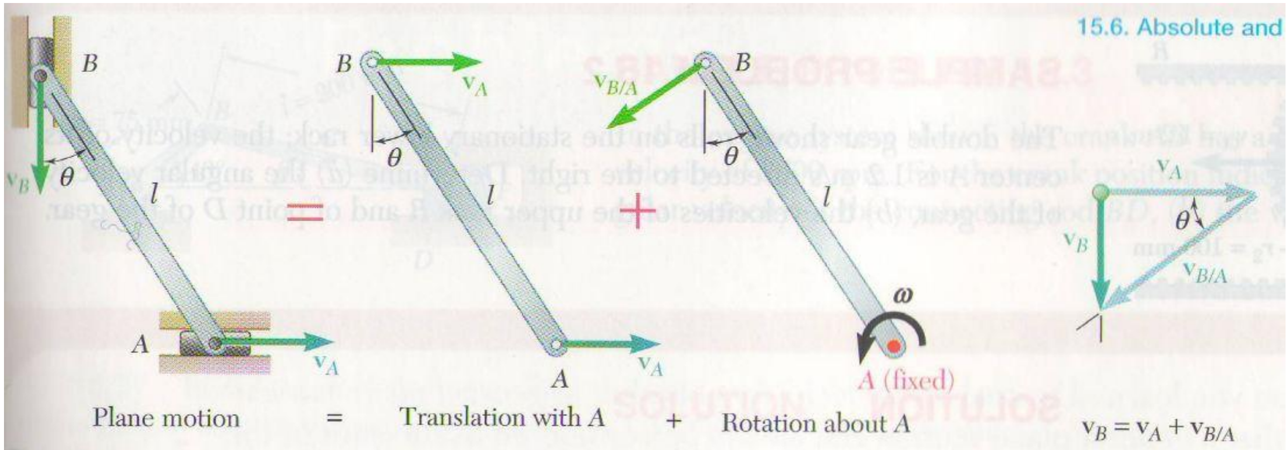


Fig. 15.16

15.2B Absolute velocity of a particle of the slab

---- same result obtained by using B as a point of ref. (Fig.15.17.)

$\rightarrow \quad \vec{v}_{A/B}, \ \vec{v}_{B/A}$: same magnitude, opposite sense

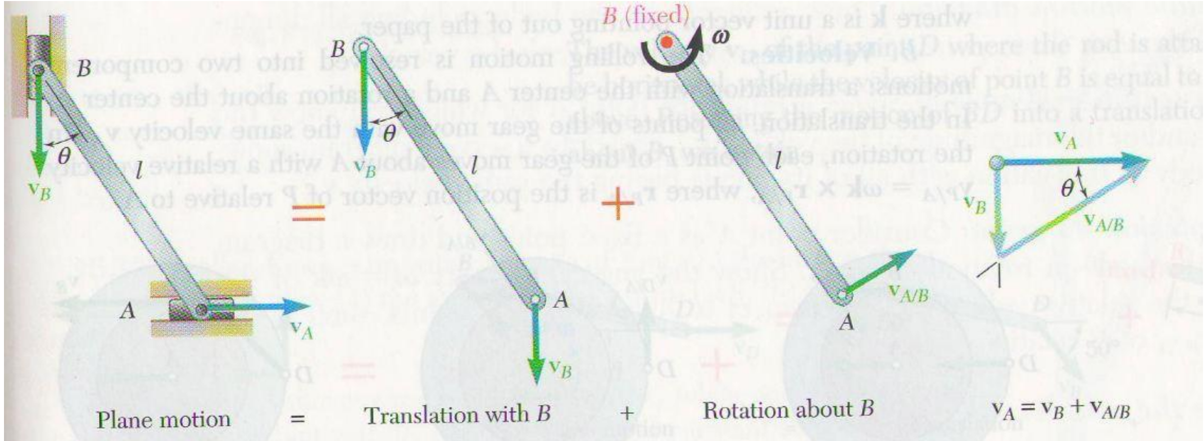


Fig. 15.17

Angular velocity $\vec{\omega}$ --- same no matter which the ref. is at a or b.

→ angular velocity $\vec{\omega}$ of a rigid body in plane motion is independent of its ref. point.

15.2B Absolute velocity of a particle of the slab

---- same result obtained by using B as a point of ref. (Fig.15.17.)

Several moving parts --- pin- connected…

i) The points where two parts are connected must have the same absolute velocity
ii) Gears---- teeth in contact must have the same absolute velocity
iii) Parts which slide on each other --- relative velocity must be considered.

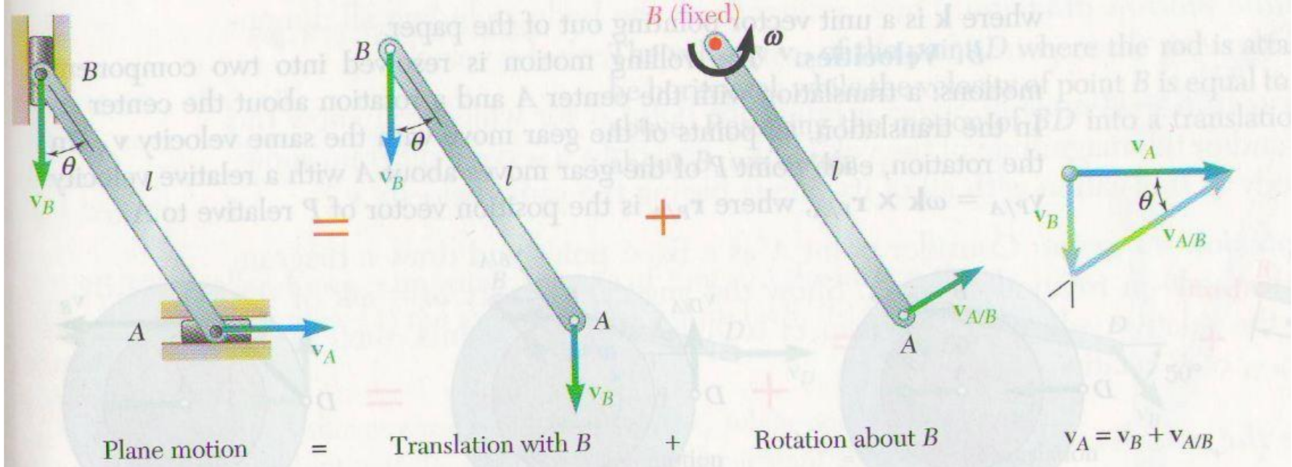


Fig. 15.17

At any given instant, the velocities of the various particles of the slab are the same as if the slab were rotating about a certain axis → " instantaneous axis of rotation",

C : "instantaneous center of rotation"

---- $\vec{v}_A, \vec{\omega}$ can be obtained by rotating the slab with $\vec{\omega}$ at a distance

$r = v_A / \omega$ from A (Fig.15.18(b))

The velocities of all the other particles would be the same as originally defined.

→ As far as the velocities are concerned , the slab seems to rotate about the instant considered.

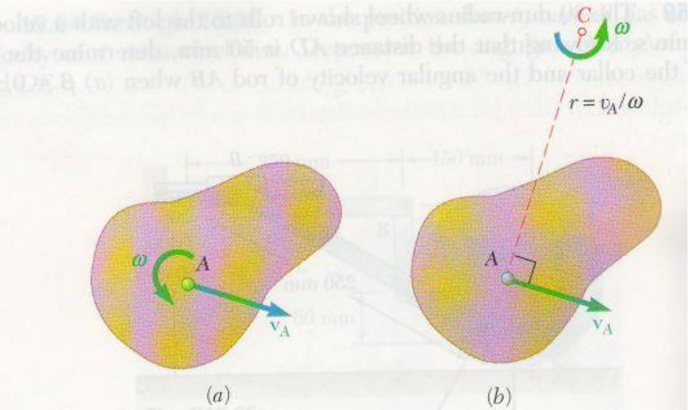


Fig. 15.18

15.3 Instantaneous center of rotation in plane motion

Two ways to define the instantaneous center C

i) Directions of $\vec{v}_A$, $\vec{v}_B$ are known and different –--- drawing perpendicular lines to $\vec{v}_A$ and $\vec{v}_B$, the point where the two lines intersect (Fig.15.19(a))

ii) $\vec{v}_A$, $\vec{v}_B$ are perpendicular to $\overline{AB}$ and their magnitudes are known ---- by intersecting $\overline{AB}$ with the line joining the extremities of $\vec{v}_A$, $\vec{v}_B$ (Fig.15.19(b))

iii) IF $\vec{v}_A$ and $\vec{v}_B$ were parallel in Fig.15.19(a)) or if $\vec{v}_A$, $\vec{v}_B$ had the same magnitude in Fig.15.19(b)) ------ C would be at an infinite distance, $\vec{\omega}$ would be zero.

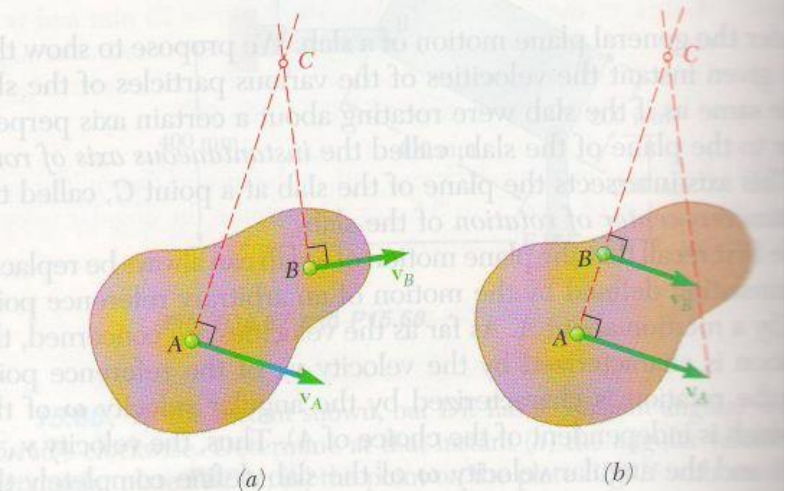


Fig. 15.19

15.3 Instantaneous center of rotation in plane motion

Sliding rod with the instantaneous center

C can be obtained by drawing perpendicular to $\vec{v}_A$ and $\vec{v}_B$

The velocities of all the particles are the same as if the rod rotated about C.

if v_A is known , $\omega = \dfrac{v_A}{AC} = \dfrac{v_A}{l\cos\theta}$

$$v_B = (BC)\omega = l\sin\theta\frac{v_A}{l\cos\theta} = v_A\tan\theta$$

Only absolute velocities are involved.

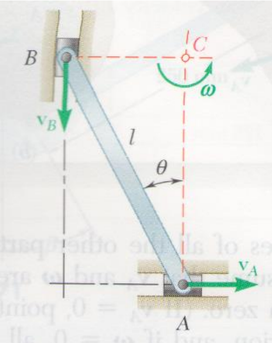


Fig. 15.20

C inside the rigid body ---- at that instant, its velocity is zero. But will probably different from zero at $t + \Delta t$

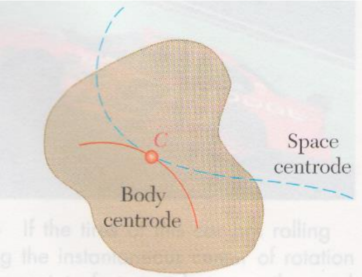


Fig. 15.21

⇒ C doesn't have zero acceleration.

⇒ Acceleration can not be determined as if the slab were rotating about C

15.4A Abs. and Rel. Acceleration in Plane Motion

Any plane motion = a translation of an arbitrary ref. point + simultaneous rotation

→ determine the acceleration of the points of the slab

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad (15.21.)$$

relative acceleration $\vec{a}_{B/A}$ — tangential component $(\vec{a}_{B/A})_t$, normal component $(\vec{a}_{B/A})_n$ (Fig 15.22.)

$$(\vec{a}_{B/A})_t = \alpha\vec{k} \times \vec{r}_{B/A}, \quad |(\vec{a}_{B/A})_t| = r\alpha \qquad (15.22.)$$

$$(\vec{a}_{B/A})_n = -\omega^2 \vec{r}_{B/A}, \quad |(\vec{a}_{B/A})_n| = r\omega^2 \qquad (15.22.)$$

$$(15.21.) \rightarrow \quad \vec{a}_B = \vec{a}_A + \alpha\vec{k} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A} \qquad (15.21')$$

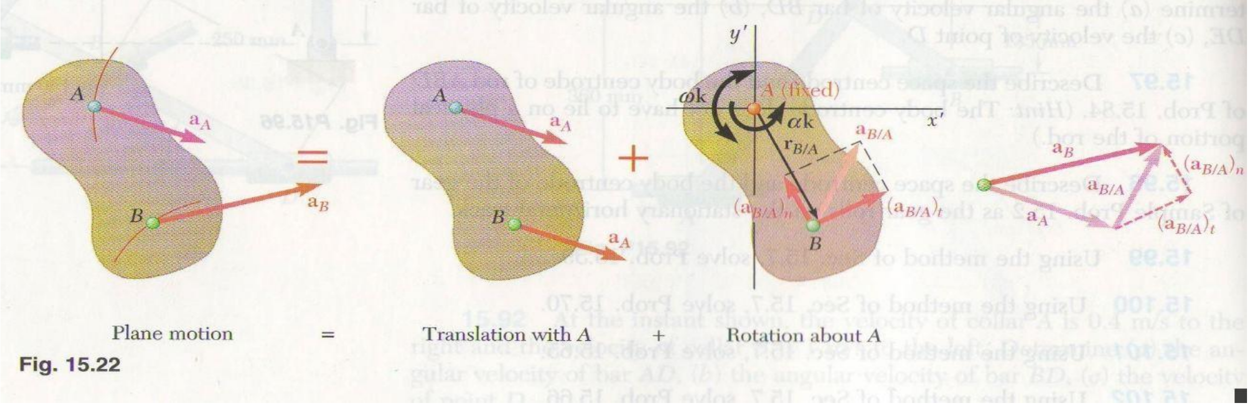


Fig. 15.22

15.4A Abs. and Rel. Acceleration in Plane Motion

Sliding AB(Fig. 15.23.)

$\vec{v}_A$, $\vec{a}_A$ known , determine $\vec{a}_B$, $\vec{\alpha}$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_A + (\vec{a}_{B/A})_n + (\vec{a}_{B/A})_t \quad (15.23.)$$

$(\vec{a}_{B/A})_n$: $l\omega^2$ Toward A

$(\vec{a}_{B/A})_t$: $l\alpha$,but no way to tell it is directed to left or right

Both possible sense of $\vec{a}_B$ in Fig. 15.23

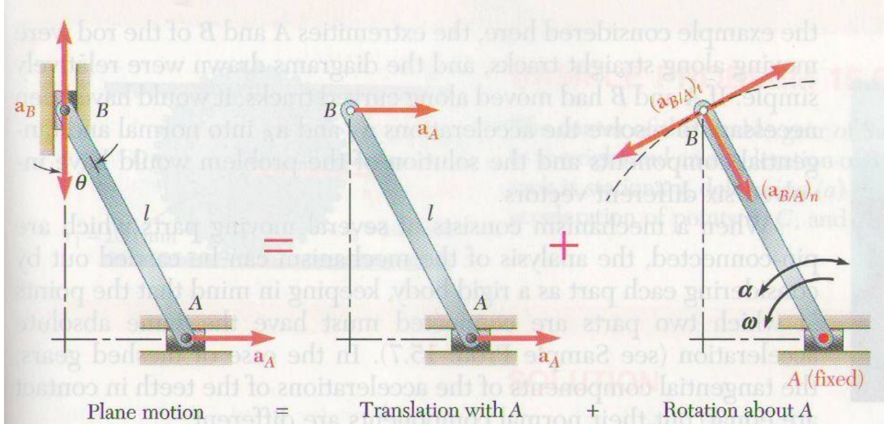


Fig. 15.23

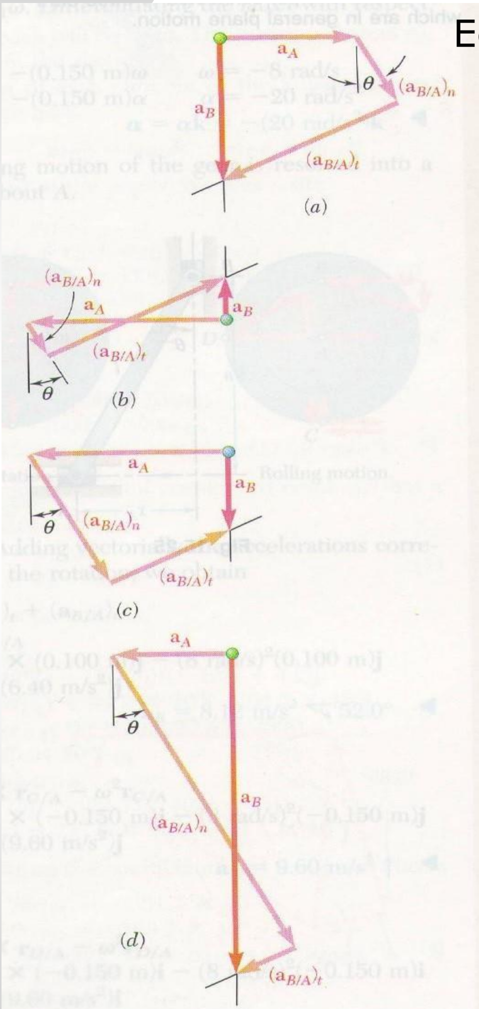


Fig. 15.24

Eq. (15.23) → Fig. 15.24

four possible vector polygons --- - depending upon the sense of $\vec{a}_A$

and relative magnitude of a_A and $(a_{B/A})_n$

ω also has to be known ← either method from Sec. 15.2 or 15.3.

Then, a_B and α can be obtained by x and y components.

From Fig.15.24(a),

$$\xrightarrow{+}\ x\ component\ :\quad 0 = a_A + l\omega^2 \sin\theta - l\alpha\cos\theta$$

$$+\uparrow\quad y\ component\ :\quad -a_B = -l\omega^2\cos\theta - l\alpha\sin\theta$$

Or, direct measurement on the vector polygon.

(careful on $\vec{a}_A$ and $\overrightarrow{(a_{B/A})_n}$)

15.4A Abs. and Rel. Acceleration in Plane Motion

If the extremities were moving along curved tracks, necessary to resolve

To normal, tangential components → six different vectors.

Several moving parts ------ the point where two parts are connected

→ must have the same absolute acceleration

* Meshed gears ----- teeth in contact → { the same tangential acceleration; normal component --- different }

15.5A Rate of change of a vector w.r.t. a Rotating Frame

Sec 11.4B --- rate of change of a vector is the same w.r.t. { a fixed frame / a frame in translation }

how about w.r.t. a rotating frame of reference?

Fig.15.26. ---- fixed frame *OXYZ*

rotating frame *Oxyz*, $\vec{\Omega}$: angular velocity

$\vec{Q}(t)$: vector function

$$\begin{cases} (\dot{\vec{Q}})_{OXYZ} \\ (\dot{\vec{Q}})_{Oxyz} \end{cases}$$

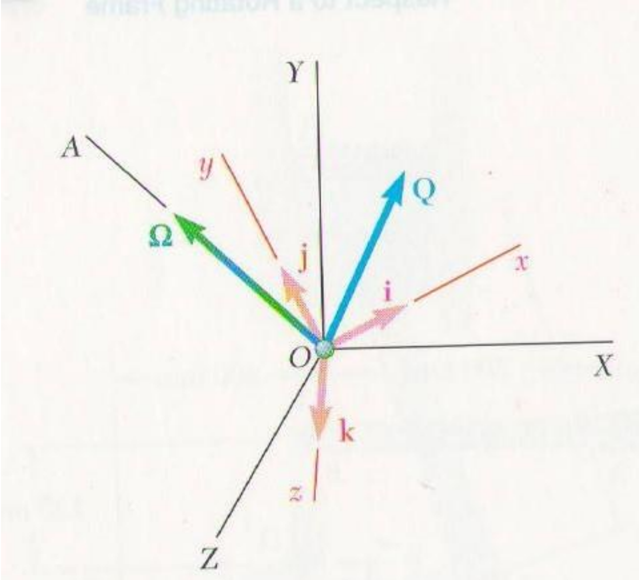


Fig. 15.26

15.5A Rate of change of a vector w.r.t. a Rotating Frame

$$\vec{Q} = Q_x \vec{i} + Q_y \vec{j} + Q_z \vec{k} \quad (\vec{i},\ \vec{j},\ \vec{k} : unit\ \ vectors) \qquad (15.27)$$

Differentiate, rate of change of $\vec{Q}$, w.r.t. rotating frame *Oxyz*

$$(\dot{\vec{Q}})_{Oxyz} = \dot{Q}_x \vec{i} + \dot{Q}_y \vec{j} + \dot{Q}_z \vec{k} \qquad (15.28)$$

Differentiate $\vec{Q}$, w.r.t. the fixed frame *OXYZ*, $\vec{i}$, $\vec{j}$, $\vec{k}$ are variable

$$(\dot{\vec{Q}})_{OXYZ} = \underbrace{\dot{Q}_x \vec{i} + \dot{Q}_y \vec{j} + \dot{Q}_z \vec{k}}_{(\dot{\vec{Q}})_{Oxyz}} + \underbrace{Q_x \frac{d\vec{i}}{dt} + Q_y \frac{d\vec{j}}{dt} + Q_z \frac{d\vec{k}}{dt}}_{\text{velocity of a particle at the tip of } \vec{Q}} \qquad (15.29)$$

velocity of a particle at the tip of $\vec{Q}$

$$= \vec{\Omega} \times \vec{Q} \qquad (15.30)$$

$$(\dot{\vec{Q}})_{OXYZ} = (\dot{\vec{Q}})_{Oxyz} + \vec{\Omega} \times \vec{Q} \qquad (15.31)$$

- $(\dot{\vec{Q}})_{Oxyz}$: Rate of change of $\vec{Q}$ *w.r.t. rotating frame Oxyz*
- $\vec{\Omega} \times \vec{Q}$: Induced by the rotation of Oxyz

15.5B Plane Motion of a Particle Relative to a Rotating Frame

(15.29)의 부연설명

$(\dot{\vec{Q}})_{OXYZ}$ 는 만일, 벡터 Q가 frame $Oxyz$에 fixed 되어 있어 $(\dot{\vec{Q}})_{Oxyz} = 0$

이 된다면 (15.29)식의 마지막 3개 항과 같이 됨.

그러한 경우 $(\dot{\vec{Q}})_{OXYZ}$ 는 $\vec{Q}$ 이 tip에 위치하여 있는 particle의 velocity를 나타내게 되며,

이는 frame Oxyz에 rigidly attached 되어 있는 질점의 velocity를 의미함.

Frame $Oxyz$는 Frame $OXYZ$ 에 대하여 $\vec{\Omega}$ 의 angular velocity로 회전하고 있으므로

마지막 3 개 항$= \vec{\Omega} \times \vec{Q}$

15.5A Rate of change of a vector w.r.t. a Rotating Frame

When $\vec{Q}$ is represented by the unit vectors in *Oxyz*, Eq.(15.31) simplifies the determination of $(\dot{\vec{Q}})_{OXYZ}$, since no need to separate computation of the derivatives of the unit vectors defining the rotating frame.

15.5B Plane Motion of a Particle Relative to a Rotating Frame

Fig. 15.27. ---- two frames of ref, in the plane of the figure

fixed frame *OXY*, rotating frame *Oxy*

$(\dot{\vec{r}})_{OXY}$: rate of change of $\vec{r}$, w.r.t. a fixed frame

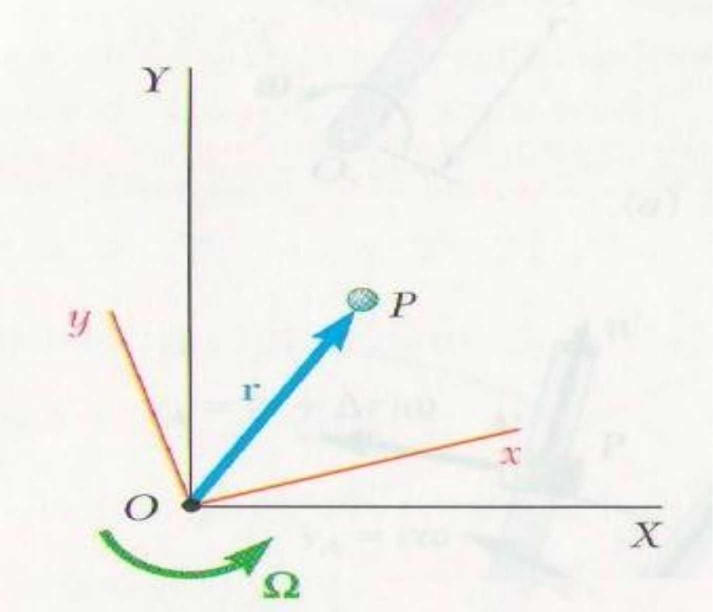


Fig. 15.27

15.5B Plane Motion of a Particle Relative to a Rotating Frame

$(\vec{r})_{Oxy}$: w.r.t. the rotating frame Oxy

$\vec{\Omega}$: angular velocity of the frame *Oxy* w.r.t. *OXY*

$$\vec{v}_P = (\dot{\vec{r}})_{OXY} = \vec{\Omega} \times \vec{r} + (\dot{\vec{r}})_{Oxy} \qquad (15.32)$$

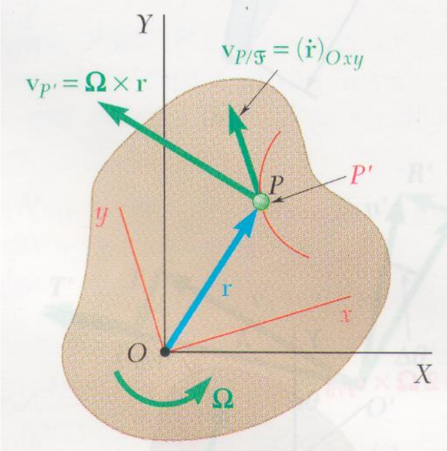


Fig. 15.28

$\vec{\Omega} \times \vec{r} = \vec{v}_{P'}$: *P′* velocity in the slab which coincides with *P* at the instant

$(\dot{\vec{r}})_{Oxy} = \vec{v}_{P/F}$

F : *rotating frame*

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/F} \qquad (15.33)$$

- $\vec{v}_P$: abs. velocity of *P*
- $\vec{v}_{P'}$: vel. of *P′* of moving frame *F* coinciding with *P*
- $\vec{v}_{P/F}$: vel. of *P* relative to *F*

Absolute acceleration --- rate of change of $\vec{v}_P$, w.r.t. OXY

$$\vec{a}_P = \dot{\vec{v}}_P = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \dot{\vec{r}} + \frac{d}{dt}[(\dot{\vec{r}})_{Oxy}] \quad (15.34)$$

$\dot{\vec{r}} = \vec{v}_P = \vec{\Omega} \times \vec{r} + (\dot{\vec{r}})_{Oxy}$ by Eq.(15.32)

$\frac{d}{dt}[(\dot{\vec{r}})_{Oxy}] = (\ddot{\vec{r}})_{Oxy} + \vec{\Omega} \times (\dot{\vec{r}})_{Oxy}$

$$= \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + 2\vec{\Omega} \times (\dot{\vec{r}})_{Oxy} + (\ddot{\vec{r}})_{Oxy}$$

$\vec{a}_{P'}$; $\vec{a}_C$: { complementary acceleration / Coriolis acceleration ; $\vec{a}_{P/F}$

(15.36)

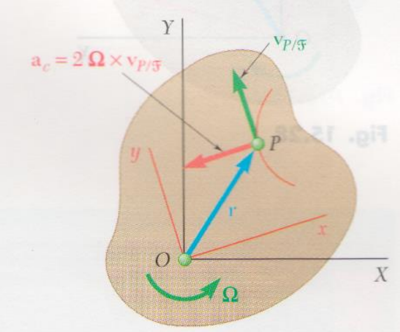


Fig. 15.29

$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/F} + \vec{a}_C$$

- $\vec{a}_P$ → abs. accel of *P*
- $\vec{a}_{P'}$ → accel. of *P′* of moving frame *F* coinciding with *P*
- $\vec{a}_{P/F}$ → accel. of *P′* of relative to *F*
- $\vec{a}_C$ → $2\vec{\Omega} \times \vec{v}_{P/F}$

Compared with Eq. (15.21) $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{a}_{B/A}$ → accel. w.r.t. a frame in translation

→ if the frame is rotating, necessary to include Coriolis' acceleration $\vec{a}_C$

Direction of $\vec{a}_C$

$\left|\vec{a}_C\right| = 2\Omega v_{P/F}$, rotating $\vec{v}_{P/F}$ through 90°

in the sense of rotation of the moving frame (Fig. 15.29)

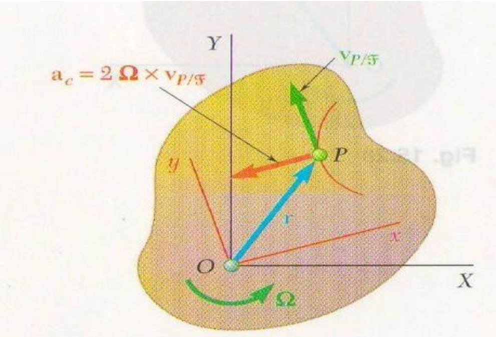


Fig. 15.29

15.5B Plane Motion of a Particle Relative to a Rotating Frame

Significance of $\overrightarrow{a_C}$

Abs. velocity of *P* at time t and $t + \Delta t$ (Fig. 15.30(b))

At t, velocity components $\overrightarrow{u}$, $\overrightarrow{v_A}$, at $t + \Delta t$ $\overrightarrow{u'}$,$\overrightarrow{v_{A'}}$

- Fig 15.30(c) , change in velocity during $\Delta t \rightarrow \overrightarrow{RR'}$, $\overrightarrow{TT''}$, $\overrightarrow{T''T'}$
- $\overrightarrow{TT''}$ ---- change in the direction of $\overrightarrow{v_A}$, $\overrightarrow{TT''}/\Delta t$ represents

$\overrightarrow{a_A}$ as $\Delta t \rightarrow 0$

$$\lim_{t\to 0}\frac{TT''}{\Delta t} = \lim_{t\to 0} v_A \frac{\Delta\theta}{\Delta t} = r\omega\omega = r\omega^2 = a_A$$

$\overrightarrow{RR'}$ --- change in direction of $\vec{u}$ due to the rotation

$\overrightarrow{T''T'}$ ---change in magnitude of $\overrightarrow{v_A}$ due to the motion of *P* along the rod

"combined effect of the relative motion of *P* and of the rotation of the rod

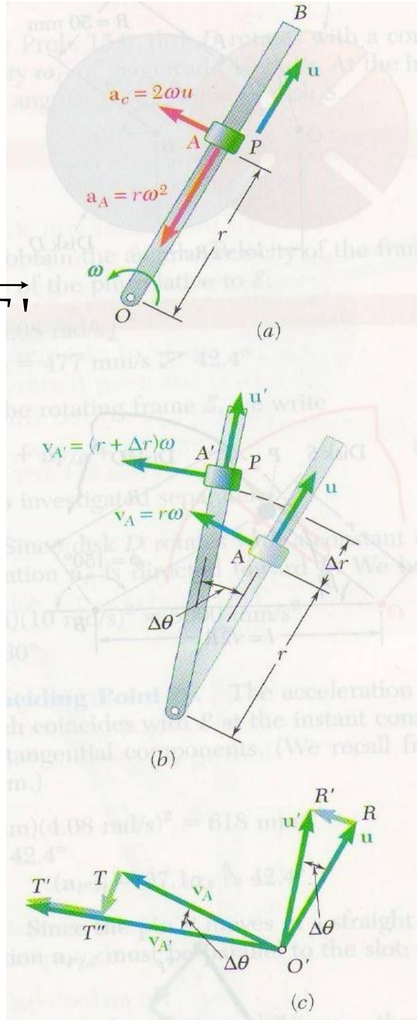


Fig. 15.30

15.5B Plane Motion of a Particle Relative to a Rotating Frame

- sum of these two → $\overrightarrow{a_C}$

$$\overrightarrow{RR'} = u\Delta\theta, \; T''T' = v_{A'} - v_A = (r+\Delta r)\omega - r\omega = \omega\Delta r$$

$$\lim_{t\to 0}\left(\frac{RR'}{\Delta t} + \frac{T''T'}{\Delta t}\right) = \lim_{t\to 0}\left(u\frac{\Delta\theta}{\Delta t} + \omega\frac{\Delta r}{\Delta t}\right) = u\omega + \omega u = 2\omega u$$

Eqs. (15.33.),(15.36) → mechanism which contain parts sliding on each other

abs. and relative motions of sliding pins and collars.

$\overrightarrow{a_C}$ ---- useful in long-range projectiles, appreciably affected by the earth rotation.

* system of axes attached to the earth--- not truly a Newtonian frame.

→ rotating frame of ref., formulas derived in this section facilitate the study of the motion w.r.t. axes attached to the earth.