Frequency Domain Techniques and Loop Shaping

Francis Ch.1-7
Chen Ch. 9

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Course Info

- Control Systems I

  Introductory graduate-level control course, mainly focusing on linear systems, bridge between System Control Theory (undergrad. classical control) and Control System II (nonlinear/optimal control).
- Instructor: Dongjun Lee (djlee@snu.ac.kr, 301-1517, 880-1724)
- Teaching Assistants:
  - Nguyen Hai-Nguyen (Lead: hainguyen@snu.ac.kr, 301-211, 880-1690)
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- Prerequisites
  - System Control Theory (ME2794.002100) or equivalent; or by the consent of instructor
- Grading
  1. HW 20% (score 0/1, 0.5/1, 1/1)
  2. Mid-term exam 40% 4/19/2017 W 7-9:30pm
  3. Final exam 40% 6/14/2017 W 7-10pm

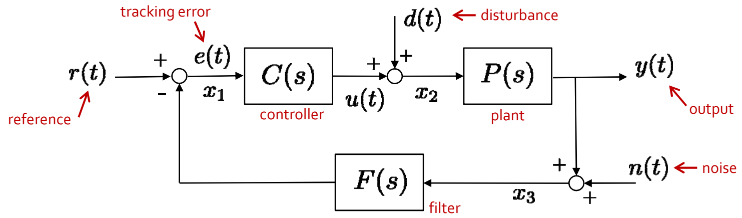
Course Topics

1. Loop shaping and fundamental limitations
2. Control design using Youla parameterization, coprime factorization
3. Linear system state-space representation and solution
4. Stability
5. Controllability, observability, canonical decomposition
6. Minimal realization, balanced realization
7. State-space control design, state estimation, separation principle
8. Linear quadratic regulator (LQR)
9. Kalman-Bucy filter and linear quadratic Gaussian (LQG)
10. Kalman estimation and extended Kalman filtering (EKF)

Course Info

- Textbooks
  - Linear System Theory and Design, Chen, Oxford University Press
  - Feedback Control Theory, Doyle, Francis & Tannenbaum, Dover
  - Control System Design, Friedland, Dover
- References
  - Multivariable Feedback Control, Skogestad & Postlethwaite, Willey
  - Control System Design, Goodwin, Gradebe & Salgado, Prentice Hall
  - Linear System Theory, Rugh & Kailath, Prentice Hall
  - Principles of Robot Motion, Choset et al, MIT Press
  - Optimal Control: Linear Quadratic Methods, Anderson & Moore, Dover

Basic Feedback Loop

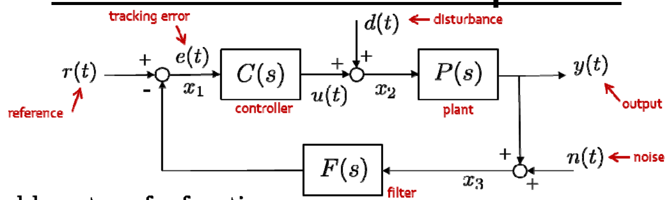


- Plant $P(s)$, controller $C(s)$, filter $F(s)$, all SISO.
- Exogeneous signals: reference $r(t)$, disturbance $d(t)$, noise $n(t)$.
- Signal signals: tracking error $e(t)$, output $y(t)$, control $u(t)$ (or x_1, x_2, x_3).
- Closed-loop transfer functions:

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{1}{1+PCF} \begin{bmatrix} 1 & -PF & -F \\ C & 1 & -CF \\ PC & P & 1 \end{bmatrix} \begin{pmatrix} r \\ d \\ n \end{pmatrix}}$$

- **Well-posedness** (stronger than stability): all closed-loop TFs exist $\Leftrightarrow$ $1 + P(s)C(s)F(s) \neq 0 \; \forall s \in \mathcal{C}$ (e.g., $P = C = 1, F = -1$).

Well-Posedness and Proper TFs

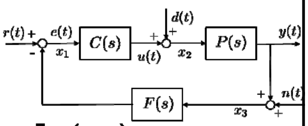


- Closed-loop transfer functions:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{1}{1+PCF} \begin{bmatrix} 1 & -PF & -F \\ C & 1 & -CF \\ PC & P & 1 \end{bmatrix} \begin{pmatrix} r \\ d \\ n \end{pmatrix}$$

- Ill-posedness arises due to the lack of time to response (cf. integrator).
- CL system well-posed if $P(s)$ is strictly proper and $C(s), F(s)$ are proper.
- $G(s)$ is (strictly) stable if analytic in closed-RHP (i.e., $\mathrm{Re}(s) \geq 0$).
- $G(s)$ is proper if $G(j\infty)$ is finite (i.e., $\deg(N) = \deg(D)$).
- $G(s)$ is strictly proper if $G(j\infty) = 0$ (i.e., $\deg(D) > \deg(N)$).
- CL system well-posed if $1 + P(s)C(s)F(s)$ is not strictly proper.

Internal Stability



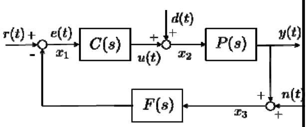
- Closed-loop transfer functions:

$$\begin{pmatrix} e \\ y \\ u \end{pmatrix} = \frac{1}{1+PCF} \begin{bmatrix} 1 & -PF & -F \\ PC & P & -PCF \\ C & -PCF & -CF \end{bmatrix} \begin{pmatrix} r \\ d \\ n \end{pmatrix}$$

- **Def:** The CL system is **internally stable** if all nine TFs are stable.
  - Not enough just to ensure stability of input-output TF (i.e., $\frac{PC}{1+PCF}$).
  - Internal instability with unstable pole-zero cancelation (e.g., with $P = \frac{1}{s^2-1}$, $C = \frac{s-1}{s+1}$, $F = 1$, $r \to y$ stable, yet, $d \to y$ unstable)

- **Th. 1-1:** CL system is internally stable iff no CL poles in $\mathrm{Re}(s) \geq 0$.
  - Write P, C, F as coprime polynomials (i.e., with no common factors): $P = \frac{N_P}{M_P}$, $C = \frac{N_C}{M_C}$, $F = \frac{N_F}{M_F} \Rightarrow \frac{1}{1+PCF} = \frac{M_P M_C M_F}{N_P N_C N_F + M_P M_C M_F}$.
  - **CL poles** = zeros of $N_P N_C N_F + M_P M_C M_F = 0$ (w/o PZ-cancelation).
  - CL TF matrix: $\frac{1}{N_P N_C + M_P M_C} \begin{bmatrix} M_P M_C & -N_P M_C & -M_P M_C \\ M_P N_C & M_P M_C & -M_P N_C \\ N_P N_C & -N_P M_C & M_P M_C \end{bmatrix}$
  - ($\Leftarrow$) obvious; ($\Rightarrow$) if not, that unstable pole should be canceled in all nine TFs $\to$ impossible w/ coprimeness among $M_\star, N_\star$.

Internal Stability



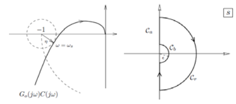
- Feedback system transfer functions:

$$\begin{pmatrix} e \\ y \\ u \end{pmatrix} = \frac{1}{1+PCF} \begin{bmatrix} 1 & -PF & -F \\ PC & P & -PCF \\ C & -PCF & -CF \end{bmatrix} \begin{pmatrix} r \\ d \\ n \end{pmatrix}$$

- **Th. 1-2:** The CL system is internally stable if and only if:
  1. $1 + PCF = \frac{N_P N_C N_F + M_P M_C M_F}{M_P M_C M_F}$ has no zeros in $\mathrm{Re}(s) \geq 0$; and
  2. No unstable pole-zero cancelation when the loop-TF PCF is formed.

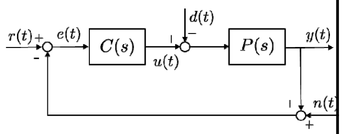
  - ($\Rightarrow$) First condition obvious. Also, from Th. 1-1, no unstable zeros of $N_P N_C N_F + M_P M_C M_F \to$ no unstable pole-zero cancelation possible.
  - ($\Leftarrow$) Will show that, under two conditions, CL poles necessarily stable. Suppose not. Then, $\exists s_o$, s.t., $[N_P N_C N_F + M_P M_C M_F](s_o) = 0$, $\mathrm{Re}(s_o) \geq 0$.
    1. Suppose $[M_P M_C M_F](s_o) \neq 0 \to [\frac{M_P M_C M_F + N_P N_C N_F}{M_P M_C M_F}](s_o) = [1 + PCF](s_o) = 0 \to$ contradict to condition 1.
    2. Suppose $[M_P M_C M_F](s_o) = 0 \to [N_P N_C N_F](s_o) = 0 \to$ unstable PZ-cancelation $\to$ contradict to condition 2.

Unit Feedback System



- Unit-feedback CL transfer functions with $F(s) = 1$:

$$\begin{pmatrix} e \\ y \\ u \end{pmatrix} = \frac{1}{1+PC} \begin{bmatrix} 1 & -P & -1 \\ PC & P & -PC \\ C & -PC & -C \end{bmatrix} \begin{pmatrix} r \\ d \\ n \end{pmatrix}$$



- **Nyquist stability theorem:** Construct Nyquist plot of $L = PC$, indenting to RHP around poles on jw-axis. Let n denote unstable poles of P and C. Then, CL system is internally stable iff Nyquist plot doesn't pass through $(-1, 0)$ and encircles it exactly n-times counter-clockwise.

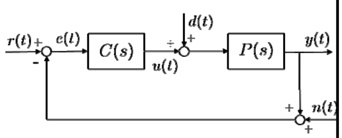
- Sensitivity function $\boxed{S := \frac{e}{r} = \frac{1}{1+L} = \frac{1}{1+PC} = \frac{M_P M_C}{N_P N_C + M_P M_C}.}$
  - Step tracking if S has at least one zero at $s = 0$; ramp tracking if two zeros at $s = 0$ (cf. system type, smaller $\to$ better performance).

- Complementary sensitivity function $\boxed{T := \frac{y}{r} = \frac{L}{1+L} = \frac{PC}{1+PC} = \frac{N_P N_C}{N_P N_C + M_P M_C}.}$
  - Total transfer function from reference r to output y.
  - S is sensitivity of total TF T against ΔP: $S = \frac{\partial T/T}{\partial P/P}$.
  - $\boxed{S(s) + T(s) = 1}$ with T relevant to robust stability.

Control Parameterization: Stable P

- Unit-feedback transfer functions:

$$\begin{pmatrix} e \\ y \\ u \end{pmatrix} = \frac{1}{1+PC} \begin{bmatrix} 1 & -P & -1 \\ PC & P & -PC \\ C & -PC & -C \end{bmatrix} \begin{pmatrix} r \\ d \\ n \end{pmatrix}$$

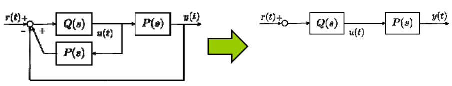


- **Control synthesis:** given P, design C s.t., feedback system (1) is internally stable and (2) acquires some desired properties (e.g., tracking).

- Parameterize stabilizing $C \to$ tweak C to achieve desired properties.

- **Th. 1-3:** Suppose $P \in \mathcal{S}$, where $\mathcal{S}$ is the set of all stable, proper, real-rational functions. Then, all the stabilizing controllers are given by

$$C = \frac{Q}{1 - PQ}, \quad \forall Q \in \mathcal{S}$$



- Affine relation: $S = 1 - PQ$, $T = PQ \to$ not work for unstable P.

- (Ex) Design ramping tracking control for $P(s) = \frac{1}{(s+1)(s+2)}$.
  - S with at least two zeros at $s = 0 \to Q = \frac{as+b}{s+1} \to$ CL stable.
  - $S = 1 - PQ = \frac{s^3+4s^2+(5-a)s+(2-b)}{(s+1)^2(s+2)} \to a = 5, b = 2$ (increase order$[C]$).

Coprime Factorization

- **Bezout identity:** given polynomials $N(s), M(s)$, $\exists$ polynomials $X(s), Y(s)$ s.t.,
$$N(s)X(s) + M(s)Y(s) = 1$$
iff $N(s), M(s)$ are coprime ($X = s+1, Y = s^2 + s$; also for integers).
- **Coprimeness of TFs:** we say $N(s), M(s) \in \mathcal{S}$ are coprime if $\exists X(s), Y(s) \in \mathcal{S}$ s.t.,
$$N(s)X(s) + M(s)Y(s) = 1$$
This coprimeness of N, M holds is iff $N(s), M(s)$ have no common zero in $\text{Re}(s) \geq 0$ and at $s \to \infty$ (i.e., both N, M can't be strictly proper).
- **Coprime factorization:** for any real-rational TF $G(s)$, we can always write its coprime factorization over $\mathcal{S}$ s.t.,
$$\boxed{G(s) = \frac{N(s)}{M(s)}, \quad NX + NY = 1,\ N, M, X, Y \in \mathcal{S}}$$
- (Ex) $G(s) = \frac{1}{s-1} \to N(s) = \frac{1}{s+1}$, $M(s) = \frac{s-1}{s+1}$ (all pass). What if $(N, M) = (\frac{1}{(s+1)^2}, \frac{s-1}{(s+1)^2})$ or $(N, M) = (\frac{(s-2)}{(s+1)^2}, \frac{(s-1)(s-2)}{(s+1)^2})$?
- (Ex: Euclid algorithm) $G(s) = \frac{1}{(s-1)(s-2)} \to N = \frac{1}{(s+1)^2}$, $M = \frac{(s-1)(s-2)}{(s+1)^2}$, $X = \frac{19s-11}{s+1}$, $Y = \frac{s+6}{s+1}$.

Youla Parameterization

- **Th. 1-4:** Consider unit feedback system with possibly unstable $P = \frac{N}{M}$, where $N, M \in \mathcal{S}$ are coprime with $NX + MY = 1$, $X, Y \in \mathcal{S}$. Then, all the stabilizing controllers are given by
$$C = \frac{X + MQ}{Y - NQ}, \quad \forall Q \in \mathcal{S}$$
- Main idea of proof:
  - (Lemma 1-1) Let $C = \frac{N_C}{M_C}$ be a comprime factorization. Then, the feedback system is internally stable iff $(NN_C + MM_C)^{-1} \in \mathcal{S}$.
  - ($\Leftarrow$) Given $Q \in \mathcal{S}$ and C as above, define $N'_C := X + MQ$, $M'_C := Y - NQ$ (not necesarily coprime) $\to$ put these into $NX + MY = 1$, we have $\boxed{NN'_C + MM'_C = 1}$, implying N'_C, M'_C are coprime. Further, CL stable from Lem. 1-1 w/ $NN'_C + MM'_C \in \mathcal{S}$.
  - ($\Rightarrow$) Suppose internal stability with C and let $C = \frac{N_C}{M_C}$ its coprime factorization. Define $V := (NN_C + MM_C)^{-1} \in \mathcal{S}$ (Lem. 1-1). Define Q s.t., $\boxed{M_C V =: Y - NQ}$ $\to$ put this into $NN_C V + MM_C V = 1$ $\to NN_C V + M(Y - NQ) = 1$. Also, from $NX + NY = 1$, $N(X + MQ) + M(Y - NQ) = 1$. Thus, $\boxed{N_C V = X + MQ}$, and $C = \frac{N_C V}{M_C V} = \frac{X + MQ}{Y - NQ}$. Further, $(NX + MY)Q = YN_C V - XM_C V \to Q \in \mathcal{S}$.

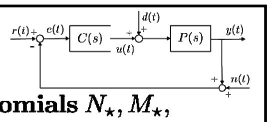
Q-Parameterization: Example

- **Th. 1-4:** Consider unit feedback system with possibly unstable $P = \frac{N}{M}$, where $N, M \in \mathcal{S}$ are coprime with $NX + MY = 1$, $X, Y \in \mathcal{S}$. Then, all the stabilizing controllers are given by

$$C = \frac{X + MQ}{Y - NQ}, \quad \forall Q \in \mathcal{S}$$

- $S = M(Y - NQ)$ and $T = N(X + MQ)$, still all affine in Q.
- (Ex) Design control C for $P = \frac{1}{(s-1)(s-2)}$ s.t., (1) CL system internally stable; (2) perfect step tracking when $d = 0$; and (3) rejection of sinusoid disturbance d of 10rad/s.
  - From Th. 1-4, $C = \frac{X+MQ}{Y-NQ}$ with $Q \in \mathcal{S}$. With this, $H_{r\to y} = T = N(X + MQ)$ and $H_{d\to y} = N(Y - NQ)$.
  - Step tracking/stabilization (dc-gain): $H_{r\to y}(0) = 1 = N(0)[X(0) + M(0)Q(0)] = 1 \to Q(0) = 6$.
  - Disturbance rejection (zero at jw): $H_{d\to y}(jw) = 0 = N(10j)[Y(10j) - N(10j)Q(10j)] \to Q(10j) = -94 + 70j$.
  - Three constraints for Q: $Q = a + \frac{b}{s+1} + \frac{c}{(s+1)^2} \to C(s)$.

Pole Placement Control



- Unit feedback system: $P = \frac{N_P}{M_P}$, $C = \frac{N_C}{M_C}$ **coprime polynomials** $N_\star, M_\star$,

$$\begin{pmatrix} e \\ y \\ u \end{pmatrix} = \frac{1}{N_P N_C + M_P M_C} \begin{bmatrix} M_P M_C & -N_P M_C & -M_P M_C \\ N_P N_C & N_P M_C & -N_P N_C \\ N_C M_P & -N_P N_C & -N_C M_P \end{bmatrix} \begin{pmatrix} r \\ d \\ n \end{pmatrix}$$

- **Pole-placement control:** given desired CL-CE $F(s)$ and $N_P(s), M_P(s)$, design $N_C(s), M_C(s)$ s.t.,

$$N_C(s)N_P(s) + M_C(s)M_P(s) = F(s)$$

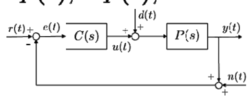
  - From Bezout's identity, there exist coprime polynomials X, Y s.t., $N_P X + M_P Y = 1$ iff N_P, M_P are coprime.
  - Thus, we can always find a solution $\bar{N}_C(s) = F(s)X(s)$, $\bar{M}_C(s) = F(s)Y(s)$ iff N_P, M_P are coprime (may not stabilizing though).
  - General solution: $N_C(s) = \bar{N}_C(s) + M_P(s)Q(s)$, $M_C(s) = \bar{M}_C(s) - N_P(s)Q(s)$ for any polynomial $Q(s)$ with

$$C(s) = \frac{X(s)F(s) + M_P(s)Q(s)}{Y(s)F(s) - N_P(s)Q(s)}$$

  - $H_{r\to y} = \frac{N_C N_P}{N_P N_C + M_P M_C}$ implying that unit-feedback cannot alter open-loop zeros $\to$ two-DOF control for model (i.e., pole-zero) matching.

Pole Placement via Linear Algebra

- **Pole-placement control:** given desired CL-CE $F(s)$ and $N_P(s), M_P(s)$, design $N_C(s), M_C(s)$ s.t.,



$$N_C(s)N_P(s) + M_C(s)M_P(s) = F(s)$$

- Given $\deg(M_P) = n$, choose $\deg(M_C) = m$ to match $\deg(F) = n + m$. The larger m is (i.e., more DOFs), the more likely can match $F(s)$. Then,

$$N_P(s) = a_{p0} + a_{p1}s + ...a_{pn}s^n, \quad M_P(s) = b_{p0} + b_{p1}s + ...b_{pn}s^n,$$
$$N_C(s) = a_{c0} + a_{c1}s + ...a_{cm}s^m, \quad M_C(s) = b_{c0} + b_{c1}s + ...b_{cm}s^m,$$
$$F(s) = f_0 + f_1 s + ...f_{n+m}s^{n+m}$$

with $a_{pn} = 0$, $b_{pn} \neq 0$, $b_{cm} \neq 0 \to f_{n+m} = b_{pn}b_{cm} + a_{pn}a_{cm} = b_{pn}b_{cm} \neq 0$.

- The pole-placement can then be written by linear equation (Chern Ch.9):

$$\boxed{[S_m]_{(n+m+1)\times 2(m+1)} \left[b_{co}; a_{co}; ...; b_{cm}; a_{cm}\right] = [f_o; ...; f_{n+m}]}$$

which has a solution if $2(m+1) \geq n + m + 1$ due to the structure of S_m.

- **Th. 1-5:** For the above problem, for any $F(s)$ with $\deg(F) = n + m$, there always exist a proper $C = \frac{N_C}{M_C}$ with $\deg(C) = m$, if $m \geq n - 1$.

Example: Chen 9.2

- For $P = \frac{s-2}{s^2-1}$, design a proper compensator C so that y can track any step response.

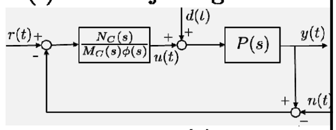
1. Choose desired CL poles: $F(s) = (s+2)(s^2+2s+2) = s^3+4s^2+6s+4$.
2. Set up linear pole-placement equation:

$$\begin{bmatrix} -1 & -2 & 0 & 0 \\ 0 & 1 & -1 & -2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} b_o \\ a_o \\ b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 4 \\ 1 \end{pmatrix}$$

   with S_m full column rank (i.e., solution exists if $m \geq n-1$).
3. Compute the control law: $C = -\frac{22s+23}{3s+34}$.
4. Analyze dc-tracking performance: $T(0) = \frac{P(0)C(0)}{1+P(0)C(0)} = \frac{23}{6}$.
5. Design pre-compensator $P(s) = \frac{6}{23}\bar{P}(s)$, $\bar{P}(s) \in \mathcal{S}$ with $P(0) = 1$.

Internal Model Control

- Suppose we want to track a class of reference signal $r(t)$ while rejecting a class of disturbance $d(t)$, given by



$$\boxed{r(s) = \frac{\hat{N}_r(s)}{D_r(s)}, \quad d(s) = \frac{\hat{N}_d(s)}{D_d(s)}}$$

where D_r, D_d are **known**, yet, $\hat{N}_r, \hat{N}_d$ unknown (e.g., any ramp $r(s) = \frac{\hat{N}_r(s)}{s^2}$, any step+sinusoid with known w $d(s) = \frac{\hat{N}_d(s)}{s(s^2+w^2)}$).

- Incorporate this information of r, d into the controller to achieve robust tracking and disturbance rejection $\rightarrow$ **internal model control.**

- Define $\phi(s)$ to be least common denominator of unstable (i.e., non-vanishing) poles of $r(s), d(s)$ (e.g., $\phi = s^2(s^2+w^2)$) and incorporate into control.

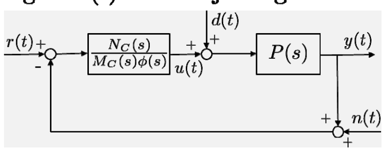
- $H_{d\rightarrow y}(s) = \frac{N_P M_C \phi}{N_P N_C + M_P M_C \phi} \rightarrow y(s) = \frac{N_P M_C \phi}{N_P N_C + M_P M_C \phi(s)} \frac{\hat{N}_d(s)}{D_d(s)} = \boxed{\frac{N_P M_C \phi(s)}{F(s)} \frac{\hat{N}_d(s)}{D_d(s)}}$
$\rightarrow$ with CL-CE $F(s)$ Hurwitz, $y(\infty) = \lim_{s\rightarrow 0} s \frac{N_P M_C \phi(s)}{F(s)} \frac{\hat{N}_d(s)}{D_d(s)} = 0.$

- $H_{r\rightarrow e}(s) = \frac{M_P M_C \phi}{N_P N_C + M_P M_C \phi} \rightarrow e(s) = \frac{M_P M_C \phi}{N_P N_C + M_P M_C \phi(s)} \frac{\hat{N}_r(s)}{D_r(s)} = \boxed{\frac{M_P M_C \phi(s)}{F(s)} \frac{\hat{N}_r(s)}{D_r(s)}}$
$\rightarrow$ with CL-CE $F(s)$ Hurwitz, $e(\infty) = \lim_{s\rightarrow 0} s \frac{M_P M_C \phi(s)}{F(s)} \frac{\hat{N}_r(s)}{D_r(s)} = 0.$

Pole Placement for IMC

- Suppose we want to track a class of reference signal $r(t)$ while rejecting a class of disturbance $d(t)$, given by

$$r(s) = \frac{\hat{N}_r(s)}{D_r(s)}, \quad d(s) = \frac{\hat{N}_d(s)}{D_d(s)}$$



- **CL pole placement with IMC:**

$$\boxed{N_P(s)N_C(s) + M_P(s)M_C(s)\phi(s) = F(s)}$$

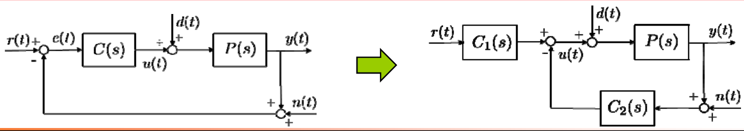
where N_P, M_P are coprime. Thus, if no common zeros between $\phi(s)$ and $N_P(s)$, N_P and $M_P\phi$ are coprime $\rightarrow$ from **Th.1-5**, $\exists$ a proper $C(s) = \frac{N_C}{M_C}$ if $m \geq n + n_d - 1$, $n = \deg(P)$, $n_d = \deg(\phi)$, $m = \deg(C)$.

- A simplest form of IMC is I-control to reject constant disturbance.

- (Chen Ex 9.3) Design control for $P = \frac{s-2}{s^2-1}$ to track any step refernece.
  - IMC $\phi(s) = s$. Then, $m \geq 2 \rightarrow \deg[F(s)] = 5$.
  - Choose $F(s) = (s+2)(s^2+4s+5)(s^2+2s+5)$.
  - Solve LA to obtain N_C, M_C: $C(s) = \frac{N_C(s)}{M_C(s)\phi(s)} = -\frac{96.3s^2+118.7s+25}{s(s^2+127.3)}$.
  - $T(0) = \frac{P(0)C(0)}{1+P(0)C(0)} = 1$ (dc-tracking). $H_{d\rightarrow y}(0) = \frac{P(0)}{1+P(0)C(0)} = 0$ (constant disturb. rejection)

Model Matching Control

- So far, we have been working on CL pole placement control (i.e., $F(s)$), yet, CL system behavior also (possibly severely) depends on zeros as well.
- **Model matching problem:** given P, design a proper C s.t., the CL behavior $H_{r\to y}(s)$ matches with a desired TF $H_o(s)$.
- Unit feedback can do arbitrary pole-placement $F(s)$, yet, not general model matching $H_o(s) \to$ FF pre-compensator C_1 + FB control C_2.
- Even with two-DOF control, not arbitrary $H_o(s)$ implementable due to OL-dynamics $P(s)$ (e.g., $H_{r\to y} = \frac{N_P N_1}{N_P N_2 + M_P M_2}$ w/ unmovable zeros of P).
- **Th. 1-6:** Consider $P = \frac{N_P}{M_P}$. Then, $H_o(s) = \frac{E(s)}{F(s)}$ is **implementable** iff:
  1. CL-CE $F(s)$ is Hurwitz.
  2. $\deg(F) - \deg(E) \geq \deg(M_P) - \deg(N_P)$ (relative degree can't decrease).
  3. All zeros of $N_P(s)$ with zero/positive real parts are also zeros of $E(s)$ (non-minimum phase zeros not removable).



Model Matching Procedure

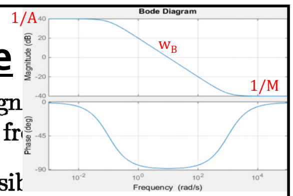
Given $P = \frac{N_P}{M_P}$ and $H_o = \frac{E}{F}$ (satisfying **Th. 1-6**), design proper $C_1 = \frac{N_1}{M_C}$ and $C_2 = \frac{N_2}{M_C}$ s.t., $H_{r\to y}(s) = H_o(s)$ while ensuring CL internal stability.

1. Define coprime $\bar{E}, \bar{F}$ s.t., $\boxed{\frac{H_o}{N_P} = \frac{E}{FN_P} = \frac{\bar{E}}{\bar{F}}}$.
   - May attempt $H = \frac{PC_1}{1+PC_2} = \frac{N_P N_1}{M_P M_C + N_P N_2} = H_o = \frac{\bar{E} N_P}{\bar{F}} \to N_1 = \bar{E}$ and solve M_C, N_2 from $M_P M_C + N_P N_2 = \bar{F}$ with coprime N_P, M_P $\to$ typically, $\deg(\bar{F})$ not enough (e.g., improper C_2 w/ HO P).
2. Augment $\bar{F}(s)$ w/ a Hurwitz $\hat{F}(s)$ s.t., $\deg(\bar{F}\hat{F}) \geq 2n - 1$, $n = \deg(M_P)$.
3. Rewrite $H_o = \frac{\bar{E}\hat{F} N_P}{\bar{F}\hat{F}} = H = \frac{N_P N_1}{M_P M_C + N_P N_2}$, and choose/solve for N_2, M_C:
$$\boxed{N_1(s) = \bar{E}(s)\hat{F}(s), \quad M_P(s) M_C(s) + N_P(s) N_2(s) = \bar{F}(s)\hat{F}(s)}$$
   which has a solution N_2, M_C, since N_P, M_P coprime and $m = \deg(M_C) = \deg(\bar{F}\hat{F}) - \deg(M_p) \geq n - 1$ (cf. **Th. 1-5**).
4. $C_2 = \frac{N_2}{M_C}$ is proper (**Th. 1-5**) Also, for $C_1 = \frac{\bar{E}\hat{F}}{M_C} = \frac{N_1}{M_C}$, from item 2 of **Th. 1-6**, $\deg(\bar{F}\hat{F}) - \deg(\bar{E}\hat{F} N_P) \geq \deg(M_P) - \deg(N_P) \to \deg(\bar{E}\hat{F}) = \deg(N_1) \leq \deg(\bar{F}\hat{F}) - \deg(M_P) = n + m - n = m \to C_1$ also proper.

Model Matching: Example 9.8 Chen

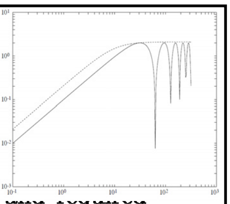
- (Ex 9.8) Given $P = \frac{s-2}{s^2-1}$, match $H_o(s) = \frac{-(s-2)(4s+2)}{(s+2)(s^2+2s+2)}$.
  - $H_o(s)$ ensures step and ramp tracking: $H_o = \frac{-4s^2+6s+4}{s^3+4s^2+6s+4}$
  - $H_o(s)$ is implementable:
  - Compue $\bar{E}(s), \bar{F}(s)$: $\frac{H_o}{N_P} = \frac{\bar{E}}{\bar{F}} = \frac{-(4s+2)}{s^3+4s^2+6s+4}$.
  - Compute $\hat{F}(s)$: $\deg(\bar{F}\hat{F}) \geq 2n - 1 = 3 \to \hat{F} = 1$.
  - Compute C_1 via direct substitution and Bezout: $N_1 = -(4s+2)$, $M_C = s + 34/3 \to C_1 = \frac{-(4s+2)}{s+34/3}$.
  - Compute C_2 via Bezout: $C_2 = \frac{-(22s+23)}{3s+34}$.

Nominal Performance



- We have considered tracking of step, ramp or sinusoid sign
  freq. Yet, in practice, often, need to track signals with fr
- For tracking, ideally, we want $||S||_\infty = \frac{e}{r} \equiv 0 \to$ impossil
- **Nominal performance:** $||W_pS||_\infty < 1$, where $W_p(s) \in \mathcal{S}$ is frequency-dependent performance weight:
  - $W_p(s) = \frac{100}{(s+1)^3}$ for tracking up to 1rad/s with error less than 1/100.
  - $W_p(s) = \frac{s/M + w_B^*}{s + w_B^* A}$ for CL bandwidth w_B^* (w/ $|W_p(jw_B^*)| \approx 1$), LF track $|e_{ss}| < A < 1$ and HF max peak of $|S(jw)| < M$ (stability).
- Signal norm of $u(t) \in \Re$:
  - Defining properties: (1) $||u|| \geq 0$; (2) $||u|| = 0$ iff $u(t) \equiv 0$; (3) $||au|| = |a| \cdot ||u||$; (4) $||u+v|| \leq ||u|| + ||v||$.
  - 2-norm: $||u||_2 := \sqrt{\int_{-\infty}^{+\infty} |u(t)|^2 dt}$; ∞-norm $||u||_\infty := \sup_t |u(t)|$,
- System norm of $H(s)$ (or $h(t)$):
  - $\boxed{\infty\text{-norm: } ||H||_\infty := \sup_w |H(jw)|}$ with $||y||_2 = ||H||_\infty ||u||_2$ (via Parseval's identity); if $u(t) = A \sin wt$, $w \in \Re$, $||y||_\infty = ||H||_\infty A$.

Multiplicative Uncertainty



- So far, we have rather neglected uncertainty in system, w only degrade performance but also destabilize CL system.
- Goal: maintain CL-system stability (i.e., robust stability) and required performance (i.e., robust performance) in the presence of uncertainty.
- **Multiplicative uncertainty:** plant perturbed by uncertainty given by

$$\tilde{P} = (1 + W_u\Delta)P, \quad \frac{\tilde{P}-P}{P} = W_u\Delta \;\Rightarrow\; \left|\frac{\tilde{P}(jw)-P(jw)}{P(jw)}\right| \le |W_u(jw)|$$

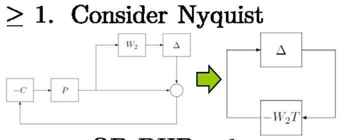
where $W_u \in \mathcal{S}$ is uncertainty weight (e.g., large for HP), $\Delta \in \mathcal{S}$ is (any) nominal uncertainty with $||\Delta||_\infty < 1$ w/o unstable pole cancelation of P (i.e., allowable uncertainty).

  - Given scattered gain/phase measurements (M_{ik}, ϕ_{ik}) at w_i: $\left|\frac{M_{ik}e^{j\phi_{ik}}}{M_i e^{j\phi_i}} - 1\right| < |W_u(jw_i)|$
  - Nominal $P(s) = 1/s^2$ perturbed by delay up to $\tau = 0.1$s with $\tilde{P} = e^{-\tau s}\frac{1}{s^2} \Rightarrow |\frac{\tilde{P}}{P} - 1| = |e^{-j\tau w} - 1| < |W_u(jw)|$ for $W_u = \frac{0.21s}{0.1s+1}$.

Robust Stability

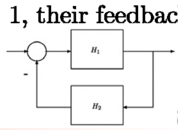
- Given P and uncertainty information W_u, design control law C to ensure **robust stability** for any plant $\tilde{P} \in \mathcal{P} := \{(1 + W_u\Delta)P\}$.
- **Th. 1-7:** Assume C stabilizes nominal plant P, with PC and $\tilde{P}C$ having same number of RHP-poles. Then, C provides robust stability for any plant $\tilde{P} \in \mathcal{P}$ iff $||W_uT||_\infty < 1$, where $T = \frac{PC}{1+PC}$.
  - $(\Leftarrow)$ Convert feedback diagram and apply small-gain theorem.
  - $(\Rightarrow)$ Suppose not, i.e., $|W_u(jw)T(jw)| \ge \gamma \ge 1$. Consider Nyquist plot of $\tilde{L} = \tilde{P}C$ w.r.t. $(-1, 0)$-point:

$$1 + \tilde{L} = 1 + \tilde{P}C = (1 + L)(1 + \Delta W_u T)$$



where $1 + L \neq 0$ with # of encirclement same as OP RHP-poles. Then, can find Δ w/ $|\Delta(jw)| = 1/\gamma \le 1$, $\angle\Delta(jw) = -\pi + \angle W_u T$ at $w \to \tilde{L}$ touches $(-1, 0) \to$ unstable $\to$ contradiction.

- **Small gain theorem:** Let H_1 and H_2 be (possibly nonlinear) stable systems with finite IO-gains $||H_1||, ||H_2||$. If $||H_1|| \cdot ||H_2|| < 1$, their feedback system is also stable.



Robust Performance

- Given P and uncertainty W_u, design control C to ensure **robust stability** and **robust performance** for any plant $\tilde{P} \in \mathcal{P} := \{(1 + W_u\Delta)P\}$, i.e.,

$$||W_uT||_\infty < 1, \quad ||W_p\tilde{S}||_\infty < 1, \quad \forall \tilde{P} \in \mathcal{P}$$

where $\tilde{S} = \frac{1}{1+\tilde{L}} = \frac{1}{1+\tilde{P}C}$. Then, $||W_p\tilde{S}||_\infty = \left\| \frac{W_pS}{1+\Delta W_uT} \right\|_\infty < 1$

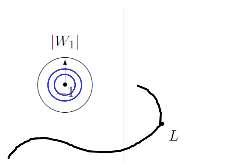
- **Th. 1-8:** A necessary and sufficient condition for RP (also RS) is

$$\| \, |W_pS| + |W_uT| \, \|_\infty < 1$$

  - ($\Leftarrow$) $||W_uT||_\infty < 1$ obvious. Also, $|||W_pS|+|W_uT|||_\infty < 1 \to |W_pS| + |W_uT| < 1 \to \frac{|W_pS|}{1-|W_uT|} < 1 \to \left\| \frac{W_pS}{1+\Delta W_uT} \right\|_\infty \le \left\| \frac{|W_pS|}{1-|W_uT|} \right\|_\infty < 1.$
  - ($\Rightarrow$) $\left\| \frac{W_pS}{1+\Delta W_uT} \right\|_\infty < 1 \to \left\| \frac{W_pS}{1-|W_uT|} \right\|_\infty \le \left\| \frac{W_pS}{1+\Delta W_uT} \right\|_\infty < 1.$

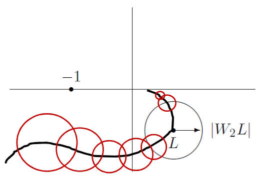
- Maximum tolerable uncertainty $||\Delta||_\infty \le \beta$:
  - RS: $1 + \tilde{L} = (1 + L)(1 + \Delta W_uT) \to |\beta W_uT| < 1 \to \beta < \frac{1}{||W_uT||_\infty}$.
  - RP: $\left\| \frac{W_pS}{1+\Delta W_uT} \right\|_\infty < 1 \to \left| \frac{W_pS}{1-\beta|W_uT|} \right| < 1 \to \boxed{\beta < \left\| \frac{1-|W_pS|}{W_uT} \right\|_\infty}$.

Graphical Representations



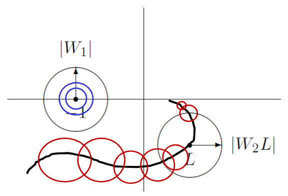
- Nominal performance: $||W_pS||_\infty < 1$

  $|1 + L(jw)| > |W_p(jw)|, \ \forall w$



- Robust stability: $||W_uT||_\infty < 1$

  $|1 + L(jw)| > |W_u(jw)L(jw)|, \ \forall w$

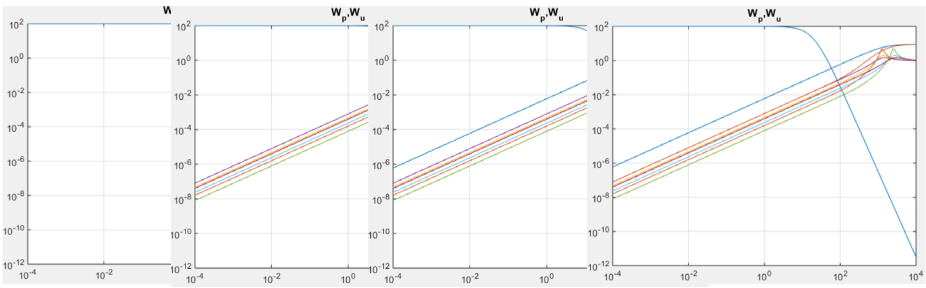


- Robust performance: $|||W_pS| + |W_uT|||_\infty < 1$

  $|1 + L(jw)| > |W_p(jw)| + |W_u(jw)L(jw)|, \ \forall w$

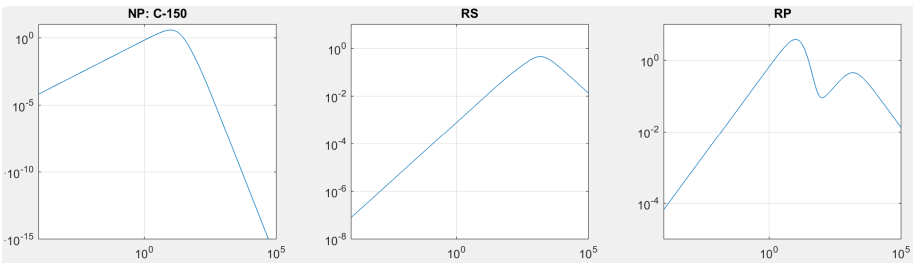
RP Design Example

- Nominal plant (integrator): $P = \frac{1}{s}$.
- Perturbed plant: $\tilde{P} = \frac{1}{s}\frac{k}{ms^2+bs+k} \to \frac{\tilde{P}-P}{P} = \frac{s^2+2\zeta w_n s}{s^2+2\zeta w_n s + w_n^2}$.
- Performance specification: $w_c = 20$rad/s, dc-tracking error $\leq 1\%$
- Design performance weight function: $W_p = \frac{100 w_c^5}{(s+w_c)^5}$.
- Uncertainty: $w_n \in [200, 500]$Hz, $\zeta \in [0.1, 0.5]$.
- Design uncertainty weight function: $W_u = \frac{9s}{s+1500} \to W_u = \frac{9s(s+200)}{(s+1500)^2}$.



RP Design Example

- Perturbed plant: $\tilde{P} = \frac{1}{s}\frac{k}{ms^2+bs+k}$. $W_u = \frac{9s(s+200)}{(s+1500)^2}$, $W_p = \frac{100}{(\frac{s}{20}+1)^5}$.
- $w_n \in [200, 500]$Hz, $\zeta \in [0.1, 0.5]$; $w_c = 20$rad/s, dc-tracking error $\leq 1\%$.
- Design P-control $C(s) = K$ s.t., $\| |W_p S| + |W_u T| \|_\infty < 1$.
  - Design K for NP: $K = 600$ and check RS and RP.
  - Decrease K improves RS, yet, degrades RP $\to$ P-control can't satisfy both (or reduce performance W_p $--$ or improve system-ID $W_u$$--$).
  - More complicated control $\to$ Loop shaping.



Loop Shaping

- Loop shaping: graphical technique to shape loop-transfer function $L(s) = P(s)C(s)$ to satisfy RP and internal stability:

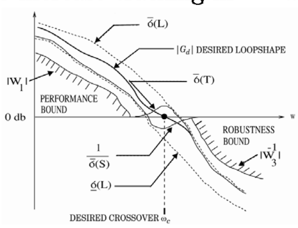
$$\| |W_p S| + |W_u T| \|_\infty < 1$$

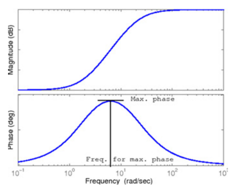
  if such control law exists, where $S = \frac{1}{1+L}$ and $T = \frac{L}{1+L}$.

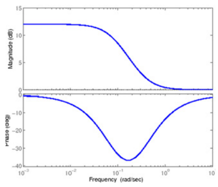
- Typically, L should have large gain in LF (for performance) and low gain in HF (for robust stability). More precisely, **basic loop shaping condition:**

$$\boxed{|L(jw)| > \frac{|W_p|}{1-|W_u|} \text{ (LF)}, \qquad |L(jw)| < \frac{1-|W_p|}{|W_u|} \text{ (HF)}}$$

- Simplest loop shaping: lead (to increase w_c while improving PM) and lag (to increase tracking in LF w/o affecting GM/PM).







Loop Shaping: Derivation

- **Stability-performance trade-off:** A necessay condition for $L(s)$ to satisfy RP is

$$\boxed{\min\{|W_p(jw)|, |W_u(jw)|\} < 1, \quad \forall w \geq 0}$$

can't allow high performance & high uncertainty at same band

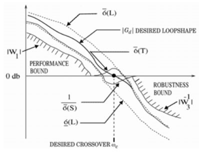
  i.e., both $|W_p|$ and $|W_u|$ can't be larger than 1 at the same time.

  - Suppose not, i.e., at w', $|W_p| \geq 1$ and $|W_u| \geq 1$. WLG, assume $|W_p| \geq |W_u|$. Then, at w',

$$|W_p S| + |W_u T| \geq |W_u S| + |W_u T| = |W_u S| + |W_u(1-S)| \geq |W_u| \geq 1$$

- Now, define $\Gamma(w) := \frac{|W_p|}{|1+L|} + \frac{|W_u L|}{|1+L|}$. Then, RP iff $\boxed{\Gamma(w) < 1\ \forall w.}$ Also,

$$\boxed{\frac{|W_p| + |W_u| \cdot |L|}{1+|L|} \leq \Gamma(w) \leq \frac{|W_p| + |W_u| \cdot |L|}{|1-|L||}}$$



  - (LF) $|W_p| >> 1 > |W_u|$ and $|L| >> 1 \rightarrow$

$$|L| > \frac{|W_p| - 1}{1-|W_u|} \approx \frac{|W_p|}{1-|W_u|}, \quad |L| > \frac{|W_p| + 1}{1-|W_u|} \approx \frac{|W_p|}{1-|W_u|}$$

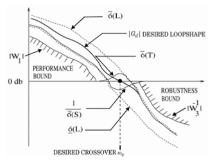
  - (HF) $|W_u| >> 1 > |W_p|$ and $|L| << 1 \rightarrow$

$$|L| < \frac{1-|W_p|}{|W_u| - 1} \approx \frac{1-|W_p|}{|W_u|}, \quad |L| < \frac{1-|W_p|}{1+|W_u|} \approx \frac{1-|W_p|}{|W_u|}$$

Loop Shaping: Conditions

- We assume stable/minimum-phase $P(s)$ (i.e., no RHP pole/zero) to avoid unstable/non-minimum-phase $L = PC$ (general case $\to$ Doyle Ch. 8).
- A necessay condition for W_p, W_u: $\min\{|W_p(jw)|, |W_u(jw)|\} < 1$.
- HF roll-off of L should be at least as fast as that of P (proper C).
- Slop of $|L|$ at crossover frequency w_c (i.e., $|L(jw_c)|$) should be as gentle as possible (-20[dB/dec] to -40[dB/dec]).
- Loop shaping is based on approximation: NS, RP shoul be checked *a posterior* (RP assumes RS, RS assumes NS).
- Bode's gain formula: For a non-minimum phase stable L with all positive coefficients, its phase $\angle L(jw_o)$ is uniquely given by: with $\nu := \ln(w/w_o)$,

$$\boxed{\angle L(jw_o) = \frac{1}{\pi}\int_{\infty}^{+\infty} \frac{d\ln|L|}{d\nu} \ln\frac{|w+w_o|}{|w-w_o|} d\nu}$$

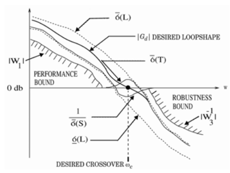


where, if constant slop $\frac{d\ln|L|}{d\nu} = c$ at w_o, $\angle L(jw_o) = -\frac{c\pi}{2}$.

- The stiffer the slop of $|L(jw_c)|$ is, the less the PM is.
- For system w/ RHP-zeros, phase angle larger than minimum angle above.

Loop Shaping: Procedure

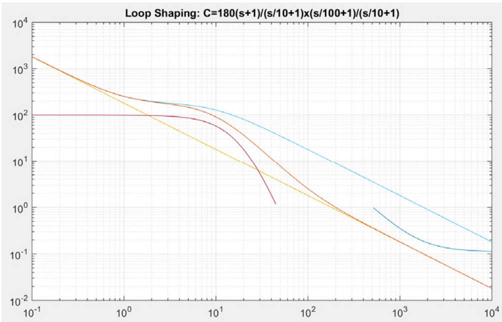
1. For stable/minimum-phase nominal plant P, design W_p, W_u s.t.,
$$\min\{|W_p|, |W_u|\} < 1, \quad \forall w$$
2. On $(\log w, 20\log|L|)$ plane, plot LF and HF bounds:
$$\frac{|W_p|}{1-|W_u|},\quad |W_p| >> 1 > |W_u| \text{ (LF)} \qquad \frac{1-|W_p|}{|W_u|},\quad |W_u| >> 1 > |W_p| \text{ (HF)}$$
3. Construct a desired loop-TF candidate $L = PC$ s.t.,
   - $|L|$ is above (or below) the LF (or HF) bounds.
   - Roll-off of L at HF at least as fast as P.
   - Slop of L at crossover frequency as gentle as possible (< -40dB/dec).



4. Check RP by observing if $|W_pS| + |W_uT| < 1$.
5. Check NS by ensuring roots of $1 + L(s) = 0$ in LHP.
6. Determine the controller $C(s) = L(s)/P(s)$.

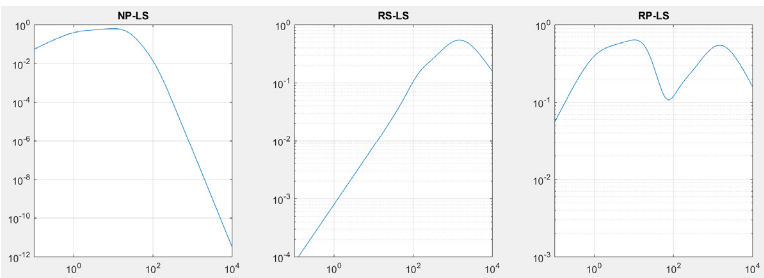
Loop Shaping: Example

- Nominal plant; $P = \frac{1}{s}$. perturbed plant: $\tilde{P} = \frac{1}{s}\frac{k}{ms^2+bs+k}$.
- $w_n \in [200, 500]$Hz, $\zeta \in [0.1, 0.5]$; $w_c = 20$rad/s, dc-tracking error $\leq 1\%$
- Do loop shaping to design $C(s)$ s.t., $\| |W_p S| + |W_u T| \|_\infty < 1$.
  - Check $L(s)$ with $C(s) = 180$, i.e., $L(s) = 180\frac{1}{s}$ → violate LF bound.
  - Shape $L(s)$ w/ Lead → $L(s) = 180\frac{1}{s}\frac{s+1}{\frac{s}{10}+1}$ → violate HF bound.
  - Shape $L(s)$ w/ Lag → $L(s) = 180\frac{1}{s}\frac{s+1}{\frac{s}{10}+1}\frac{\frac{s}{100}+1}{\frac{s}{10+1}}$ → LF/HF bounds satisfied.



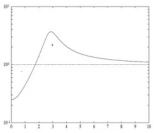
Loop Shaping: Example

- Nominal plant; $P = \frac{1}{s}$. perturbed plant: $\tilde{P} = \frac{1}{s}\frac{k}{ms^2+bs+k}$.
- $W_u = \frac{9s(s+200)}{(s+1500)^2}$, $W_p = \frac{100}{(\frac{s}{20}+1)^5}$.
- Do loop shaping to design $C(s)$ s.t., $\| |W_p S| + |W_u T| \|_\infty < 1$.
  - With $C(s) = 180\frac{s+1}{\frac{s}{10}+1}\frac{\frac{s}{100}+1}{\frac{s}{10+1}}$ → LF/HF bounds satisfied.
  - Check RP via Bode plot of $|W_p S| + |W_u T|$.
  - Check NS via CL CE: CL poles $-0.9925, -99.5023 \pm 90.45j$.



Waterbed Effects

- **Bode's integral theorem for sensitivity function:** Suppose $L(s)$ has relative-degree ≥ 1, has M RHP poles p_i (with $\mathrm{Re}(p_i) > 0$), and $k = \lim_{s\to\infty} sL(s)$ (e.g., $k = 0$ if relative-degree ≥ 2). Then,



$$\boxed{\int_0^\infty \ln|S(jw)|dw = -k\frac{\pi}{2} + \pi \cdot \sum_{i=1}^{M} \mathrm{Re}(p_i)}$$

  - If you "push down" $|S(jw)|$ at some frequency-band, it is lead into "swelling-up" at another frequency-band.
  - Overall level of $|S(jw)|$ will increase if open-loop system is unstable with fast poles (difficult to stabilize).

- **Bode's integral theorem for complementary sensitivity function:** Suppose $L(s)$ has at least 1 pole at 0, M RHP zeros z_i, with $k_v = \lim_{s\to 0} sL(s)$ (e.g., $k_v = \infty$ if type ≥ 2). Then,

$$\boxed{\int_0^\infty \frac{1}{w^2}\ln|T(jw)|dw = -\frac{\pi}{2k_v} + \pi \cdot \sum_{i=1}^{M} \frac{1}{z_i}}$$

  - Waterbed effect for $|T(jw)|$.
  - Overall level of $|T(jw)|$ increase w/ RHP-zeros (bad for performance).

RHP Poles and RHP Zeros

- **Effect of combined RHP zeros/poles:** Suppose $L(s)$ has N_z RHP-zeros z_j and N_p RHP-poles p_i. Then, $\forall j = 1, .., N_z, i = 1, .., N_p$,

$$\boxed{||W_pS||_\infty \geq \prod_{i=1}^{N_p} \frac{|z_j+\bar{p}_i|}{|z_j-p_i|}|W_p(z_j)|, \quad ||W_uT||_\infty \geq \prod_{i=1}^{N_p} \frac{|\bar{z}_j+p_i|}{|z_j-p_i|}|W_p(z_j)|}$$

- It would be extremely difficult to control if RHP-pole and RHP-zero are close with each other (i.e., unstable mode nearly uncontrollable/unobservable).

  1. Suppose $L(s)$ has RHP-pole p and RHP-zero z. Then, $S(p) = 0, T(p) = 1$ and $S(z) = 1, T(z) = 0$.
  2. Then, we can write $S = S_{ap}S_{mp} = \frac{s-p}{s+p}S_{mp}$ where S_{ap} is all-pass with $|S_{ap}(jw)| = 1$ and $|S_{mp}(jw)| = |S(jw)|$. Further, $|S_{mp}(z)| = |S(z)|/|S_{zp}(z)| = |\frac{z+p}{z-p}|$.
  3. Moreover, from maximum modulus theorem,

$$||W_pS||_\infty = \sup_w |W_p(jw)S_{mp}(jw)| \geq \sup_{\mathrm{Re}(s)\geq 0} |W_p(s)S_{mp}(s)|$$
$$\geq W_p(z)||S_{mp}(z)| = |W_p(z)|\frac{|z+p|}{|z-p|}$$

- If $W_p = W_u = 1$, $||S||_\infty \geq \frac{|z+p|}{|z-p|}$ and $||T||_\infty \geq \frac{|z+p|}{|z-p|}$, again, shows difficulty of control.

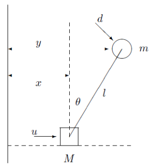
Bandwidth Limitation with RHP Poles/Zeros

- RHP-poles typically require aggressive/fast control to stabilize. RHP-zeros typically require non-aggressive/slow control due to inverse response.

- **Effect of RHP zeros:** approximate bound for the open-loop bandwidth w_B of $L(s)$ is given by

$$\boxed{w_B \approx w_C \leq \begin{cases} |z|/4 & \text{if } \mathrm{Re}(z) >> \mathrm{Im}(z) \\ |z|/2.8 & \text{if } \mathrm{Re}(z) = \mathrm{Im}(z) \\ |z| & \text{if } \mathrm{Re}(z) << \mathrm{Im}(z) \end{cases}}$$

  - CL-BW w_c is limited by z and should be slower w.r.t. z.
  - RHP-zeros close to origin is bad.



- **Effect of RHP-poles:** $w_C \approx w_B > 2p$, i.e. should be fast enough to stabilize RHP-pole.

- It would be extremely difficult to control system w/ RHP-poles and RHP-zeros close with each other; w/ slow RHP-zeros and fast RHP-poles.

- Ex) inverted pendulum: $G_1(s) = \frac{-g}{s^2(Mls^2-(M+m)g)}$ and $G_2(s) = \frac{ls^2-g}{s^2(Mls^2-(M+m)g)}$ (e.g., short/light rod, small m/M, large m).