Chapter 11.
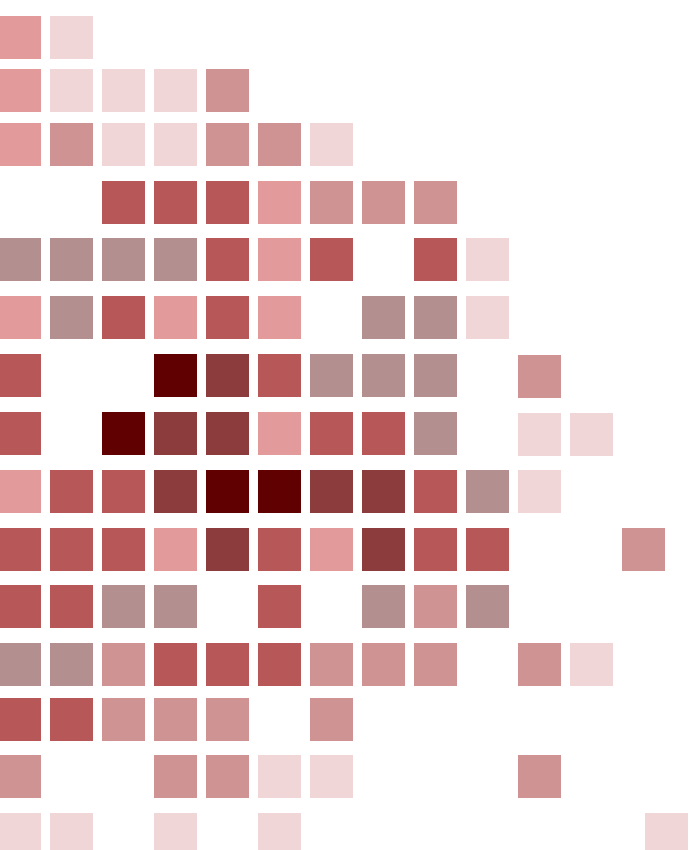# Fatigue Crack Growth

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CONTENTS

11.1 Introduction

- **Objectives**

  - Apply the stress intensity factor K of fracture mechanics to fatigue crack growth and to environmental crack growth, and understand test methods and trends in behavior.

  - Explore fatigue crack growth rate curves, da/dN versus ΔK, including fitting common equations and evaluating R-ratio (mean stress) effects.

  - Calculate the life to grow a fatigue crack to failure, including cases requiring numerical integration and cases of variable amplitude loading. Employ such calculations to evaluate safety factors and inspection intervals.

- **Types of Crack Growth**

  - *Fatigue crack growth*
    : crack growth caused by cyclic loading (11.3-11.9)

  - *Environmental crack growth*
    : crack growth when a hostile environment is present (11.10)

- **11.2.1 Need for Crack Growth Analysis**
  - If the number of cycles expected in actual service is $\widehat{N}$, then **the safety factor** on life is

$$X_N = \frac{N_{if}}{\widehat{N}} \tag{11.2}$$

  - The critical strength for brittle fracture of the member is determined by the current crack length and the fracture toughness K_c for the material and thickness involved:
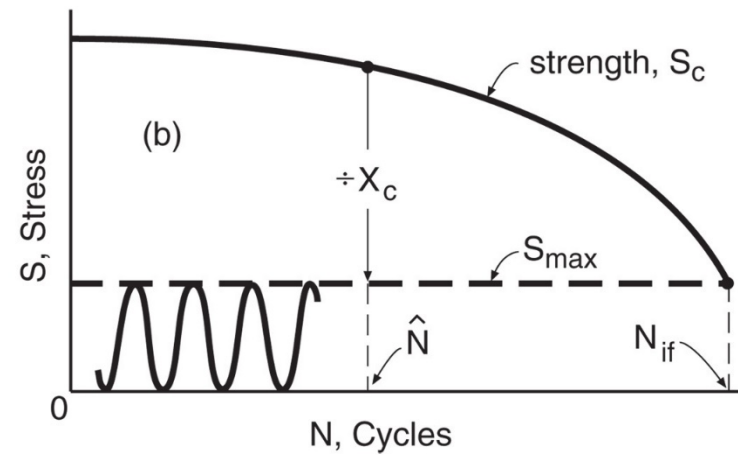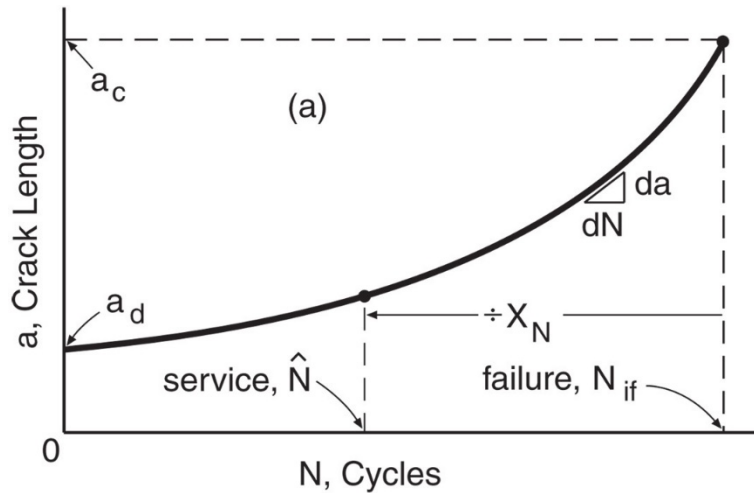
$$S_c = \frac{K_c}{F\sqrt{\pi a}} \tag{11.3}$$

  - The safety factor on stress against sudden brittle fracture due to the applied cyclic load is

$$X_c = \frac{S_c}{S_{\max}} \tag{11.4}$$

  - Predicted failure prior to reaching the actual service life, $X_N < 1$, ***Periodic inspections*** for cracks are then necessary. The safety factor on life is then determined by the inspection period:

$$X_N = \frac{N_{if}}{N_p} \tag{11.5}$$

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Figure 11.1 Growth of a worst-case crack from the minimum detectable length a_d to failure (a), and the resulting variation in worst-case strength (b).
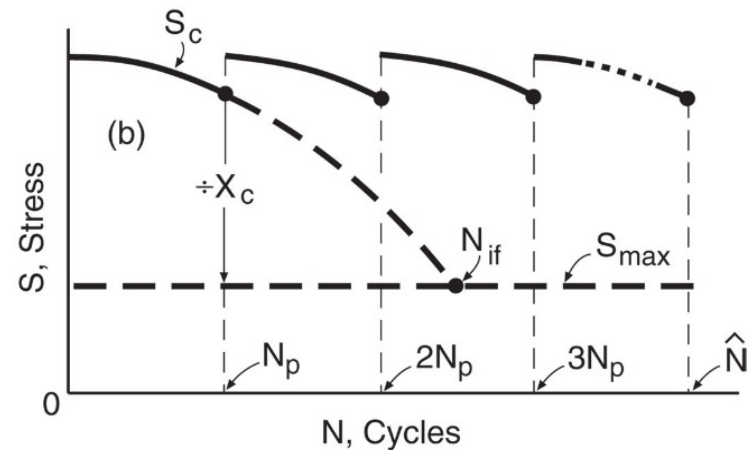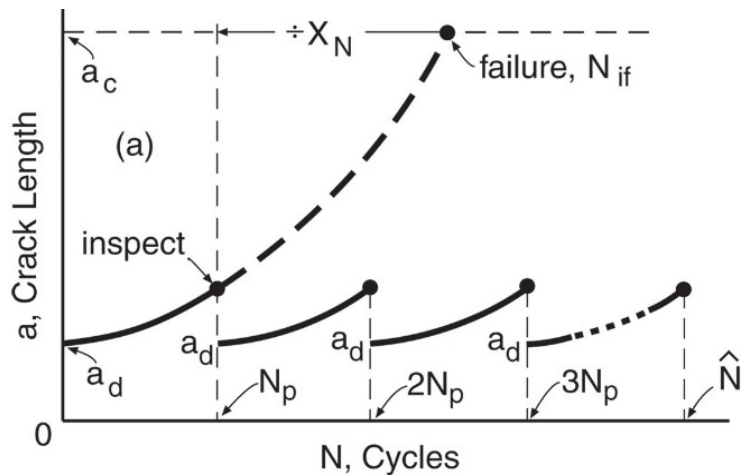
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Figure 11.2 Variation of worst-case crack length (a), and strength (b), where periodic inspections are required.

Definition for Fatigue Crack Growth

- *Fatigue crack growth rate*: the slope at a point on an a versus N curve.
- The primary variable affecting the growth rate of a crack is the range of the stress intensity factor, K.

$$\Delta K = F \, \Delta S \, \sqrt{\pi a} \tag{11.7}$$

where,

$$\Delta S = S_{\max} - S_{\min} \, , \quad R = \frac{S_{\min}}{S_{\max}} \tag{11.6}$$

- Also, it may be convenient, especially for laboratory test specimens, to use the alternative expression of K in terms of applied force P, as discussed in Ch. 8, relative to Eq. 8.13.

$$\Delta K = F_P \, \frac{\Delta P}{t\sqrt{b}} \, , \qquad R = \frac{P_{\min}}{P_{\max}} \tag{11.9}$$

Describing Fatigue Crack Growth Behavior of Materials (1)

- **The crack growth behavior can be described by the relationship between cyclic crack growth rate da/dN and stress intensity range ΔK.**

$$\frac{da}{dN} = C(\Delta K)^m \qquad (11.10)$$

where C is a constant and m is the slope on the log-log plot.

- ***Fatigue crack growth threshold*, ΔK_{th}**: At low growth rates, the curve generally becomes steep and appears to approach a vertical asymptote denoted ΔK_{th}.

- **At high growth rates, the curve may again become steep, due to rapid unstable crack growth. In this case the curve approaches an asymptote corresponding to $K_{max} = K_c$, the fracture toughness for the material and thickness of interest.**

- **The value of the stress ratio R or mean stress affects the growth rate. For a given ΔK, increasing R increases the growth rate, and vice versa.**
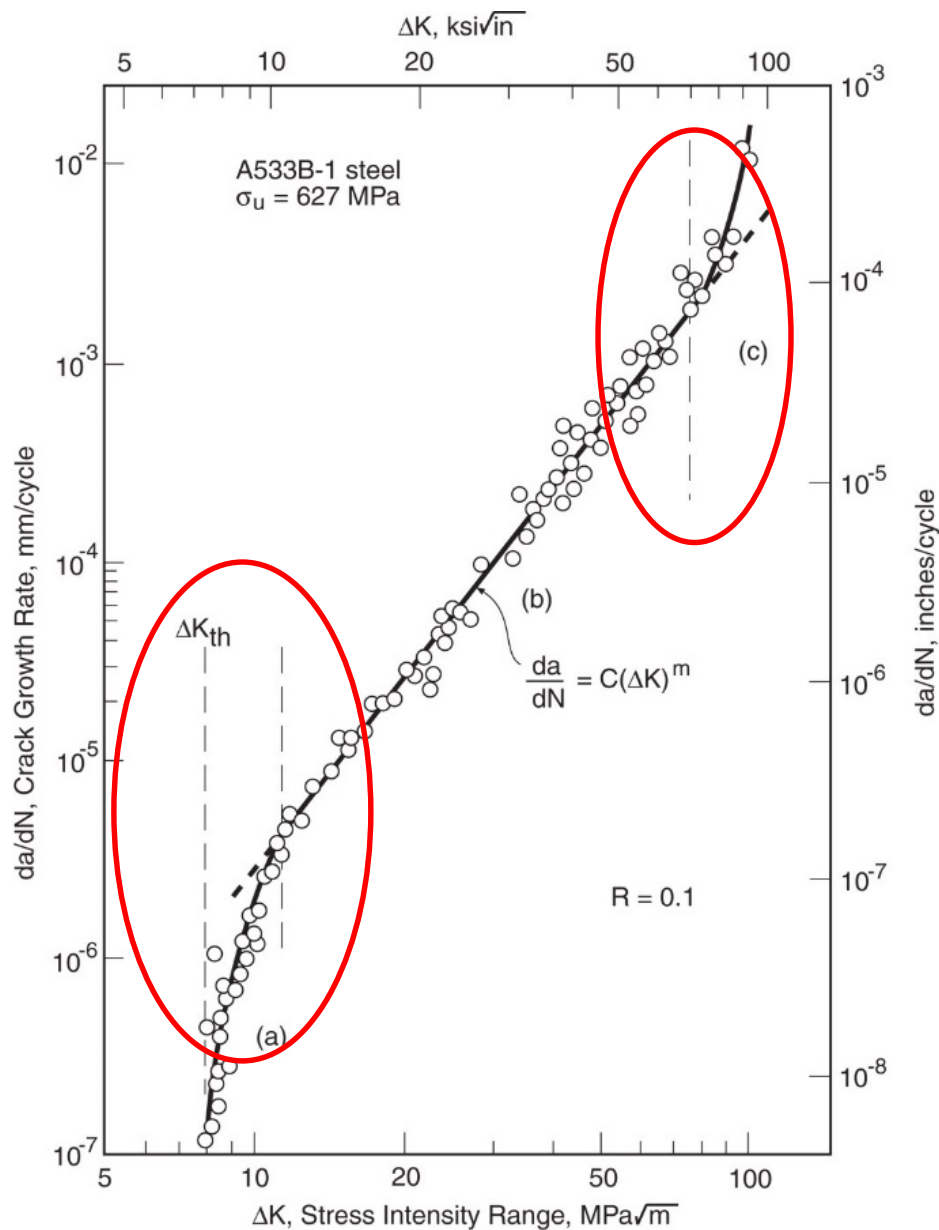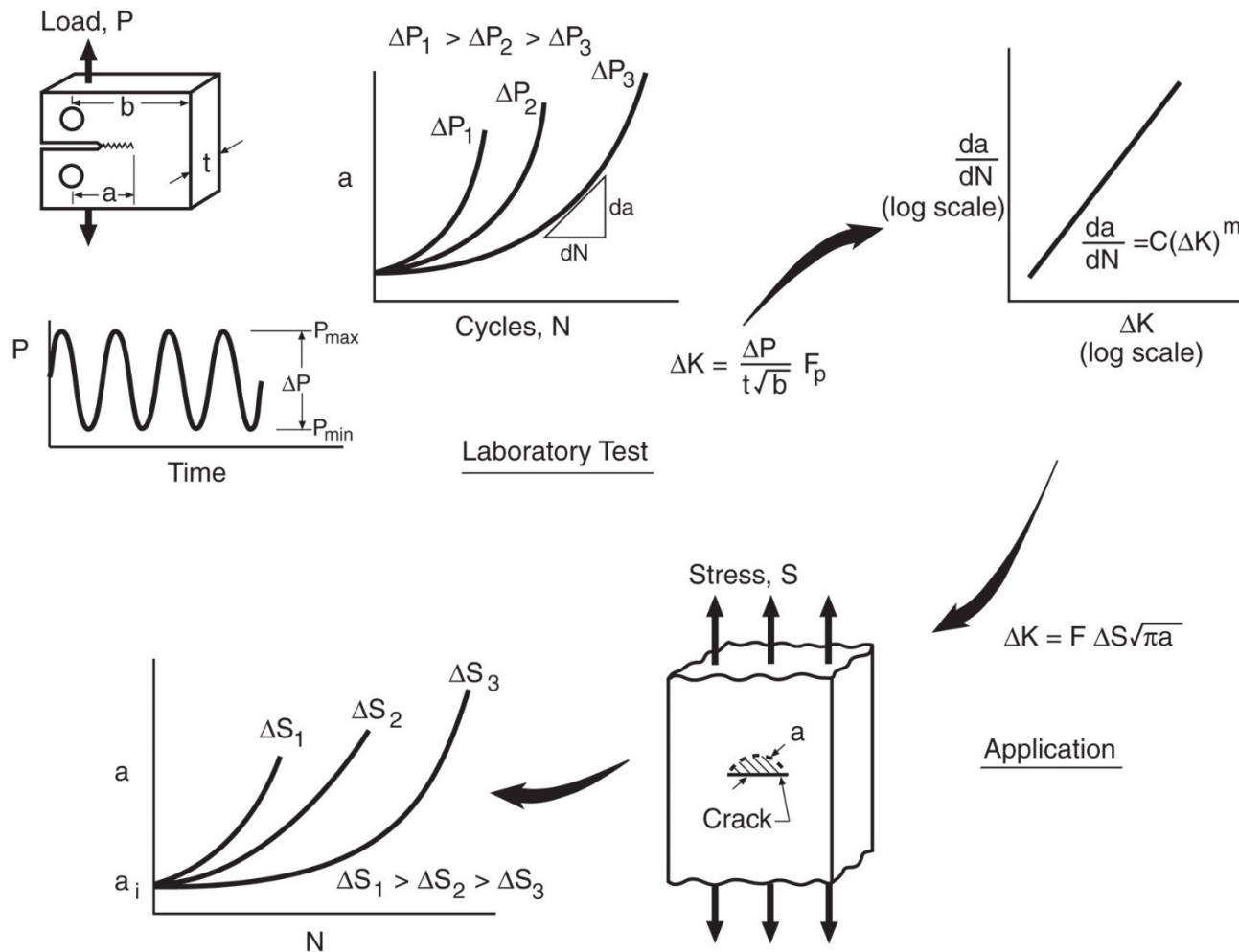
Figure 11.3 Fatigue crack growth rates over a wide range of stress intensities for a ductile pressure vessel steel. Three regions of behavior are indicated: (a) slow growth near the threshold ΔK_{th}, (b) intermediate region following a power equation, and (c) unstable rapid growth. (Plotted from the original data for the study of [Paris 72].)
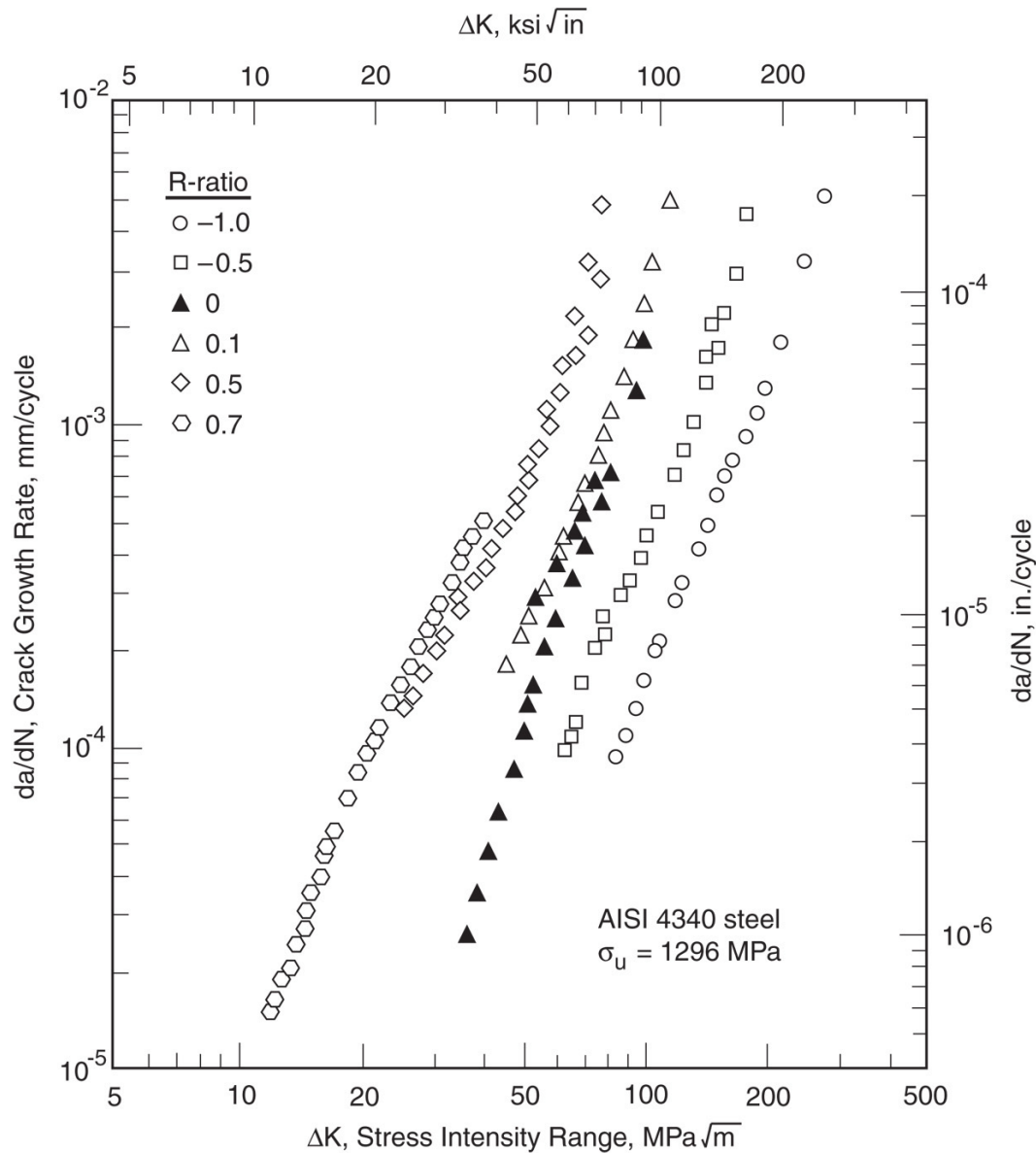
Figure 11.4 Effect of *R*-ratio on crack growth rates for an alloy steel. For *R* < 0, the compressive portion of the load cycle is here included in calculating Δ*K*. (Data from [Dennis 86].)

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Figure 11.5 Steps in obtaining da/dN versus ΔK data and using it for an engineering application. (Adapted from [Clark 71]; used with permission.)

11.3 Fatigue Crack Growth Rate Testing

- **Standard methods for conducting fatigue crack growth test: ASTM E647**
- **Two commonly used test specimen geometries are the standard compact specimen and center-cracked plates.**
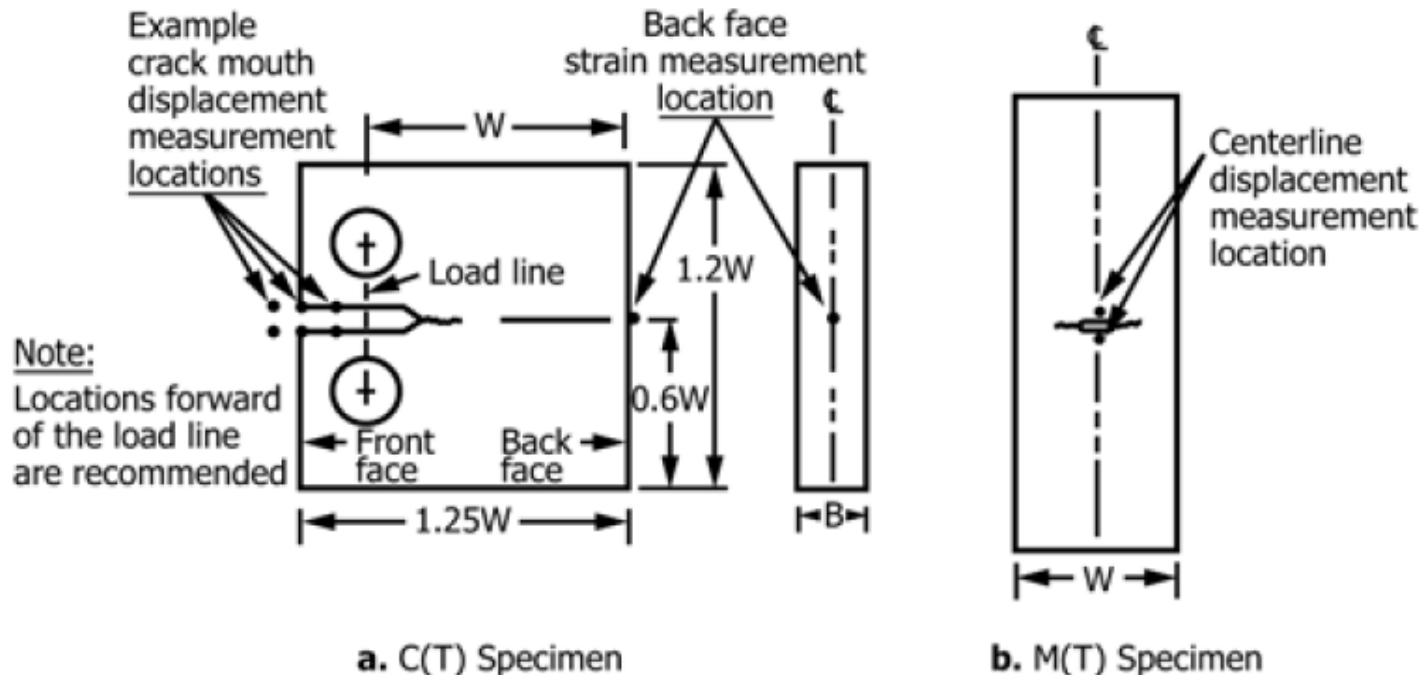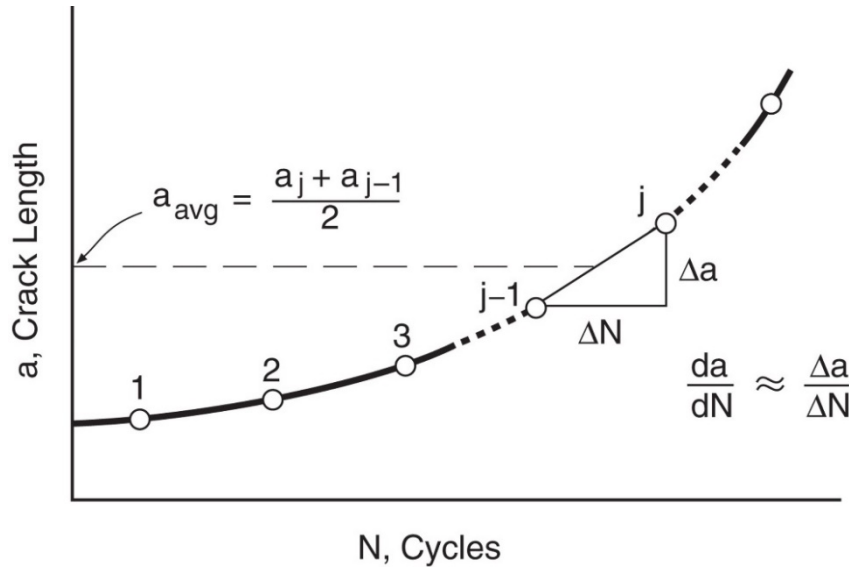

FIG. X2.1 Recommended Displacement and Strain Measurement Locations for Determination of Fatigue Crack Opening Load on C(T) and M(T) Specimens

Test Methods and Data Analysis

$$\left(\frac{da}{dN}\right)_j \approx \left(\frac{\Delta a}{\Delta N}\right)_j = \frac{a_j - a_{j-1}}{N_j - N_{j-1}} \qquad (11.11)$$

$$\Delta K_j = F\Delta S\sqrt{\pi a_{\text{avg}}}, \;\; \Delta K_j = F_P \frac{\Delta P}{t\sqrt{b}} \qquad (11.12)$$

Figure 11.6 Crack growth rates obtained from adjacent pairs of a versus *N* data points.
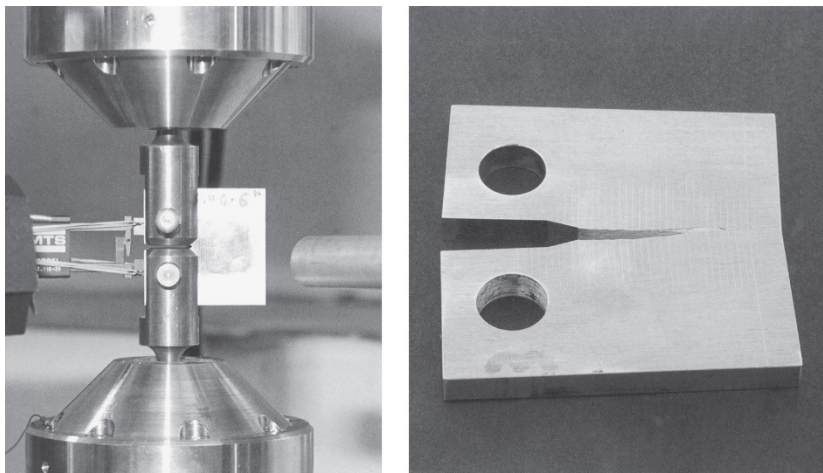
Figure 11.7 Crack growth rate test under way (left) on a compact specimen (*b* = 51 mm), with a microscope and a strobe light used to visually monitor crack growth. Cycle numbers are recorded when the crack reaches each of a number of scribe lines (right). (Photos by R. A. Simonds.)

Test Variables

- **Variations of R in the range 0 to 0.2 have little effect on most materials, and tests in this range are accepted by convention as the standard basis for comparing the effects of various materials, environments, etc.**

- **Test conditions may be selected to include situations that resemble the anticipated service use of the material:** Wide range of variables: Temperature, frequency of the cyclic load, hostile chemical environment

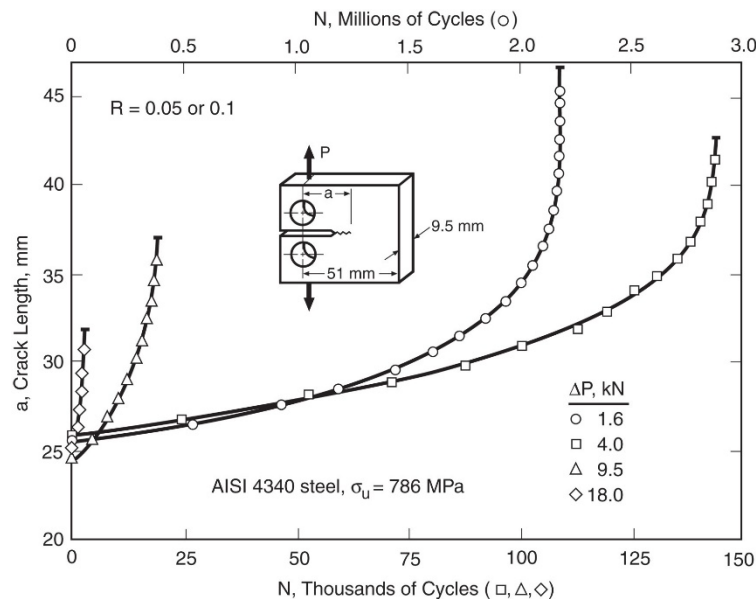**Figure 11.8** Crack length versus cycles data for four different levels of cyclic load applied to compact specimens of an alloy steel.
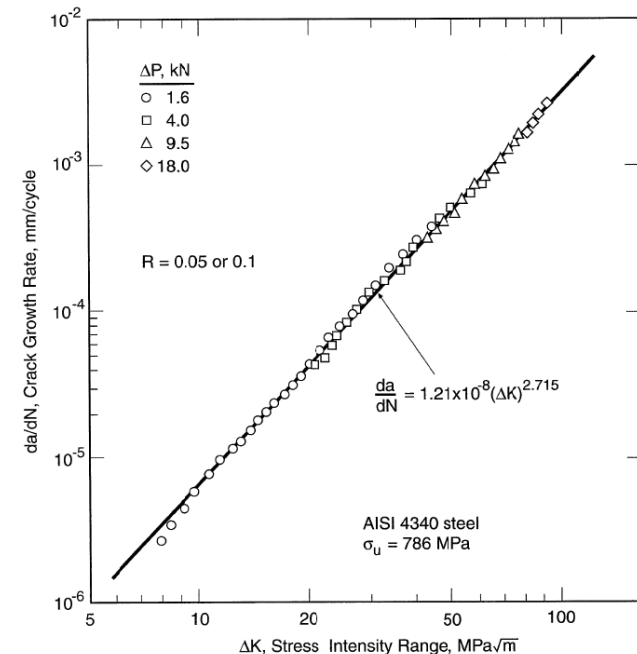
Figure 11.9 Data and least-squares fitted line for *da/dN* vs *ΔK* from the a versus *N* data of Fig. 11.8.

Geometry Independence of da/dN versus ΔK Curves

- For a given material and set of test conditions, such as a particular R value, test frequency, and environment, **the growth rates should depend only on ΔK.**

- Regardless of the load level, crack length, and specimen geometry, all *da/dN* vs ΔK data for a given set of test conditions should fall together along a single curve.

- However, difficulty with the applicability of ΔK can occur if there is excessive yielding, or for very small cracks (Section 11.9)
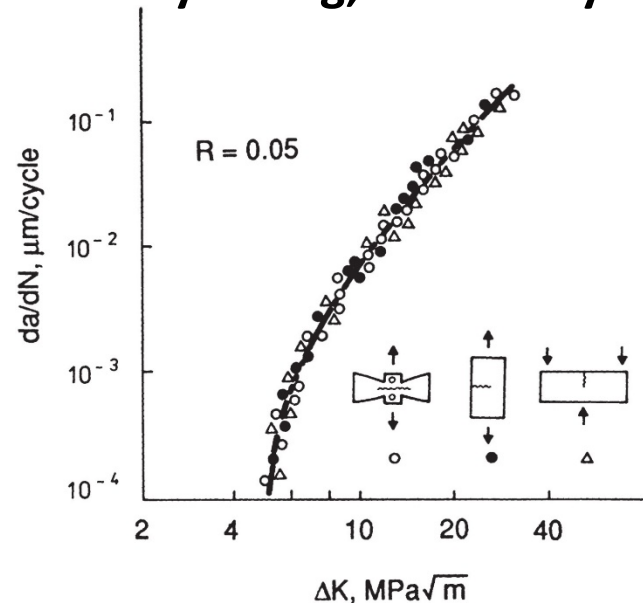
Figure 11.10   Fatigue crack growth rate data for a 0.65% carbon steel, demonstrating geometry independence. (Adapted from [Klesnil 80] p. 111; used with permission.)

- **The effect is usually more pronounced for more brittle materials.**

- **In contrast, mild steel and other relatively low-strength, highly ductile, structural metals exhibit only a weak R effect in the intermediate growth rate region of the da/dN versus ΔK curve.**
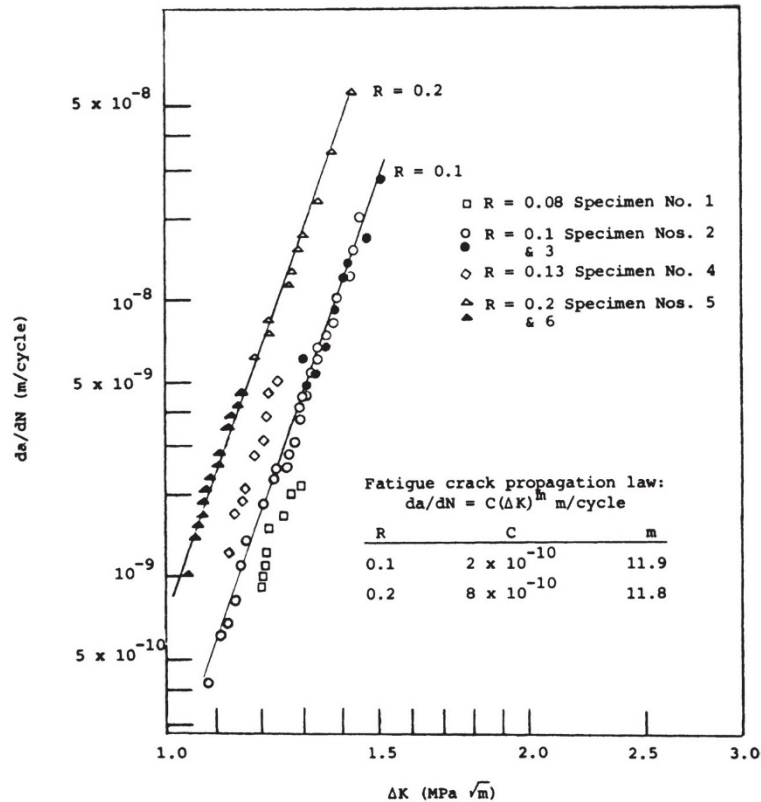
Figure 11.11 Effect of R-ratio on fatigue crack growth rates for Westerly granite, tested in the form of three-point bend specimens. (From [Kim 81]; copyright © ASTM; reprinted with permission.)

- **The Walker Equation**
  - Based on the Walker relationship (Section 10.6.4)

$$\overline{\Delta K} = K_{\max}(1 - R)^{\gamma} \qquad (11.15)$$

  - γ is a constant for the material, $\overline{\Delta K}$ is an equivalent zero-to-tension stress intensity
  - From Eq. 11.10,

$$\frac{da}{dN} = \frac{C_0}{(1-R)^{m(1-\gamma)}}(\Delta K)^m \qquad (11.19)$$

  - However, it is primarily employed for intermediate growth rates where Eq. 11.10 does apply

- **The Forman Equation**
  - The equation has the attractive feature of predicting accelerated growth near the final toughness failure, while approaching Eq. 11.10 at low ΔK

$$\frac{da}{dN} = \frac{C_2(\Delta K)^{m_2}}{(1-R)K_c - \Delta K} = \frac{C_2(\Delta K)^{m_2}}{(1-R)(K_c - K_{\max})} \qquad (11.22)$$
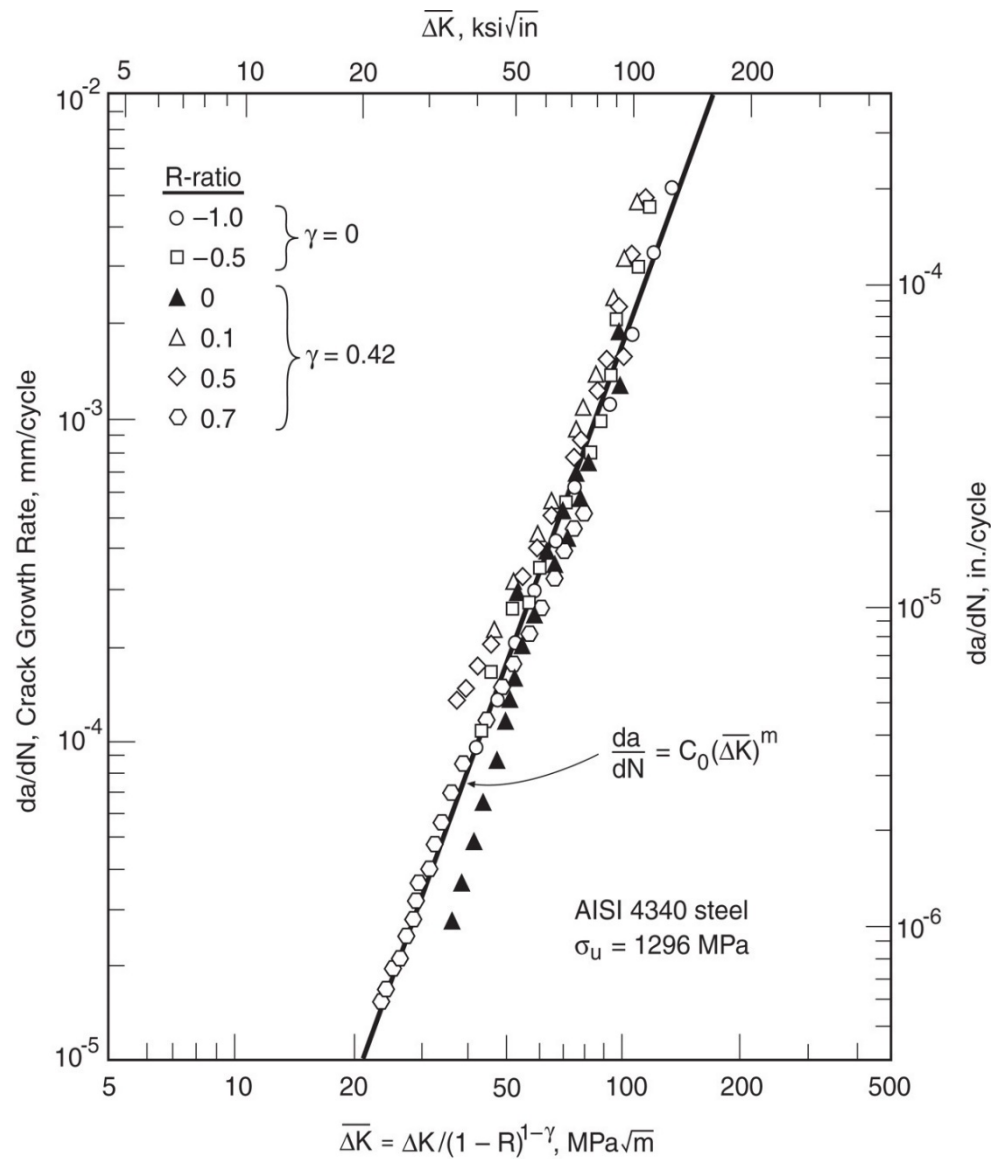
Figure 11.12 Representation of the data of Fig. 11.4 by a single relationship based on the Walker equation. (Data from [Dennis 86].)
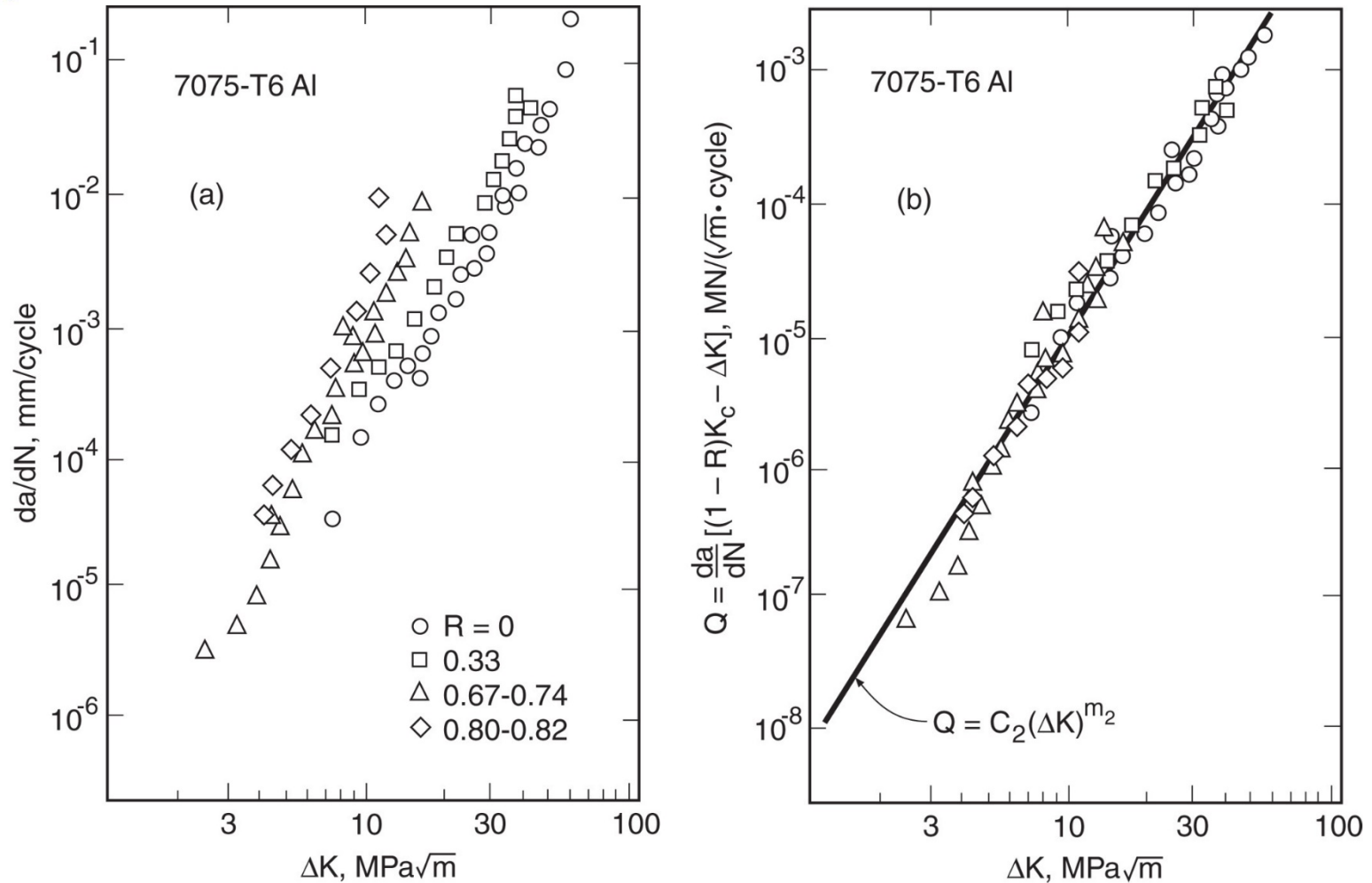
Figure 11.13 Effect of *R*-ratio on growth rates in 7075-T6 aluminum (a), and correlation of these data (b) on the basis of the Forman equation, with constants as listed in Table 11.3. (Data from [Hudson 69].)

- **11.5.1 Trends with Material**
  - If various major classes of metals are considered, such as steels, aluminum alloys, and titanium alloys, crack growth rates differ considerably when compared on a da/dN versus ΔK plot.
  - However, the ΔK values corresponding to a given growth rate scale roughly with the elastic modulus E.
  - Polymers exhibit a wide range of growth rates which are considerably higher than for most metals
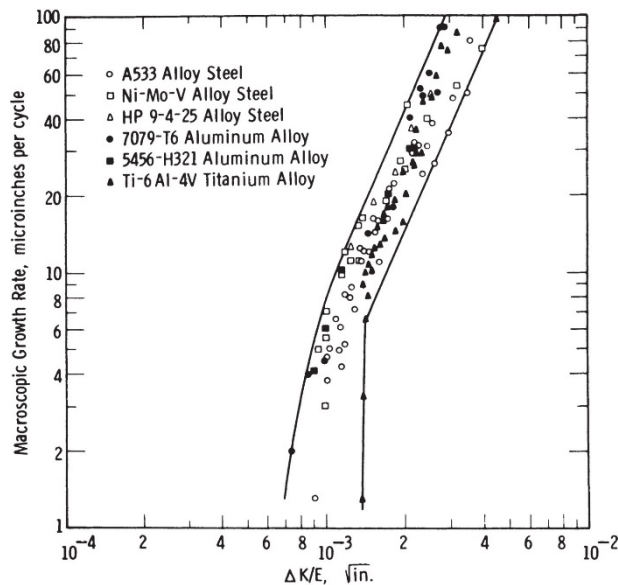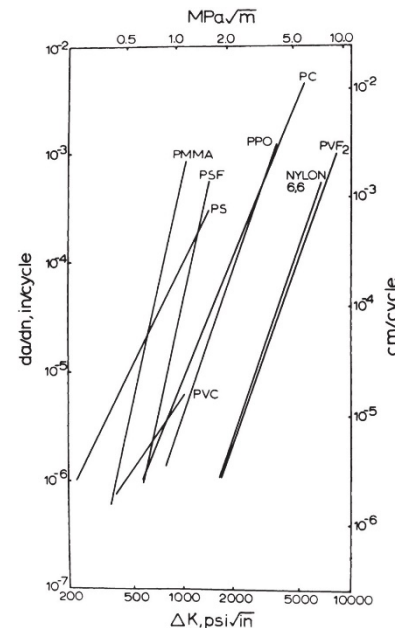
(left) Figure 11.18 Fatigue crack growth trends for various metals correlated by plotting $\Delta K/E$. (From [Bates 69]; used with permission.)

(right) Figure 11.19 Fatigue crack growth trends for various crystalline and amorphous polymers. (From [Hertzberg 75]; used with permission.)

11.5 Trends in Fatigue Crack Growth Behavior

- **11.5.2 Trends with Temperature and Environment**
  - Changing the temperature usually affects the fatigue crack growth rate, with higher temperature often causing faster growth.
  - However, an opposite trend can occur in BCC metals due to the cleavage mechanism contributing to fatigue crack growth at low temperature. (Section 8.6)
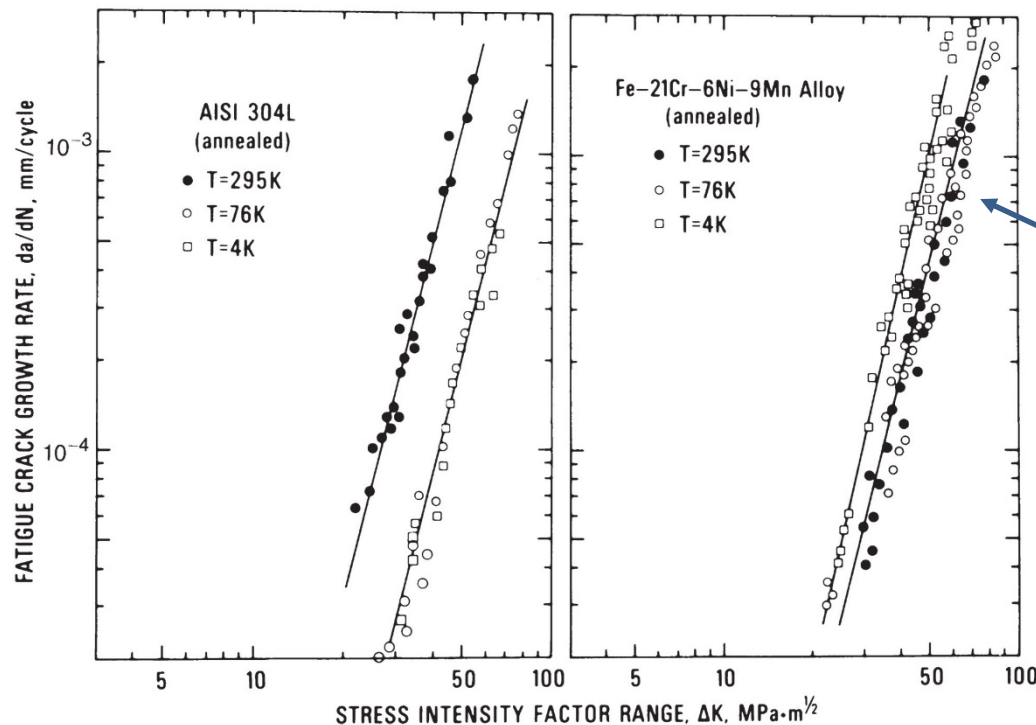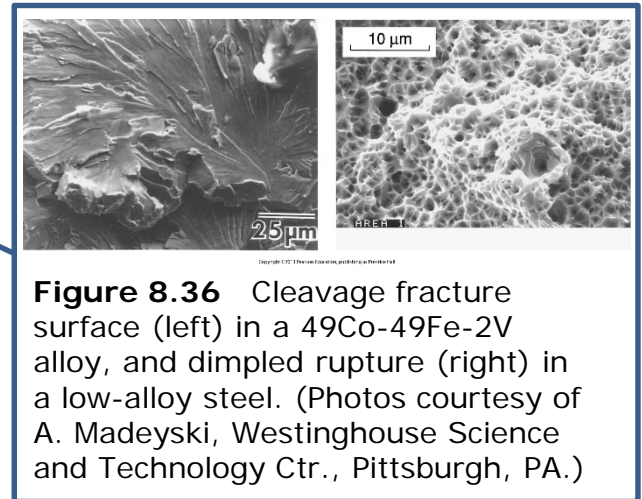
Figure 8.36 Cleavage fracture surface (left) in a 49Co-49Fe-2V alloy, and dimpled rupture (right) in a low-alloy steel. (Photos courtesy of A. Madeyski, Westinghouse Science and Technology Ctr., Pittsburgh, PA.)

Figure 11.21 Effect of temperature on fatigue crack growth rates in two metals. (From [Tobler 78]; used with permission.)

- **11.5.2 Trends with Temperature and Environment**

Figure 11.23 Contrasting sensitivity to corrosion fatigue crack growth of two strength levels of an alloy steel. (Adapted from [Imhof 73]; copyright © ASTM; reprinted with permission.)
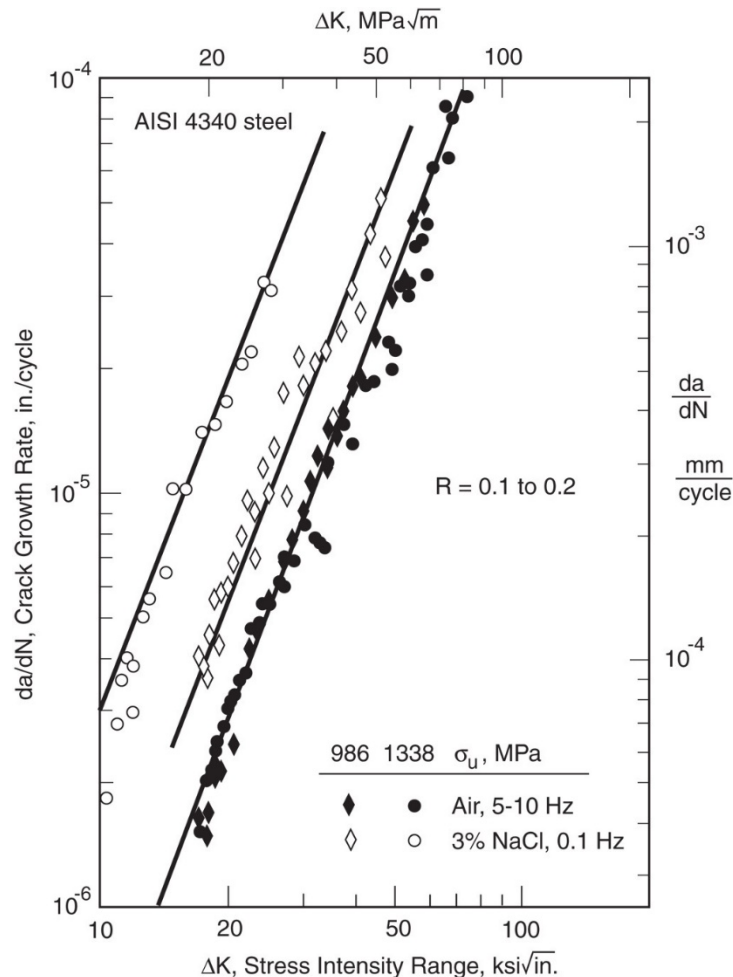
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- – Hostile chemical environments often increase fatigue crack growth rates, with certain combinations of material and environment causing especially large effects.

- – The term *corrosion fatigue* is often used when the environment involved is a corrosive medium, such as seawater even the gases and moisture in air.

- **Crack growth rates, da/dN**
  - Crack growth rates da/dN for a given combination of material and R-ratio are given as a function of ΔK by Eqs. 11.10, 11.18, and 11.22

$$\frac{da}{dN} = f(\Delta K, R) \tag{11.26}$$

  - where any effects of environment, frequency, etc., are assumed to be included in the material constants involved.
  - The life in cycles required for crack growth may be calculated by solving this equation for dN and integrating both sides:

$$\int_{N_i}^{N_f} dN = N_f - N_i = N_{if} = \int_{a_i}^{a_f} \frac{da}{f(\Delta K, R)} \tag{11.27}$$
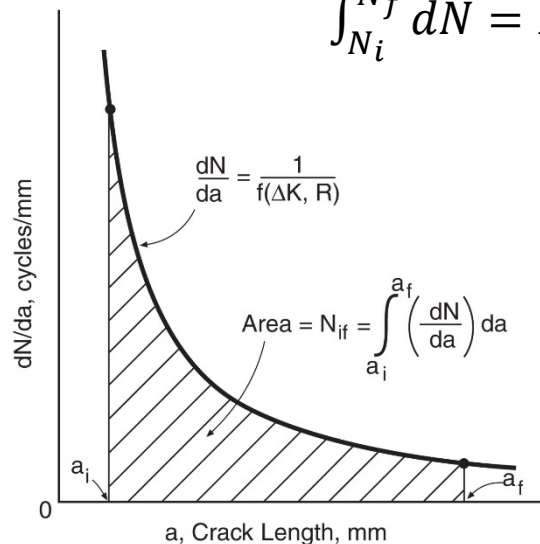
$$N_{if} = \int_{a_i}^{a_f} \left(\frac{dN}{da}\right) da \tag{11.29}$$

Figure 11.26 Area under the *dN/da* versus a curve used to estimate the number of cycles to grow a crack from initial size a_i to final size a_f.

- **Closed-Form Solutions**

  - Consider a situation where growth rates are given by Eq. 11.10 and where $F = F(a/b)$ in Eq. 11.7 can be approximated as constant over the range of crack lengths a_i to a_f:

$$\frac{da}{dN} = f(\Delta K, R) = C(\Delta K)^m, \quad \Delta K = F\Delta S\sqrt{\pi a} \qquad (11.30)$$

  - Assume that $S_{\max}$ and $S_{\min}$ are constant, so that ΔS and R are also both constant. Substituting this particular $f(\Delta K, R)$ into Eq. 11.27 and then substituting for ΔK gives

$$N_{if} = \int_{a_i}^{a_f} \frac{da}{C(\Delta K)^m} = \int_{a_i}^{a_f} \frac{da}{C(F\Delta S\sqrt{\pi a})^m} = \int_{a_i}^{a_f} \frac{1}{C(F\Delta S\sqrt{\pi})^m} \frac{da}{a^{m/2}} \qquad (11.31)$$

  - Since $C, F, \Delta S,$ and m are all constant, the only variable is a, and integration is straightforward, giving

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\Delta S\sqrt{\pi})^m (1-\frac{m}{2})} \quad (m \neq 2) \qquad (11.32)$$

- **Solutions by Numerical Integration**
  - When F changes excessively between the initial and final crack lengths, a_i and a_f, closed-form integration of Eq. 11.27 is seldom possible, numerical integration becomes necessary.

$$\text{Area} = \int_{a_j}^{a_{j+2}} \left(\frac{dN}{da}\right) da$$
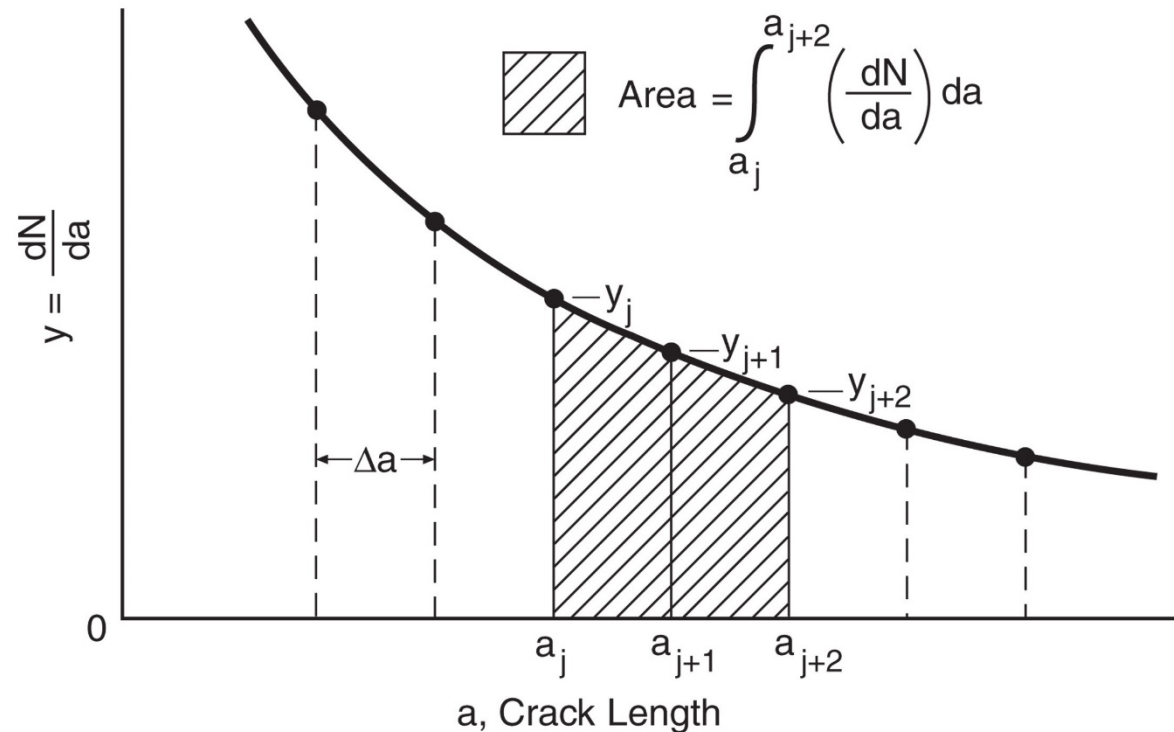
Figure 11.27   Area under the *dN/da* versus a curve over two intervals Δ*a* as estimated by Simpson's rule.

11.7 Life Estimates for Variable Amplitude Loading

- **One simple approach – assume that growth for a given cycle is not affected by the prior history –that is, *sequence effects* are absent.**

- **11.7.1 Summation of crack Increments**

$$a_{j+1} = a_j + \Delta a_j = a_j + \left(\frac{da}{dN}\right)_j \qquad (11.39)$$

where a_j is the current crack length, Δa_j is the increment, a_{j+1} is the new value of crack length.

Denoting the initial crack length as a_i, we find that the crack length after N cycle is

$$a_N = a_i + \sum_{j=1}^{N}\left(\frac{da}{dN}\right)_j \qquad (11.40)$$

For highly irregular loading, rainflow cycle counting as described in Chapter 9 can be used to identify the cycles.

- **11.7.2 Special Method for Repeating or Stationary Histories**
  - In some cases, it may be reasonable to approximate the actual service load history by assuming that it is equivalent to repeated applications of a loading sequence of finite length.
  - This can be useful where some repeated operation occurs, such as lift cycles for a crane, or flights of an aircraft, and also for random loading with characteristics that are constant with time, called *stationary* loading.

$$\Delta a_j = C_0\big(\overline{\Delta K_j}\big)^m \qquad\qquad (11.41)$$

where different R-ratios are handled by calculating an equivalent zero-to-tension ($R = 0$) value $\overline{\Delta K}$, as in the Walker approach using Eq. 11. 15. Note that the coefficient C_0 corresponding to $R = 0$ applies due to the use of $\overline{\Delta K}$.

11.7 Life Estimates for
Variable Amplitude Loading (optional)

- **11.7.2 Special Method for Repeating or Stationary Histories (cont'd)**

  If the repeating load history contains N_B cycles, the increase in crack length during one repetition is obtained by summing:

  $$\Delta a_B = \sum_{j=1}^{N_B} \Delta a_j = \sum_{j=1}^{N_B} C_0 \left(\overline{\Delta K_j}\right)^m \qquad (11.42)$$

  The average growth rate per cycle during one repetition of the history is thus

  $$\left(\frac{da}{dN}\right)_{\text{avg.}} = \frac{\Delta a_B}{N_B} = C_0 \sum_{j=1}^{N_B} \left(\overline{\Delta K_j}\right)^m \qquad (11.43)$$

  Note that C_0 is constant and so can be factored from the summation:

  $$\left(\frac{da}{dN}\right)_{\text{avg.}} = C_0 \left(\left[\frac{\sum_{j=1}^{N_B}\left(\overline{\Delta K_j}\right)^m}{N_B}\right]^{1/m}\right)^m = C_0 \left(\Delta K_q\right)^m \qquad (11.44)$$

  where

  $$\Delta K_q = \left[\frac{\sum_{j=1}^{N_B}\left(\overline{\Delta K_j}\right)^m}{N_B}\right]^{1/m} \qquad (11.45)$$

- **11.7.2 Special Method for Repeating or Stationary Histories (cont'd)**

$$\Delta K_q = \left[\frac{\sum_{j=1}^{N_B}\left(\overline{\Delta K}_j\right)^m}{N_B}\right]^{1/m} \tag{11.45}$$

The quantity ΔK_q can be interpreted as an equivalent zero-to-tension stress intensity range that is expected to cause the same crack growth as the variable amplitude history when applied for the same number of cycles N_B.

$$\Delta S_q = \frac{\Delta K_q}{F\sqrt{\pi a}} = \left[\frac{\sum_{j=1}^{N_B}\left(\overline{\Delta S}_j\right)^m}{N_B}\right]^{1/m} \tag{11.46}$$

Since ΔS_q is independent of crack length, it can be applied throughout the life as the crack grows. Hence, we can make a life estimate by using ΔS_q just as if it were a constant amplitude loading at $R = 0$, for example, by using Eq. 11.32.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\left(F\Delta S\sqrt{\pi}\right)^m\left(1-\frac{m}{2}\right)} \quad (m \neq 2) \tag{11.32}$$

- **Example 11.6**
  - A center-cracked plate of the AISI 4340 steel of Table 11.2 has dimensions, as defined in Fig. 8.12(a), of $b = 38$ and $t = 6mm$, and the initial crack length is $a_i = 1\ mm$. It is repeatedly subjected to the axial force history of Fig. E11.6. How many repetitions of this history can be applied before fatigue failure is expected?

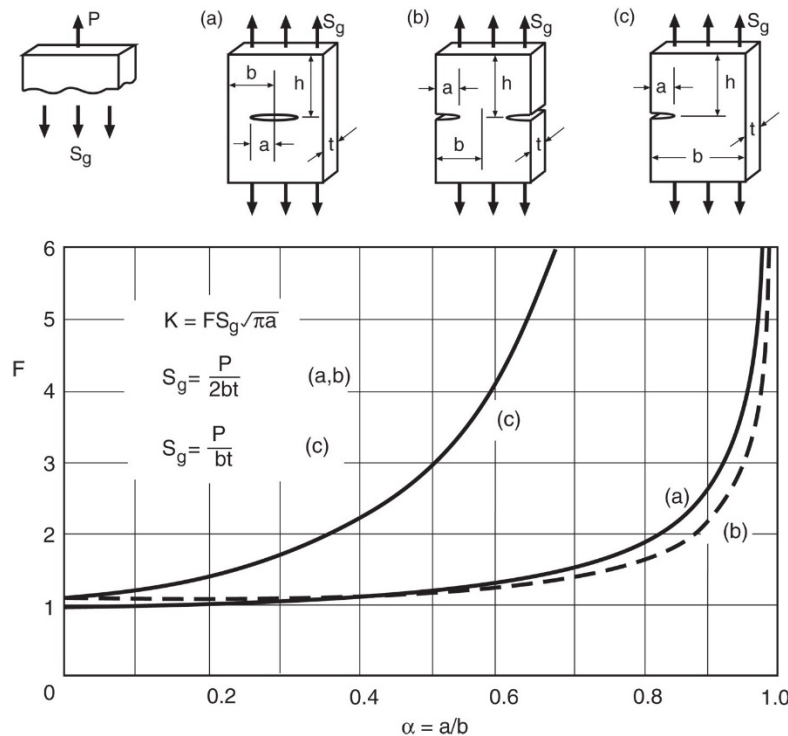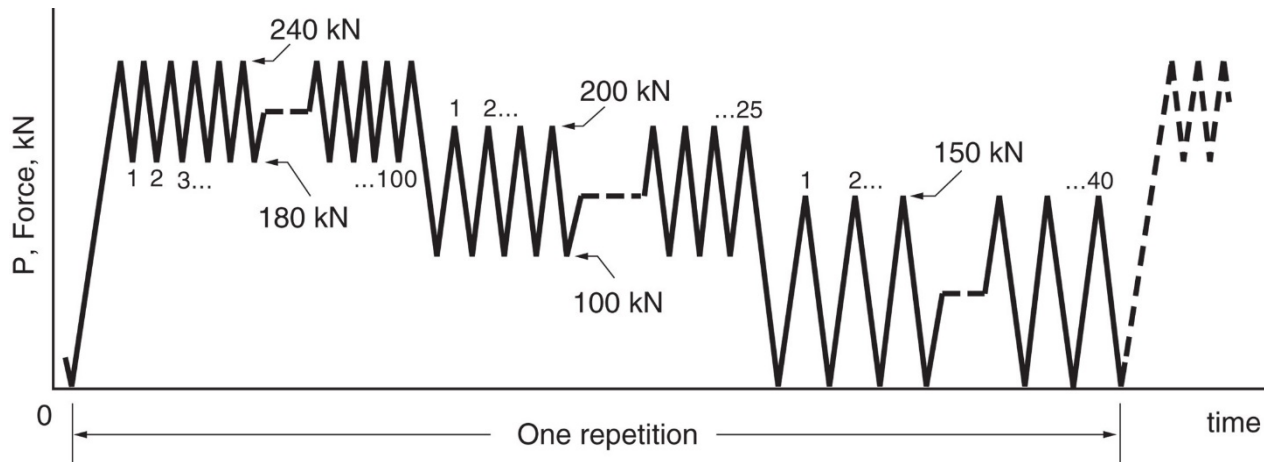Copyright ©2013 Pearson Education, publishing as Prentice Hall

Figure 8.12 Stress intensity factors for three cases of cracked plates under tension. Geometries, curves, and equations labeled (a) all correspond to the same case, and similarly for (b) and (c).

Table 11.2 Constants for the Walker Equation for Several Metals

Material	Yield σ_o MPa (ksi)	Toughness K_{Ic} MPa $\sqrt{m}$ (ksi$\sqrt{in}$)	Walker Equation				
			C_0 $\dfrac{\text{mm/cycle}}{(\text{MPa}\sqrt{m})^m}$	C_0 $\dfrac{\text{in/cycle}}{(\text{ksi}\sqrt{in})^m}$	m	γ $(R \geq 0)$	γ $(R < 0)$
Man-Ten steel	363 (52.6)	200^1 (182)	3.28×10^{-9}	1.74×10^{-10}	3.13	0.928	0.220
RQC-100 steel	778 (113)	150^1 (136)	8.01×10^{-11}	4.71×10^{-12}	4.24	0.719	0
AISI 4340 steel ($\sigma_u = 1296$ MPa)	1255 (182)	130 (118)	5.11×10^{-10}	2.73×10^{-11}	3.24	0.420	0
17-4 PH steel (H1050, vac. melt)	1059 (154)	120^1 (109)	3.29×10^{-8}	1.63×10^{-9}	2.44	0.790	—

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Figure E11.6

- **Solution: Example 11.6**

From rainflow counting of the given force history, we obtain the results presented in the first four columns of Table E11.6. The single cycle for $j = 4$ arises from rainflow cycle counting as the major cycle between the highest peak and lowest valley.

Since multiple cycles occur at each of k=4 load levels, the summation for Eq. 11.46 may be done in the form

$$\sum_{j=1}^{N_B}\left(\overline{S_j}\right)^m = \sum_{j=1}^{k} N_j\left(\overline{\Delta S_j}\right)^m$$

Noting that $N_B = \sum N_j = 166 \ cycles$, we may now calculate ΔS_q :

$$\Delta S_q = \left[\frac{\sum_{j=1}^{k} N_j\left(\overline{\Delta S_j}\right)^m}{N_B}\right]^{1/m} = \left[\frac{1.986\times10^{10}}{166}\right]^{1/3.24} = 311.3 \ \text{Mpa}$$

$$N_{if} = \frac{a_f^{1-m/2}-a_i^{1-m/2}}{C\left(F\Delta S\sqrt{\pi}\right)^m\left(1-\frac{m}{2}\right)} = \frac{0.0158^{-0.62}-0.001^{-0.62}}{5.11\times10^{-13}\left(1.00\times\ 311.3\sqrt{\pi}\right)^{3.24}(-0.62)} = 2.45\times10^5 \ cycles$$

Finally, the number of repetitions to failure is

$$B_{if} = \frac{N_{if}}{N_B} = \frac{2.45\times10^5}{166} = 1477 \ repetitions$$

Table E11.6

j	N_j cycles	$P_{\max}$ kN	$P_{\min}$ kN	R	$S_{\max}$ MPa	$\overline{\Delta S}_j$ MPa	$N_j(\overline{\Delta S}_j)^m$
1	100	240	180	0.75	526.3	294.0	9.94×10^9
2	25	200	100	0.5	438.6	327.8	3.54×10^9
3	40	150	0	0	328.9	328.9	5.72×10^9
4	1	240	0	0	526.3	526.3	6.56×10^8
Σ	166						1.986×10^{10}

- **11.7.3 Sequence Effects**
  - Assumption: the crack growth in a given cycle is unaffected by prior events in the load history ≠ **sometimes lead to significant error**
  - In case C, after high tensile overload is applied the growth rate during the lower level cycles is decreased
  - This beneficial effect of tensile overloads is called *crack growth retardation*
  - Overload sequence effects are likely to be important where high overloads occur **predominantly in one direction**
  - Less effect is expected if overloads occur in both directions, if the history is highly irregular, or if the overloads are relatively mild.
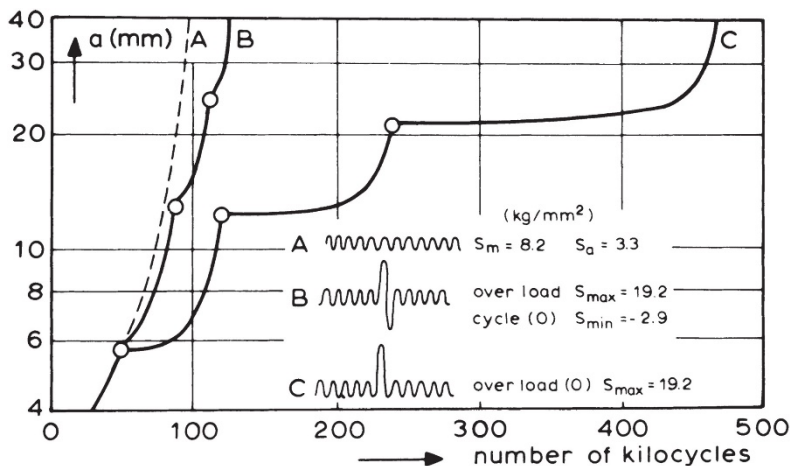

Figure 11.28   Effect of overloads on crack growth in center-cracked plates ($b = 80$, $t = 2$ mm) of 2024-T3 aluminum. (From [Broek 86] p. 273, based on data in [Schijve 62]; reprinted by permission of Kluwer Academic Publishers.)
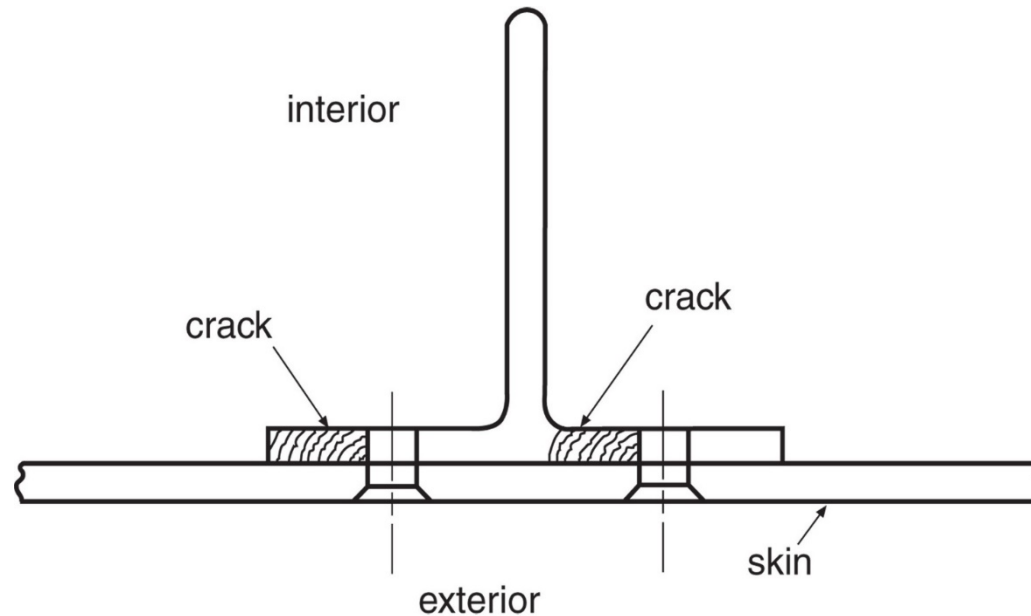
11.8 Design Considerations

- A *damage-tolerant* approach is critically dependent on initial and sometimes periodic inspections for cracks.

- Inspections of cracks, especially small ones, is an expensive process and is not generally feasible for inexpensive components that are made in large numbers

- Failures are minimized by careful attention to design detail and to manufacturing quality control, including initial inspection to eliminate any obviously flawed parts

- All approaches to ensuring adequate life are subject to additional uncertainties, such as

    1) Estimates of the service loading being too low
    2) Accidental substitution during manufacturing of the wrong material
    3) Undetected manufacturing quality control problems
    4) Hostile environmental effects that are more severe than forecast

- **Example: Aircraft skin**
  - Cracks at fastener (rivet or bolt) holes are of concern in aircraft structure, and access to the interior of the skin of the fuselage or wing structure may be needed for situations. Design must accommodate the disassembly when this is necessary for inspection.
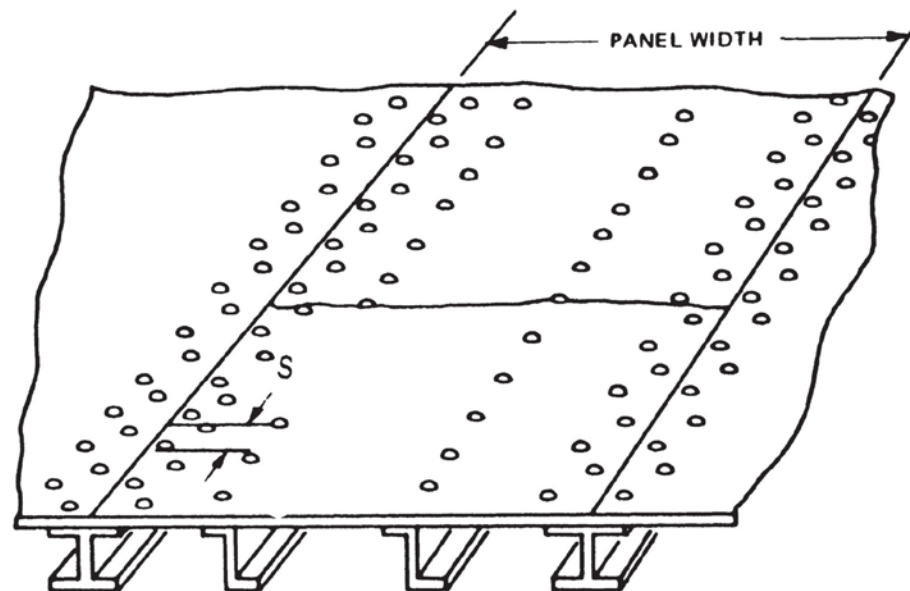

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Figure 11.29 Cracks in the interior of an aircraft skin structure. (Adapted from [Chang 78].)

11.8 Design Considerations

- **Example: Stiffened panel**
  - Specific measures can also be taken by the designer to allow structures to function without sudden failure even if a large crack does develop. Stiffeners retard crack growth, and joints in skin panels may be intentionally introduced so that a crack in one panel has difficulty growing into the next.
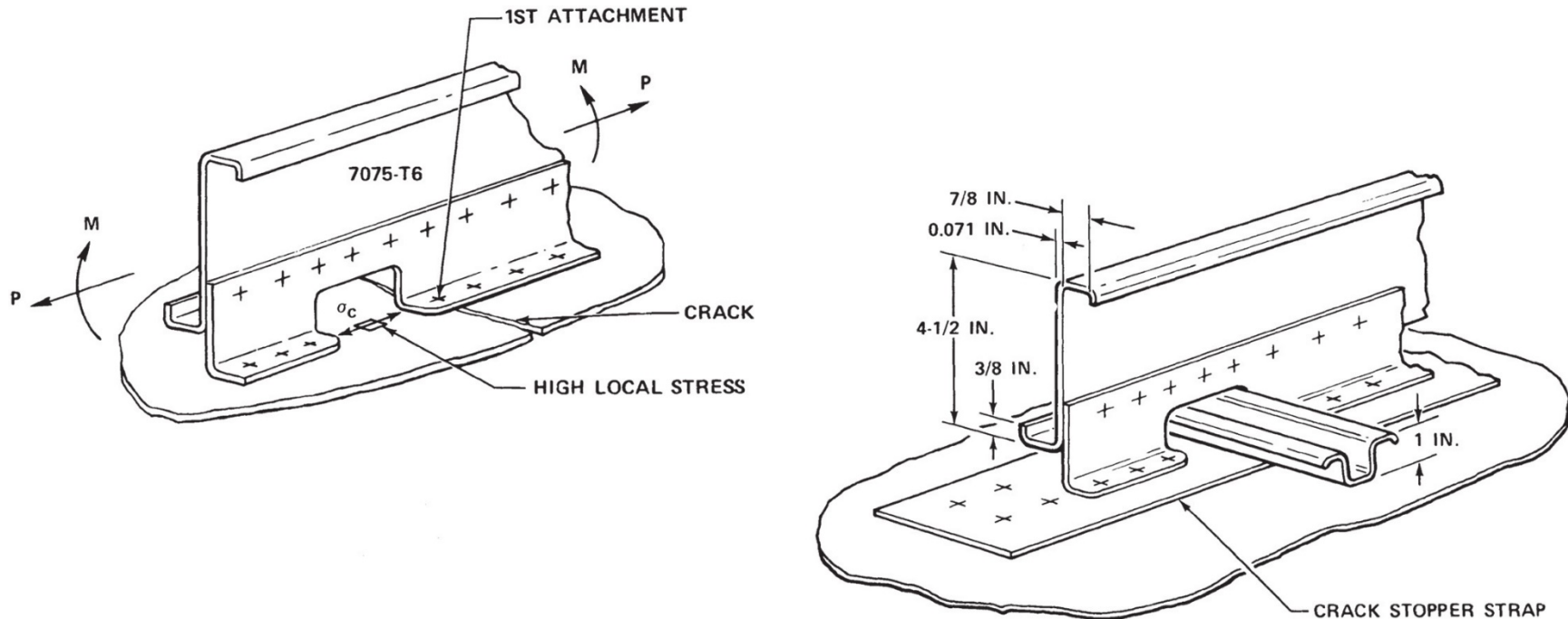
Figure 11.30 Stiffened panel in aircraft structure with a crack delayed before growing into adjacent panels. The rivet spacing dimensioned is 38 mm. (From the paper by J. P. Butler in [Wood 70] p. 41.)

- **Example: Crack stopper strap**

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Figure 11.31 Crack (left) in a DC-10 fuselage in the longitudinal direction, due to cabin pressure loading, and (right) a crack stopper strap. Rivet locations are indicated by (+), and the longeron member with a hat-shaped cross section is omitted on the left for clarity. (From [Swift 71]; copyright © ASTM; reprinted with permission.)

- *Transgranular fracture*:
    - In ductile metals, the process of crack advance during a cycle is thought to be similar to Fig 11.32.
    - Another mechanism is crack growth by small increments of brittle cleavage during each cycle. It is not uncommon in metals

- *Intergranular fracture*:
    - In other cases, the boundaries between grains are the weakest regions in the material so that the crack grows along grain boundaries.
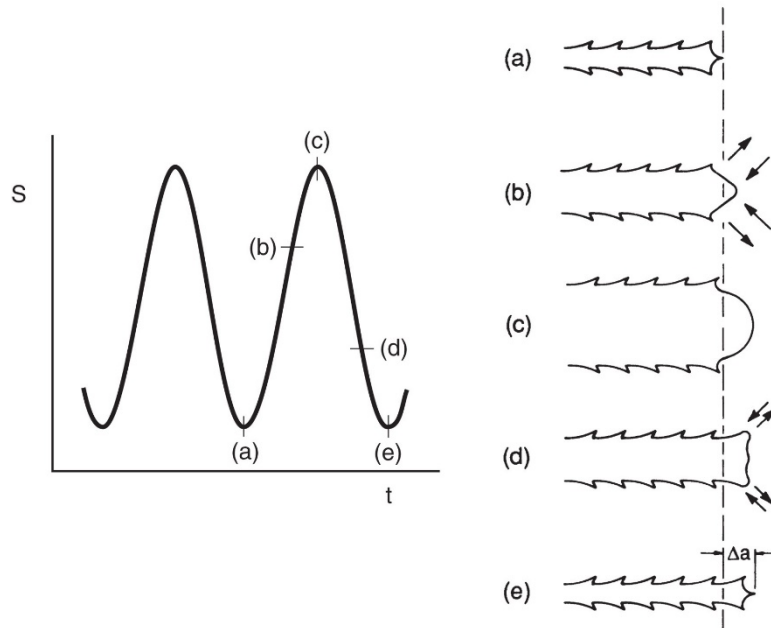
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Figure 11.32 Hypothesized plastic deformation behavior at the tip of a growing fatigue crack during a loading cycle. Slip of crystal planes along directions of maximum shear occurs as indicated by arrows, and this plastic blunting process results in one striation (Δa) being formed for each cycle.
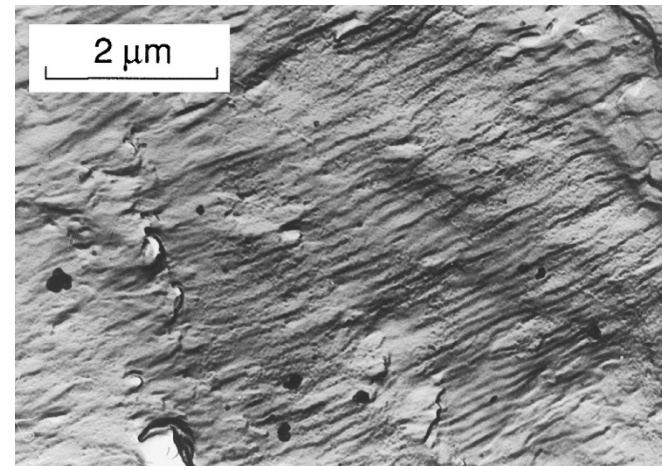

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Figure 9.22 Fatigue striations spaced approximately 0.12 μm apart, from a fracture surface of a Ni-Cr-Mo-V steel.

- **11.9.2 Thickness Effects and Plasticity Limitations**
  - If the monotonic plastic zone is not small compared with the thickness, then plane stress exists, and fatigue cracks may grow in a shear mode, with the fracture inclined about 45° to the surface.
  - Since K and hence the plastic zone size increase with crack length, a transition to this behavior can occur during the growth of a crack.
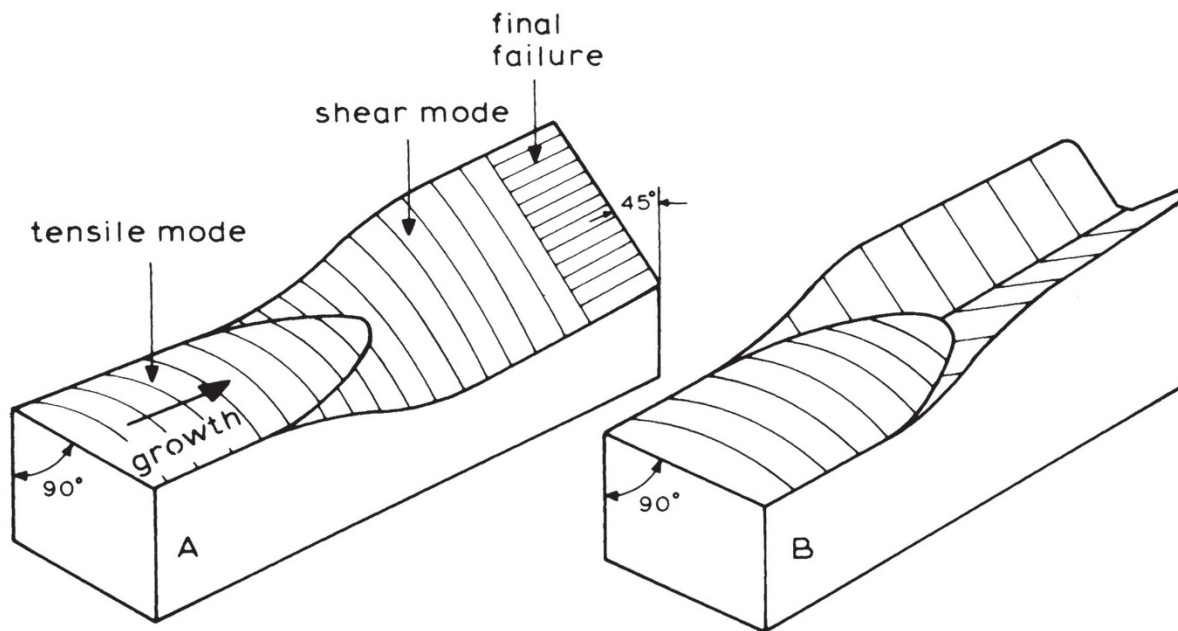

Figure 11.35 Schematic of surfaces of fatigue cracks showing transition from a flat tensile mode to an angular shear mode. The shear growth can (A) occur on a single sloping surface, or (B) form a *V*-shape. (From [Broek 86] p. 269; reprinted by permission of Kluwer Academic Publishers.)

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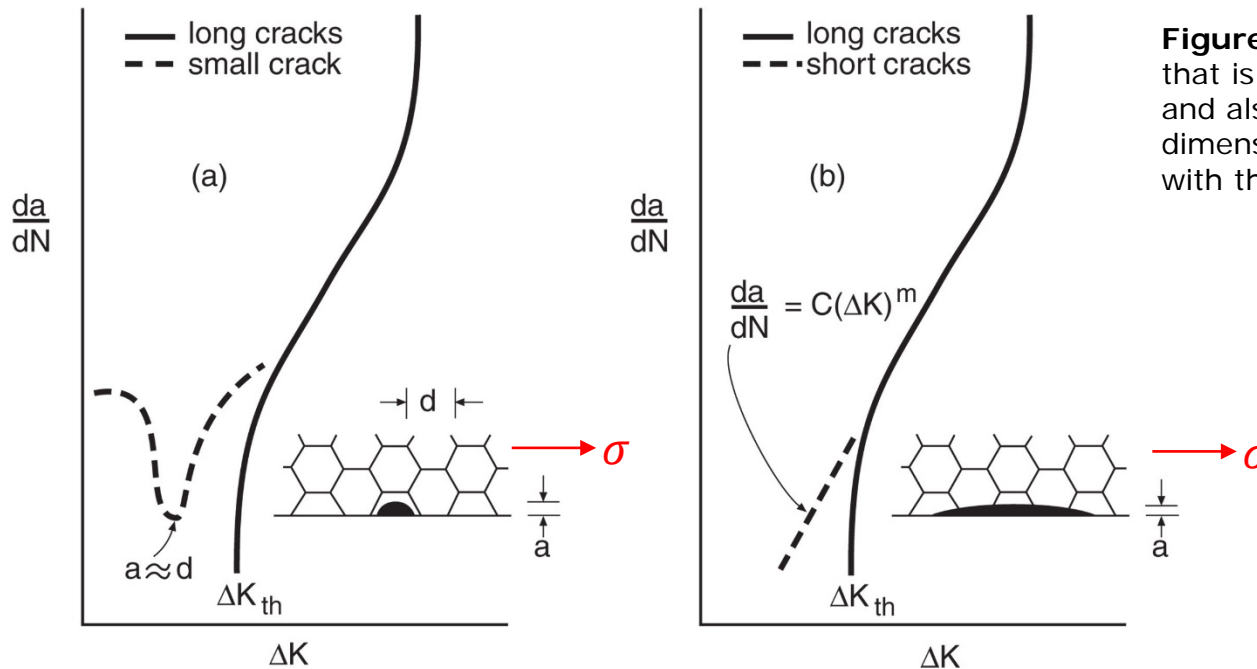
- **11.9.3 Limitations for small cracks**

Figure 11.36 Behavior for a crack that is small in all dimensions (left) and also for a crack with one dimension that is large compared with the microstructure (right).
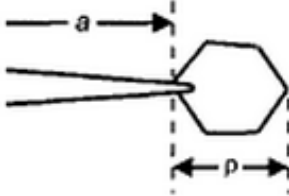
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- *Small cracks*: The crack is within a single crystal grain in a metal, the growth rate is much higher than expected from usual da/dN versus ΔK curve

- *Short cracks*: Growth rates for such cracks in metals are similar to the da/dN versus ΔK curve, except at low ΔK
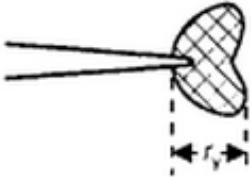
- **11.9.3 Limitations for small cracks**

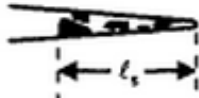**Major Classes of Small Cracks**

- **MICROSTRUCTURALLY SMALL:** $a \sim \rho$ (microstructural dimension)
  - continuum limitation
  - statistical sampling of microstructure
  - need to use probabilistic approach

- **MECHANICALLY SMALL:** $a \sim r_y$ (plastic zone size)
  - LEFM/driving force limitation
  - K-field inappropriate
  - need to use inelastic analysis, e.g., ΔJ

- **PHYSICALLY SMALL:** $a \sim \ell_s$ (size of crack-tip shielding zone)
  - similitude limitation
  - *local* driving force inappropriate
  - need to correct for closure/shielding

Fig. 1. Schematic illustration of the three major classes of small fatigue cracks.

Ravichandran, K. S., Yukitaka Murakami, and Robert O. Ritchie, eds. *Small fatigue cracks: mechanics, mechanisms and applications: mechanics, mechanisms and applications.* Elsevier, 1999.

- **11.9.3 Limitations for small cracks (cont'd)**
  - The crack length a_s, where the ΔK_{th} prediction exceeds the unnotched-specimen fatigue limit, in the intersection of the lines for the two equations

    $$\Delta S = \Delta\sigma_e, \ \Delta K_{th} = \Delta S\sqrt{\pi a} \quad (11.51)$$

  - where the completely reversed fatigue limit is given as a stress range $\Delta\sigma_e = 2\sigma_{er}$, where ΔK_{th} is the value for $R = -1$, and where the geometry factor is approximated as $F = 1$.
  - Combining these and solving for a gives

$$a_s = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_e}\right)^2 \quad\quad (11.52)$$
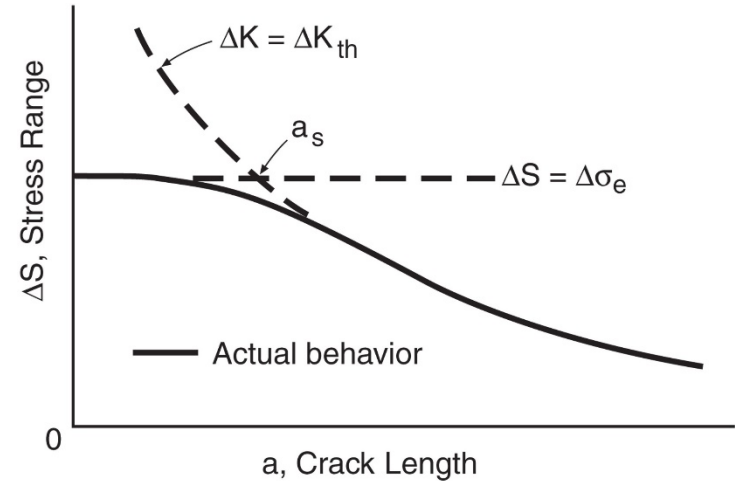
Figure 11.37 Fatigue limit stress as a function of crack length, and the transition length as, below which special small crack effects are expected.

11.10 Environmental Crack Growth

- **Time-based growth rate, or *crack velocity***

$$\dot{a} = \frac{da}{dt} = AK^n \qquad\qquad (11.53)$$

- A and n are material constants that depend on the particular environment and are affected by temperature.
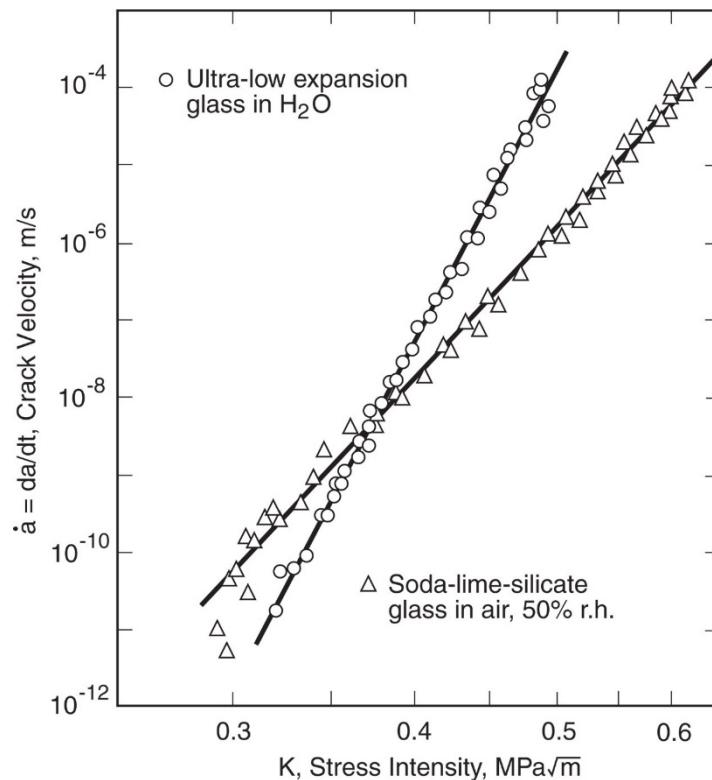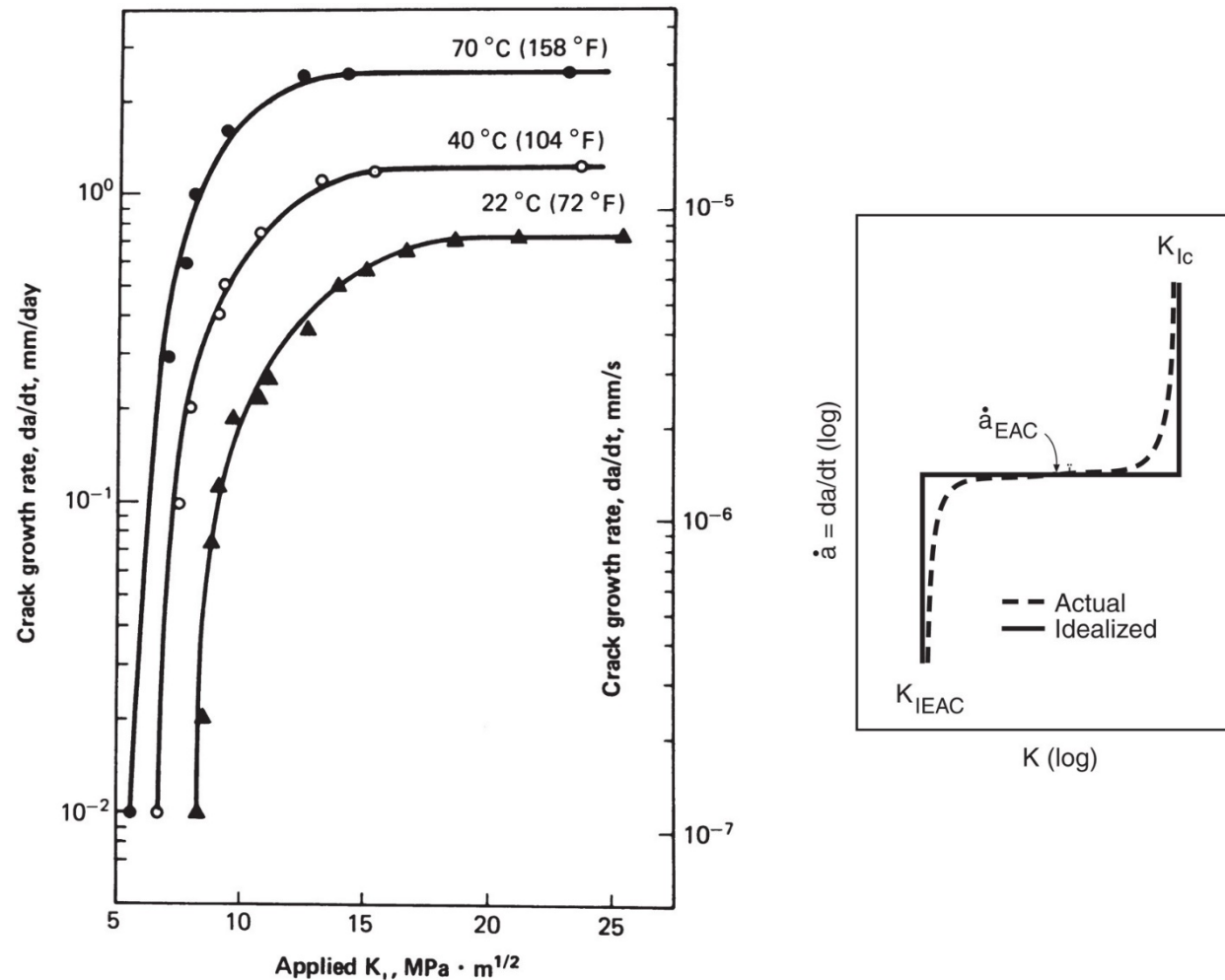

Figure 11.38 Crack velocity data for two silica glasses in room temperature environments as indicated. (Data from [Wiederhorn 77].)

11.10 Environmental Crack Growth

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Figure 11.39 Crack velocity data (left) for 7075-T6 aluminum in a 3.5% NaCl solution similar to seawater, and approximation of such behavior (right) by use of a constant a between K_{IEAC} and K_{Ic}. (Left from [Campbell 82] p. 20; used with permission.)

11.11 Summary

- The crack growth behavior can be described by the relationship between cyclic crack growth rate da/dN and stress intensity range ΔK.

$$\frac{da}{dN} = C(\Delta K)^m$$

- For a given applied stress, material, and component geometry, the crack growth life N_{if} depends on both the initial crack size a_i and the final crack size a_f

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\left(F\Delta S\sqrt{\pi}\right)^m \left(1-\frac{m}{2}\right)} \quad (m \neq 2)$$

- For variable amplitude loading, the da/dN versus ΔK curve can be used to estimate increments in crack length Δa for each cycle

- Limitations on the use of LEFM due to excessive plasticity can be set on the basis of plastic zone size according to Eq. 11.50.

- For static loading in a hostile chemical environment, time-dependent crack growth may occur.