Chapter 10. Stress-Based Approach to Fatigue: Notched Members

Mechanical Strengths and Behavior of Solids

Contents

10.1 Introduction: What are Notches?

- **Geometric discontinuities are unavoidable in engineering design; e.g., holes, fillets, grooves, and keyways.**
- **They are termed notches (stress raisers) for brevity.**
- **The existence of notches:**
  - Increases stress locally.
  - Reduces the resistance of a component to fatigue failure.

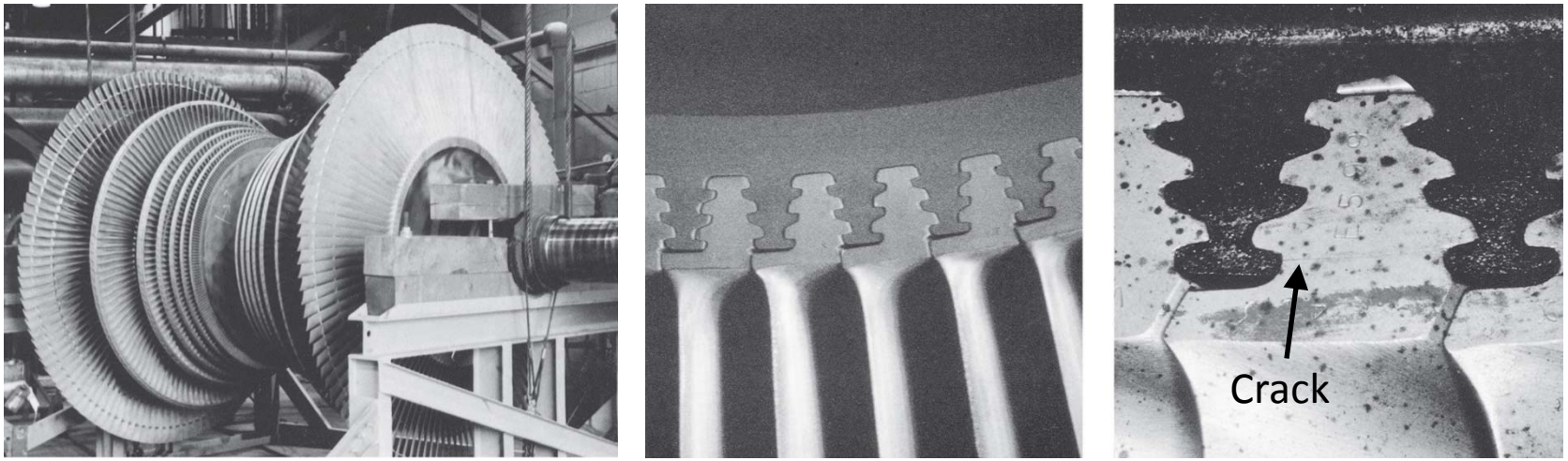


Figure 10.1 Steam turbine rotor with blades and the fir tree type of connection at the blade root

10.1 Introduction: Effect of a Notch on S-N Curve

- **The fatigue strength is lowered substantially by the notch.**
- **Such effects should be considered in engineering design and analysis.**

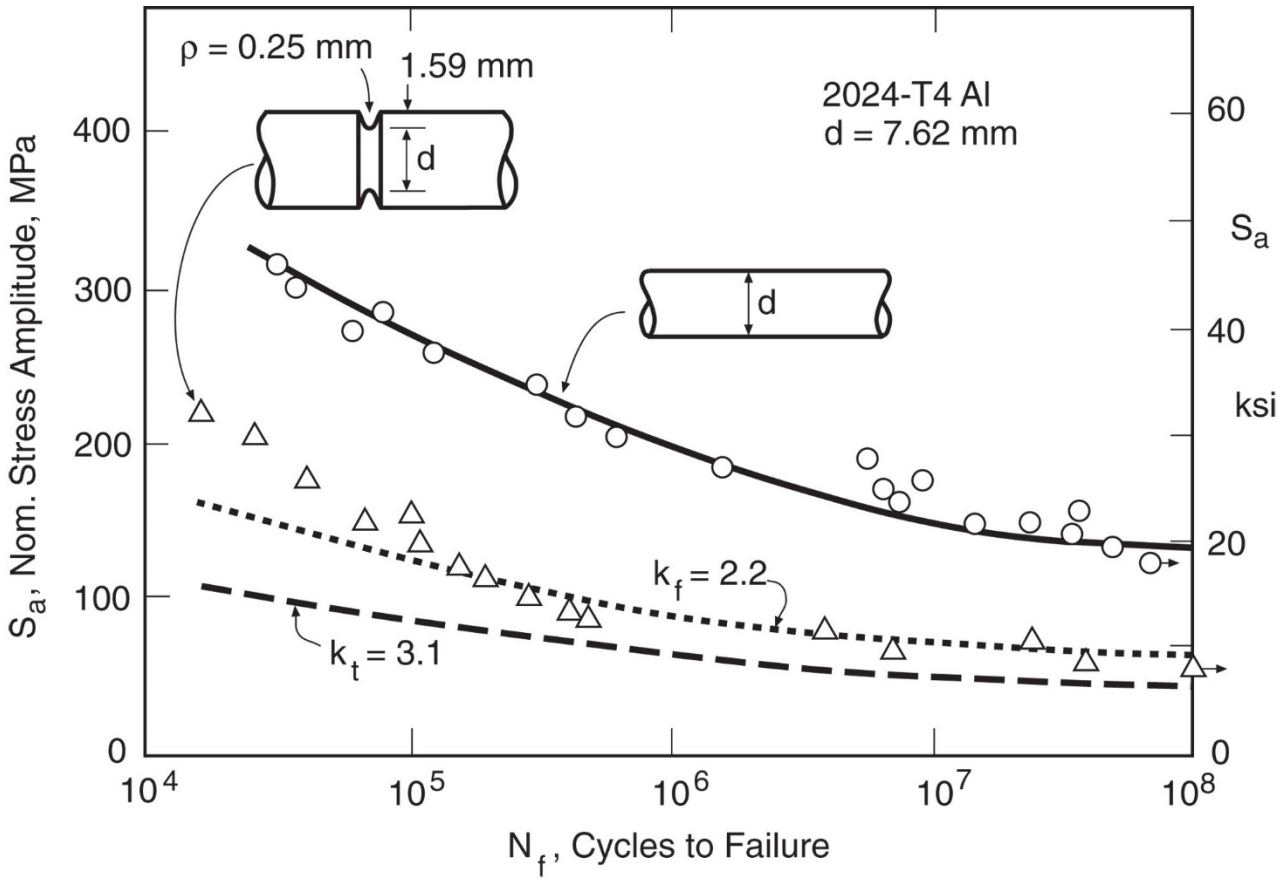


Figure 10.2 Effect of a notch on the rotating bending S-N behavior of an aluminum alloy

10.2 Notch Effects

- **The "theoretical" or "geometric" stress concentration factor (k_t) can be employed to characterize the severity of a notch.**

$$k_t = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch}-\text{free specimen}} = \frac{\sigma_a}{S_a}$$

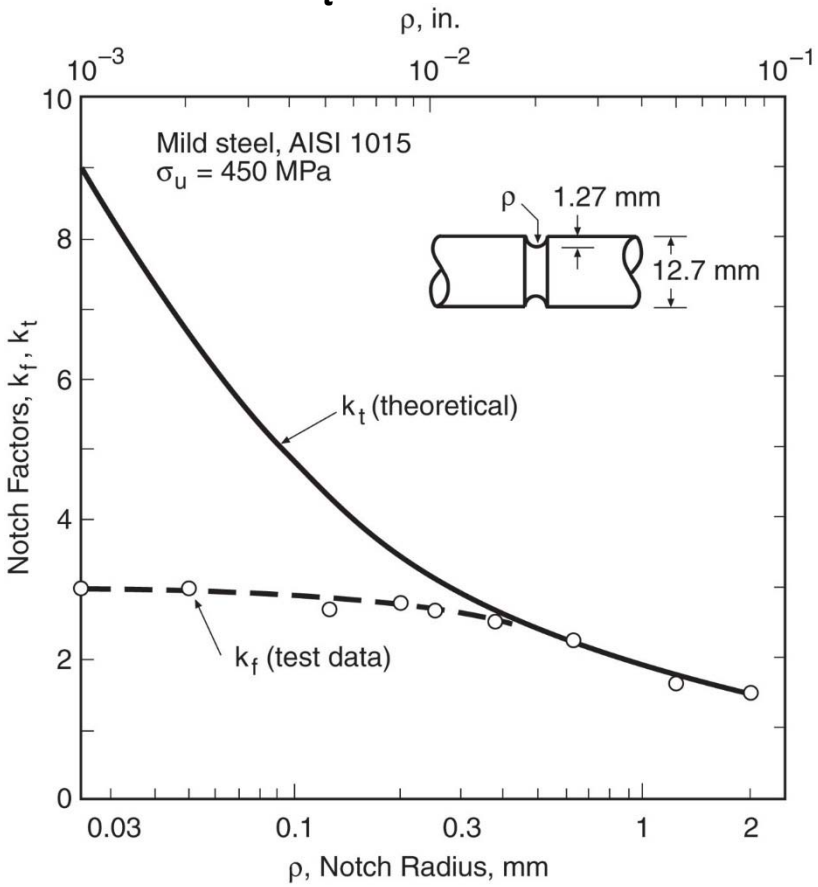
k_t depends for its value only on the geometry of the part. That is, the particular material used has no effect on the value of k_t.

- **The actual test data shows that the notch has less effect than expected on the basis of K_t. To account for this, we define fatigue notch factor (k_f; fatigue stress concentration factor).**

$$k_f = \frac{\text{complete reverse stress for smooth member}}{\text{complete reverse stress for notched member}} = \frac{\sigma_{ar}}{S_{ar}}$$

10.2.1 k_f is Smaller Than k_t

- **If the notch has a large radius ρ at its tip, k_f may be essentially equal to k_t.**
- **For small ρ, the discrepancy may be quite large, so that k_f is considerably smaller than k_t.**



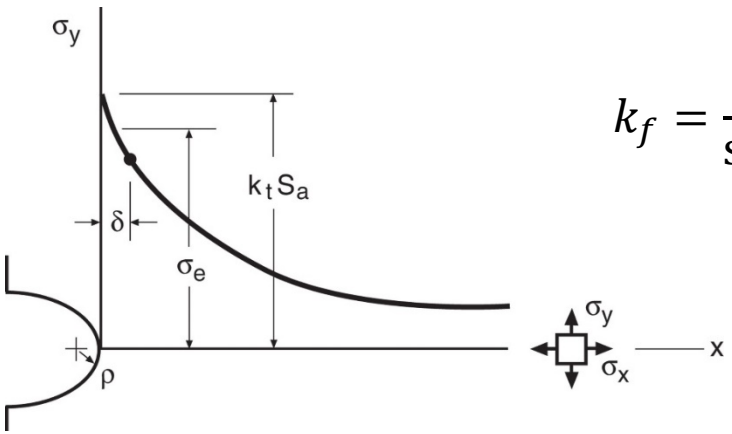
Four possible explanations for $k_f < k_t$

- Stress gradient effect
  - Process zone size
  - Weakest line effect
- Crack growth effect
- Reversed yielding effect

Figure 10.3 Fatigue notch factors for various notch radii from rotating bending of mild steel

10.2.2 Process Zone Size

- **Stress gradient effect:** The material is not sensitive to the peak stress, but rather to the average stress that acts over a region of small, but finite, size.
- **The size of the active region can be characterized by a dimension δ, called the process zone size.**
- **The stress that controls the initiation of fatigue damage is not the highest stress at x=0, but rather the somewhat lower value that is the average out to a distance x=δ.**

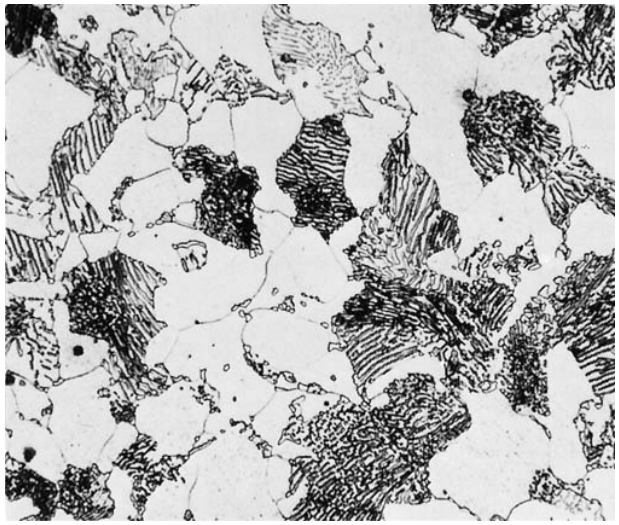


$$k_f = \frac{(\text{average } \sigma_y \text{ out to x=}\delta)}{\text{stress in notch}-\text{free specimen}} = \frac{\sigma_e}{S} < k_t$$

Figure 10.4 Interpretation of the fatigue limit as the average stress over a finite distance δ

10.2.2 Weakest Link Effect

- **The explanation of the weakest link effect to $k_f < k_t$ is based on the concept of statistics.**
  - The fatigue damage process may initiate in a crystal grain that has an unfavorable orientation of its slip planes or in other cases at an inclusion, void, or other microscopic stress raiser.
  - Many potential damage initiation sites occur within the volume of a smooth specimen. However, at a sharp notch, there is a possibility that no such damage initiation site occurs in the small region where the stress is near its peak value.
- **On the average, the notched member will be more resistant to fatigue than expected if the comparison is made on the basis of the local notch stress σ_a/S_a.**



Crystal grain structure in steel alloy

10.2.3 Crack Growth Effect

- **A crack may start quickly in a sharply notched member during cyclic loading, so that the fatigue behavior is dominated by crack growth. That is, the duration for crack initiation is much shorter than that for crack propagation.**
  - In the notched member, the crack is growing into a region of rapidly decreasing stress (not the shortest path), as in Fig. 10. 4. As a result, the crack in the notched member will require more cycles to grow than the one in the smooth member, extending the life in the notched member.
- **In sharply notched members, cracks are observed to start due to the high stress and strain at the notch tip, but then to fail to grow. The stress below which such non-propagating cracks exist determines the fatigue limit of the sharply notched member.**

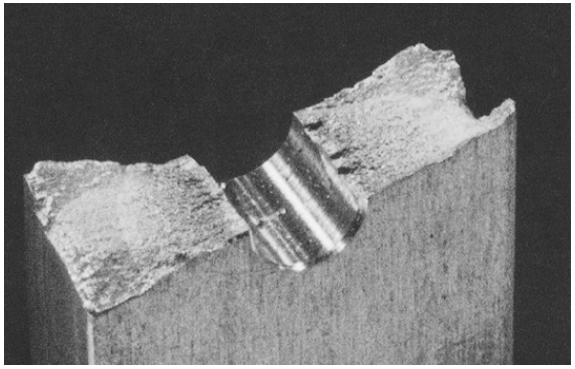


Figure 9.16 Photographs of broken 7075-T6 aluminum fatigue test specimen

10.2.4 Reversed Yielding Effect

- **The plastic strains cause the actual stress amplitude at the notch to be less than the k_tS. This gives a longer life than expected from k_tS.**
- **Such behavior occurs at high stress levels corresponding to short fatigue lives in most engineering metals, and it occurs even at long lives in a few very ductile metals. However, there is little yielding at long lives for most engineering materials.**

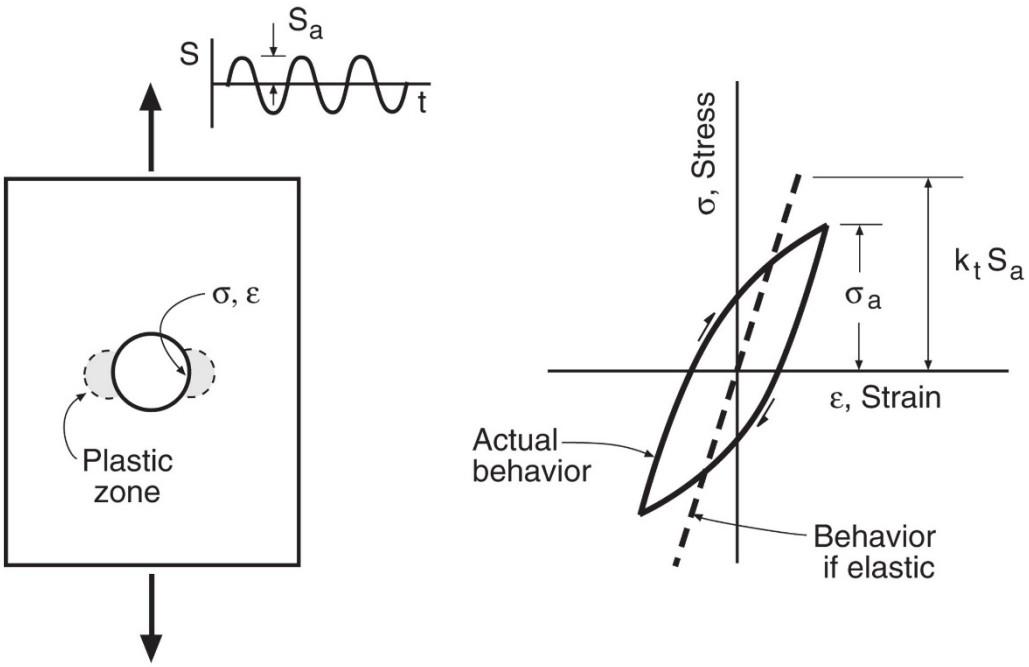


Figure 10.5 Effect of reversed yielding in a small region near the notch on the stress amplitude

10.2.4 Discussion on the $k_f < k_t$ Effect

- In the view point of fracture mechanics, the presence of cracks is a major cause of the effect.
- Reversed yielding is a factor at short lives, and the process zone and weakest-link effects may play a further role.
- In general, however, the situation is quite complex and is not completely understood. This circumstance has resulted in the development of several empirical approaches.

10.3 Notch Sensitivity

- **A useful concept in dealing with notch effect is the notch sensitivity (*q*).**

$$q = \frac{k_f - 1}{k_t - 1}$$

  - If $k_f = k_t$, then q=1.
  - If $k_f < k_t$, the value of q decreases from unity.
  - The value of q between 0 and 1 is a convenient measure of how severely a given member is affected by a notch.
  - If k_f=1, the value of q becomes the minimum 0, which corresponds to the notch having no effect.
- **The discrepancy between k_f and k_t is greatest for highly ductile materials and for sharp notches, and least for low-ductile materials and blunt notches.**

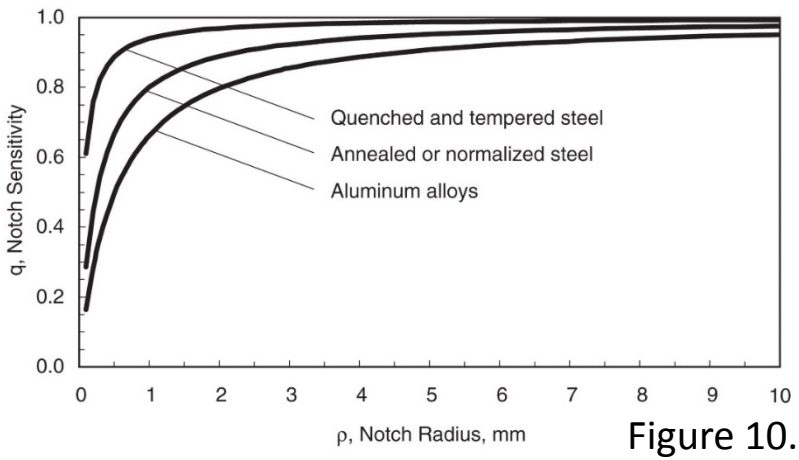


Figure 10.6 Notch sensitivity

10.3 Empirical Estimates of k_f: Peterson Method

- **Values of *q* may be estimated from empirical material constants that are independent of notch radius. Peterson (1974) employs:**

$$q = \frac{1}{1 - \frac{\alpha}{\rho}}$$

where α is a material constant having dimensions of length. Some typical values are:

  - α = 0.51 mm for aluminum alloys
  - α = 0.25 mm for annealed or normalized low-carbon steels
  - α = 0.064 mm for quenched and tempered steels

- **Peterson provides more detail on the variation of α with strength for steels. Values of α can be curve-fitted:**

$$\log\alpha = 2.654 \times 10^{-7}{\sigma_u}^2 - 1.309 \times 10^{-3}\sigma_u + 0.01103$$

10.3 Empirical Estimates of k_f: Peterson Method

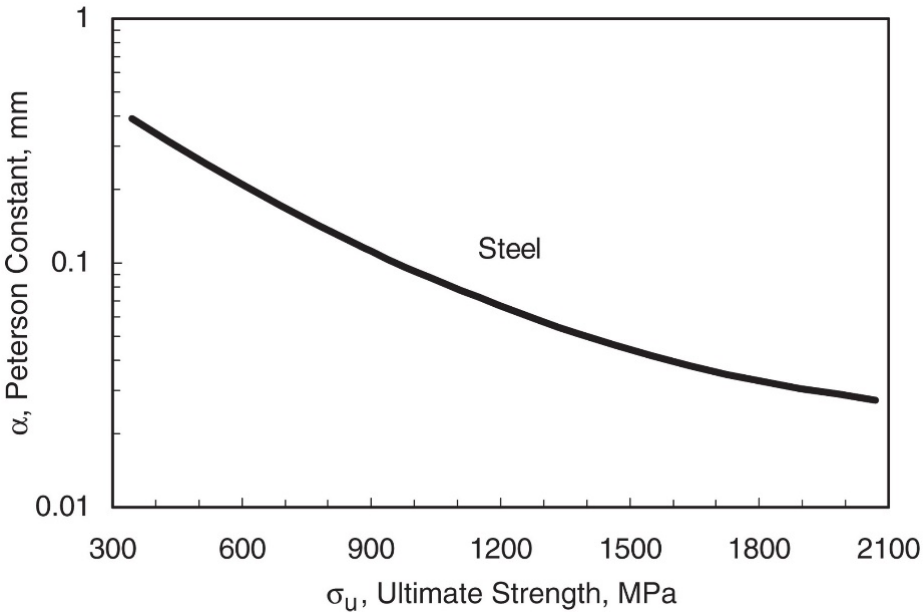


Figure 10.7 Peterson constant as a function of ultimate tensile strength for carbon and low-alloy steels

- **The empirical curve giving either *q* or α can be used to obtain k_f.**

$$k_f = 1 + \frac{k_t - 1}{1 + \frac{\alpha}{\rho}}$$

10.3 Empirical Estimates of k_f: Neuber Method

- **Another frequently used empirical formulation for q and k_f are**

$$q = \frac{1}{1+\sqrt{\frac{\beta}{\rho}}} \qquad k_f = 1 + \frac{k_t - 1}{1+\sqrt{\frac{\beta}{\rho}}}$$

- **Neuber provides the variation of β with strength for steels and for heat-treated aluminum. Values of β can be curve-fitted:**

$$\log\beta = -1.079 \times 10^{-9}{\sigma_u}^3 + 2.740 \times 10^{-6}{\sigma_u}^2 - 3.740 \times \sigma_u + 0.6404$$

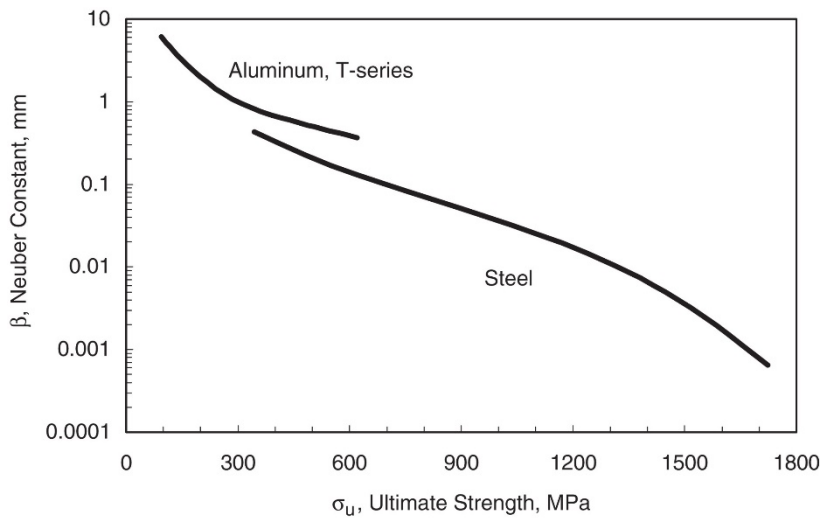


Figure 10.8 Neuber constant as a function of ultimate tensile strength for carbon and low-alloy steels and for solution treated and aged aluminum alloys.

10.4 Estimating Long-Life Fatigue Strengths

- **Fatigue strength at long lives on the order of 10^6 to 10^8 cycles are often available in the literature from rotating bending fatigue test on smoothly polished samples.**
  - For situations that differ from standard test conditions as to type of loading, size, surface finish, etc. modified values are often estimated on the basis of trends observed in existing data.
- **For many materials, S-N curves generally continue to decrease slowly at the longest lives.**
  - e.g., aluminum, magnesium, copper, some stainless steels
- **For some materials, S-N curves appear to become horizontal at long lives.**
  - e.g., low-strength carbon and alloy steels, irons, molybdenum alloys, polymers

10.4.1 Estimation of Smooth Specimen Fatigue Limits

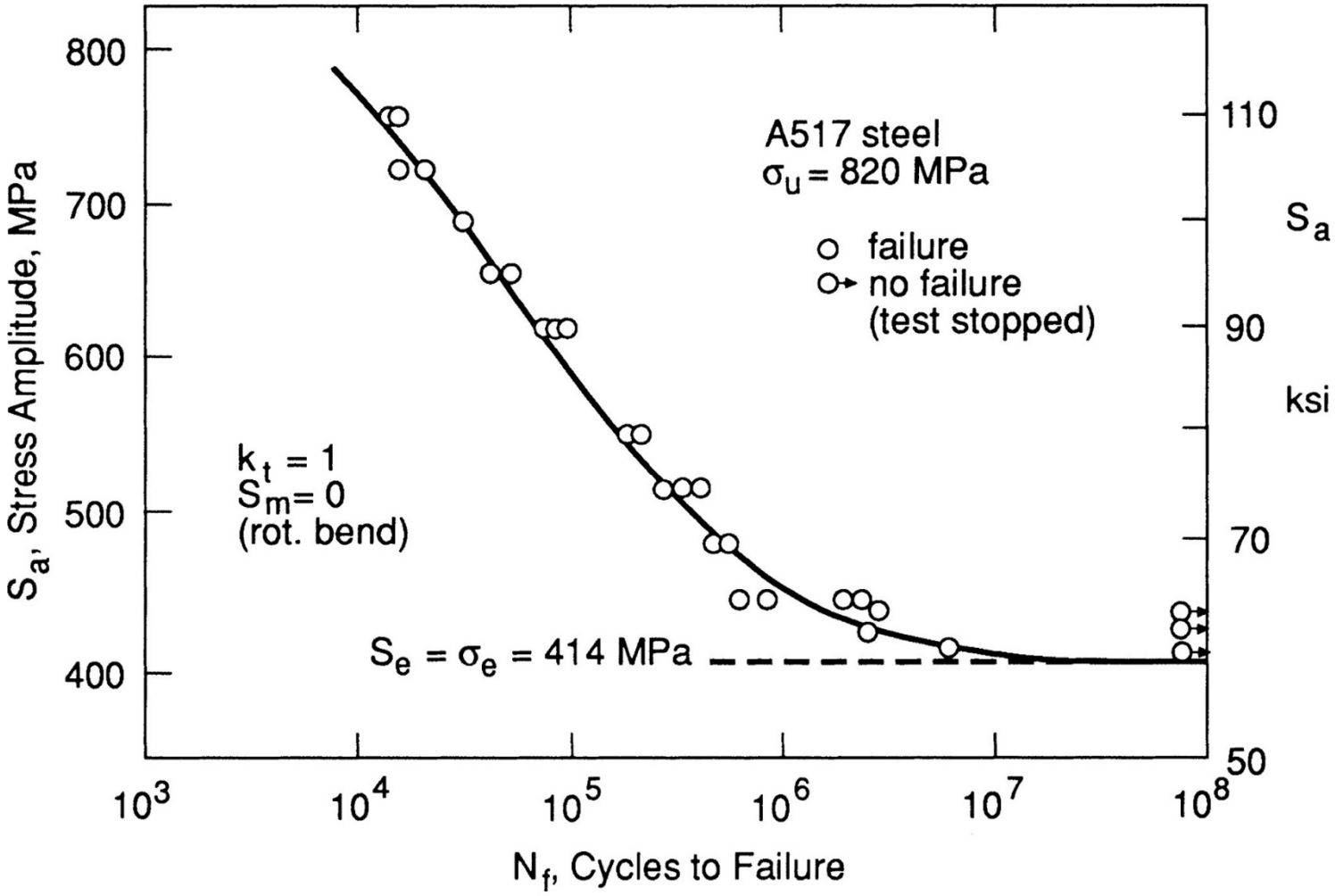


Figure 9.25 Rotating bending S-N curve for unnotched specimens of a steel with a distinct fatigue limit.

10.4.1 Estimation of Smooth Specimen Fatigue Limits

- **The ratio of the fatigue limit to the ultimate tensile strength is defined:**

$$\sigma_{erb} = m_e \sigma_u \quad \text{or} \quad m_e = \frac{\sigma_{erb}}{\sigma_u}$$

where σ_{erb} is the polished specimen fatigue limit for completely reversed loading in bending; and σ_u is the ultimate tensile strength.

  - m_e=0.5 for ferrous metals and m_e=0.4 for aluminum alloys
  - For both steels and aluminums, m_e decreases beyond a certain ultimate tensile strength level. A degree of ductility is helpful in providing fatigue resistance., and high-strength alloys generally have limited ductility.

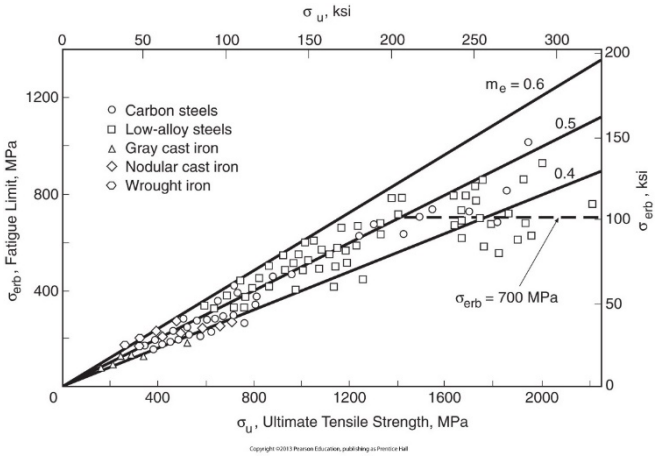


Figure 9.24: Ferrous metals

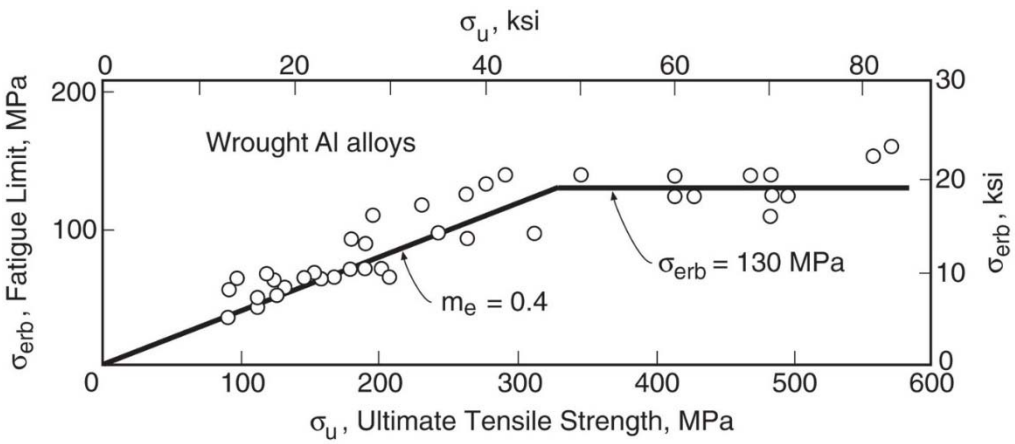


Figure 9.25: Aluminum alloys

10.4.2 Factors Affecting Long-Life Fatigue Strength

- **A variety of additional factors affects the long-life fatigue strength:**
  - Load effect: axial loading produces a lower strength than bending by 10% or more due to the stress gradient effect, such as process zone, weakest link, etc.
  - Size effect
  - Surface finish

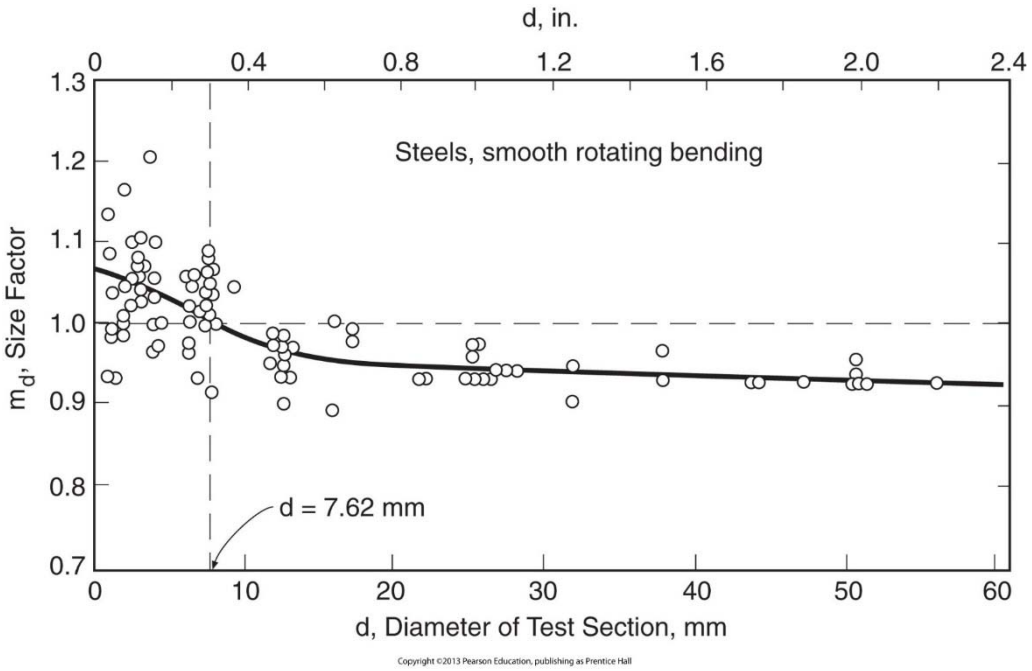


Figure 10.9 Effect of size on the fatigue limit of smoothly polished specimens of steels tested in rotating bending.

10.4.3 Reduction Factors for the Fatigue Limit

- **An adjusted fatigue limit (σ_{er}) is:**

$$\sigma_{er} = m_t m_d m_s m_o \sigma_{erb} = m_t m_d m_s m_o m_e \sigma_u = m\sigma_u$$

where m_t is the type of loading; m_d is the size; m_s is the surface finish; m_o is the any other effects that may be involved, such as elevated temperature, corrosion, etc; and m is the combined reduction factor.

  – e.g., m_t=0.58 for torsion; m_d=0.95 for diameters around 25 mm. m_s=0.8 for a machined surface in low-strength steel.

- **If the starting point for the estimate is the ultimate tensile strength, then the fatigue limit, S_{er} , as a nominal stress for notched member is obtained:**

$$S_{er} = \frac{\sigma_{er}}{k_f} = \frac{m\sigma_u}{k_f}$$

10.4.3 Reduction Factors for the Fatigue Limit

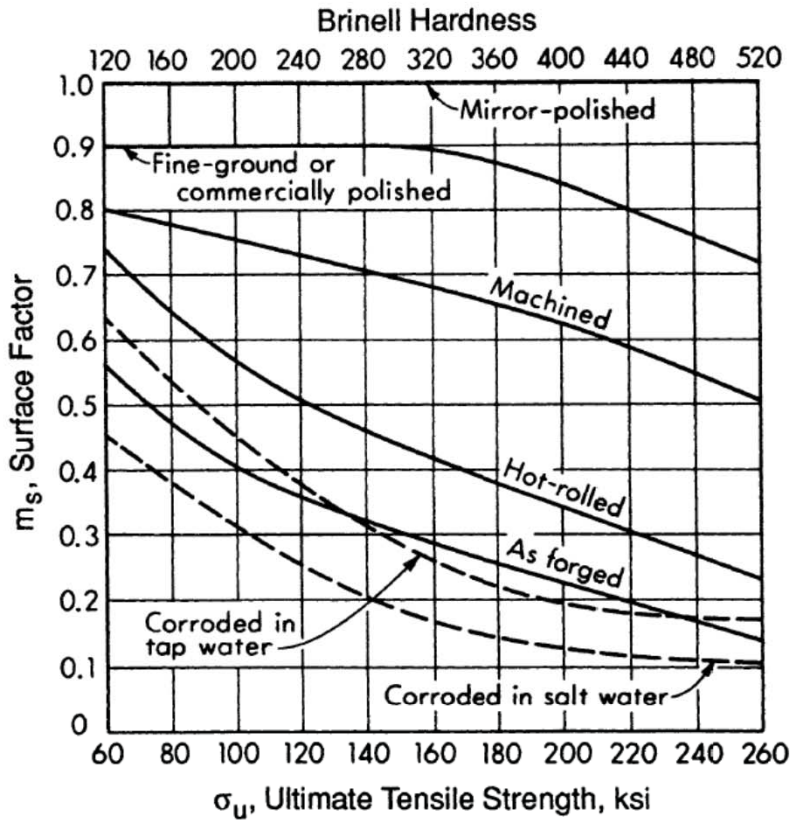


Figure 10.10 Effect of various surface finishes on the fatigue limit of steel. Values are plotted of m_s, the ratio of the fatigue limit to that for polished specimens. (Adapted from R. C. Juvinall, *Stress, Strain, and Strength*, 1967; [Juvinall 67] p. 234; reproduced with permission; © 1967 the McGraw-Hill Companies, Inc.)

10.4.3 Reduction Factors for the Fatigue Limit

Table 10.1 Parameters for Estimating Fatigue Limits

Parameter	Applicability	Juvinall (2006)	Budynas (2011)
Bending fatigue limit factor: m_e	Steels, $\sigma_u \leq 1400\,\text{MPa}$[1]	0.5	0.5
	High-strength steels	≤ 0.5	$\sigma_{erb} = 700\,\text{MPa}$
	Cast irons; Al alloys if $\sigma_u \leq 328\,\text{MPa}$	0.4	—
	Higher strength Al	$\sigma_{erb} = 131\,\text{MPa}$	—
	Magnesium alloys	0.35	—
Load type factor: m_t	Bending	1.0	1.0
	Axial	1.0	0.85
	Torsion	0.58	0.59
Size (stress gradient) factor: m_d	Bending or torsion[2,3,4]	1.0 ($d < 10\,\text{mm}$)	$1.24d^{-0.107}$
		0.9 ($10 \leq d < 50$)	($3 \leq d \leq 51\,\text{mm}$)
	Axial[2,3]	0.7 to 0.9 ($d < 50$)[5]	1.0

10.5 Notch Effect at Intermediate and Short Lives

- **At intermediate and short fatigue lives in ductile materials, the reversed yielding effect in Fig. 10.5 becomes increasingly important as higher stresses.**
- **The ratio of the smooth specimen to notched specimen fatigue strengths becomes even less than k_f, so that it is useful to define a fatigue notch factor k'_f that varies with life.**

$$k'_f = f(N_f) = \frac{\sigma_{ar}}{S_{ar}}$$

where σ_{ar} and S_{ar} are the nominal stresses for smooth and notched specimens, respectively.

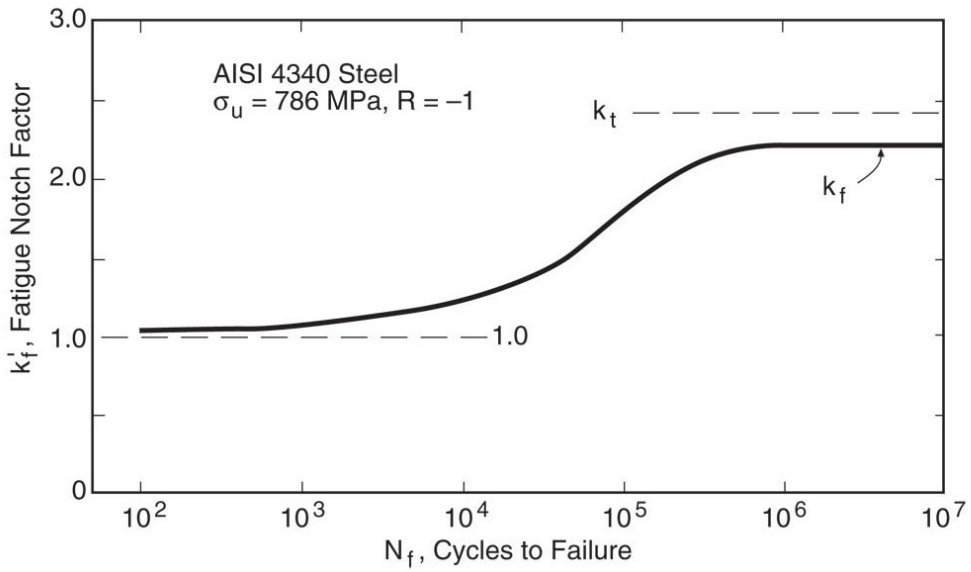


Figure 10.11 Variation of the fatigue notch factor with life.

10.5 Notch Effect at Intermediate and Short Lives

- **Considering completely reversed loading, the trend of k'_f can be rationalized by idealizing the behavior of the material as (a) no yielding, (b) local yielding, and (c) full yielding.**

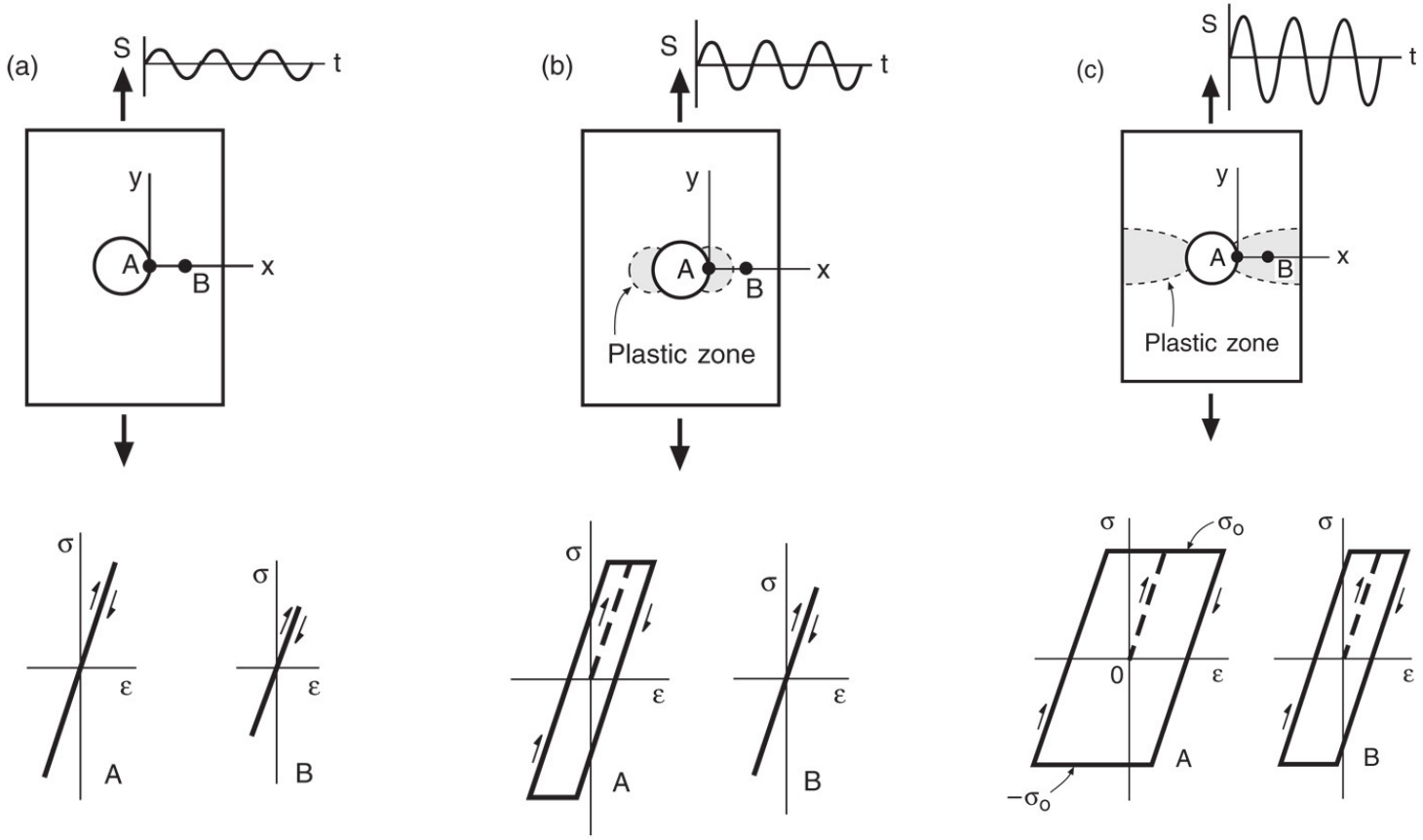


Figure 10.12(a), (b), (c)

10.5 Notch Effect at Intermediate and Short Lives

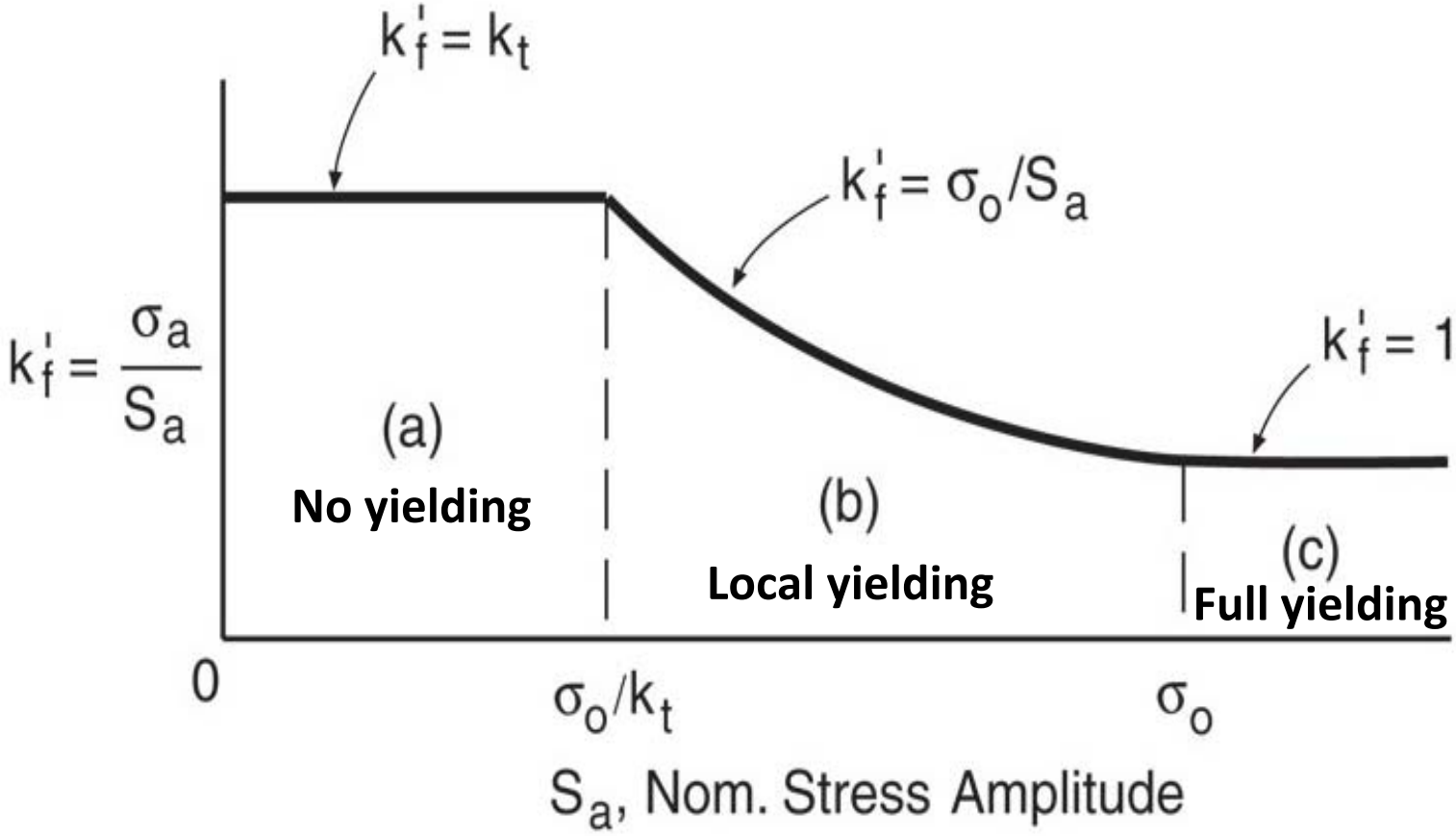


Figure 10.12(d)

10.6 Combined Effects of Notches and Mean Stress

- **The empirical expressions and curves for k_f and k'_f are based on trends and data observed under completely reversed loading. Therefore, they cannot be applied directly if mean stresses are present.**
- **The most common approach to handling mean stresses for notched members is to apply the Goodman relationship. Despite its predominance, its application is limited. Some reasons are:**
  - The brittle versus ductile choice is problematical.
  - Plotting S_{ar} versus life N_f for test data usually gives a poor correlation due to deviation of the data.
  - Nominal stress S is an arbitrarily defined quantity and, for complex geometries, there may be more than one possible choice for defining S or no clear choice at all.
- **Sometimes, the Smith, Watson, and Topper (SWT) method and the Walker relationship offer useful alternatives.**

10.6.1 Goodman Equation for Notched Members (Optional)

- **If the material is quite ductile, redistribution of stresses results in an approximately uniform stress at failure. Hence, the ultimate strength of the notched member is not strongly affected by the notch.**
  - The equation for ductile materials: $\frac{S_m}{\sigma_u} + \frac{S_a}{S_{ar}} = 1$ or $S_{ar} = \frac{S_a}{1 - \frac{S_m}{\sigma_u}}$
- **However, for low-ductility materials, the redistribution does not occur and the ultimate strength of the notched member is reduced.**
  - The equation for brittle materials: $\frac{S_m}{\sigma_u / k_{fm}} + \frac{S_a}{S_{ar}} = 1$ or $S_{ar} = \frac{S_a}{1 - \frac{k_{fm} S_m}{\sigma_u}}$

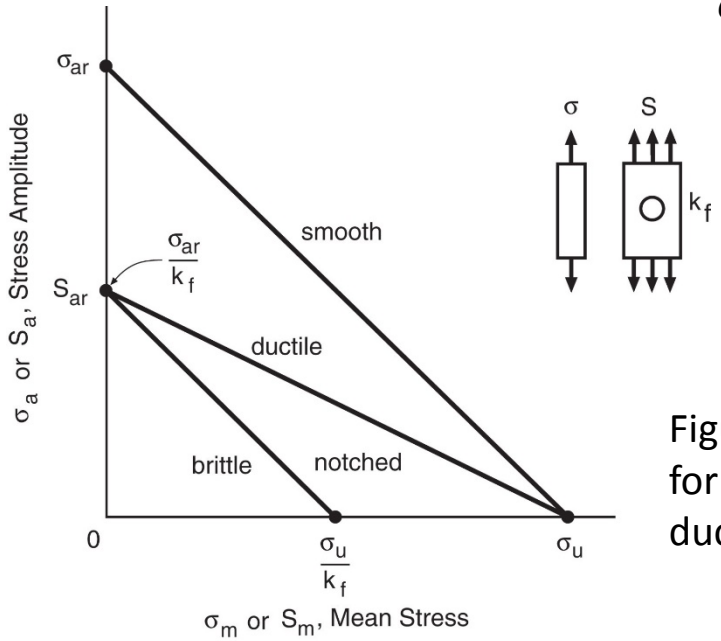


Figure 10.14 Goodman amplitude-mean plots for smooth and notched members of brittle and ductile materials.

10.7 Estimating S-N Curves (Optional)

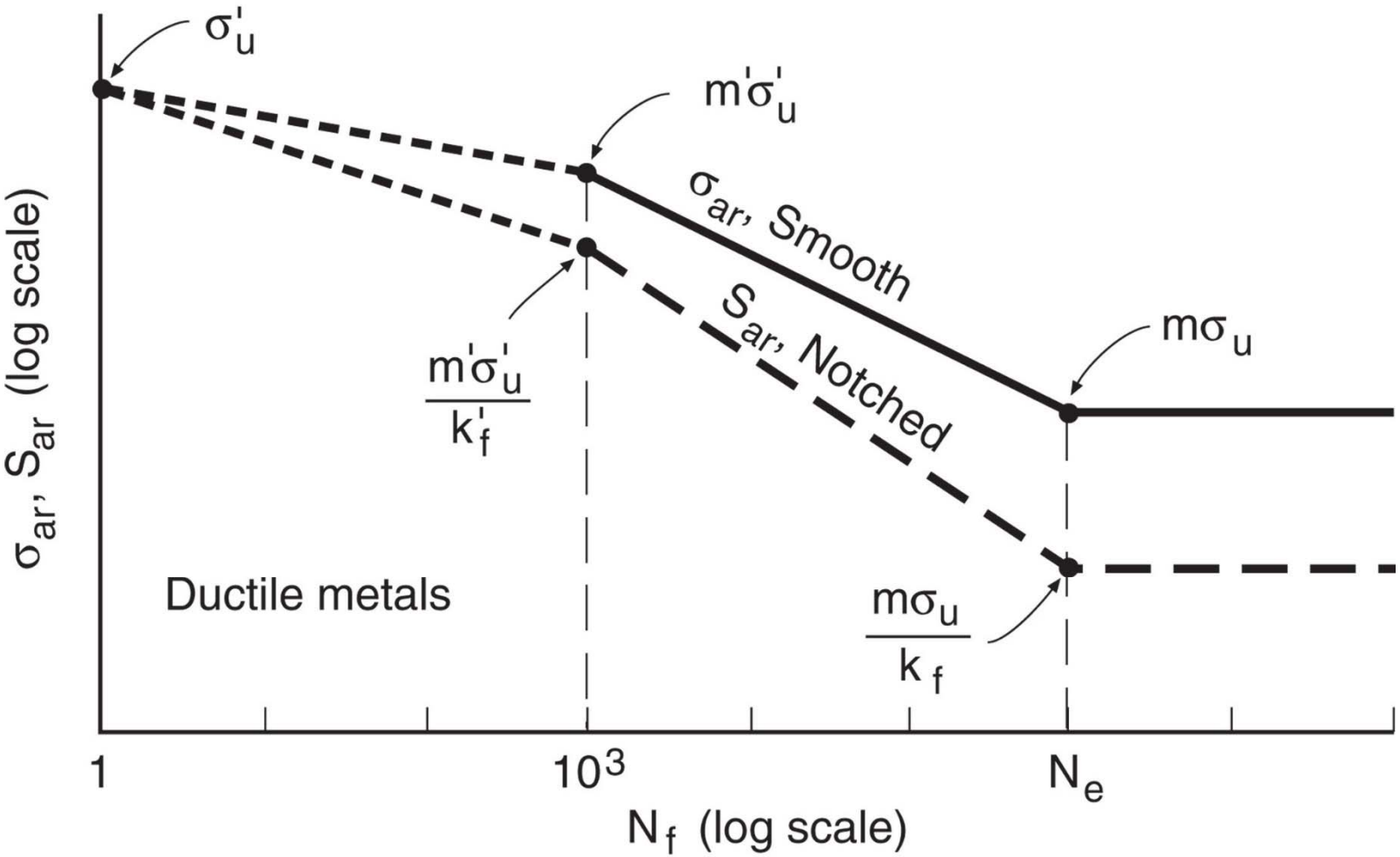


Figure 10.16 Estimating completely reversed S-N curves for smooth and notched members according to procedures suggested by Juvinall or Budynas.

10.7.1 Methodology for Estimating S-N Curves (Optional)

- **For smooth members,**
  - A stress-life curve for smooth material at zero mean stress is estimated.
  - Using the ultimate tensile stress (σ_u), the fatigue limit ($\sigma_{er}= m\sigma_u$) is obtained for long life N_e such as 10^6 cycles for steels.
  - A point $\sigma'_{ar}=m'\sigma'_u$ is established at $N_f=10^3$ cycles, where the quantity σ'_u is the ultimate tensile strength σ_u for tension or bending, or the ultimate strength in shear τ_u for torsion.
  - The points at 10^3 and N_e are connected with a straight line on a log-log plot, giving a relationship of the form $\sigma_{ar}=AN^B_f$.
  - Another straight line may be employed to connect the 10^3 cycles point with σ'_u at $N_f=1$ cycle.

$$(\sigma'_u, 1) \qquad (\sigma'_{ar}, N_f)=(m'\sigma'_u, 10^3) \qquad (\sigma_{er}, N_f)=(m\sigma_u, N_e)$$

- **For notched members,**

$$(\sigma'_u, 1) \qquad (S'_{ar}, N_f)=(m'\sigma'_u/k'_f, 10^3) \qquad (S_{er}, N_f)=(m\sigma_u/k_f, N_e)$$

10.7.2 Estimates by the Methods of Juvinall or Budynas (Optional)

Table 10.2 Estimates of the *S-N* Curve Point at 10^3 Cycles

Juvinall (2006)[1]	$m' = 0.9,\ k'_f = k_f$ (bending; torsion with τ_u replacing σ_u)
	$m' = 0.75,\ k'_f = k_f$ (axial)
Budynas (2011)[2]	$m' = 0.90$ ($\sigma_u <$ 483 MPa)
(steel only)	$m' = 0.2824x^2 - 1.918x + 4.012,\quad x = \log \sigma_u$ ($\sigma_u \geq$ 483 MPa)
	$k'_f = k_f$

Notes: [1] Use the estimate $\tau_u \approx 0.8\sigma_u$ for steel, and $\tau_u \approx 0.7\sigma_u$ for other ductile metals. [2] The equation for m' is a fit to the curve given in Budynas (2011).

- **The two methods should not be regarded as providing anything more than very rough curves for use in design.**
  - Juvinall's is the most complete.
  - Budynas estimate is more detailed where it does apply for steels.
- **Actual fatigue data from tests are always preferable to an estimated S-N curves. Such data should be used where possible to aid in estimating the S-N curve, or even to replace the estimate entirely.**

10.8. Use of Component S-N Data

- **It is often advantageous to employ S-N data from actual tests.**
- **The fatigue test program for Bailey bridge panel observed:**
  - Cracks generally started at a weld near the “slot for sway brace”.
  - Cracks were visibly growing for at least half of the life before failure.
- **Actual tests on members automatically account for the effects of details such as complex geometry, surface finish, residual stress from fabrication and the unusual metallurgy at welds.**

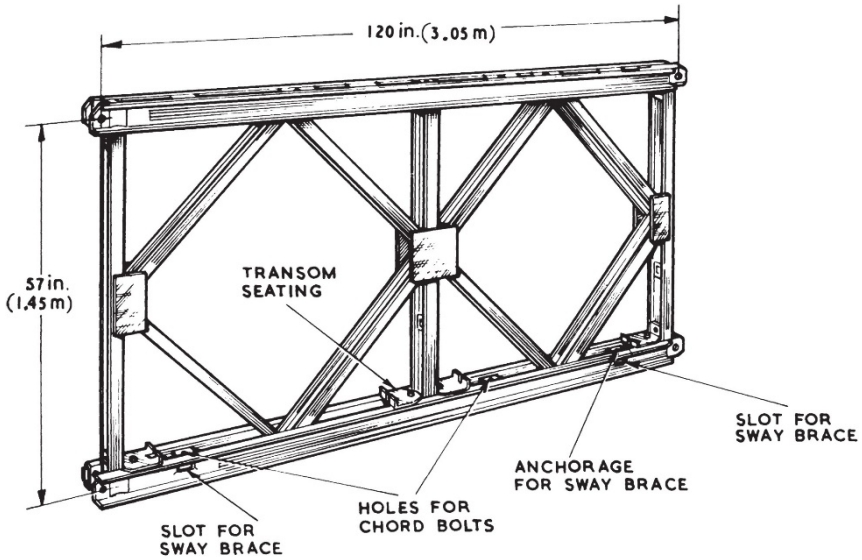


Figure 10.17 Bailey bridge panel

Bailey bridge over the White Nile, Juba, South Sudan

10.8 Bailey Bridge Example

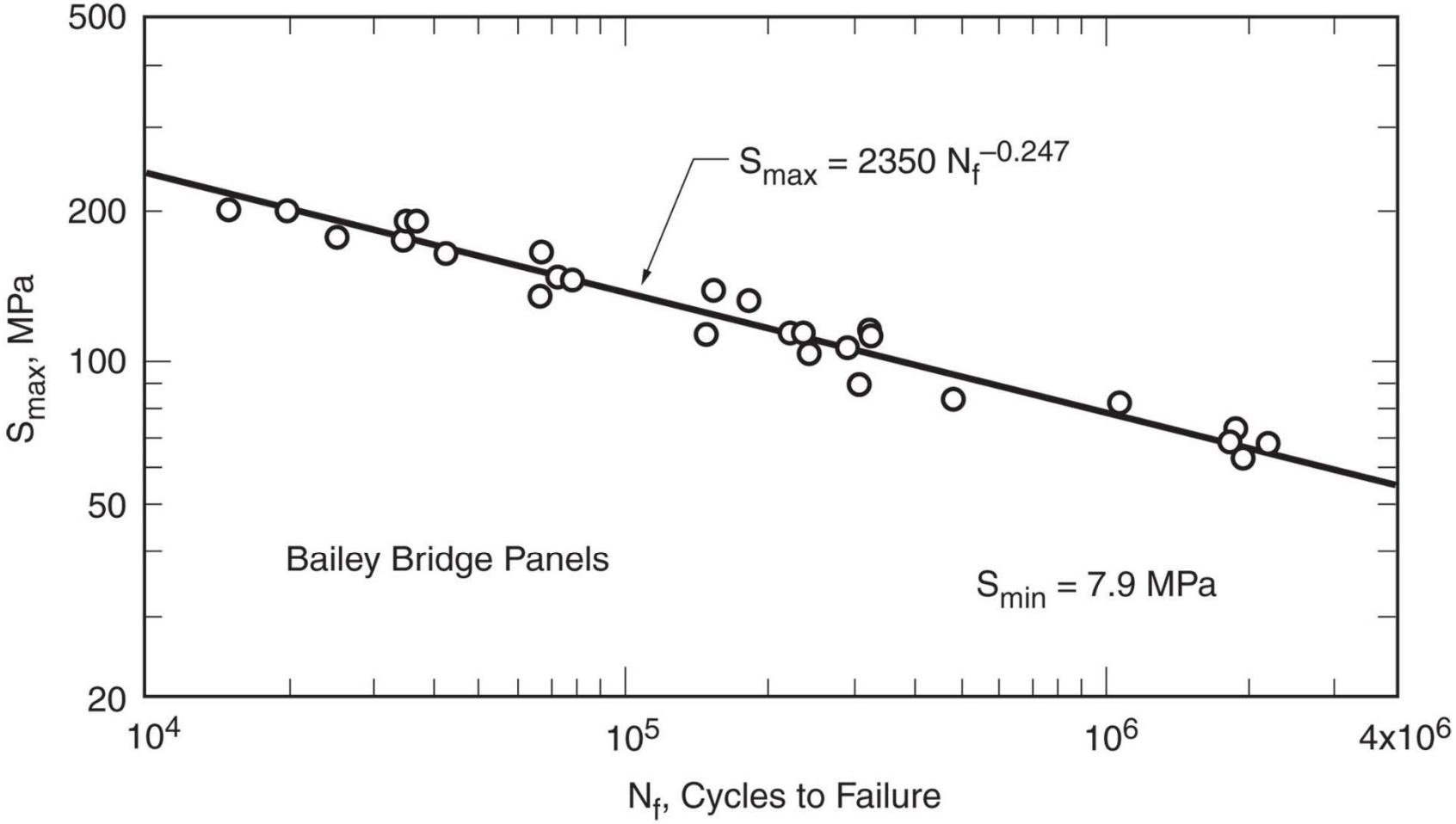


Figure 10.18 Fatigue data at constant S_{min} and fitted line, for Bailey bridge panels in vertical bending, with failure defined as complete separation of a truss member

10.8 Component S-N Curves for Welded Members

- **Welded structural members have complex geometry and metallurgy in the vicinity of the weld, and they may contain porosity or other defects.**
- **Defects make it difficult to determine stress concentration factors or to relate the behavior to that of any non-welded test member.**
- **As a result , mode design codes that cover welded structural members employ component S-N curves based on extensive testing of actual welded members.**

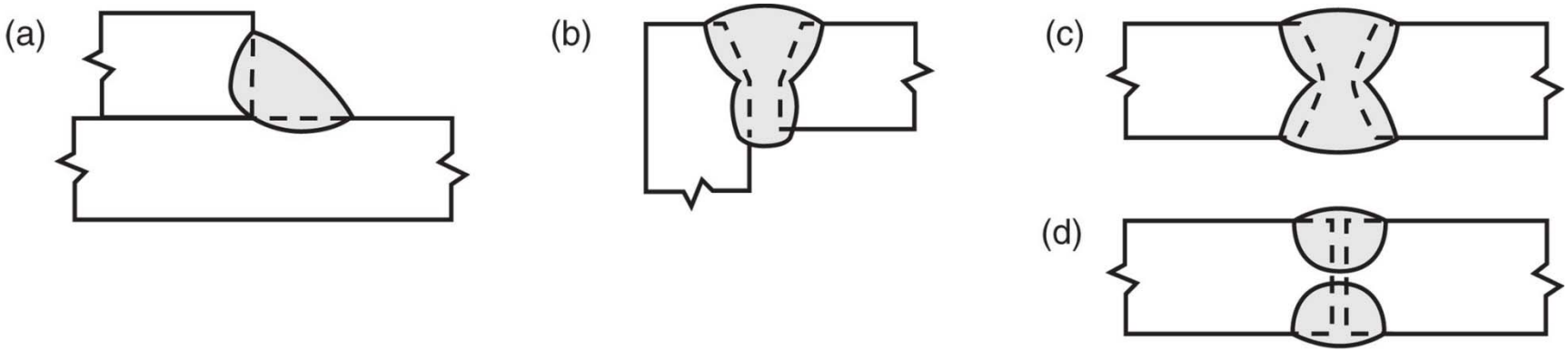


Figure 10.19 Typical welds: (a) fillet weld, (b) corner joint formed by a single-V-groove weld, (c) butt joint formed by a double-V-groove weld, and (d) butt joint formed by a square-groove weld with partial penetration. Dashed lines indicate the base metal shape before welding, and shaded areas indicate the melted and resolidified material.

10.8 Component S-N Curves for Welded Members

- **Although data for three different structural steels are plotted, the results are insensitive to the particular structural steel involved, but highly sensitive to the geometric detail.**

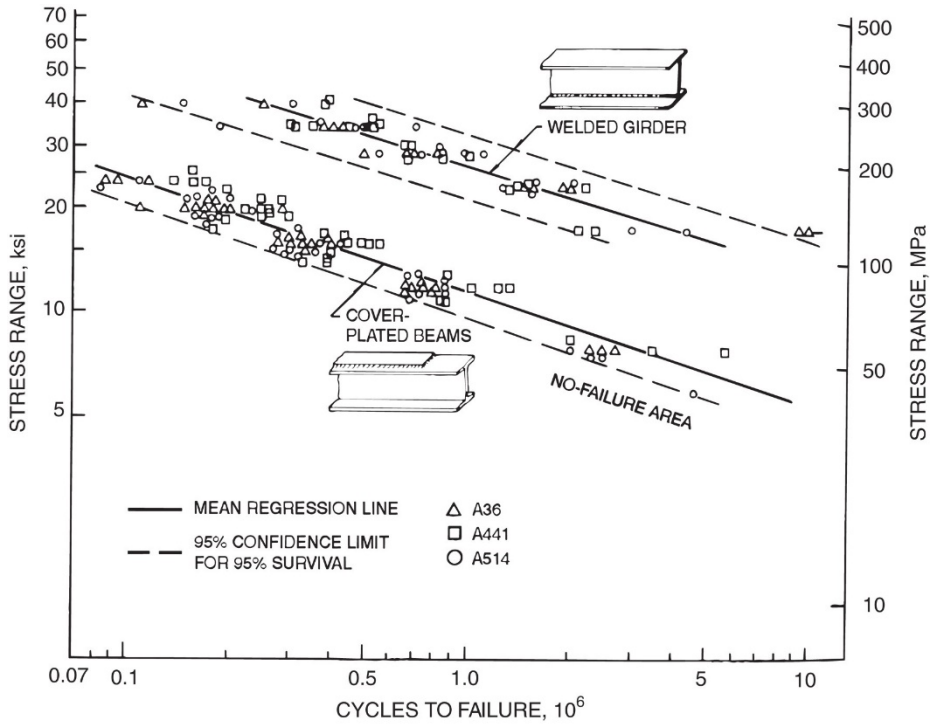


Figure 10.21 Nominal stress range versus millions of cycles to failure for bending tests on welded structural members of three different structural steels. The data for longitudinal welds (top) correspond to AWS category B, and that for cover-plated beams (bottom) to category E.

10.8 Component S-N Curves for Welded Members

- **According to the American Welding Society (AWS),**
  - Category A: plain structural steel with no welding or other stress raiser.
  - Category B: only with mild stress concentration (See Fig. 10.20(a).)
  - Category C: somewhat more severe cases (See Fig. 10.20(b).)
  - Category D: see Fig. 10.20(d) as an example.
  - Category E: severe stress concentration is caused by a transverse weld at the end of a partial-length cover plate (See Fig 10.20(c).)

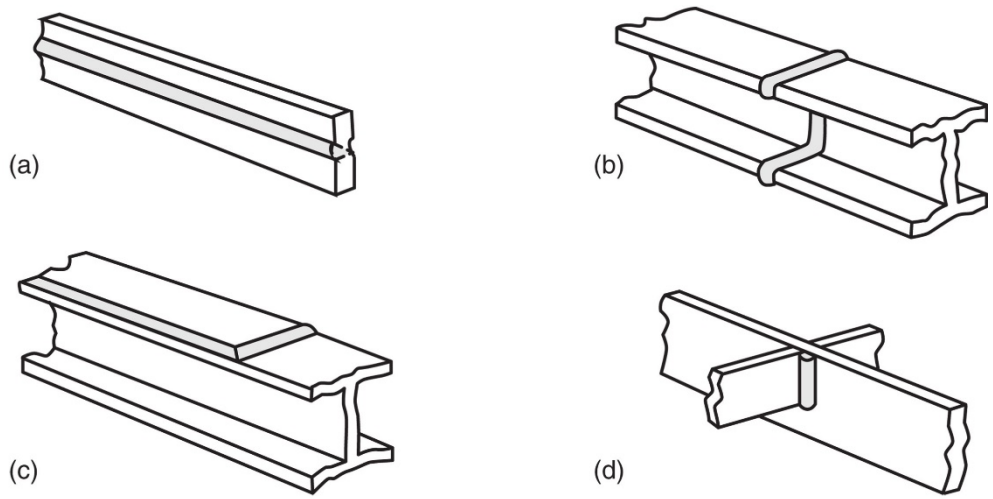


Figure 10.20 A sampling of the numerous structural details involving welding: (a) longitudinal butt weld, (b) transverse butt weld, (c) cover plate attached with fillet welds, and (d) lateral attachment with fillet welds.

10.8 Component S-N Curves for Welded Members

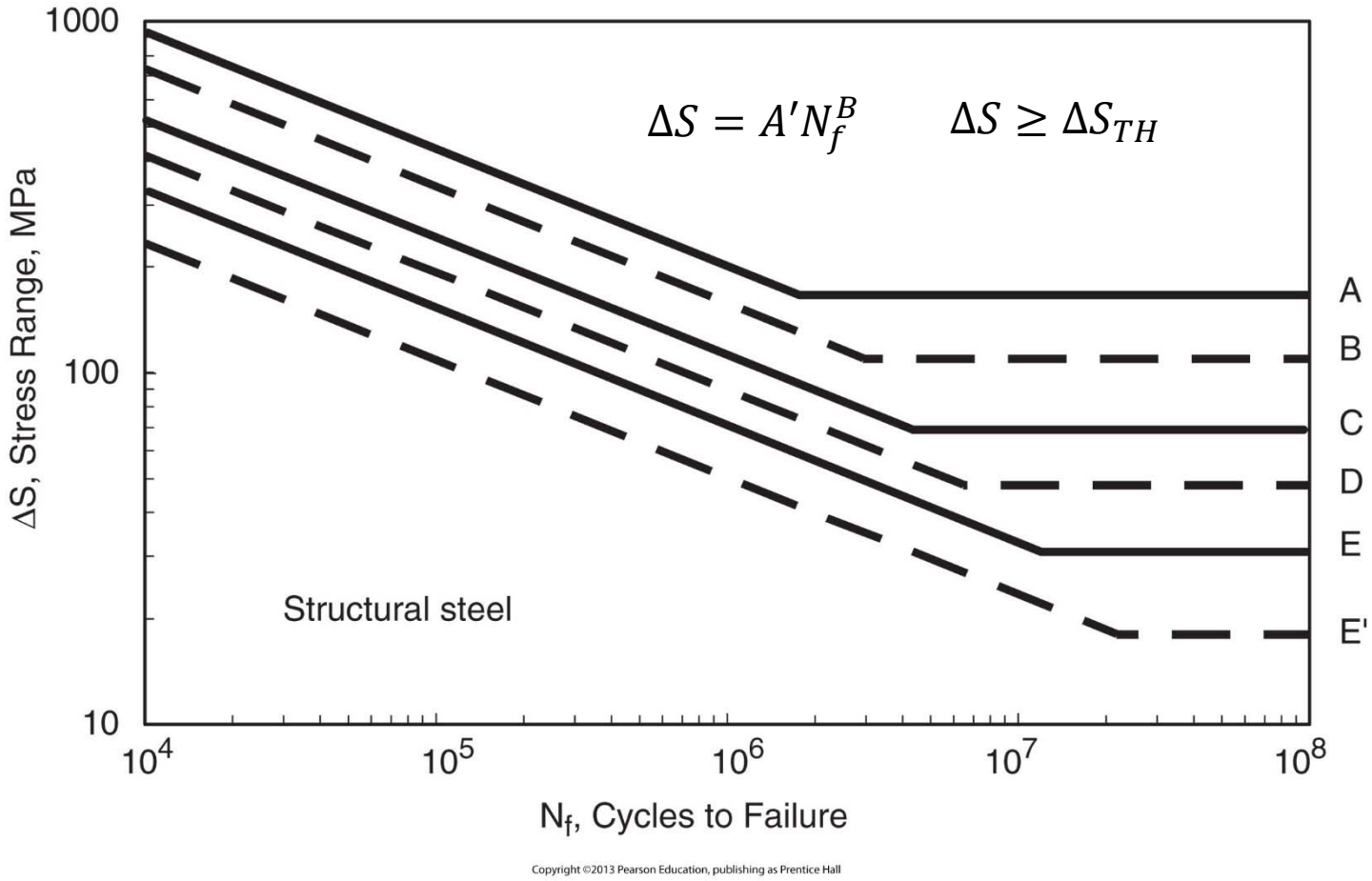


Figure 10.22 Stress–life curves with 95% survival from the AWS design code for various categories of non-tubular connections.

10.8 Component S-N Curves for Welded Members

Table 10.3 Constants for AWS Fatigue Curves for Nontubular Sections

Category	C, cycles	B	A', MPa	ΔS_{TH}, MPa
A	2.50×10^{10}	−0.3330	19 987	166.0
B	1.20×10^{10}	−0.3330	15 653	110.0
C	4.40×10^{9}	−0.3330	11 207	69.0
D	2.20×10^{9}	−0.3330	8 897	48.0
E	1.10×10^{9}	−0.3330	7 063	31.0
E′	3.90×10^{8}	−0.3330	5 001	18.0

10.9 Designing to Avoid Fatigue Failure

- **To avoid fatigue failure, one way is to minimize the severity of stress raisers.**
  - Minimizing the elastic stress concentration factor k_t
  - Decreasing the fatigue notch factor k_f
  - Raising the fatigue limit and the overall S-N curve
- **Changes in geometric detail, such as reducing eccentricity that causes bending, may be beneficial.**
- **Another strategy is to exploit the mean stress effect by introducing residual stress.**
  - Shot peening
  - Surface hardening treatment: carburizing or nitriding of steels
  - Thermal treatment: rapid quenching of steel shafts

10.9.1 Design Details: Shaft

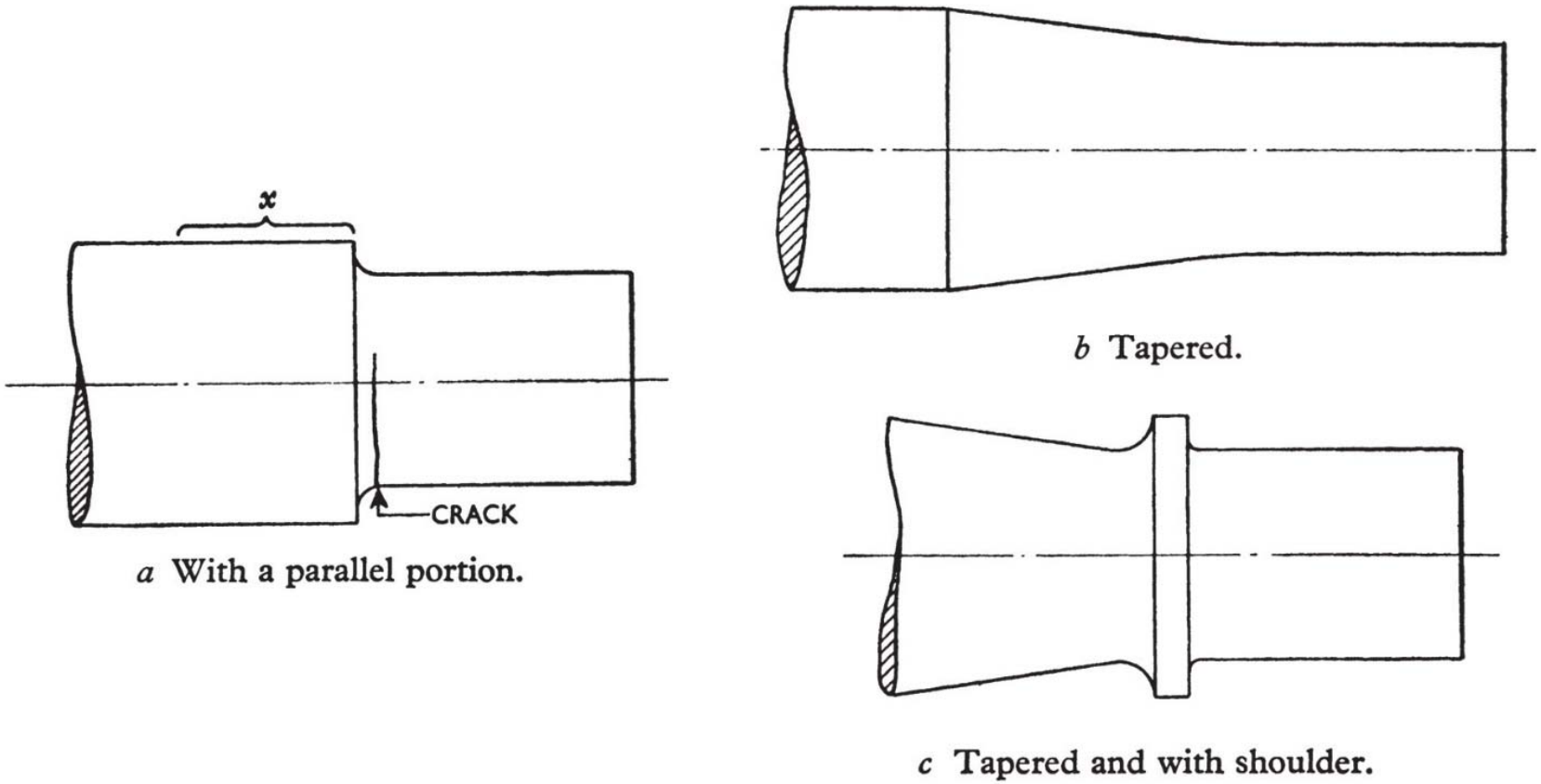


Figure 10.23 The common location (a) of fatigue cracks in a stepped shaft, and (b) reducing the stress raiser effect by using a taper, or (c) by using a taper with a shoulder.

10.9.1 Design Details: Keyhole and Fitting

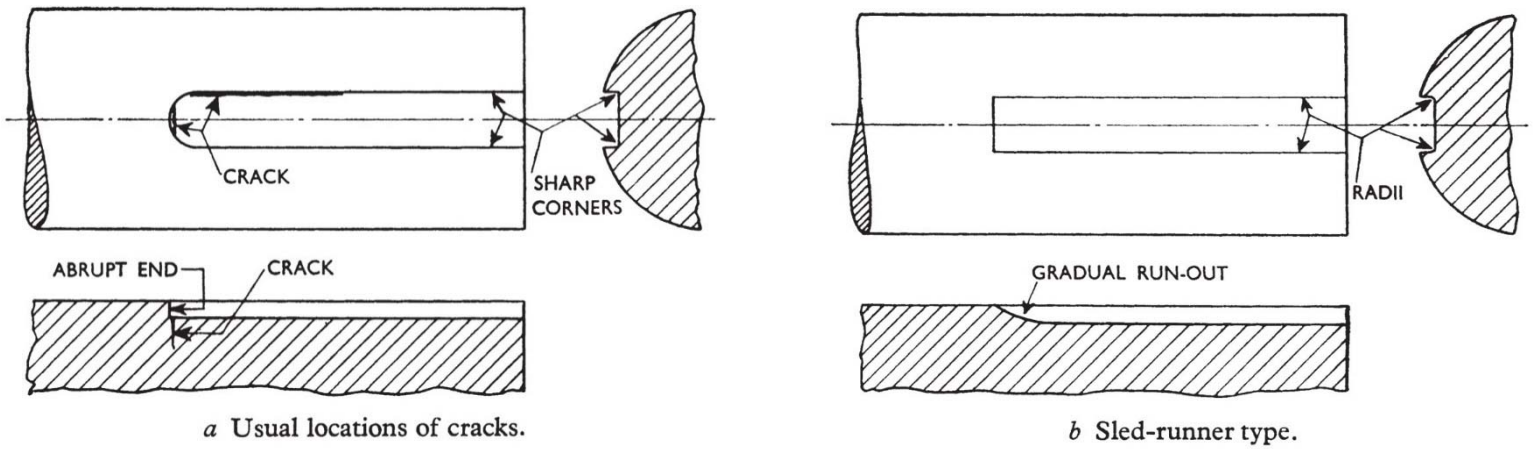


Figure 10.24 Usual location (a) of fatigue cracks in keyways, and an improved design (b) called a sled-runner keyway.

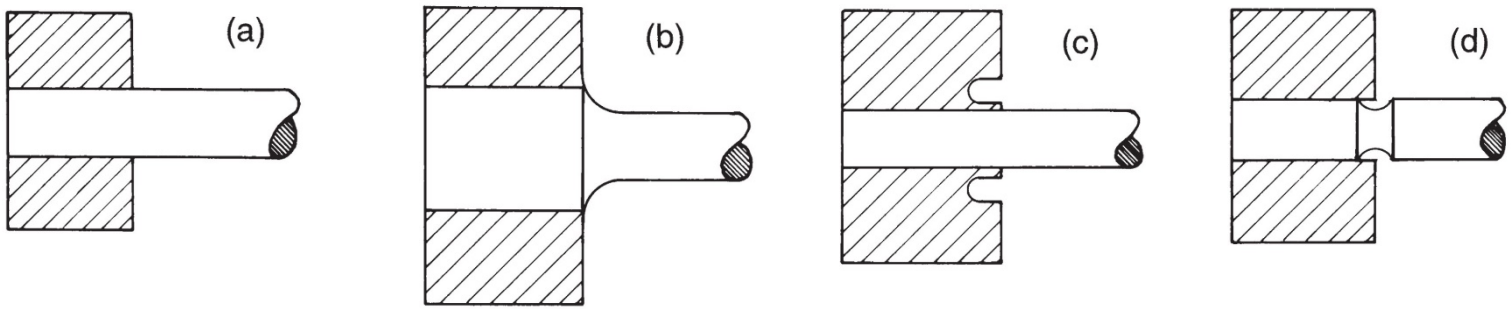
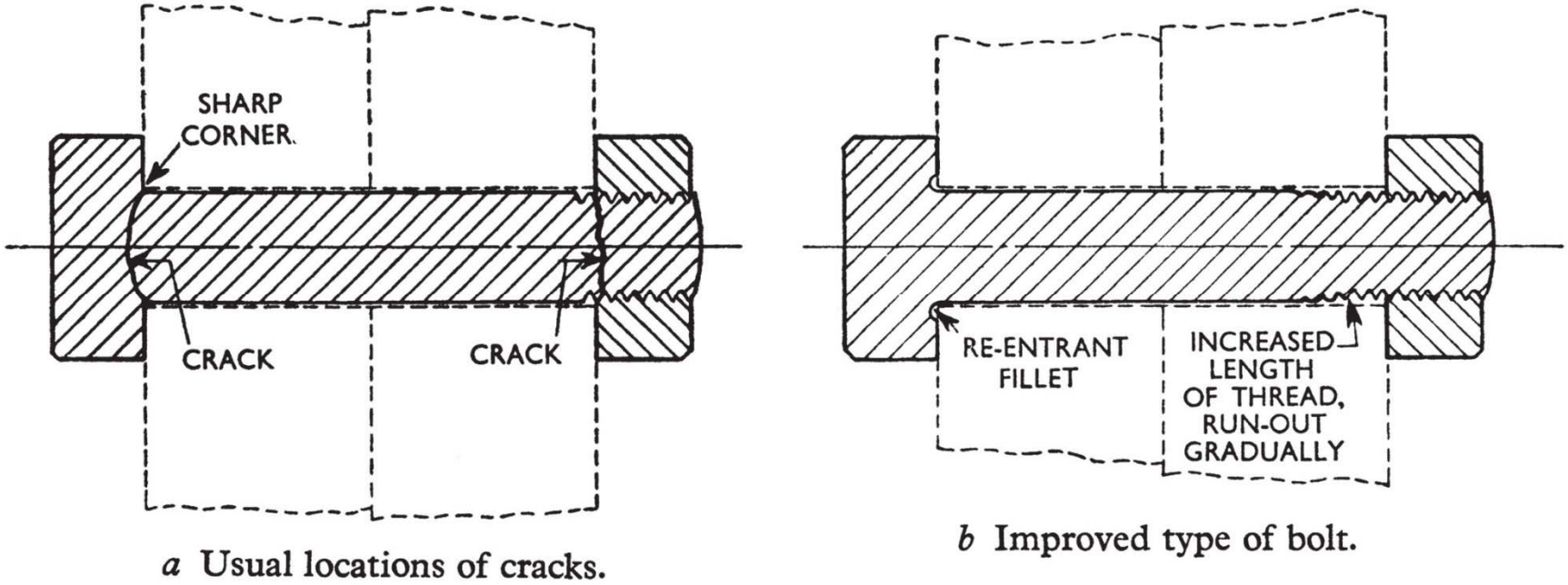


Figure 10.25 Some designs to alleviate stress concentration at a press-fitted shaft. The plain shaft (a) involves a severe stress raiser and is susceptible to fretting. Some possible improvements are (b) enlarging the shaft end, (c) modifying the collar, or (d) grooving the shaft.

10.9.1 Design Details: Bolting



a Usual locations of cracks.

b Improved type of bolt.

Figure 10.26 Usual location (a) of fatigue cracks in a bolt and (b) some measures to improve fatigue resistance.

10.9.1 Design Details: Bolting

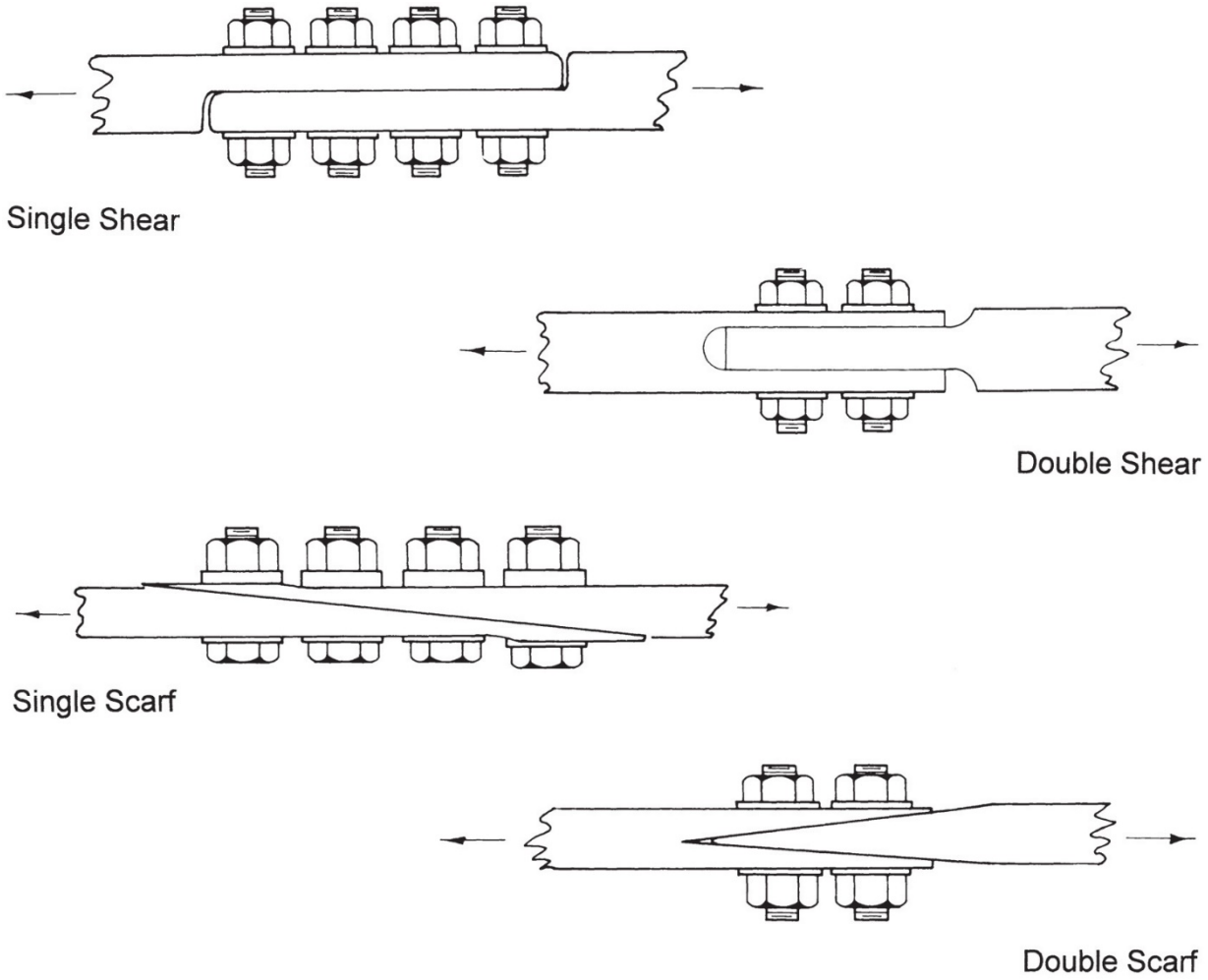


Figure 10.27 Some bolted joint details. A single shear joint can be improved by introducing a taper (scarf). Double shear joints minimize bending and can also be tapered.

10.10 Discussion

- **k_t values are based on elastic behavior, and most finite element analysis done in a design environment also assumes elastic behavior.**
  - Little or no yielding: reasonable to use stress-based approach
- **When localized yielding due to sharp notch or complexities occurs,**
  - Estimated S-N curves are recommended only for obtaining rough initial estimates. These should be replaced by S-N data for actual components or for notched members similar to the component.
  - The crack-growth approach based on fracture mechanics (Ch. 11) and the strain-based approach (Ch. 14) can be used.
  - For welded members, the crack-growth approach has room for further exploitation. The strain-based approach is not easy to apply due to the geometric complexity. For those reasons, the stress-based approach is preferred.