Chapter 14. Fatigue of Materials:

Strain-Based Approach to Fatigue

Seoul National University System Health & Risk Management

Jong Moon Ha

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CHAPTER 14 Objectives

- Explore **strain versus fatigue life curves** and equations, including trends with material and adjustments for surface finish and size
- Extend strain-life curves to cases of **nonzero mean stress** and **multi-axial stress**
- Apply the **strain-based method** to make **life estimates** for engineering components, especially members with **geometric notches**, including cases of **irregular variation of load** with time

Contents

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p. 63.

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Review: Crack Initiation and Propagation*

- The fatigue life of a component is made up of **initiation** and **propagation** stages
- The size of crack is usually unknown and often depends on the size of the component being analyzed (e.g. A: 0.1mm-crack, B:0.15mm-crack)
- **Stress & Strain-based Approach:** To determine crack initiation life
- **Fracture Mechanics:** To estimate the crack propagation life

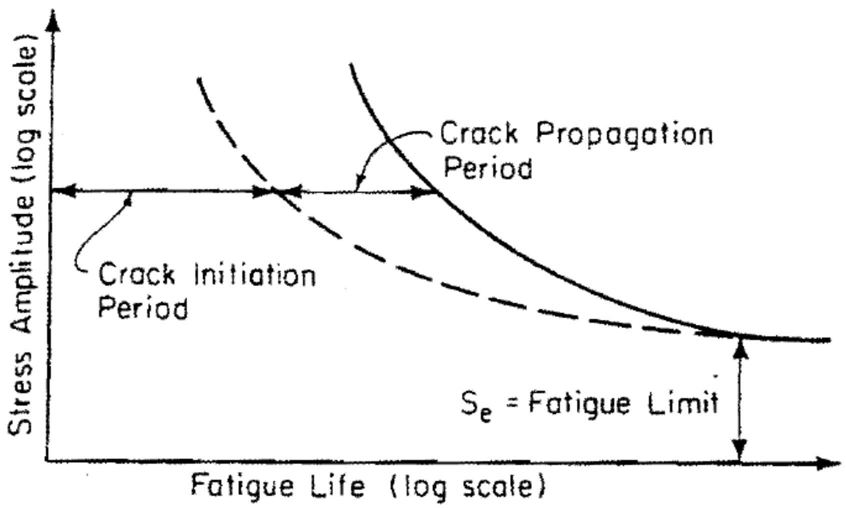

Figure 3.1* Initiation and propagation portions of fatigue life

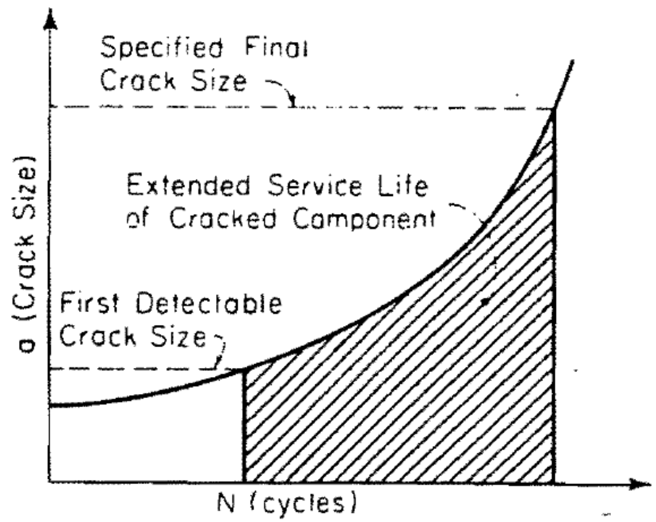

Figure 3.9* Extended service life of a cracked component

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273.

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14.1 Introduction – Why strain-based approach?

Brief Review of Stress-based Approach

- Key Idea is to **develop stress-life relationship** by employing *elastic stress concentration factors* and *empirical modifications thereof*.
- Stress-based approach emphasizes **nominal stress**, rather than local stresses and strains
- Stress-based approach **does not account for plastic strain**.

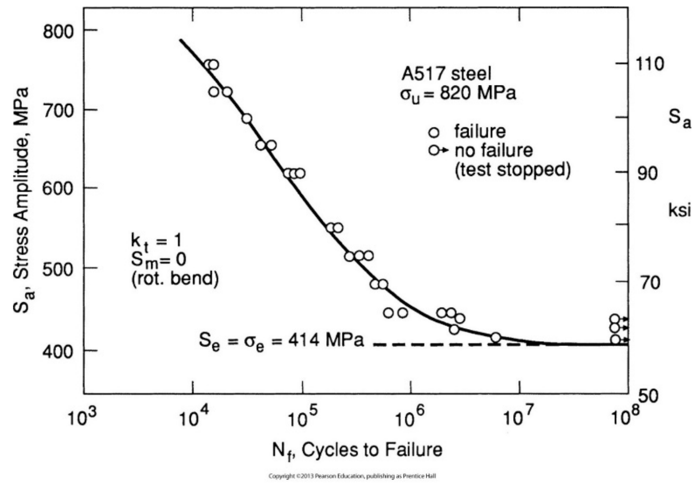

Figure 9.5

Stress-based Approach vs Strain-based Approach

High cycle fatigue	Low cycle fatigue
Elastic range of the material (Does not account for plastic strain)	Account for plastic strain
More than 1000 cycles (~N_t)	Less than 1000 cycles (~N_t)
Stress-Life (Stress controlled)	Strain-Life (Strain controlled)

Source: http://www.public.iastate.edu/~gkstarns/

14.1 Introduction

- Employment of ***cyclic stress-strain curve*** is a unique feature of the strain-based approach, as is the use of a ***strain versus life curve***, instead of a nominal stress versus life (*S*-*N*) curve

- Strain-based method gives **improved estimates** for intermediate and especially **short fatigue lives**.

- Strain-based approach employs the **local mean stress at the notch**, rather than the mean nominal stress.

- Certain concepts employed will be related to those introduced in **Chapters 9 and 10**. Also, we will draw upon the information in **Chapters 12 and 13** on plastic deformation.
  ➔ Be familiar with the contents in Chapters 9, 10, 12, and 13 before studying Chapter 14.

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14.2 Strain Versus Life Curves

- A strain versus life curve is a plot of **strain amplitude** versus **cycles to failure** (See Fig. 14.3)

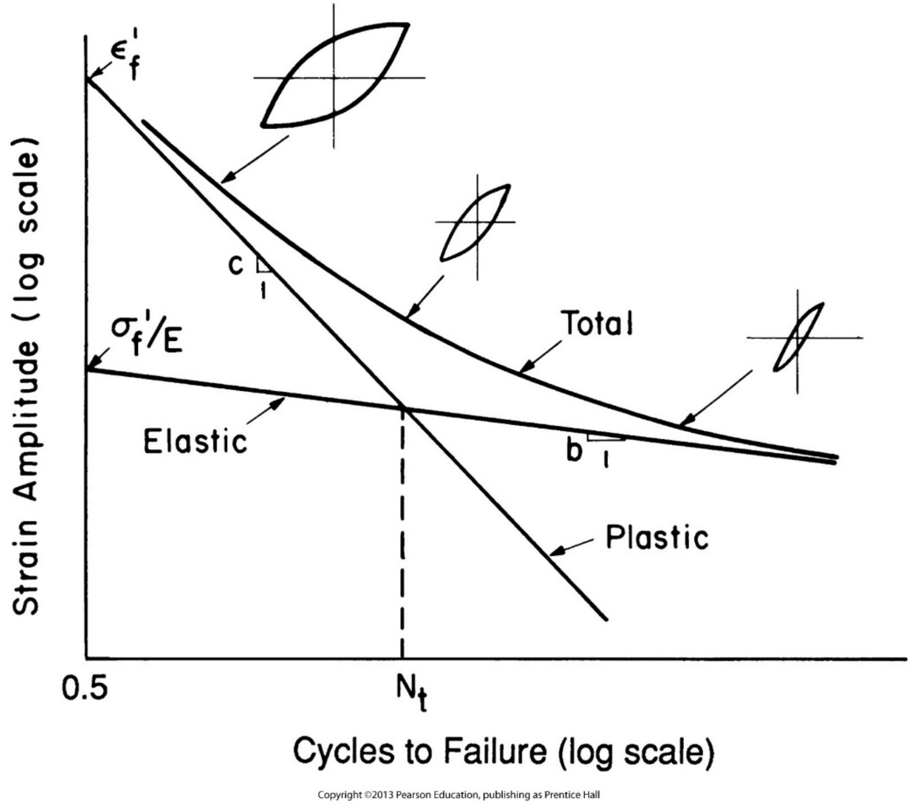


Figure 14.3 Elastic, plastic, and total strain versus life curves. (Adapted from [Landgraf 70]; copyright © ASTM; reprinted with permission.)

14.2.1 Strain-Life Tests and Equations

- Of particular relevance is the cyclic stress-strain curve (Also can be found in Eq. 12.54)

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'} \qquad (14.1)$$

- Note that the strain amplitude can be divided into elastic and plastic parts as

$$\varepsilon_a = \underline{\varepsilon_{ea}} + \underline{\varepsilon_{pa}} \qquad (14.2)$$

$$\varepsilon_{ea} = \frac{\sigma_a}{E}$$

Measure of the half-width of the stress-strain hysteresis loop

- Strain-life curves are derived from fatigue tests under completely reversed cyclic loading between constant strain limits (See Fig. 14.2)

Measurement:

$\sigma_a, \varepsilon_a, \varepsilon_{pa}$

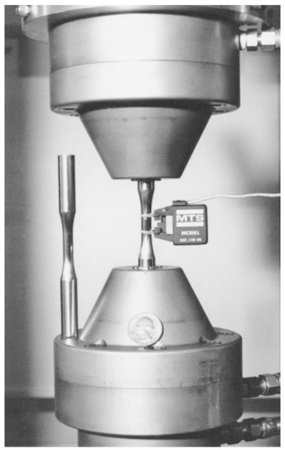


Figure 14.2

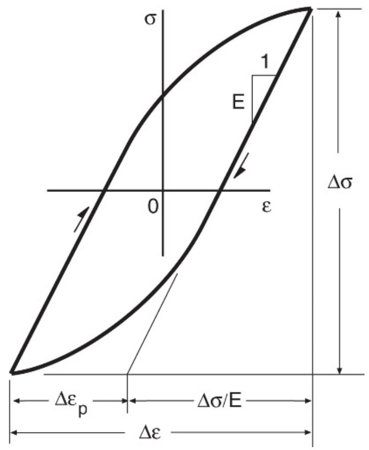


Figure 12.17

$\sigma_a = \Delta\sigma/2$

$\varepsilon_a = \Delta\varepsilon/2$

14.2.1 Strain-Life Tests and Equations

- For each test, **three points** are plotted (Fig. 14.4)
- If data from several tests are plotted, the **elastic strains** often give a straight line of shallow slope on a log-log plot, and the **plastic strains** give a straight line of steeper slope.
- Equation can then be fitted to these lines

$$\varepsilon_{ea} = \frac{\sigma_a}{E} = \frac{\sigma_f'}{E}\left(2N_f\right)^b \qquad (14.3)-(a)$$

$$\varepsilon_{pa} = \varepsilon_f'\left(2N_f\right)^c \qquad (14.3)-(b)$$

- Four constants$(\sigma_f', b, \varepsilon_f', c)$ are considered to be material properties
- Coffin-Manson Relationship

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \qquad (14.4)$$

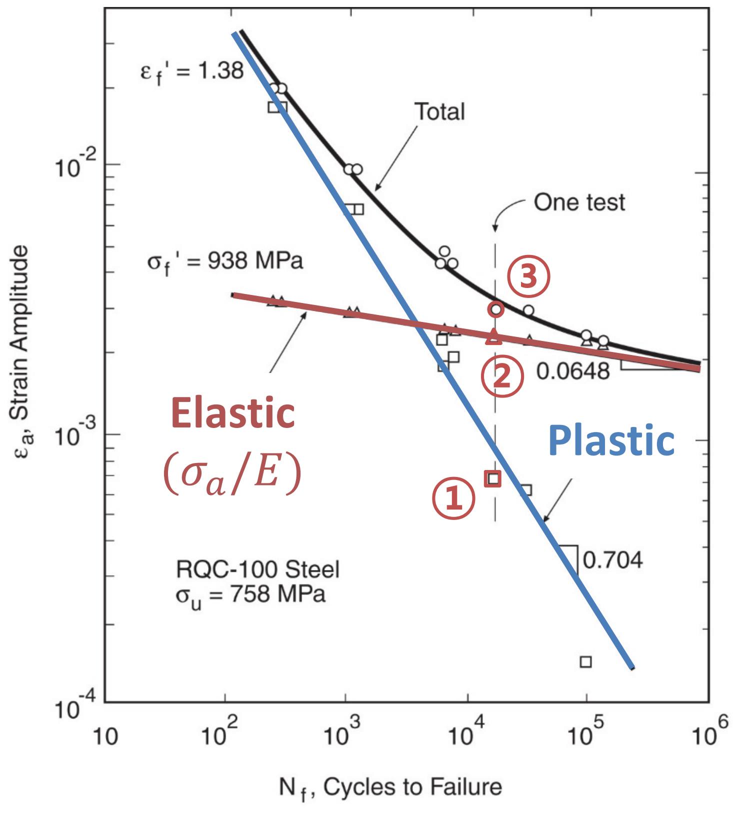


Figure 14.4 Strain versus life curves for RQC-100 steel. For each of several tests, elastic, plastic, and total strain data points are plotted versus life, and fitted lines are also shown. (From the author's data on the ASTM Committee E9 material.)

Remind - 14.2 Strain Versus Life Curves

- Coffin-Manson Relationship

$$\varepsilon_a = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \qquad (14.4)$$

Four constants $(\sigma'_f, b, \varepsilon'_f, c)$

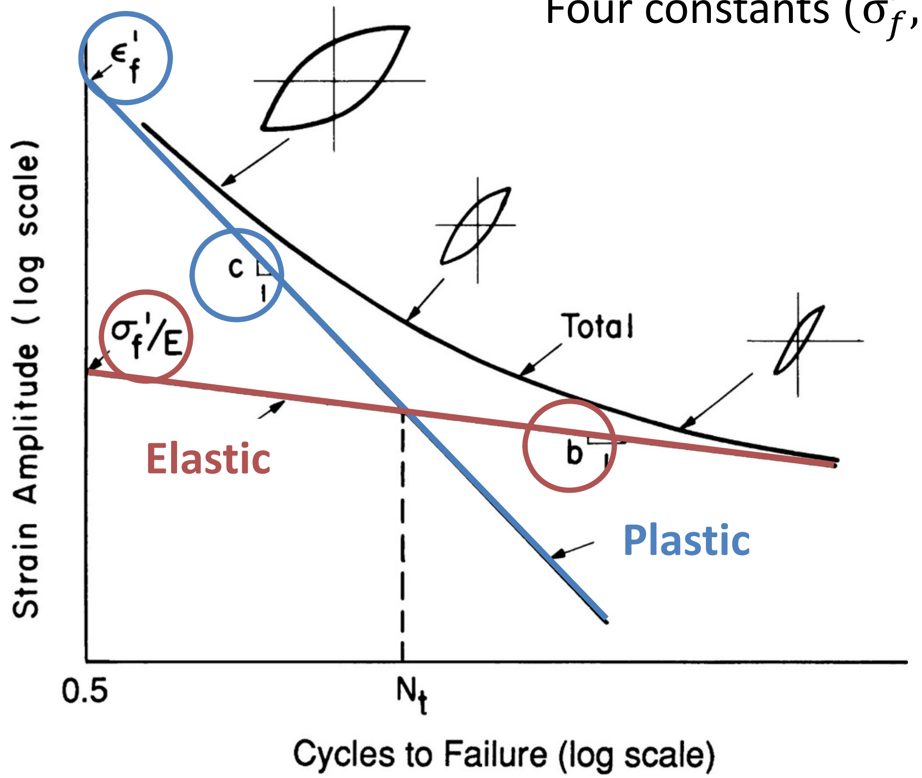


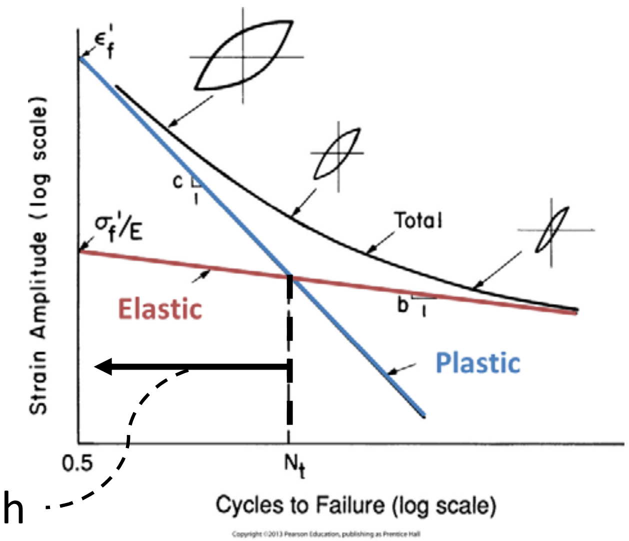
Figure 14.3 Elastic, plastic, and total strain versus life curves. (Adapted from [Landgraf 70]; copyright © ASTM; reprinted with permission.)

14.2.3 Trends for Engineering Metals

- **At long lives:** the curve approaches the **elastic strain** line
- **At short lives**: the curve approaches the **plastic strain** line
- **Near the crossing point**: the two types of strain are of similar magnitude
  → *Transition Fatigue Life*, N_t
- N_t is the most **logical point** for separating **low-cycle fatigue** and **high-cycle fatigue**
- Special analysis of **plasticity effects** by **the strain-based approach** may be needed if lives **around or less than N_t** are of interest

Let $\varepsilon_{ea} = \varepsilon_{pa}$

$$N_t = \frac{1}{2}\left(\frac{\sigma_f'}{\varepsilon_f' E}\right)^{1/(c-b)} \qquad (14.6)$$



Strain-based Approach

14.2.3 Trends for Engineering Metals

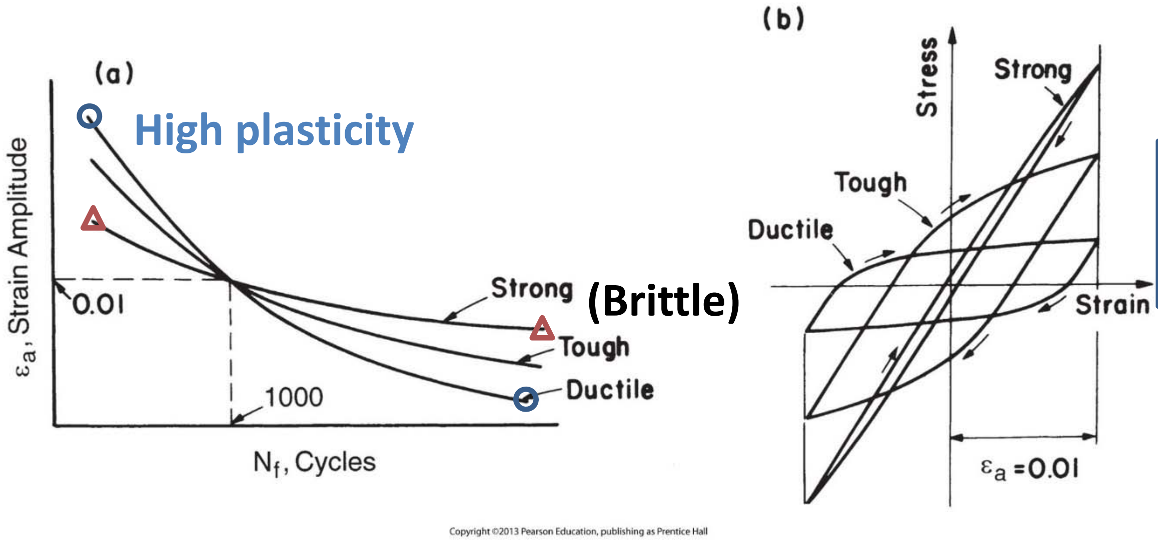
- The strain-life equation requires four empirical constants ($\sigma_f', b, \varepsilon_f', c$)
- **Several points must be considered** in attempting to obtain these constants from fatigue data*

1) **Generalization**
   Not all materials may be represented by the four-parameter strain-life equation. (Examples: some high strength aluminum alloys and titanium alloys)
2) **Data Size**
   The four fatigue constants may represent a **curve fit to a limited number of data points**. They may be changed if more data points are included in the curve fit.
3) **Range of data**
   The fatigue constants are determined from a set of data points **over a given range**. **Gross error** may occur when extrapolating fatigue life estimates **outside this range**.
4) **Physical phenomenon**
   The use of this equation is strictly a matter of mathematical convenience and is **not based on a physical phenomenon**

Nevertheless, the following approximate methods may be useful (cont.).

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p. 63.

	Fatigue Strength Coefficient, σ'_f	Fatigue Ductility Coefficient, ε'_f
Approximation*	$\sigma'_f \approx \sigma_f$ (corrected for necking) For steels (<500 BHN): $\sigma'_f \approx S_u + 50ksi$	$\varepsilon'_f \approx \varepsilon_f$, where $\varepsilon_f = \ln\left(\frac{1}{1-\mathrm{RA}}\right)$ RA is the reduction in area
A ductile Metal	Low	High
A brittle metal	High	Low



$$\varepsilon_a = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

Figure 14.6 Trends in strain–life curves for strong, tough, and ductile metals. (Adapted from [Landgraf 70]; copyright © ASTM; reprinted with permission.)

- All pass near the strain $\varepsilon_a = 0.01$ for a life of $N_f = 1000\ cycles$

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p3 64.

14.2.3 Trends for Engineering Metals

$$\varepsilon_a = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

- **Fatigue Strength Exponent, *b***
  Average: -0.085
  For soft metals: $b \approx -0.12$
  For highly hardened metals: $b \approx -0.05$
  For steels with ultimate tensile strengths below about $\sigma_u = 1400\text{MPa}$
  (Fatigue limit occurs near $N_f = 10^6$ cycles at a stress amplitude around $\sigma_{erb} = \sigma_u/2$)

$$\varepsilon_{ea} = \frac{\sigma_a}{E} = \frac{\sigma'_f}{E}(2N_f)^b \qquad (14.3)-(a)$$

$$b = -\frac{1}{log(2N_f)} log\left(\frac{2\sigma'_f}{\sigma_u}\right) = -\frac{1}{6.3} log\left(\frac{2\tilde{\sigma}_f}{\sigma_u}\right) \qquad (14.10)$$

In other cases, where the fatigue limit (or long-life fatigue strength) at N_e cycles is given by $\sigma_{erb} = m_e\sigma_u$, the estimate becomes

$$b = -\frac{1}{log(2N_e)} log\left(\frac{\tilde{\sigma}_f}{m_e\sigma_u}\right) \qquad (14.11)$$

14.2.3 Trends for Engineering Metals

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c$$

- **Fatigue Ductility Exponent**, c *
  c is not well defined as the other parameters.
  A rule of thumb approach must be followed rather than an empirical equation
  Coffin found c to be about -0.5
  Manson found c to be about -0.6
  Morrow found that c varied between -0.5 and -0.7

  Fairly ductile metals (where $\varepsilon_f \approx 1$) have average values of c=-0.6
  For strong metals (where $\varepsilon_f \approx 0.5$) have average values of c=-0.5

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p3 64.

14.2.3 Trends for Engineering Metals

- Hardness varies inversely with ductility
  so that N_t decreases as hardness is increased.

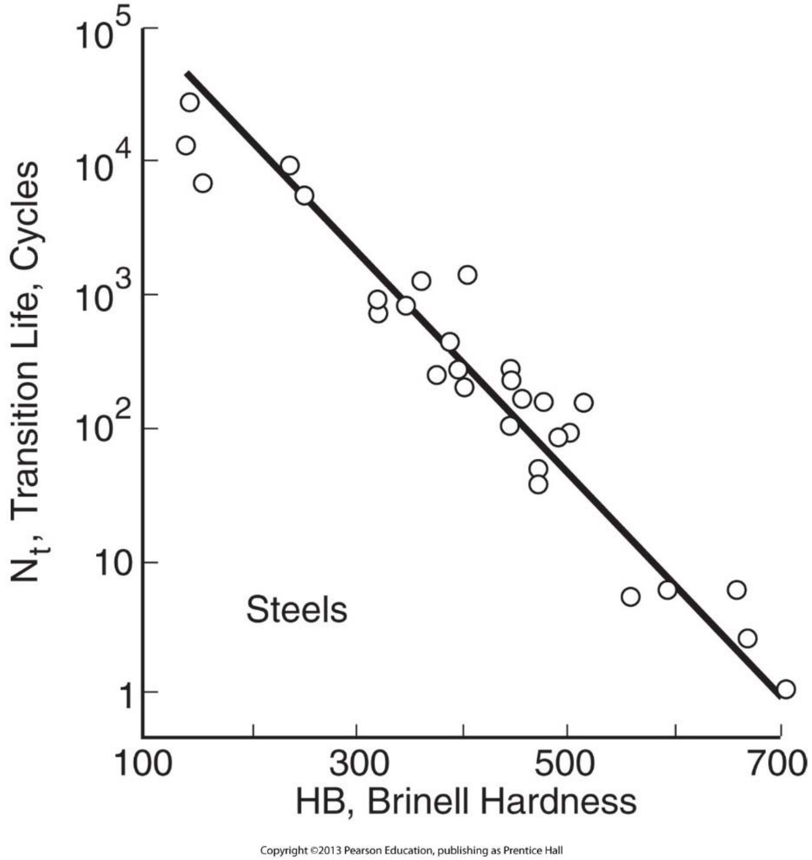


Figure 14.8 Transition fatigue life versus hardness for a wide range of steels. (Adapted from [Landgraf 70]; copyright © ASTM; reprinted with permission.)

14.2.4 Factors Affecting Strain-Life Curves

- Is a hostile chemical environment or elevated **temperature** is present, smaller numbers of cycles to failure are expected
- **Temperature**
  : At temperature exceeding about half of the absolute melting temperature of a given material, nonlinear deformation due to time-dependent creep-relaxation behavior generally become significant
- **Residual Stress**
  Residual stress are quickly removed by cycle-dependent relaxation if cyclic plastic strains are present
  ➔ These have only **limited effect at lives around and below N_t.**

Some additional comments on this topic are given in Chapter 15.

14.2.4 Factors Affecting Strain-Life Curves : Surface Finish

- **High-cycle fatigue** (most of the life is spent initiating a crack)
  ➔ Surface finish is important
- **Fatigue with significant plastic strain** (most of the life is spent growing crack size)
  ➔ Surface finish cannot have an effect
- A reasonable method to include the effect of surface finish is **to change only the elastic slope *b***.
- When the fatigue limit at N_e cycles is given by $\underline{\sigma_a = m_s \sigma_e}$,
  where $\underline{m_s}$ is a surface effect factor, as in Chapter 10

$$b_s = -\frac{1}{log(2N_e)} log\left(\frac{\tilde{\sigma}_f}{m_s \sigma_e}\right) = b + \frac{log(m_s)}{log(2N_e)} \quad (14.12)$$

$$\sigma_e = \sigma_f'(2N_e)^b$$

14.2.4 Factors Affecting Strain-Life Curves : Size Effect

- Size effect (as discussed in Chapter 10) are also a concern in applying a strain-based approach to large-size members, but experimental data are limited
- A study suggested lowering the entire strain-life curve by this factors so that the intercept constants σ'_f and ε'_f are replaced by reduced values and σ'_{fd} and ε'_{fd}

$$\sigma'_{fd} = m_d \sigma'_f, \quad \varepsilon'_{fd} = m_d \varepsilon'_f \qquad (14.14)$$

where m_d for the shafts up to 200 mm in diameter (low-carbon and low-alloy steels) was found to vary with shaft diameter as

$$m_d = \left(\frac{d}{25.4mm}\right)^{-0.093} \qquad (14.13)$$

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14.3.1 Mean Stress Effects*

- Cyclic fatigue properties of a material are obtained from **completely reversed, constant amplitude strain-controlled tests**.
  However, components seldom experience this type of loading.
- Mean stress effect can either **increase the fatigue life** with a nominally **compressive load** or **decrease it with a nominally tensile value** (Fig. 2.21*)
- **At high strain amplitude** (0.5% to 1% or above), where plastic strains are significant, mean stress relaxation occurs and the **mean stress tends toward zero**
- Modifications to the strain-life equation have been made to account for mean stress effects by 1) Morrow, 2) Smith, Watson, and Topper (SWT), and 3) Walker.

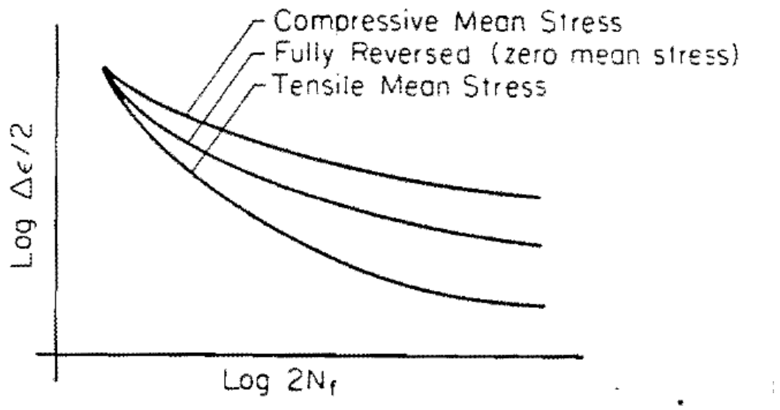

Figure 2.21* Effect of mean stress on strain-life curve

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p. 63.

14.3.3 Mean Stress Equation of Morrow*

- Morrow and Halford (1981) found that **ratio of elastic to plastic strain** in the equation proposed by Morrow (1968) is **dependent on mean stress**, which is **not true**.
- They modified both the elastic and plastic terms of the strain-life equation to **maintain the independence of the elastic-plastic strain ratio** from mean stress as:

$$\varepsilon_a = \frac{\sigma_f' - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon_f'\left(\frac{\sigma_f' - \sigma_m}{\sigma_f'}\right)^{c/b}\left(2N_f\right)^c \qquad (14.24)$$

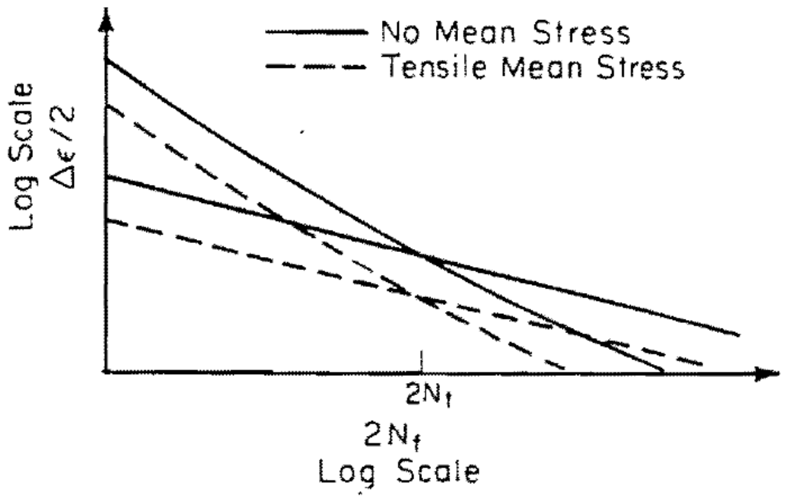

Figure 2.25* Mean stress correction for independence of elastic/plastic strain ratio from mean stress

- Refer to the Appendix for more details

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p. 63.

14.3.3 Mean Stress Equation of Morrow (modified)*

- Morrow (1968) suggested to modify the elastic term in the strain-life equation

$$\varepsilon_{ea} = \frac{\sigma_a}{E} = \frac{\sigma_f'}{E}(2N_f)^b \quad (14.3)-(a) \quad \Rightarrow \quad \varepsilon_{ea} = \frac{\sigma_a}{E} = \frac{\sigma_f' - \sigma_m}{E}(2N_f)^b$$

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \quad (14.4) \quad \Rightarrow \quad \varepsilon_a = \frac{\sigma_f' - \sigma_m}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \quad (14.27)$$

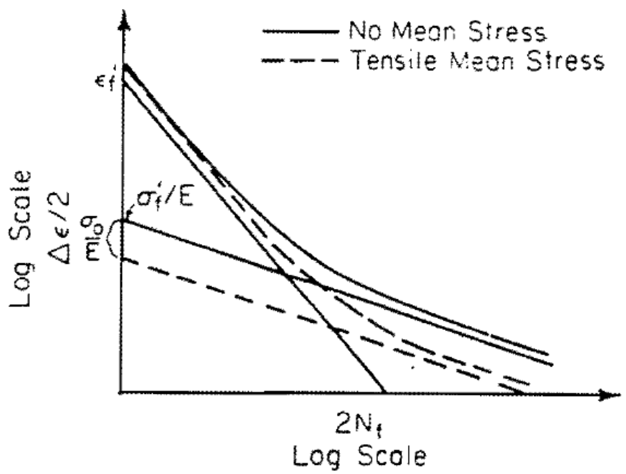

Figure 2.23* Morrow's mean stress correction to the strain-life curve for a tensile mean

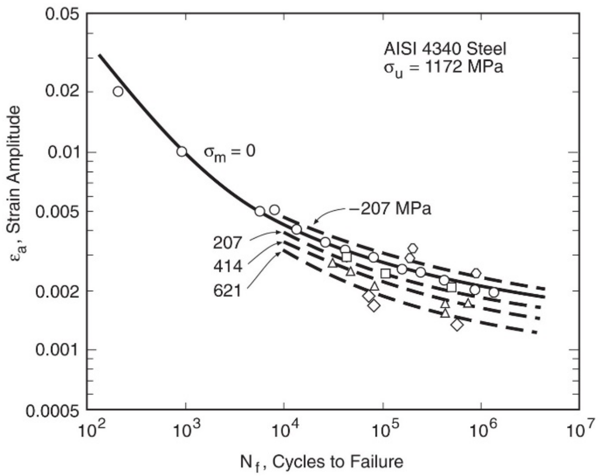

Figure 14.12 Family of strain–life curves given by the modified Morrow approach

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p. 63.

(SWT) Parameters, Walker Mean Stress Equation

14.3.5 Smith, Watson, and Topper (SWT) Parameters

- This approach assumes that the life for any situation of mean stress depends on the product

$$\sigma_{max}\varepsilon_a = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c} \qquad (14.29)$$

where $\sigma_{max} = \sigma_m + \sigma_a$

14.3.5 Walker Mean Stress Equation

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(\frac{1-R}{2}\right)^{(1-\gamma)/b}(2N_f)^b + \varepsilon_f'\left(\frac{1-R}{2}\right)^{c(1-\gamma)/b}(2N_f)^c \qquad (14.33)$$

14.3.5 Smith, Watson, and Topper (SWT) Parameters

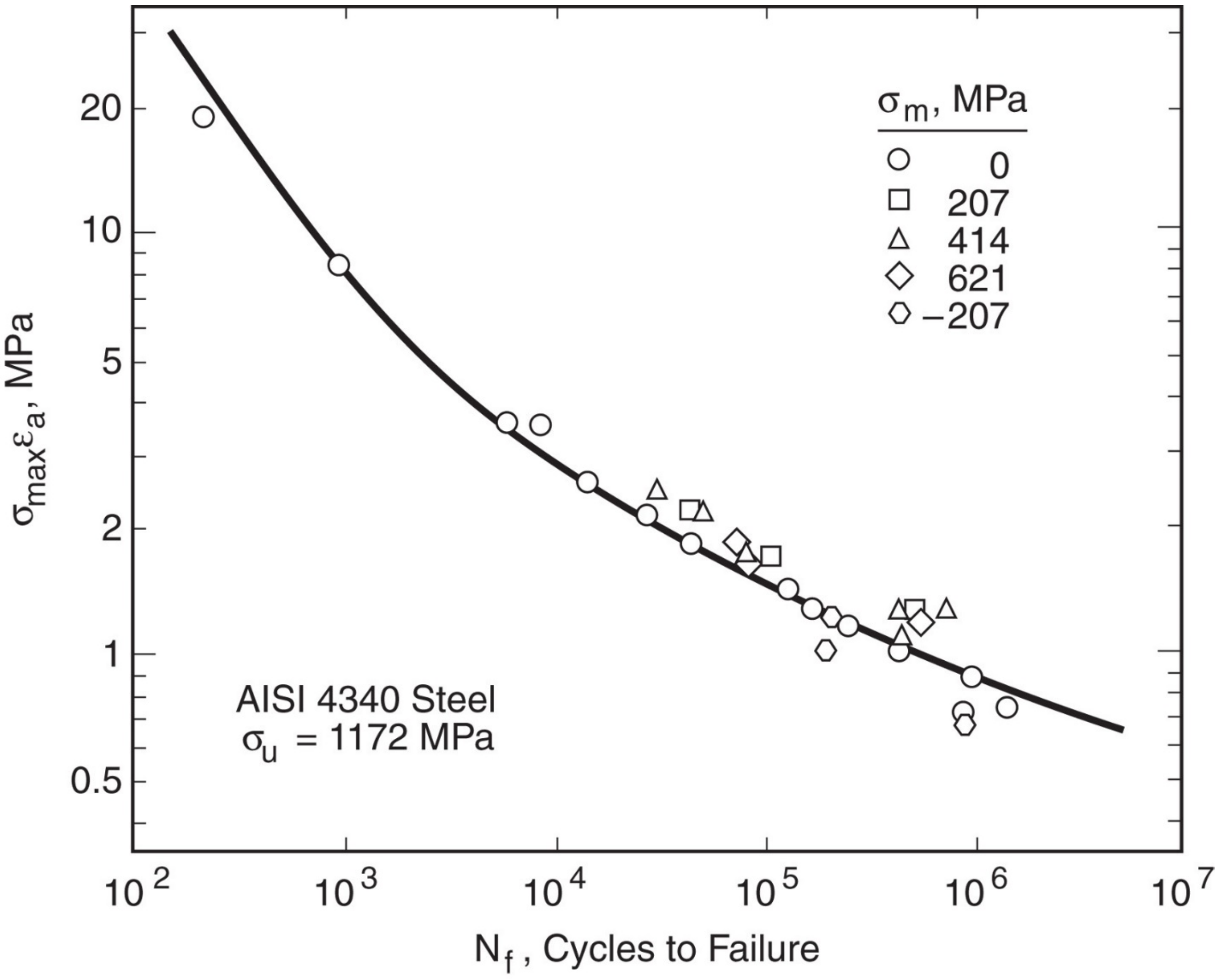

Figure 14.13 Plot of the Smith, Watson, and Topper parameter versus life for the data of Fig. 14.10.

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- Fatigue under multi-axial loading where plastic deformations occur is currently an area of active research.
- Reasonable estimates are possible for relatively simple situations

14.4.1 Effective Strain Approach

- Consider situations where all cyclic loadings have the same frequency and are either in-phase or 180° out-of-phase.
- Recall three-dimensional stress-strain relationships (Ch. 12)

$$\bar{\sigma} = 1/\sqrt{2} \times \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \qquad (12.21)$$

$$\bar{\varepsilon}_p = \sqrt{2}/3 \times \sqrt{\left(\varepsilon_{p1} - \varepsilon_{p2}\right)^2 + \left(\varepsilon_{p2} - \varepsilon_{p3}\right)^2 + \left(\varepsilon_{p3} - \varepsilon_{p1}\right)^2} \qquad (12.22)$$

$$\bar{\varepsilon} = \frac{\bar{\sigma}}{E} + \bar{\varepsilon}_p \qquad (12.23)$$

- Then, we can define an effective strain amplitude

$$\bar{\varepsilon}_a = \frac{\bar{\sigma}_a}{E} + \bar{\varepsilon}_{pa} \qquad (14.34)$$

- $\bar{\sigma}_a$ and $\bar{\varepsilon}_{pa}$ are obtained by substituting amplitudes of the principal stresses and plastic strains.
- A negative sign is employed for amplitude quantities that are 180° out-of-phase.

14.4.1 Effective Strain Approach

- For uniaxial loading ($\sigma_2 = \sigma_3 = 0$), its value reduces to the uniaxial strain amplitude ($\bar{\varepsilon}_a = \varepsilon_{1a}$)

$$\bar{\varepsilon}_a = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c \qquad (14.35)$$

$$\bar{\sigma}_a = \sigma'_f\left(2N_f\right)^b, \quad \bar{\varepsilon}_{pa} = \varepsilon'_f\left(2N_f\right)^b \qquad (14.36)$$

- Consider the special case of plane stress (Refer to Ch. 12.3.4)

$$\sigma_{2a} = \lambda\sigma_{1a}, \qquad \sigma_{3a} = 0, \qquad \varepsilon_{1a} = \varepsilon_{e1a} + \varepsilon_{p1a} \qquad (14.37)$$

- Combining Eqs. (12.19), (12.24) and (12.32) with Eqs. (14.35), (14.37)

$$\varepsilon_{1a} = \frac{\frac{\sigma'_f}{E}(1-\nu\lambda)\left(2N_f\right)^b + \varepsilon'_f(1-0.5\lambda)\left(2N_f\right)^c}{\sqrt{1-\lambda+\lambda^2}} \qquad (14.38)$$

- For the special state of plane stress that is pure shear ($\sigma_{2a} = \lambda\sigma_{1a}, \lambda = -1$)

$$\underline{\gamma_{xya}} = \frac{\sigma'_f}{\sqrt{3}G}\left(2N_f\right)^b + \sqrt{3}\varepsilon'_f\left(2N_f\right)^c \qquad (14.39)$$

Shear strain amplitude Shear modulus

Refer to Ch. 13.4 for more information

14.4.2 Discussion of the Effective Strain Approach

- Consider the hydrostatic stress not accounted for by the octahedral shear strain

$$\sigma_{ha} = \frac{\sigma_{1a} + \sigma_{2a} + \sigma_{3a}}{3} \tag{14.40}$$

- Relative value of σ_{ha} my be expressed as a *triaxiality factor* for plane stress ($\sigma_{3a} = 0$)

$$T = \frac{1+\lambda}{\sqrt{1-\lambda+\lambda^2}}, \qquad \sigma_{2a} = \lambda\sigma_{1a}, \tag{14.41}$$

  (a) Pure planar shear ($\lambda = -1$) ➔ $T = 0$
  (b) Uniaxial stress ($\lambda = 0$) ➔ $T = 1$
  (c) Equal biaxial stress ($\lambda = 1$) ➔ $T = 2$

- Marloff (1985) proposed to include this effect to strain-life equation as:

$$\bar{\varepsilon}_a = \frac{\sigma'_f}{E}\left(2N_f\right)^b + 2^{1-T}\varepsilon'_f\left(2N_f\right)^c \tag{14.42}$$

14.4.3 Critical Plane Approaches

- Critical plane approach is needed where the loading is non-proportional to a significant degree
- Stresses and strains normal to the crack plane may have a major effect on the behavior, accelerating the growth if they tend to open the crack.

1) Stresses and strains are determined for various orientations (planes) in the material.
2) Stresses and strains acting on the most severely loaded plane are used for analysis.

14.4.3 Critical Plane Approaches

- **Fatemi and Socie**

$$\gamma_{ac}\left(1+\frac{\alpha\sigma_{\mathrm{maxc}}}{\sigma_o'}\right)\bar{\varepsilon}_a=\frac{\tau_f'}{G}\left(2N_f\right)^b+\gamma_f'\left(2N_f\right)^c \quad (14.43)$$

where γ_{ac} is the largest amplitude of shear strain for any plane,
σ_{maxc} is the peak tensile stress normal to the plane of γ_{ac},
α is an empirical constant ranging from 0.6 to 1.0, and
σ_o' is the yield strength for the cyclic stress-strain curve.
τ_f', b, γ_f', c give the strain-life curve from completely reversed tests in pure shear.
(specifically torsion tests on thin-walled tubes)

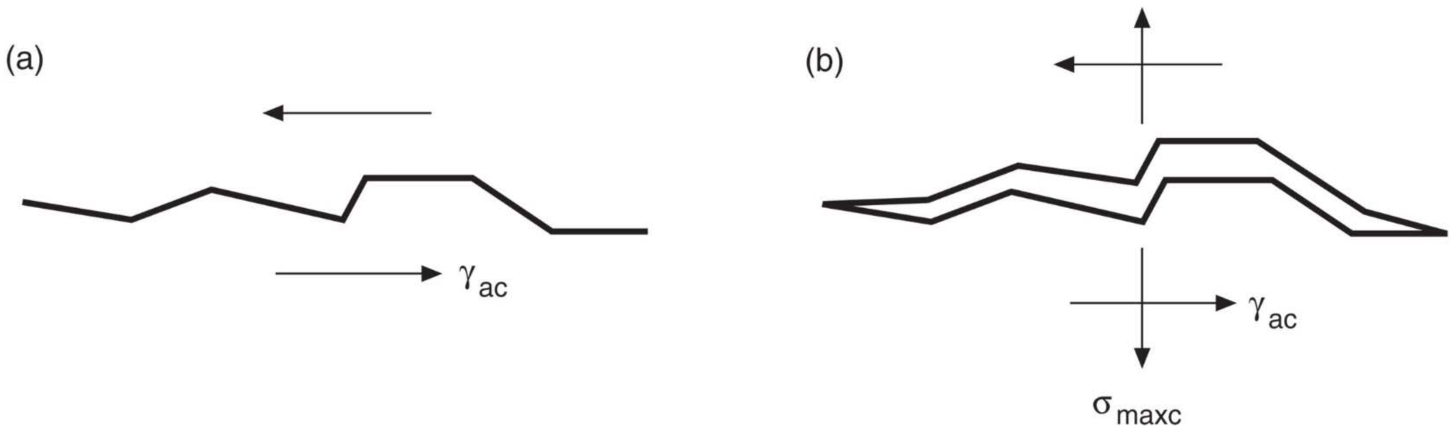

Figure 14.14 Crack under pure shear (a), where irregularities retard growth, compared with a situation (b) where a normal stress acts to open the crack, enhancing its growth. (Adapted from [Socie 87]; used with permission of ASME.)

14.4.3 Critical Plane Approaches

- **Fatemi and Socie**

$$\gamma_{ac}\left(1+\frac{\alpha\sigma_{\mathrm{maxc}}}{\sigma_o'}\right)\bar{\varepsilon}_a=\frac{\tau_f'}{G}\left(2N_f\right)^b+\gamma_f'\left(2N_f\right)^c \qquad (14.43)$$

- **Smith, Watson, and Topper (SWT) Parameters**

$$\sigma_{max}\varepsilon_a=\frac{\left(\sigma_f'\right)^2}{E}\left(2N_f\right)^{2b}+\sigma_f'\varepsilon_f'\left(2N_f\right)^{b+c} \qquad (14.35)$$

➔ The shortest life estimated from either Eqs. (14.43) or (14.35) is the final life estimate.

- A single multiaxial fatigue criterion that considers both the shear and normal stress cracking mode is that of Chu (1995)

$$2\tau_{max}\gamma_a+\sigma_{max}\varepsilon_a=f\left(N_f\right) \qquad (14.44)$$

where $f\left(N_f\right)$ can be obtained from uniaxial test data.

Contents

14.5.1 Constant Amplitude Loading

- Assuming idealized behavior for the material

$$\varepsilon = f(\sigma), \qquad \varepsilon_a = f(\sigma_a) \tag{14.45}$$

(Monotonic) (Cyclic)

$$\frac{\Delta\varepsilon}{2} = f\left(\frac{\Delta\sigma}{2}\right) \tag{14.46}$$

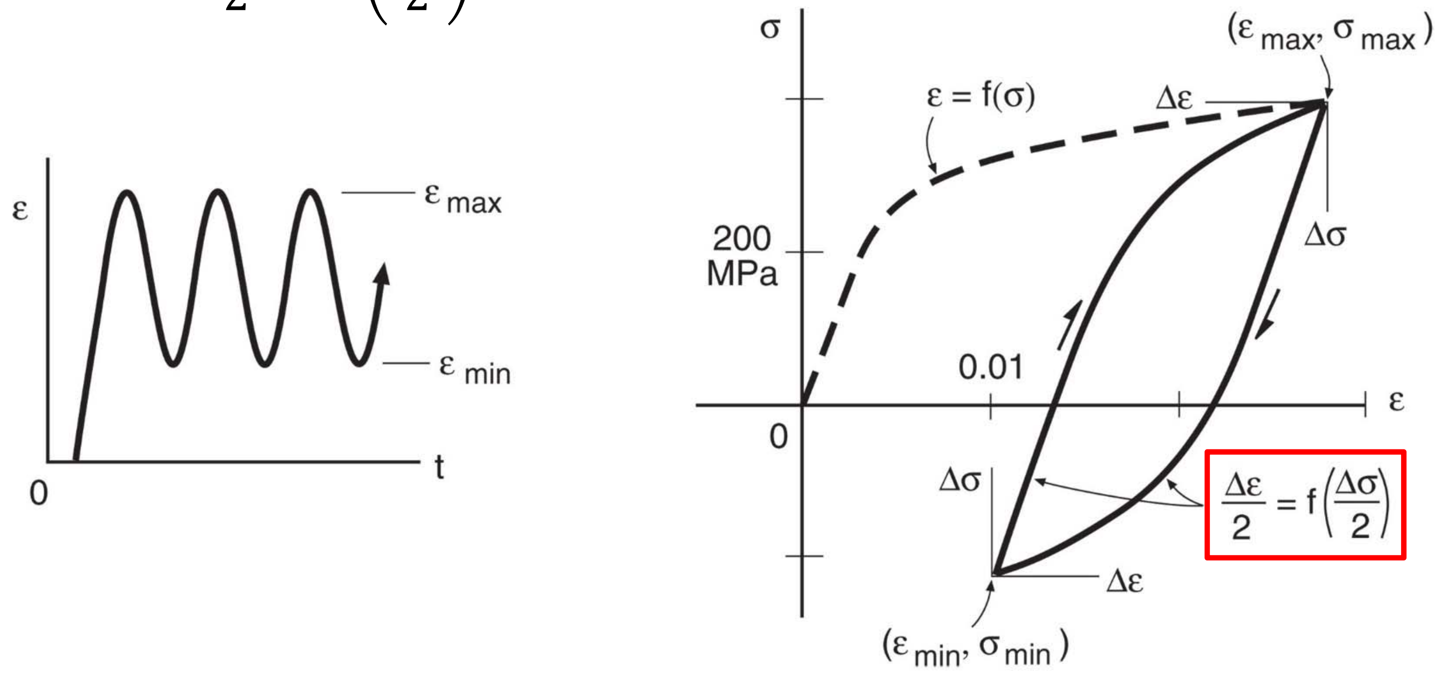

Figure 12.14 Stress–strain unloading and reloading behavior consistent with a spring and slider rheological model. The example curves plotted correspond to a Ramberg–Osgood stress–strain curve with constants as in Fig. 12.9.

14.5.1 Constant Amplitude Loading

- Consider **Neuber's rule** to analyze a notched member
  ➔ **Ch. 13** will be reviewed in the following two slides

13.5.3 Estimates of Notch Stress and Strain for Local Yielding

- Theoretical stress concentration factor, K_t, is used to relate the nominal stress or strain to the local values.*
- Upon yielding, the local values are no longer linearly related to the nominal values by K_t*
- Instead, the values are related in terms of stress and strain concentration factors as:

$$k_\sigma = \frac{\sigma \text{ (Local stress)}}{S \text{ (Nominal stress)}}, \quad k_\varepsilon = \frac{\varepsilon \text{ (Local strain)}}{e \text{ (Nominal strain)}} \tag{13.56}$$

- After yielding, the actual local stress is less than that predicted using K_t
- After yielding, the actual local strain is greater than that predicted using K_t
- This is due to residual stresses at the notch root

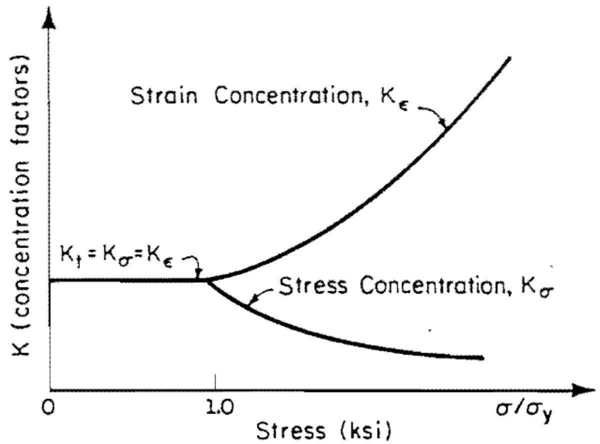

Figure 14.13* Effect of yielding on K_σ and K_ϵ

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p. 63.

13.5.3 Estimates of Notch Stress and Strain for Local Yielding

- **Neuber's rule** states simply that the geometric mean of the stress and strain concentration factors remains equal to k_t during plastic deformation
- If fully plastic yielding does not occur, $e = S/E$ applies.

$$k_t = \sqrt{k_\sigma k_\varepsilon} \qquad (13.57)$$

$$\sigma\varepsilon = \frac{(k_t S)^2}{E} \qquad (13.58)$$

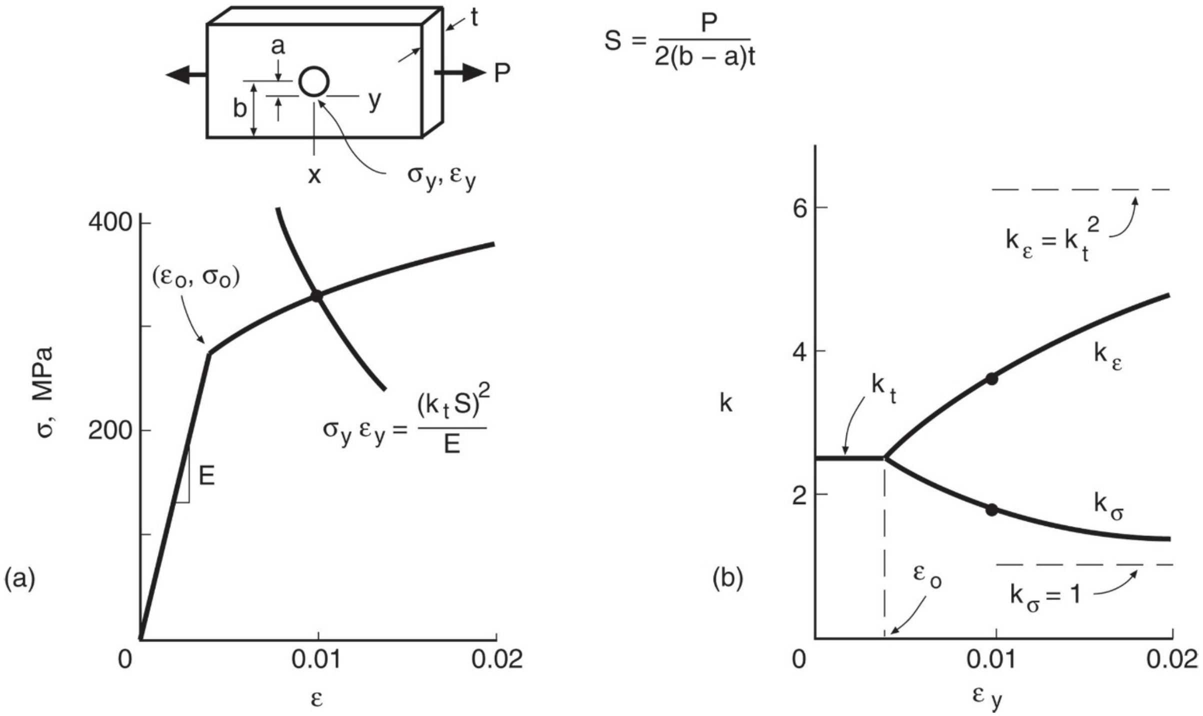

Figure 13.16 For a given notched member and stress–strain curve (a), Neuber's rule may be used to estimate local notch stresses and strains, σ and ε, corresponding to a particular value of nominal stress S. Stress and strain concentration factors vary as in (b).

14.5.1 Constant Amplitude Loading

- Consider Neuber's rule to analyze a notched member

Step 1

Step 2

Step 3

Step 4

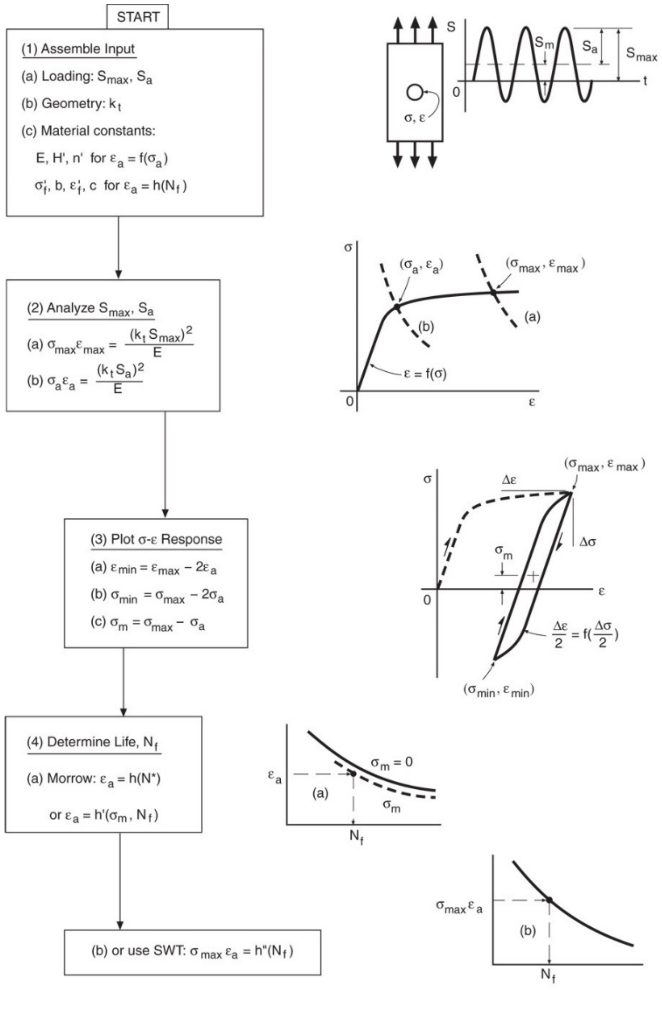

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Figure 14.15 Steps required in strain-based life prediction for a notched member under constant amplitude loading.

14.5.1 Constant Amplitude Loading

- **Step 1**

START

(1) Assemble Input

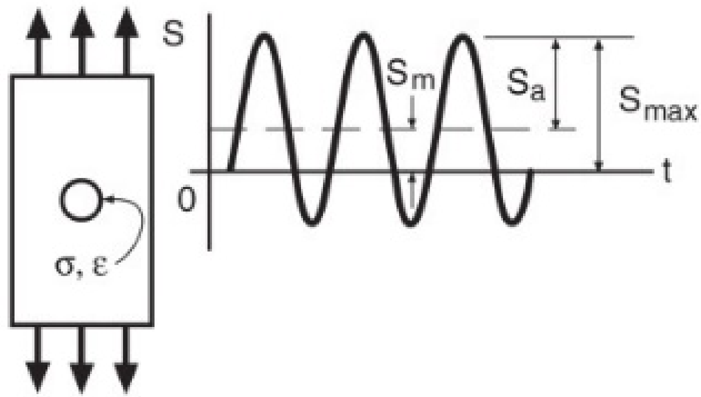
(a) Loading: S_{max}, S_a

(b) Geometry: k_t

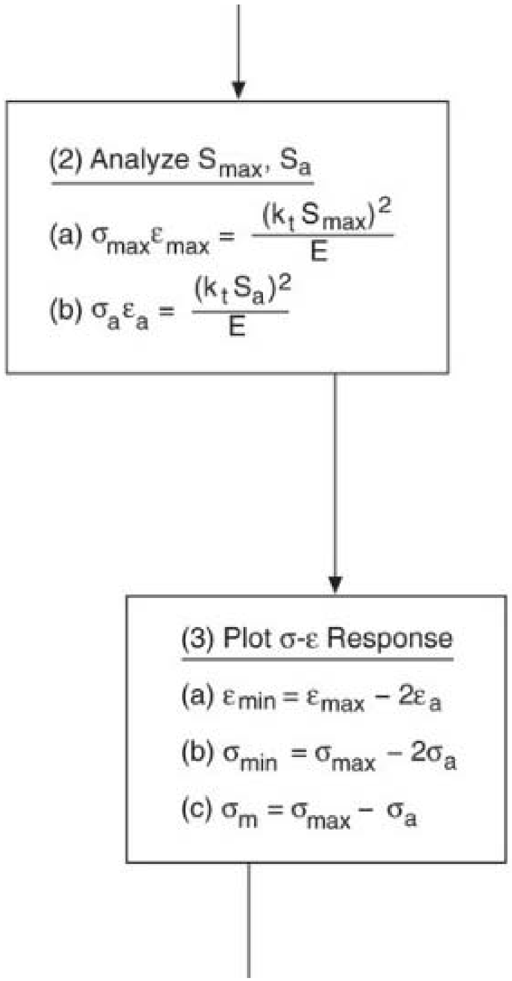
(c) Material constants:

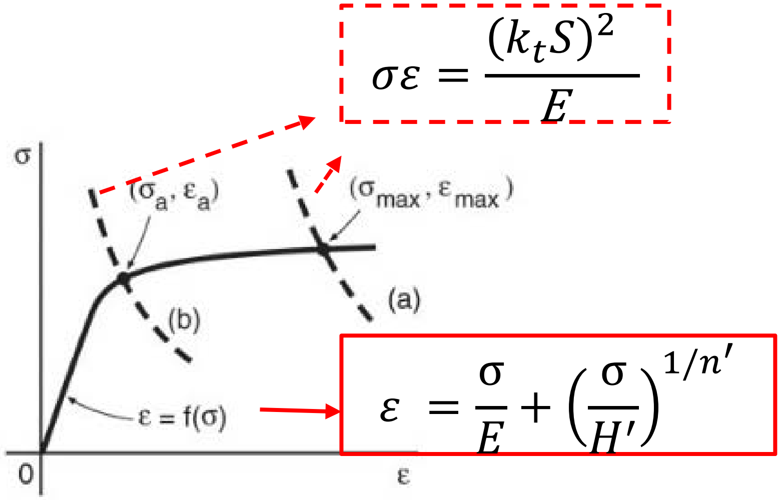
E, H', n' for $\varepsilon_a = f(\sigma_a)$

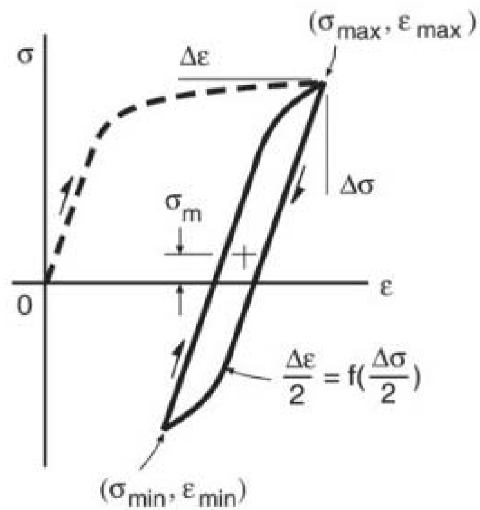
$\sigma'_f, b, \varepsilon'_f, c$ for $\varepsilon_a = h(N_f)$



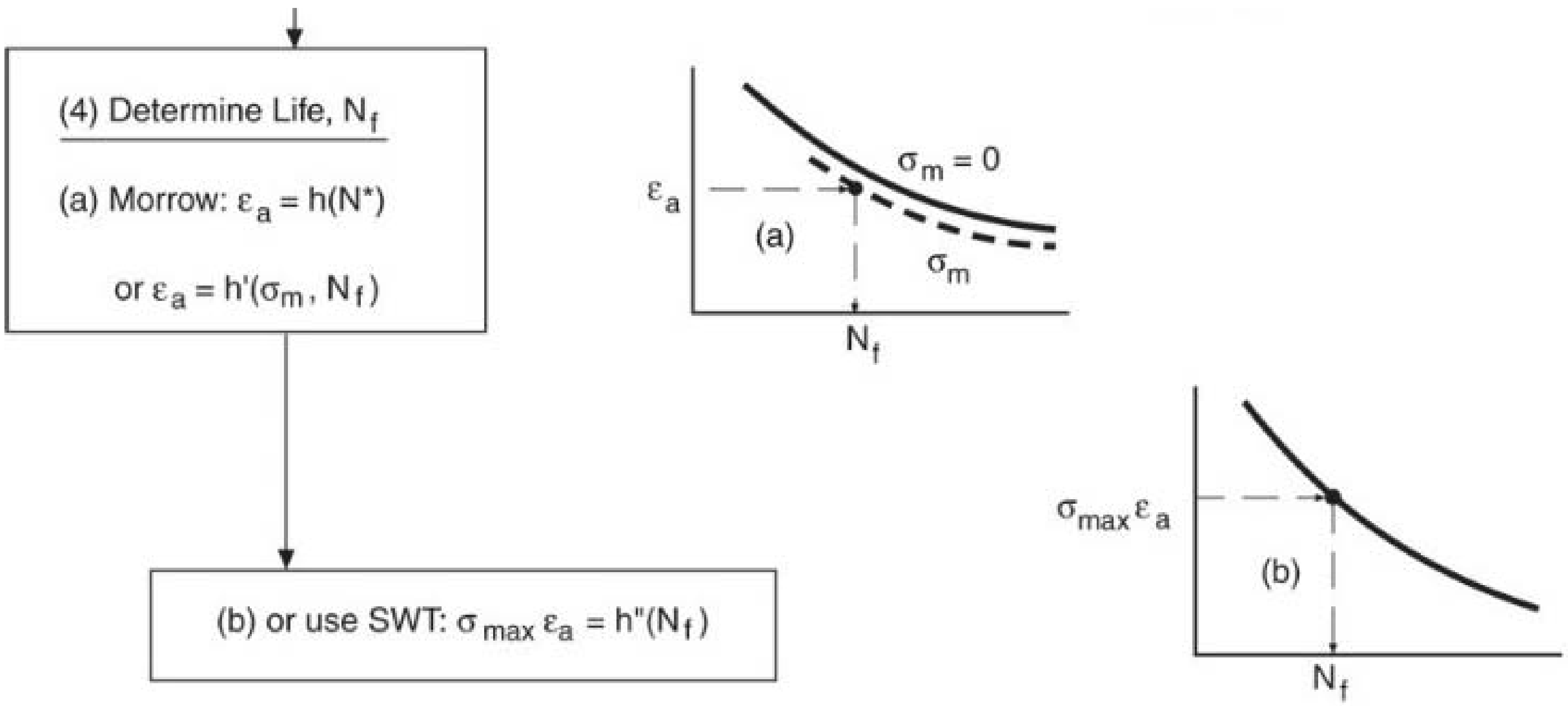
- **Step 2 & Step 3**


(2) Analyze S_{max}, S_a
(a) $\sigma_{max}\varepsilon_{max} = \frac{(k_t S_{max})^2}{E}$
(b) $\sigma_a\varepsilon_a = \frac{(k_t S_a)^2}{E}$
(3) Plot σ-ε Response
(a) $\varepsilon_{min} = \varepsilon_{max} - 2\varepsilon_a$
(b) $\sigma_{min} = \sigma_{max} - 2\sigma_a$
(c) $\sigma_m = \sigma_{max} - \sigma_a$


$\sigma\varepsilon = \frac{(k_t S)^2}{E}$
σ
$(\sigma_a, \varepsilon_a)$
$(\sigma_{max}, \varepsilon_{max})$
(a)
(b)
ε = f(σ)
$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{1/n'}$
0
ε


$(\sigma_{max}, \varepsilon_{max})$
σ
Δε
Δσ
σ_m
0
ε
$\frac{\Delta\varepsilon}{2} = f(\frac{\Delta\sigma}{2})$
$(\sigma_{min}, \varepsilon_{min})$

- **Step 4**



(4) Determine Life, N_f
(a) Morrow: $\varepsilon_a = h(N^*)$
or $\varepsilon_a = h'(\sigma_m, N_f)$
(b) or use SWT: $\sigma_{max}\,\varepsilon_a = h''(N_f)$
ε_a
$\sigma_m = 0$
(a)
σ_m
N_f
$\sigma_{max}\,\varepsilon_a$
(b)
N_f

14.5.2 Irregular Load Versus Time Histories

- The life to failure N_f corresponding to each hysteresis loop can be determined from its combination of strain amplitude and mean stress.
- If the SWT parameter is used, σ_{max} is simply the highest stress (σ_G) for F-G-F'
- Having obtained the N_f value for each loop, we can apply the Palmgren Miner rule (Ch. 9), where each closed stress-strain hysteresis loop is considered to represent a cycle.

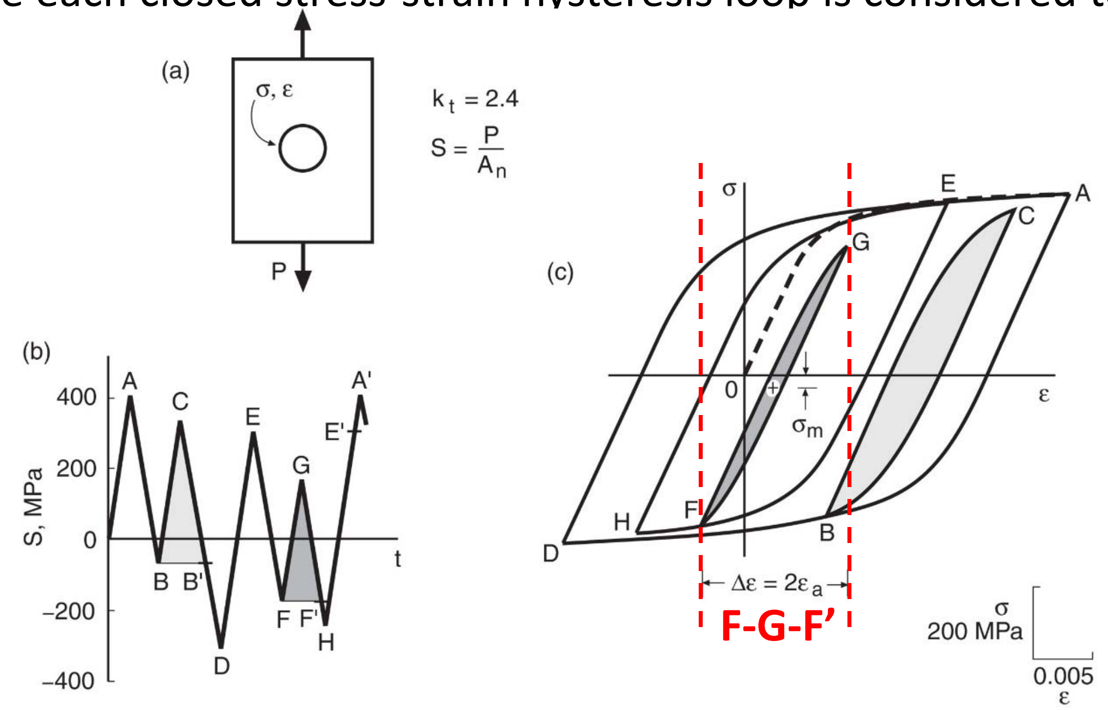


Figure 14.16 Analysis of a notched member subjected to an irregular load versus time history. Notched member (a), made of 2024-T351 aluminum, is subjected to load history (b). The resulting local stress–strain response at the notch is shown in (c).

Contents

6. Comparison of Methods*

General Points for Comparison

- Are the methods to be **used in the design cycle** or **to analyze an existing component**?
- What is the accuracy of the methods compared to input variables such as load history and material properties?
- What are the relative economics?
- What is the level of acceptance?
- Uses in design versus research.

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p. 63.

6. Comparison of Methods*

	Stress-based Approach	Strain-based Approach
Economics	**Quick and Cheap**	Expensive
Available Data Size	**Big**	Small
History	**Long (100 years)**	Short (30 years)
Amount of Confidence	**High**	Low
Complexity	**Low**	High
Physical Insight	Bad	**Good**
Plastic Strain	Not considered	**Considered**
Residual Stress	Not considered	**Considered**
Where should we use this approach?	• For constant amplitude loading and long fatigue lives. • When elastic strains are dominant (e.g. Transmission shaft, valve springs, and gears)	• For variable amplitude loading and short fatigue lives. • When plastic strains are significant • High temperature applications with fatigue-creep (e.g. Gas turbine engine)

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p. 63.

Appendix

14.3.2 Including Mean Stress Effects in Strain-Life Equation

- Let us define *equivalent completely reversed stress amplitude* σ_{ar} (as Eq. 9.22), which is repeated here:

$$\sigma_{ar} = \sigma_f'\left(2N_f\right)^b \qquad (14.15)$$

- An additional equation is needed to calculate σ_{ar} for the mean stress situation

$$\sigma_{ar} = f(\sigma_a, \sigma_m) = \sigma_a \frac{f(\sigma_a, \sigma_m)}{\sigma_a} = \sigma_f'\left(2N_f\right)^b \qquad (14.17)$$

- Then, we can define *zero-mean-stress-equivalent life, N**

$$\sigma_a = \sigma_f'\left[2N_f\left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{1/b}\right]^b = \sigma_f'(2N^*)^b$$

(14.18)

where $N^* = N_f\left(\frac{\sigma_a}{f(\sigma_a,\sigma_m)}\right)^{1/b}$

14.3.3 Mean Stress Equation of Morrow*

- Equivalent completely reversed stress amplitude (Also can be found at Eq. (9.21))

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f'}} \tag{14.21}$$

$$N_{mi}^* = N_f \left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{1/b} \tag{14.22}$$

$$N_f = N_{mi}^* \left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-1/b} \tag{14.23}$$

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(2N_f)^b + \varepsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{c/b}(2N_f)^c \tag{14.24}$$

14.3.3 Mean Stress Equation of Morrow*

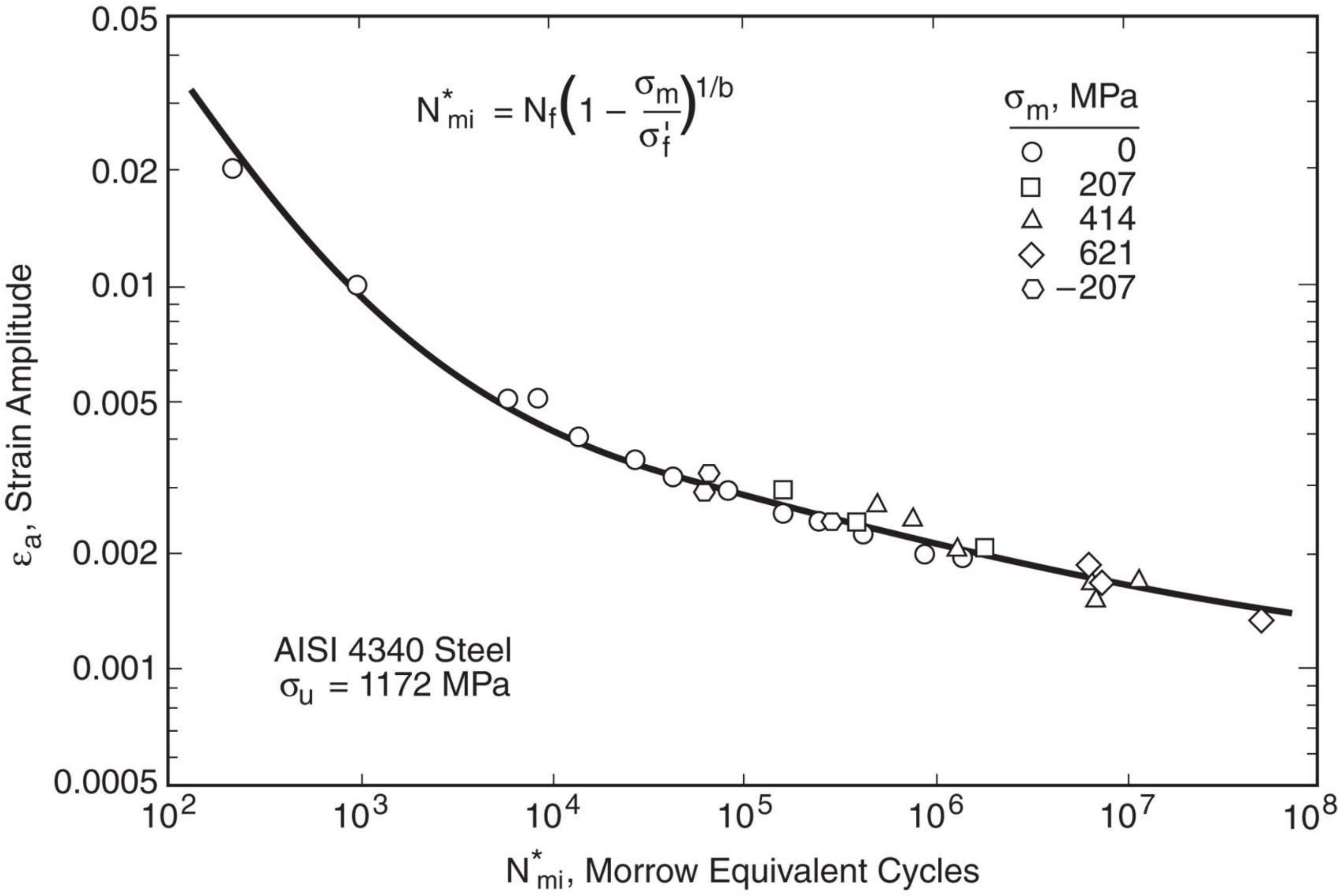


Figure 14.11 Mean stress data of Fig. 14.10 plotted versus N^* according to the Morrow equation.

14.3.5 Smith, Watson, and Topper (SWT) Parameters

- This approach assumes that the life for any situation of mean stress depends on the product

$$\sigma_{max}\varepsilon_a = h''(N_f) \tag{14.28}$$

where $\sigma_{max} = \sigma_m + \sigma_a$ and $h''(N_f)$ indicates a function of fatigue life N_f

- Life is expected to be the same as for completely reversed loading where this product has the same value.
- Let σ_{ar} and ε_{ar} be the completely reversed stress and strain amplitude that result in the same life N_f as the $(\sigma_{max},\ \varepsilon_a)$ combination.
- When $\sigma_m = 0$ and $\sigma_{max} = \sigma_{ar}$, we find $\sigma_{max}\varepsilon_a$=$\sigma_{ar}\varepsilon_{ar}$
- Then, using Eqs. 14.5 and 14.4, we can define

$$\sigma_{max}\varepsilon_a = \sigma_f'(2N_f)^b\left[\frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c\right] \tag{14.29}$$

$$\sigma_{max}\varepsilon_a = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c} \tag{14.30}$$

14.3.5 Smith, Watson, and Topper (SWT) Parameters

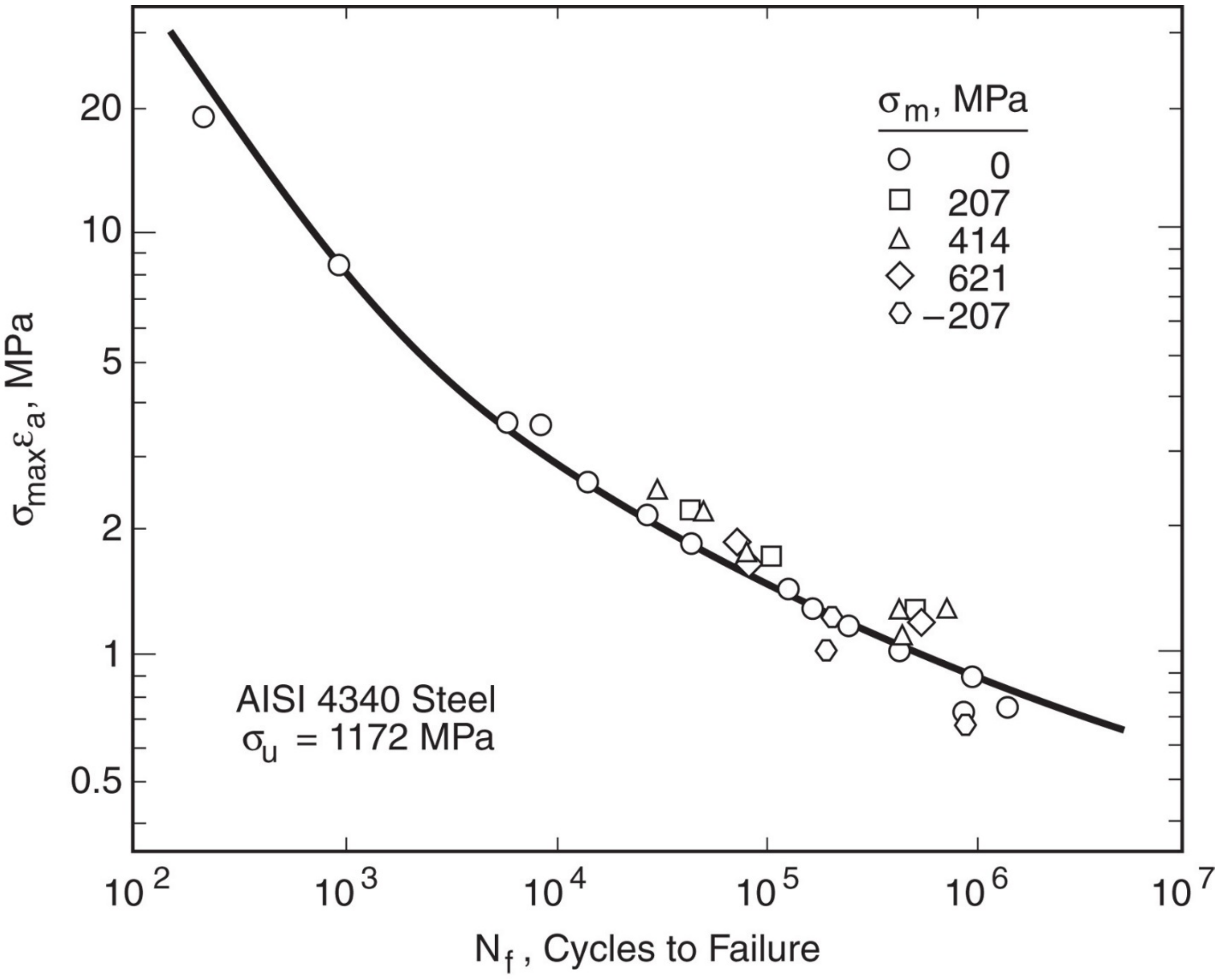


Figure 14.13 Plot of the Smith, Watson, and Topper parameter versus life for the data of Fig. 14.10.

14.3.5 Walker Mean Stress Equation

- Recall Eq. (9.19)

$$\sigma_{ar} = \sigma_{max}^{1-\gamma}\sigma_a^{\gamma}, \qquad \sigma_{ar} = \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} \tag{14.31}$$

where $R = \sigma_{min}/\sigma_{max}$

$$N_f = N_{mi}^{*}\left(1-\frac{\sigma_m}{\sigma_f'}\right)^{-1/b} \tag{14.23}$$

$$N_w^{*} = N_f\left(\frac{\sigma_a}{\sigma_{max}}\right)^{(1-\gamma)/b} \qquad N_w^{*} = N_f\left(\frac{1-R}{2}\right)^{(1-\gamma)/b} \tag{14.32}$$

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(\frac{1-R}{2}\right)^{(1-\gamma)/b}\left(2N_f\right)^{b} + \varepsilon_f'\left(\frac{1-R}{2}\right)^{c(1-\gamma)/b}\left(2N_f\right)^{c} \tag{14.33}$$

Backup

4.3 Strain-life Approach*

4.3.1 Notch Root Stresses and Strains

- Theoretical stress concentration factor, K_t, is used to relate the nominal stress or strain to the local values.
- Upon yielding, the local values are no longer linearly related to the nominal values by K_t
- Instead, the values are related in terms of stress and strain concentration factors as:

$$K_\sigma = \frac{\text{Local stress}}{\text{Nominal stress}} \quad (4.11)^*$$

$$K_\epsilon = \frac{\text{Local strain}}{\text{Nominal strain}} \quad (4.12)^*$$

- After yielding, the actual local stress is less than that predicted using K_t
- After yielding, the actual local strain is greater than that predicted using K_t
- This is due to residual stresses at the notch root

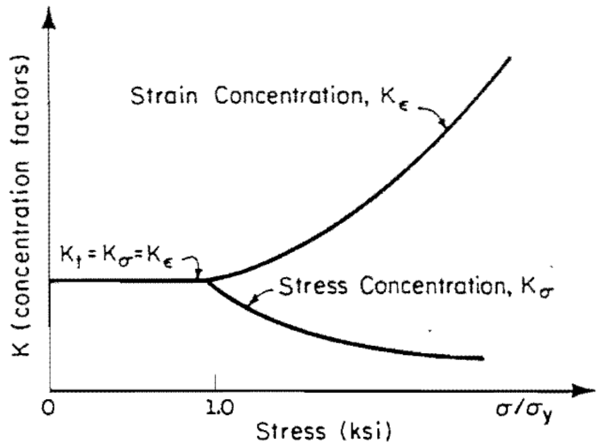


Figure 14.13* Effect of yielding on K_σ and K_ϵ

* Bannantine, Julie. "Fundamentals of metal fatigue analysis." *Prentice Hall, 1990,* (1990): 273, p. 63.

14.1 Constant Amplitude Loading

- Let strain be expressed as a function of S that denotes load, moment, nominal stress, etc.

$$\varepsilon = g(S) \qquad (14.47)$$

$$\varepsilon_{max} = g(S_{max}) = f(\sigma_{max}) \qquad \text{for tensile stress} \qquad (14.48)$$

$$\varepsilon_a = g(S_a) = f(\sigma_a) \qquad \frac{\Delta\varepsilon}{2} = g\left(\frac{\Delta S}{2}\right) = f\left(\frac{\Delta\sigma}{2}\right)$$